16th Annual Graduate School
of Particle Physics

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A SHORT COURSE ON QUANTUM CHROMODYNAMICS

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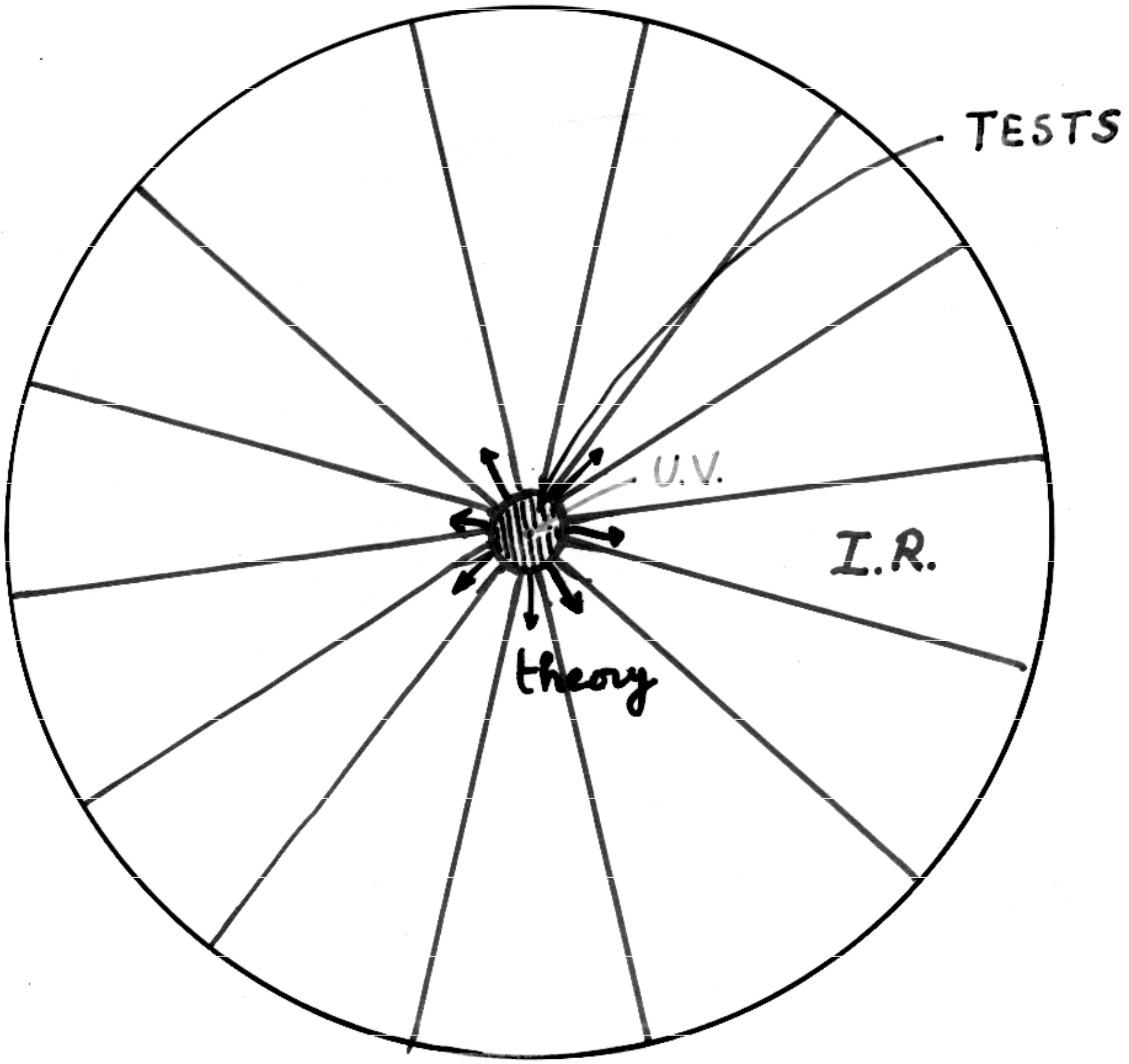
Physique théorique fondamentale

http://qcd.theo.phys.ulg.ac.be/cudell/qcd

Goals of these lectures:

- explain where we can calculate (perturbatively)
- give the main tests and tools of pQCD
- give a list of present problems and techniques



Structure of the course

1. Introduction:

- $SU(3)_F$ and quarks
- why colour
- $SU(3)_c$ gauged : QCD
- tests from QED - coloured quarks
- classical solutions

2. Perturbation theory

- Feynman rules
- colour factors
- gauges and propagators
- ghosts and cutting rules
- renormalisation and running coupling
- evolution of quark masses

3. Details on specific processes

- $e^+e^- \to q\bar{q}g$
  - I.R. divergence
  - factorisation
  - I.R. stable observables
  - modern tests of QCD

- $ep \to eX$
  - factorisation
  - parton distributions
  - evolution equations

4. a) Present developments

- DIS : small x and saturation
- Exclusive processes, diffraction
- spin

b) The roads to the I.R.

- lattice QCD
- Chiral lagrangians
- [HQET] → daenen
- Schwinger - Dyson

References : books

- Basics of perturbative QCD,
  Dokshitzer, Khoze, Mueller, Troyan
  Editions frontières, 1991

- QCD and collider physics,
  Ellis, Stirling, Webber
  Cambridge University Press, 1996

- Handbook of perturbative QCD,
  CTEQ collaboration
  Rev. Mod. Phys. 67 (1995) 157

- QCD as a theory of hadrons
  Narison
  Cambridge University Press, 2004

- The theory of quark and gluon interactions
  Yndurain
  Springer, 1999

- Quarks and leptons
  Halzen - Martin
  John Wiley

References: papers

- «Basics of QCD perturbation theory»
  Soper, hep-ph/0011256
- «Gauge theories»
  Feynman, Les Houches 1976, North Holland 77
- «Introduction to QCD»
  Mangano, http://preprints.cern.ch/yellowrep/1999/99-04/ps3.pdf
- «Lectures on the foundations of QCD»
  Smilga, hep-ph/9901412

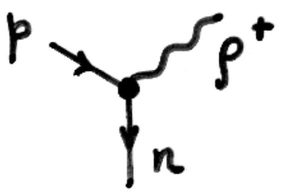
Other useful sources:

- CORE: Compendium of Relations
  Borodulin, Slabospitsky, hep-ph/9507456
- Chronology of particle physics
  http://dbserv.ihep.su/compas/contents.html

History

1954 Yang - Mills SU(2) isospin gauged

$N = \begin{pmatrix} n \\ p \end{pmatrix}$ $V = \begin{pmatrix} \rho^- \\ \rho^0 \\ \rho^+ \end{pmatrix}$ => Ex. 2



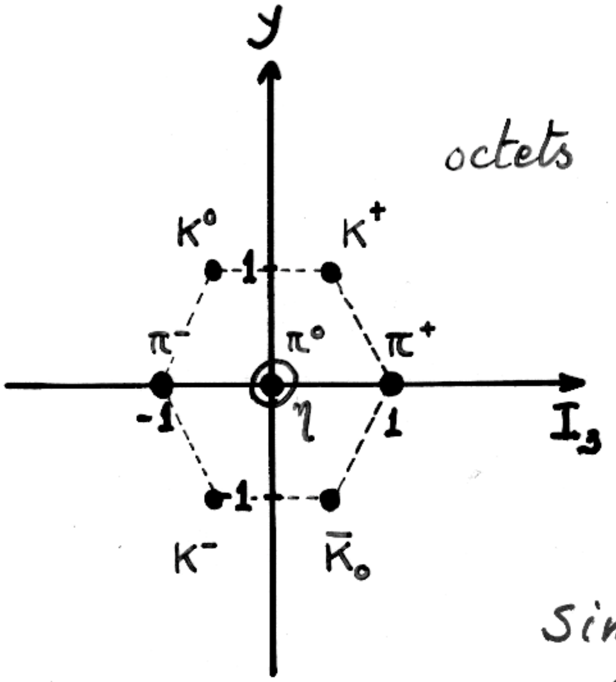
predicts : vector bosons massless !

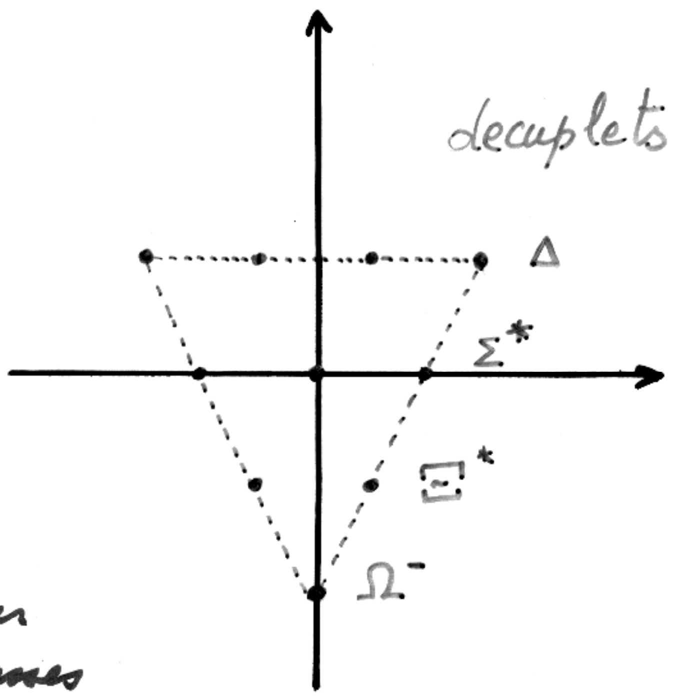
1961 Ne'eman Gell-Mann

hadrons belong to representations of $SU(3)_F$:

$Y = B + S$ I_3 = isospin // z

$Q = I_3 + Y/2$





Similar masses

Groups SU(N)

continuous transformations $\hat{U}(\vec{\theta})$: Lie group

$\hat{U}(\vec{\theta}=0) = 1$

$$\Rightarrow \hat{U}(\vec{\theta}) \approx 1 + i\,\vec{\hat{u}}\cdot\vec{\theta} \qquad \vec{\theta} \text{ small}$$

$$\Rightarrow \hat{U}(\theta) = \lim_{n\to\infty} \prod_1^n \left(1 + i\,\vec{\hat{u}}\cdot\vec{\theta}/n\right)^n$$

$$= \exp\left(i\,\vec{\hat{u}}\cdot\vec{\theta}\right)$$

$\hat{U}^{-1} = \hat{U}^{+}$ generators $\hat{u} = \hat{u}^{+}$

$\hat{U}$ unitary $\Leftrightarrow$ $\vec{u}$ hermitian

$$[\hat{u}_j, \hat{u}_k] = i \underbrace{f_{jkl}}_{\text{structure constants}} \hat{u}_l$$

(Lie algebra su(N))

fundamental representation:

N - dimensional vectors

$\hat{U} = N\times N$ matrix U with $\det U = 1$

$\Rightarrow$ N^2-1 generators $t_i = \hat{u}_i$

with $\mathrm{Tr}(t_i) = 0$

SU(2) : spin, isospin		SU(3) : colour, flavour
$\psi' = e^{i \sum_k \hat{u}_k \theta_k} \psi$ $k = 1, \dots 3$	group	$\psi' = e^{i \sum_l \hat{u}_l \theta_l} \psi$ $l = 1, \dots 8$
$[\hat{u}_k, \hat{u}_l] = i \sum_m \varepsilon_{klm} \hat{u}_m$	algebra	$[\hat{u}_k, \hat{u}_l] = i \sum_m f_{klm} \hat{u}_m$
$\varepsilon_{123} = \varepsilon_{231} = \varepsilon_{312} = 1$ $\varepsilon_{321} = \varepsilon_{213} = \varepsilon_{132} = -1$ totally antisymmetric	structure constants	$f_{123} = 1 \quad f_{458} = f_{678} = \sqrt{3}/2$ $f_{147} = f_{165} = f_{246} = f_{257} = f_{345} = f_{376} = \frac{1}{2}$ totally antisymmetric
Pauli matrices : $t_k = \sigma_k/2$ $\sigma_1 = \begin{pmatrix} \cdot & 1 \\ 1 & \cdot \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} \cdot & -i \\ i & \cdot \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$ dimension 2	fundamental representation	Gell-Mann matrices : $t_k = \lambda_k/2$ $\lambda_1 = \begin{pmatrix} \sigma_1 & \cdot \\ \cdot & \cdot \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} \sigma_2 & \cdot \\ \cdot & \cdot \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} \sigma_3 & \cdot \\ \cdot & \cdot \end{pmatrix}$ $\lambda_4 = \begin{pmatrix} \cdot & \cdot & 1 \\ \cdot & \cdot & \cdot \\ 1 & \cdot & \cdot \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} \cdot & \cdot & -i \\ \cdot & \cdot & \cdot \\ i & \cdot & \cdot \end{pmatrix} \quad \lambda_6 = \begin{pmatrix} \cdot & \cdot \\ \cdot & \sigma_1 \end{pmatrix}$ $\lambda_7 = \begin{pmatrix} \cdot & \cdot \\ \cdot & \sigma_2 \end{pmatrix} \quad \lambda_8 = \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix} / \sqrt{3}$
$(\hat{u}_a)_{mn} = -i \varepsilon_{amn}$ spin 1 dim. 3 e.g. $u_1 = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & -i \\ \cdot & i & \cdot \end{pmatrix}$	adjoint representation	$(\hat{u}_a)_{mn} = -i f_{amn}$ dimension 8

1964 Zweig, Gell-Mann

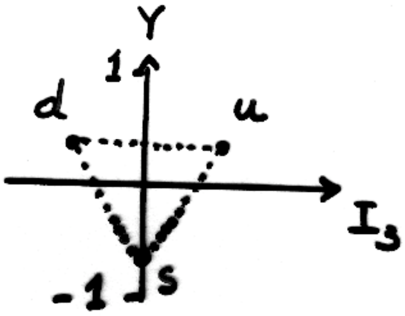
The fundamental representation of $SU(3)_F$ must be physical

"aces" or "quarks"

$$\begin{cases} B = 1/3 \quad Q = 2/3 \text{ or } -1/3 \quad \text{spin } 1/2 \\ SU(3)_F \text{ breaking due to different } m_q \end{cases}$$

$$\begin{pmatrix} u_\uparrow \\ d_\uparrow \\ s_\uparrow \end{pmatrix} \begin{pmatrix} u_\downarrow \\ d_\downarrow \\ s_\downarrow \end{pmatrix} \updownarrow SU(3)_F$$

$$\longleftrightarrow SU(2)_{spin}$$



Mesons : $3 \otimes \bar{3} = 1 \oplus 8$ flavours

$\Rightarrow$ Ex. 1 $\quad 2 \otimes 2 = 1 \oplus 3$ spin

Baryons : $3 \otimes 3 \otimes 3 = \boxed{10} \oplus 8 \oplus \boxed{8} \oplus 1$

$2 \otimes 2 \otimes 2 = \boxed{4} \oplus 2 \oplus \boxed{2}$

decuplet spin 3/2 $\qquad$ octet spin 1/2

$m_u \approx m_d \approx 300$ MeV

$m_s \approx 500$ MeV

$$\Rightarrow \begin{cases} \text{hadron magnetic moments} \quad g_p \simeq 4.79 \\ \text{Gell-Mann Okubo mass formula} \end{cases}$$

$$I_1 = \frac{\lambda_1}{2} \qquad I_2 = \frac{\lambda_2}{2} \qquad I_3 = \frac{\lambda_3}{2}$$

$$I_3 = \begin{pmatrix} 1/2 & & \\ & -1/2 & \\ & & 0 \end{pmatrix} \qquad \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

$$I_1^2 + I_2^2 + I_3^2 = I(I+1) = \begin{pmatrix} 3/4 & & \\ & 3/4 & \\ & & 3/4 \end{pmatrix}$$

$$B = \begin{pmatrix} 1/3 & & \\ & 1/3 & \\ & & 1/3 \end{pmatrix} \qquad S = \begin{pmatrix} 0 & & \\ & 0 & \\ & & -1 \end{pmatrix}$$

$$Y = B + S = \begin{pmatrix} 1/3 & & \\ & 1/3 & \\ & & -2/3 \end{pmatrix} = \lambda_8/\sqrt{3}$$

$$Q = I_3 + \frac{1}{2} Y = \begin{pmatrix} 2/3 & & \\ & -1/3 & \\ & & -1/3 \end{pmatrix}$$

Problems:

- where are the quarks? [searches ⟹ $m_q \gg \text{GeV}$]

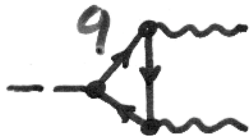
- spin-statistics violation

$$\left.\begin{array}{l} \Delta^{++} = u^\uparrow u^\uparrow u^\uparrow \\ \Omega^- = s\,s\,s \\ \Delta^- = d\,d\,d \end{array}\right\}$$ symmetric but spin $3/2$!

- dynamics?

- qqq, $q\bar{q}$ bound states?

- $\pi^0 \to \gamma\gamma$



$\Gamma_{exp.} = 7.7$ eV $\qquad \Gamma_{th} = 0{,}8$ eV

1964 Greenberg
quarks obey parastatistics
of rank 3

1965 Han-Nambu-Struminsky
quarks have a new quantum number
obey Fermi-Dirac statistics

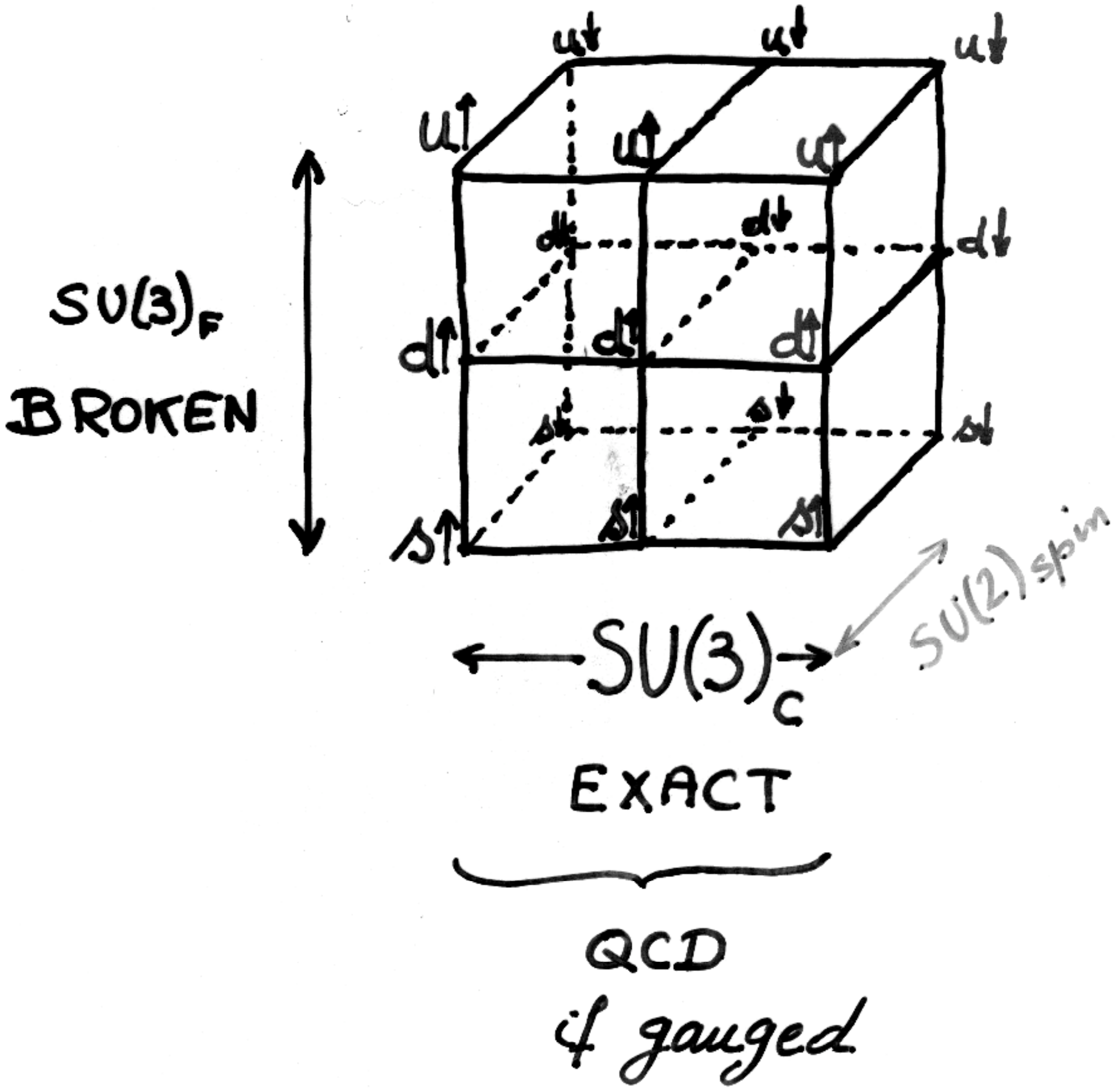
1969 SLAC + Bjorken + Feynman
photons scatter off
quasi-free constituents

1972 Bardeen-Fritzsch-Gell-Mann
quarks have SU(3) colour
as an additional symmetry

1973 Fritzsch-Gell-Mann-Leutwyler
Weinberg

$\left.\begin{array}{l}\text{Gross-Wilczek}\end{array}\right\}$ asymptotic freedom

$\mathcal{L}_{QCD}$ = Yang-Mills SU(3)



	u	d	s
charge Q	2/3	-1/3	-1/3
I_3	1/2	-1/2	0
Y	1/3	1/3	-2/3
B	1/3	1/3	1/3
S	0	0	1

How to build QED :

Dirac equation $\Rightarrow$ $\mathcal{L}_0 = \bar{\psi}(i\gamma_\mu \partial^\mu - m)\psi$

<u>global</u> phase invariance : $U(1)$

$$\psi \to e^{i\theta}\psi \qquad \theta \in \mathbb{R}$$

make it <u>local</u>:

$$\boxed{\psi(x) \to e^{i\theta(x)}\,\psi(x)}$$

introduce a field $A_\mu(x)$:

$i\partial_\mu$ is replaced by $\boxed{i\partial_\mu + eA_\mu = iD_\mu}$

$$\boxed{A_\mu(x) \to A_\mu(x) + \frac{1}{e}\partial_\mu \theta(x)}$$

$$\Rightarrow \quad iD_\mu\psi \longrightarrow e^{i\theta(x)}\, iD_\mu\psi$$

add a kinetic term: $F_{\mu\nu}F^{\mu\nu}$

$$F_{\mu\nu} = \frac{1}{e}\left[iD_\mu, iD_\nu\right] = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\Rightarrow \boxed{\mathcal{L} = \mathcal{L}_0 + e\bar{\psi}\gamma_\mu A^\mu \psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}}$$

$m_\gamma A^2$

- renormalisable
- Could add $\varepsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$ $\Rightarrow$ Ex.
- $m_\gamma = 0$

Let us build QCD : for 1 flavour

elementary fermions

$$\Rightarrow \mathcal{L}_0 = \sum_c \bar{\psi}_c (i\gamma_\mu \partial^\mu - m_c)\psi_c$$

$$\boxed{\text{global } SU(3) \quad \begin{pmatrix}\psi_R\\ \psi_G\\ \psi_B\end{pmatrix} \rightarrow e^{i\sum_1^8 \theta_a \frac{\lambda_a}{2}} \begin{pmatrix}\psi_R\\ \psi_G\\ \psi_B\end{pmatrix}}$$

$$\Rightarrow m_R = m_G = m_B = m$$

make θ_a local : $\theta_a(x)$

$$\boxed{\psi \rightarrow e^{i\theta_a(x)\frac{\lambda_a}{2}}\psi}$$

$$\partial_\mu \psi \rightarrow e^{i\theta_a(x)\frac{\lambda_a}{2}}\partial_\mu \psi + i\lambda_a \psi\, \partial_\mu \theta_a(x)$$

$\Rightarrow$ introduce 8 fields $A_\mu^a(x)$

$$iD_\mu = i\partial_\mu - g\sum_a \frac{\lambda_a}{2} A_\mu^a(x)$$

$$A_\mu^a(x) \longrightarrow A_\mu^a(x) - \frac{1}{g}\partial_\mu \theta_a(x) \quad ?$$

$$\mathcal{L} = \mathcal{L}_0 - g\sum_a \bar{\psi}\gamma^\mu \frac{\lambda_a}{2}\psi A_\mu^a \quad ?$$

problem: $\psi \rightarrow \left(1 + i\theta_b \frac{\lambda_b}{2}\right)\psi$

$$\bar{\psi}\gamma_\mu \frac{\lambda_a}{2}\psi \rightarrow \bar{\psi}\gamma_\mu \frac{\lambda_a}{2}\psi + i\theta_b \bar{\psi}\gamma_\mu \underbrace{\left(\frac{\lambda_a}{2}\frac{\lambda_b}{2} - \frac{\lambda_b}{2}\frac{\lambda_a}{2}\right)}_{f_{abc}\frac{\lambda_c}{2}}\psi$$

1 $$\boxed{A_\mu^a(x) \rightarrow A_\mu^a(x) - \frac{1}{g}\partial_\mu \theta_a(x) - f_{abc}\theta_b(x) A_\mu^c(x)}$$

The kinetic term can again be built from

$$\begin{aligned}
\frac{-1}{g}\left[iD_\mu, iD_\nu\right] &= \frac{-1}{g}\left[\partial_\mu - g\,\frac{\lambda_a}{2}A_\mu^a,\ \partial_\nu - g\,\frac{\lambda_b}{2}A_\mu^b\right] \\
&= \sum_a \frac{\lambda_a}{2}\left\{\partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f_{abc} A_\mu^b A_\nu^c\right\} \\
&= \sum_a \frac{\lambda_a}{2} F_{\mu\nu}^a
\end{aligned}$$

$$\mathcal{L}_{QCD}^{cl.} = \bar{\psi}(i\gamma_\mu\partial^\mu - m)\psi - g\left(\bar{\psi}\gamma_\mu\frac{\lambda_a}{2}\psi\right)$$

$$Tr\left(F_{\mu\nu}^a\frac{\lambda_a}{2}\right)^2 \left\{\begin{aligned}
&-\frac{1}{4}\left(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a\right)^2 \\
&+\frac{g}{2}\left(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a\right)\left(f_{abc}A_\mu^b A_\nu^c\right) \\
&+\frac{g^2}{4}\left(f_{abc}A_\mu^b A_\nu^c\right)^2
\end{aligned}\right.$$

self-interacting gauge fields:
gluons!

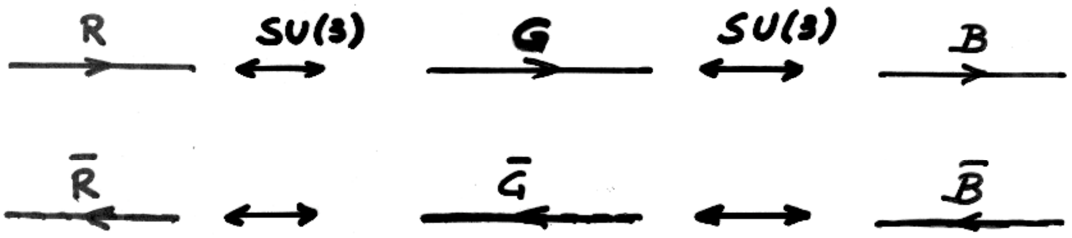
$$\mathcal{L} = i\bar{\psi}\,\mathbb{D}\cdot\gamma\,\psi - \frac{1}{2}\,Tr\{\mathbb{F}\cdot\mathbb{F}\}$$

$$\mathbb{D}_\mu = \partial_\mu - g\,\mathbb{A}_\mu$$

$$\mathbb{A}_\mu = \sum_a \frac{\lambda_a}{2} A_\mu^a$$

$$\mathbb{F}_{\mu\nu} = \partial_\mu \mathbb{A}_\nu - \partial_\nu \mathbb{A}_\mu + ig\left[\mathbb{A}_\mu, \mathbb{A}_\nu\right]$$

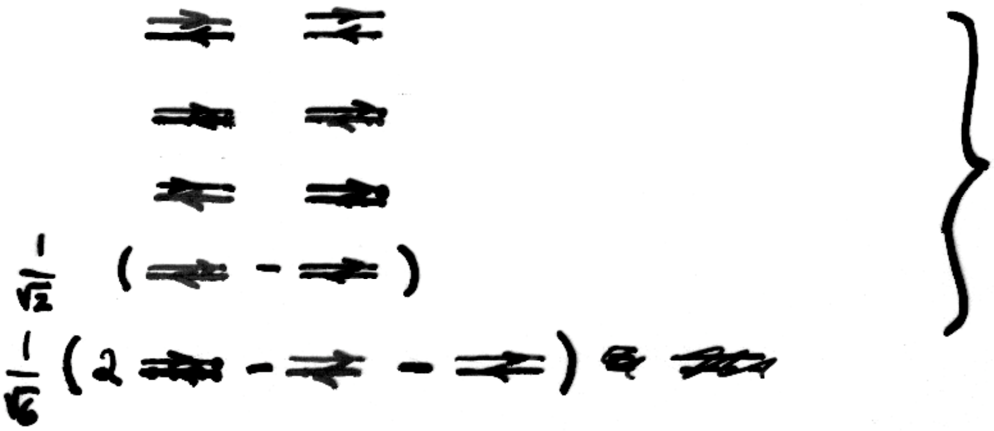
$\Rightarrow$ ex. 4



colours

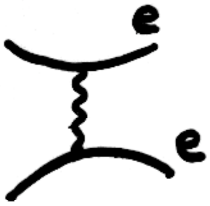
anti-colours

= invariant



8 gluons

QED :



$V(r) \sim \frac{e^2}{r}$

QCD :

not allowed

quarks repel each other

$\sim \frac{2}{\sqrt{6}} \times \frac{2}{\sqrt{6}} = \frac{2}{3} g^2$

$\sim 1 \times 1 = 1 \; g^2$

$\frac{2}{3}$

$\sim \frac{2}{\sqrt{6}} \times -\frac{1}{\sqrt{6}} = -\frac{1}{3} g^2$

antiquarks are attracted by quarks

$g \times (-g) \times 1$

gluons attract

Problems:

- too many hadrons

  uud $\rightarrow$ 9 states!

- quarks and gluons are not observed

$$\Downarrow$$

New hypothesis:

$$\boxed{\text{Only colour-singlets form asymptotic states}}$$

$$n_q - n_{\bar{q}} = 3n$$

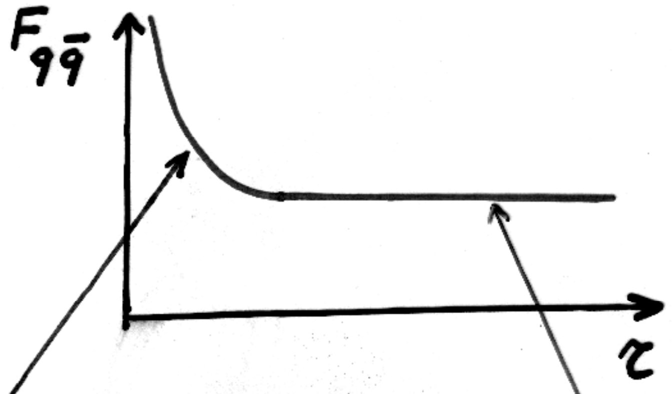
$$\Rightarrow \quad \begin{cases} n_q = n_{\bar{q}} \\ n_q = n_{\bar{q}} + 3 \\ n_q = n_{\bar{q}} + 6 \\ \vdots \end{cases}$$

- spin-statistics:

$$\Delta^{+++} = u\!\uparrow u\!\uparrow u\!\uparrow \times \text{antisymmetric } \psi$$

- $\pi^0 \rightarrow \gamma\gamma$ $\qquad \Gamma_{th} \times 9 \quad \Rightarrow$ OK

- dynamics of interactions with gluon field



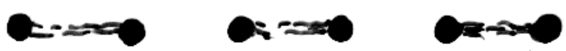
$\Rightarrow$ ex.

< Coulomb as $r \rightarrow 0$

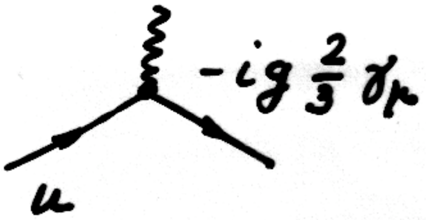
constant as $r \rightarrow \infty$

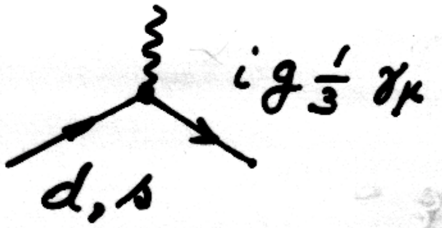
$E_{field} \sim r$

$\Downarrow$

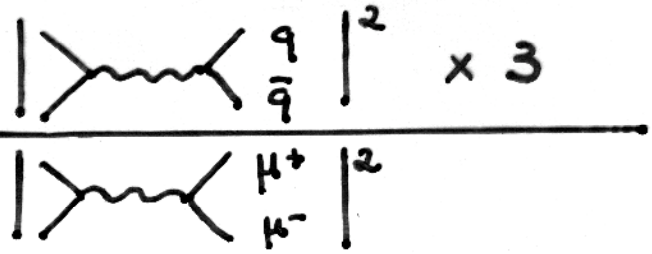


Further tests : QED





$$R = \frac{\sigma_{e^+e^- \to \text{hadrons}}}{\sigma_{e^+e^- \to \text{muons}}}$$



$$= \sum_q \frac{\left| \ \ \right|^2 \times 3}{\left| \ \ \right|^2}$$

$$= 3 \sum_q e_q^2$$

from threshold

quark	e_q	m_q
u	2/3	~ 300 MeV
d	-1/3	~ 300 MeV
s	-1/3	~ 500 MeV
c	2/3	~ 1,6 GeV
b	-1/3	~ 5 GeV
t	2/3	~ 170 GeV (other method)

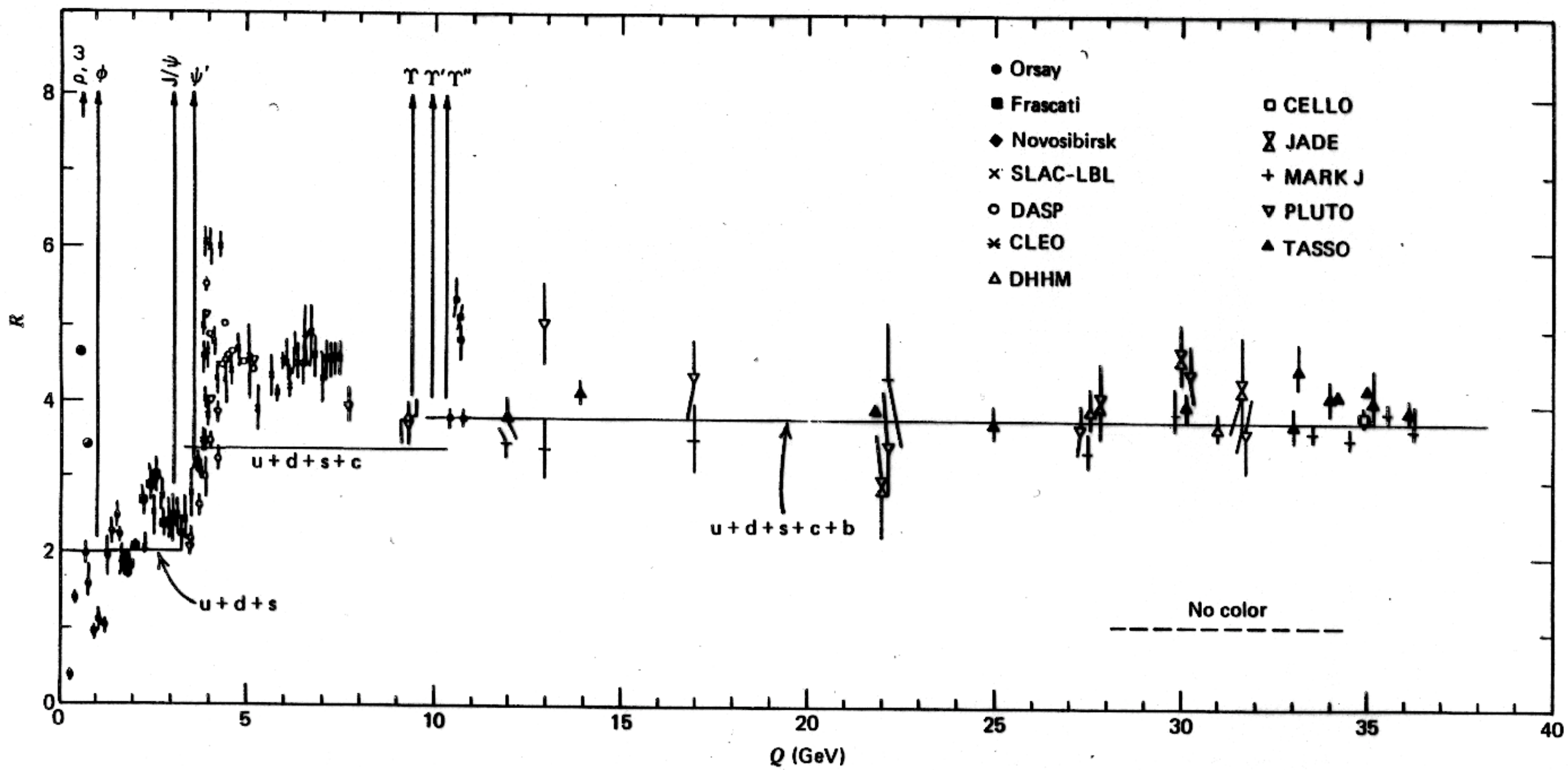


Fig. 11.3 Ratio R of (11.6) as a function of the total e^-e^+ center-of-mass energy. (The sharp peaks correspond to the production of narrow 1^- resonances just below or near the flavor thresholds.)

from Halzen - Martin

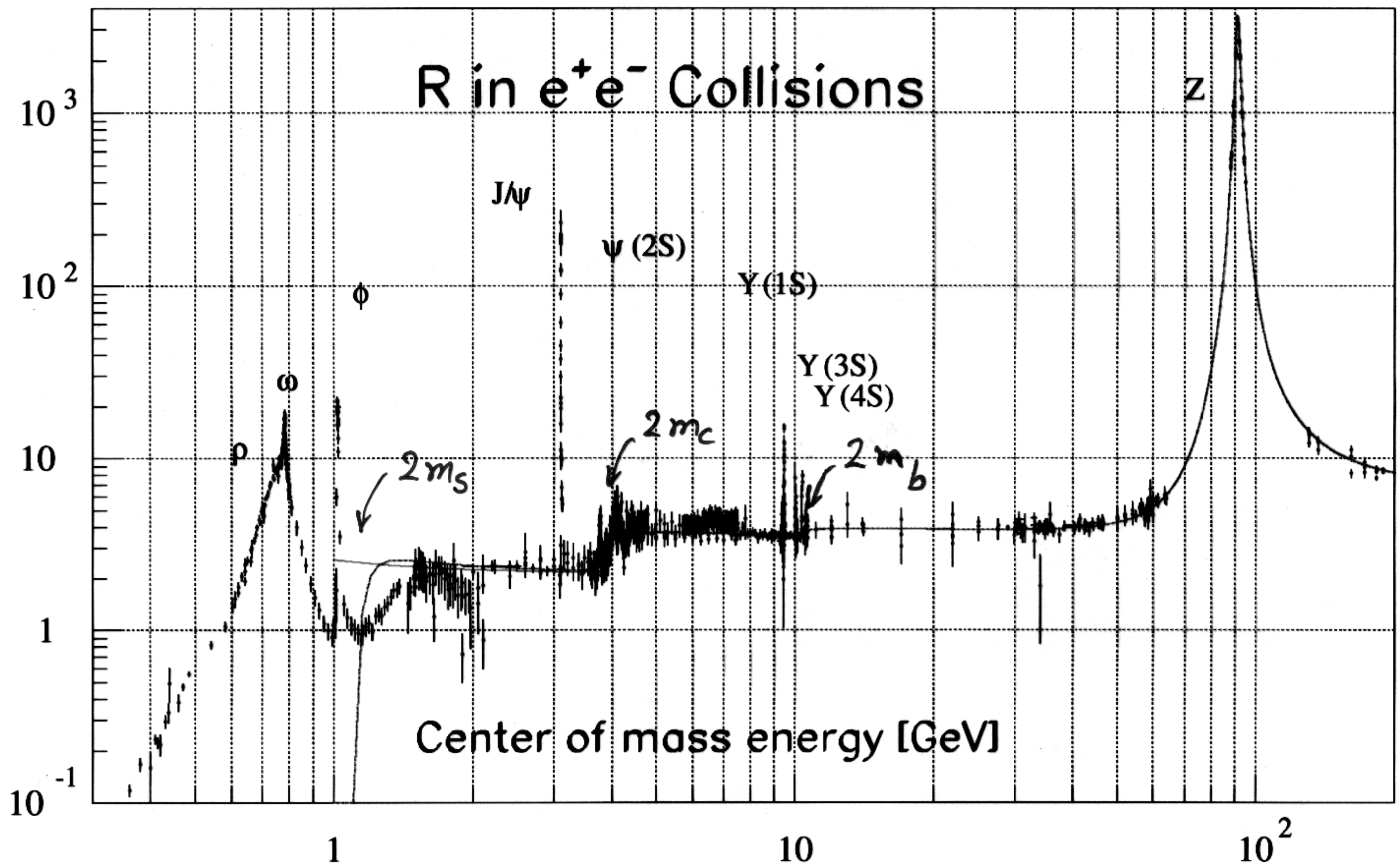
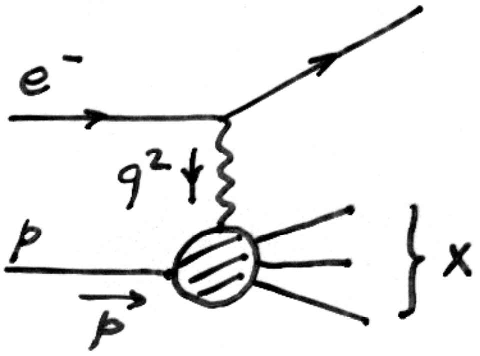
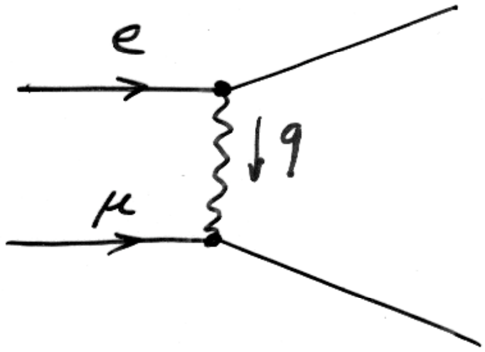


Fig. 8

COMPETE collaboration
2002

Deeply inelastic scattering:





possible iff

$$p^2 = m_p \qquad p^2 = (p+q)^2 = p^2 + 2q.p + q^2$$

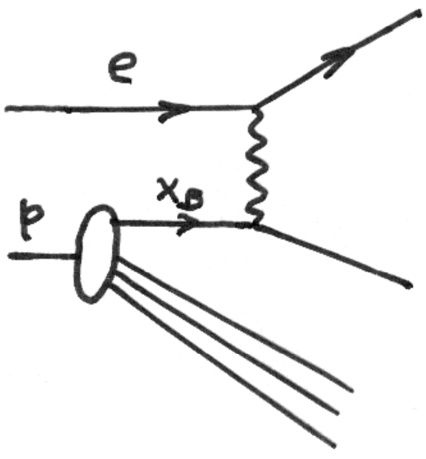
$$M_x^2 = m_p^2 + 2p.q + q^2 \qquad -q^2 = 2p.q$$

$$\text{a priori } f(q^2, p.q) \qquad \delta\left(1 + \frac{q^2}{2p.q}\right)$$

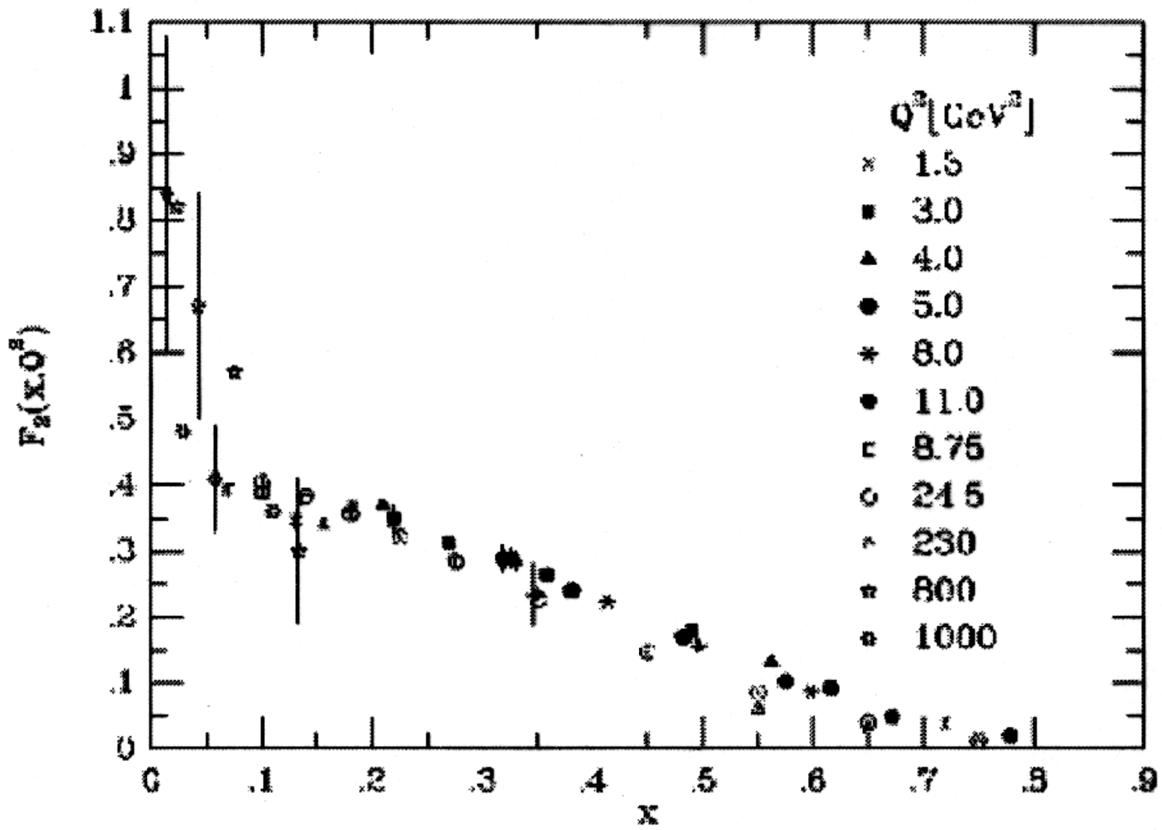
Scaling:

$$f(q^2, p.q) = f'\left(\frac{-q^2}{2p.q}\right) \sim \delta\left(1 + \frac{q^2}{2p.q}\right)$$

$$= f''(q^2, p.q)\, \delta\left(x_B + \frac{q^2}{2p.q}\right)$$

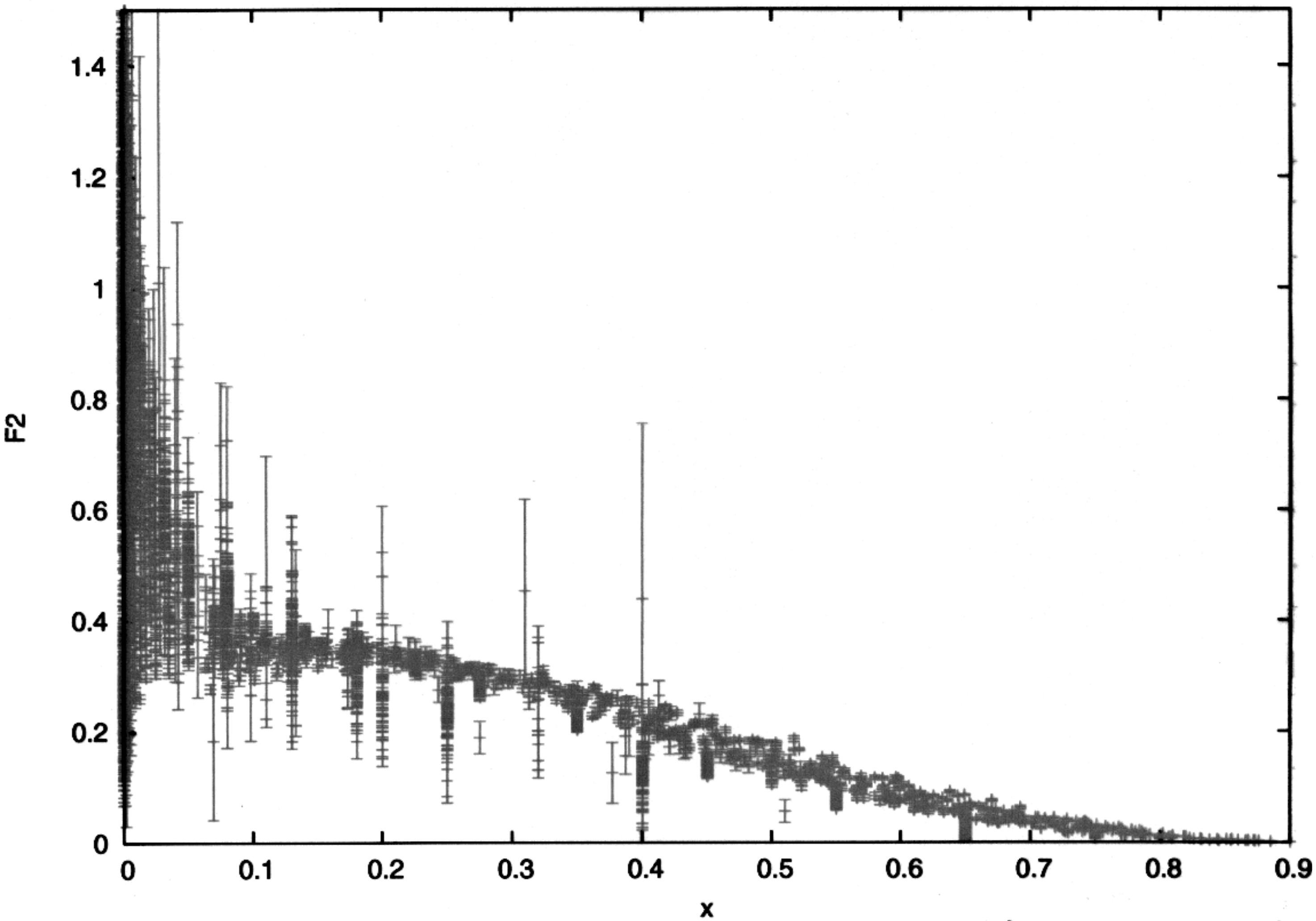


DIS =
elastic scattering
off quarks!



SLAC data

does not scale so well → high Q^2 → low x_B



all present data

Classical solutions: without quarks

$$\partial_\mu F^{\mu\nu,a} - g\, f_{abc} A_\mu^b F^{\mu\nu,c} = 0 \qquad \Rightarrow \text{ex. } 3$$

- $A_\mu^a(x) = f_\mu(x)\delta^{ad} \Rightarrow$ Maxwell $\Rightarrow$ plane waves
- no solitons (monopoles), i.e. finite E static solutions, as there is no scale in the equation
- $E = \infty$ solutions exist

Euclidean space solutions: $t \to i\tau$

$$S_E = \frac{1}{4g^2}\int d^4x\, F_a^{\mu\nu} F_{\mu\nu}^a$$

$A_\mu \to$ "pure gauge" at ∞:

$$A_\mu^a(\infty) = -\partial_\mu \alpha^a(x)$$

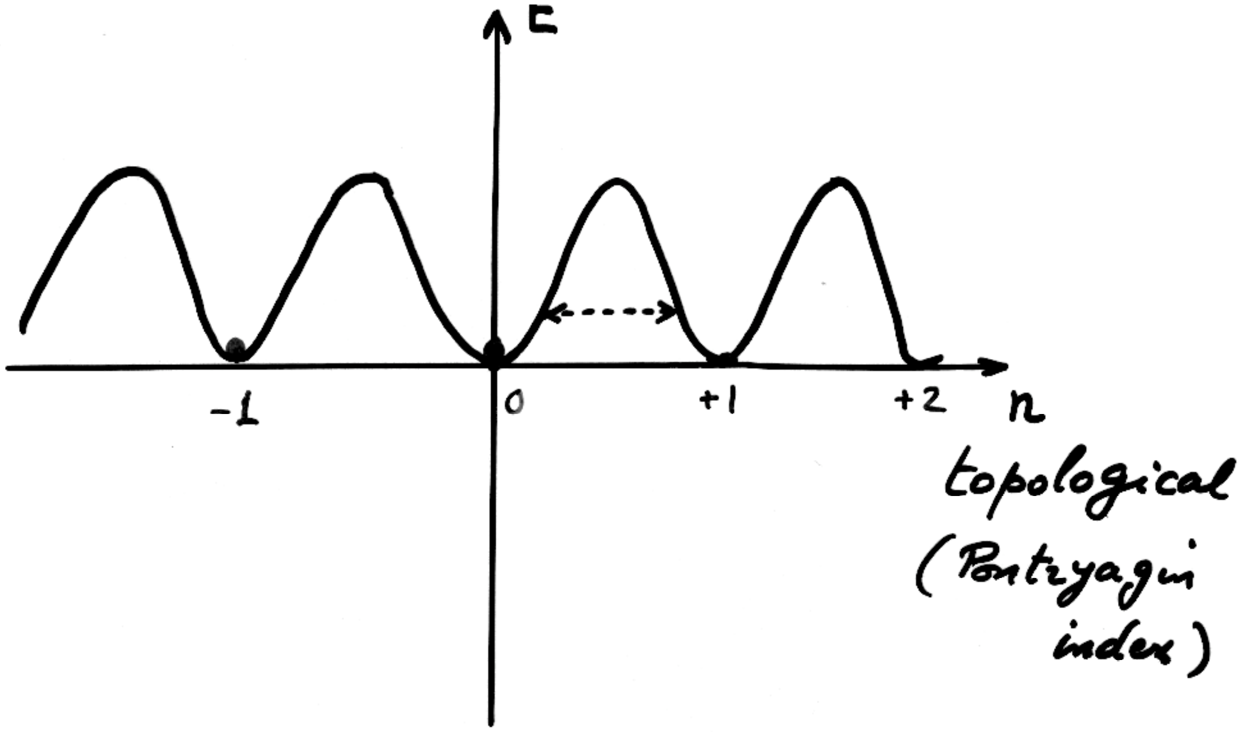
One can find such solutions: instantons

in SU(2): $A_\mu^a = \dfrac{2\eta_{\mu\nu}^a x_\nu}{x^2+\rho^2}$ $\qquad \eta_{00}^a = 0 \quad \eta_{ij}^a = \varepsilon_{aij}$

$\qquad \eta_{0i}^a = \delta_{ia} \quad \eta_{i0}^a = -\delta_{ai}$

$$S_E = \frac{8\pi^2}{g^2}$$

In general, $S_E = \dfrac{8\pi^2}{g}(\overset{\text{inst.}}{n} - \overset{\text{antiinstanton}}{\bar{n}})$



instanton = tunneling solution
between two vacua!

"true vacuum" = $\sum_n e^{in\theta} |n>$

tunneling probability amplitude

$$<0|1> \sim \exp\left(-\frac{8\pi^2}{g^2}\right)$$

g small => may be negligible.