$$
\begin{aligned}
\mathcal{L}_{QCD} = \; & \bar{\psi}\,(i\gamma^{\mu}\partial_{\mu} - m)\,\psi \\
& - g\,\bar{\psi}\,\gamma_{\mu}\,\frac{\lambda_a}{2}\,\psi \\
& - \frac{1}{4}\,(\partial_{\mu} A_{\nu}^{a} - \partial_{\nu} A_{\mu}^{a})^{2} \\
& + \frac{g}{2}\,(\partial_{\mu} A_{\nu}^{a} - \partial_{\nu} A_{\mu}^{a})(f_{abc}\, A_{\mu}^{b} A_{\nu}^{c}) \\
& + O(g^{2})
\end{aligned}
$$

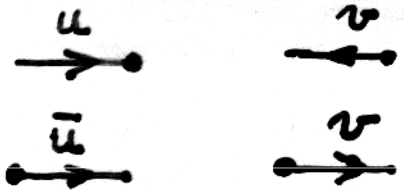
$\Rightarrow$ $\left\{ \begin{array}{l} \text{. classical solutions} \\ \text{. gluon free states} \\ \text{. quark free states} \end{array} \right.$

# Quantization of QCD ⇒ Feynman rules

## Start with free states:

$SU(3)_c$ direction

**quarks:** $u(k)\, e^{-ik.x}\, g_A$ and $v(k)\, e^{ik.x}\, g_B$   $A,B = 1\ldots3$




} same as in QED

quarks   antiquarks

**gluons:** $\varepsilon_\mu(k)\, e^{-ik.x}\, G_a$    $a = 1\ldots8$

$$\begin{cases} \text{gauge invariance} \Rightarrow \varepsilon_\mu \Leftrightarrow \varepsilon_\mu + \lambda k^\mu \\ \varepsilon^2 = -1 \end{cases}$$

⇒ 2 physical degrees of freedom

same as in QED

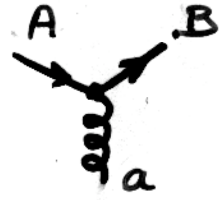
**propagation:** Green's functions

**quarks**


$$\delta_{AB} \frac{i(\not p + m)}{p^2 - m^2 + i\varepsilon} = S_F(p)$$

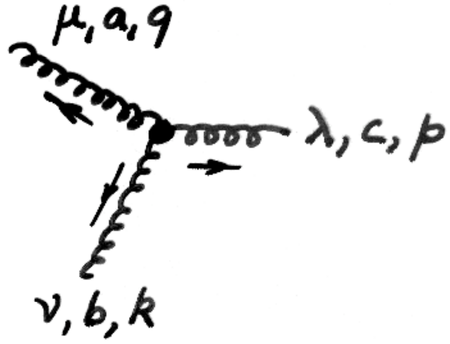
$$(i\not\partial - m)\, S_F(x) \doteq \delta^{(4)}(x - x')$$

## quark - gluon vertex:



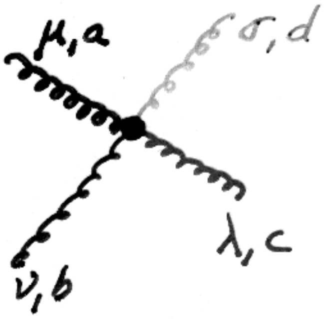

$$\boxed{-i g_s \gamma^\mu \left(\frac{\lambda_a}{2}\right)_{AB}}$$

## gluon - gluon vertices:




$$\boxed{\begin{aligned} +g_s f_{abc} \big( & (p-q)_\nu g_{\lambda\mu} \\ + & (q-k)_\lambda g_{\mu\nu} \\ + & (k-p)_\mu g_{\lambda\nu} \big) \end{aligned}}$$

+ outgoing

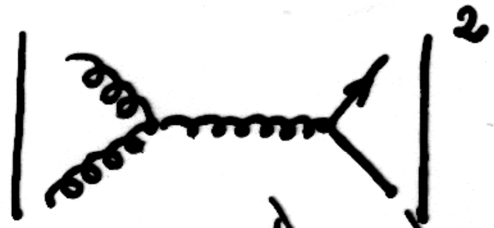



$$\boxed{\begin{aligned} i g_s^2 \sum_e \big\{ & f_{abe} f_{cde} \left( g_{\mu\sigma} g_{\nu\lambda} - g_{\mu\lambda} g_{\nu\sigma} \right) \\ + & f_{ace} f_{bde} \left( g_{\mu\sigma} g_{\lambda\nu} - g_{\mu\nu} g_{\lambda\sigma} \right) \\ + & f_{ade} f_{cbe} \left( g_{\mu\nu} g_{\sigma\lambda} - g_{\mu\lambda} g_{\sigma\nu} \right) \big\} \end{aligned}}$$

QCD calculations split:

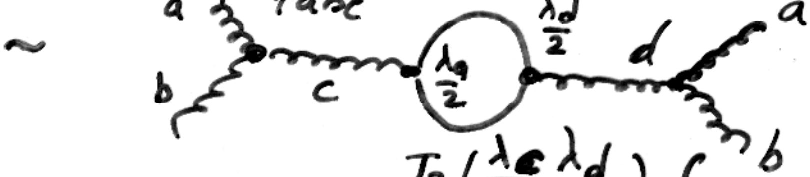
- Lorentz group
- colour group

Example:




$\sim$ 


$$\mathrm{Tr}(\gamma_\mu\, \gamma\cdot p\, \gamma_\nu\, \gamma\cdot(p-q)) \quad \underbrace{\sum \epsilon_\tau \epsilon^*_\lambda}_{\sim \text{num. of propagator}} \sum \epsilon_\sigma \epsilon^*_\xi$$

$\sim$ 


$$\mathrm{Tr}\left(\frac{\lambda_c}{2}\frac{\lambda_d}{2}\right) f_{adb}$$

$$\sim \sum_{abd} f_{abc}\, \mathrm{Tr}\left(\frac{\lambda_c}{2}\frac{\lambda_d}{2}\right) f_{abb}$$

# Colour factors in SU(N)

always sum over colour indices

$$\Rightarrow \begin{cases} Tr\left(\frac{\lambda}{2} \dots \frac{\lambda_n}{2}\right) \\ \Sigma f \dots f \\ \Sigma f \; Tr \end{cases}$$

which can be expressed as SU(N) invariants ("Casimir operators")

- $Tr\left(\frac{\lambda_A}{2} \frac{\lambda_B}{2}\right) = T_R \, \delta_{AB}$

  $T_R = \frac{1}{2}$ for fundamental

- $\sum_a \left(\frac{\lambda_a}{2}\right)_{AB} \left(\frac{\lambda_a}{2}\right)_{BC} = C_F \, \delta_{AC}$

  $C_F = \frac{N^2-1}{2N} = \frac{4}{3}$ in SU(3)

- octet : $(T^C)_{AB} = f_{ABC}$

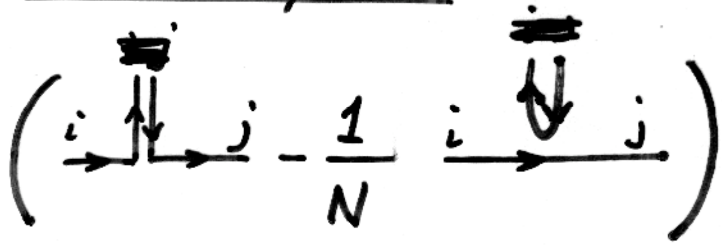
  $Tr(T^C T^D) = C_A \, \delta^{CD} \qquad C_A = N$

- $\left\{\frac{\lambda_A}{2}, \frac{\lambda_B}{2}\right\} = \frac{1}{N} \delta^{AB} + d^{ABC} \frac{\lambda_C}{2}$

  $\sum_{AB} d_{ABC} \, d_{ABD} = \frac{N^2-4}{N} \delta_{CD}$

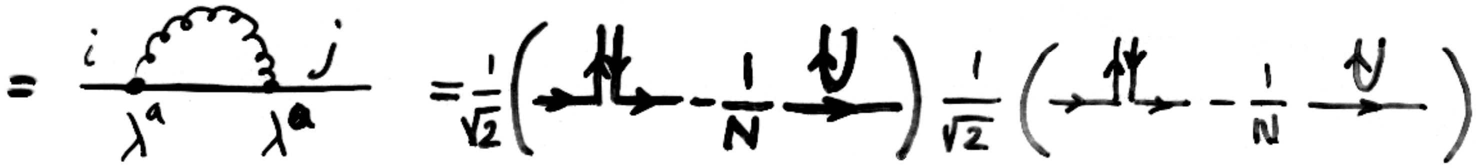
  $d_{AAC} = 0$

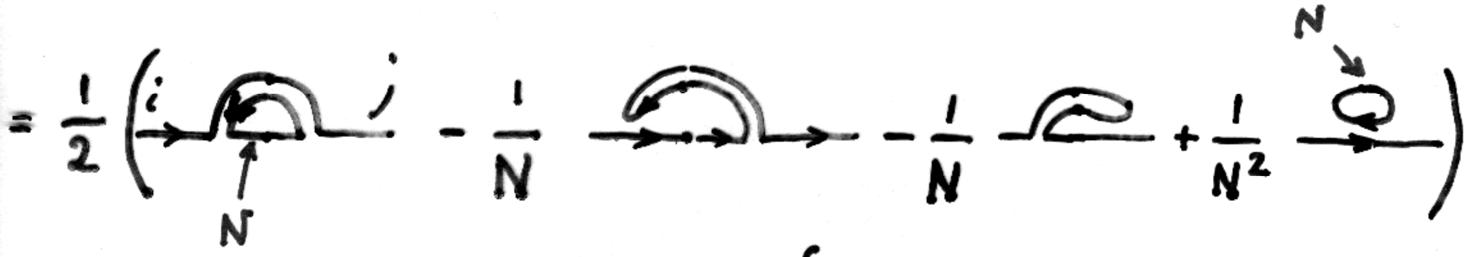
# Trick : colour flow

$\frac{\lambda_a}{2} = t^a_{ij} \Rightarrow \frac{1}{\sqrt{2}}$ 

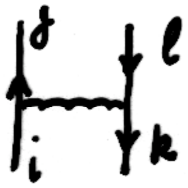
$f^{abc} \Rightarrow \frac{1}{\sqrt{2}}$ 

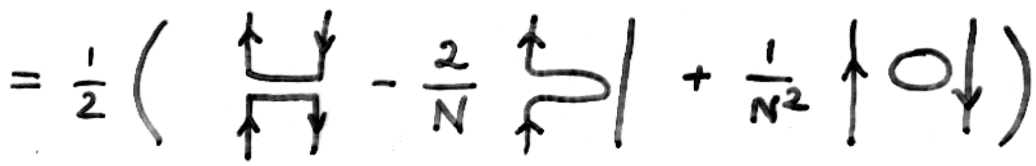
$\Sigma\left(\frac{\lambda_a}{2}\,\frac{\lambda_a}{2}\right)_{ij}$

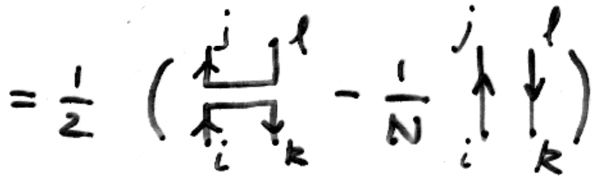




$= \delta_{ij}\left(\frac{N}{2} - \frac{1}{2N}\right) = C_F\,\delta_{ij}$

$\Sigma\left(\frac{\lambda_a}{2}\right)_{ij}\left(\frac{\lambda_a}{2}\right)_{lk} =$ 



 $= \frac{1}{2}\left(\delta_{ik}\,\delta_{jl} - \frac{1}{N}\,\delta_{ij}\,\delta_{lk}\right)$

## propagation of gluons:


we want to solve

$$\partial_\mu F^{\mu\nu} = \Box A^\nu - \partial_\mu \partial^\nu A^\mu = [\Box g^{\mu\nu} - \partial^\mu \partial^\nu] A_\nu$$

$$[\Box g^{\mu\nu} - \partial^\mu\partial^\nu]\, D_{\nu\lambda}(x-x') = j_\lambda\, \delta^{(4)}(x-x')$$

$$\langle 0 | T A_\nu(x) A_\lambda(x') | 0 \rangle$$

no solution

$\Rightarrow$ we must first FIX THE GAUGE

$\partial_\mu A^\mu = 0$ covariant

$$\boxed{D_{\mu\nu}(p) = \frac{i}{p^2}\left[-g_{\mu\nu} + (1-\lambda)\frac{p_\mu p_\nu}{p^2}\right]\delta_{ab}}$$

$\lambda = 1$ Feynman
$\lambda = 0$ Landau

$n_\mu A^\mu = 0$ axial

(light-cone if $n^2 = 0$)

$$\boxed{D_{\mu\nu}(p) = \frac{i}{p^2}\left[-g_{\mu\nu} + \frac{n_\mu p_\nu + p_\mu n_\nu}{n.p} - \frac{n^2 p_\mu p_\nu}{(n.p)^2}\right]\delta_{ab}}$$

⚠ very dangerous

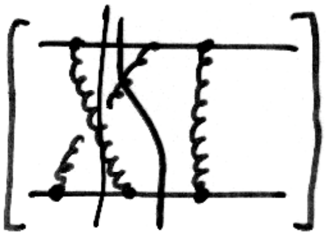
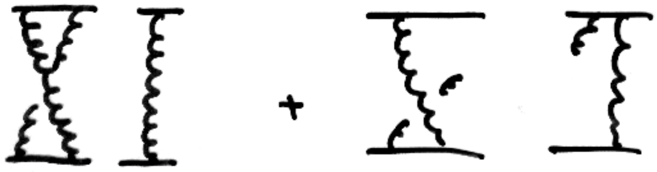
$\frac{1}{n.p}$ unphysical singularities

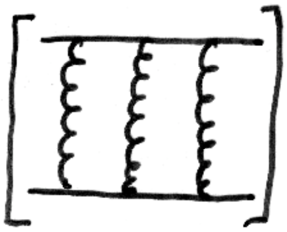
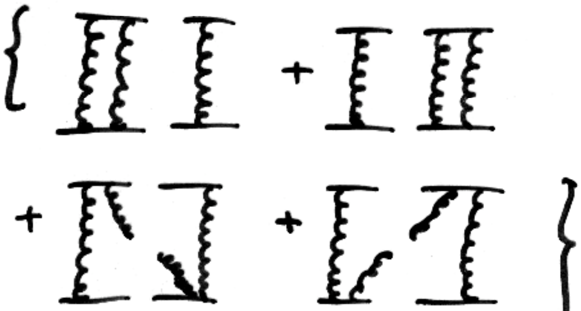
## Loop diagrams :

Unitarity $\Rightarrow$ optical theorem

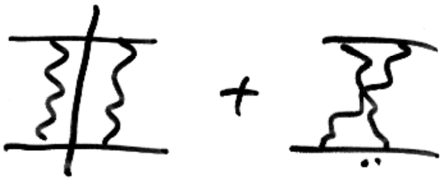
Cutkovsky's cutting rules

The imaginary part of a diagram is $\frac{1}{2}$ the sum of all cuts corresponding to 2 physical scattering processes

Im  $= \frac{1}{2}$ 

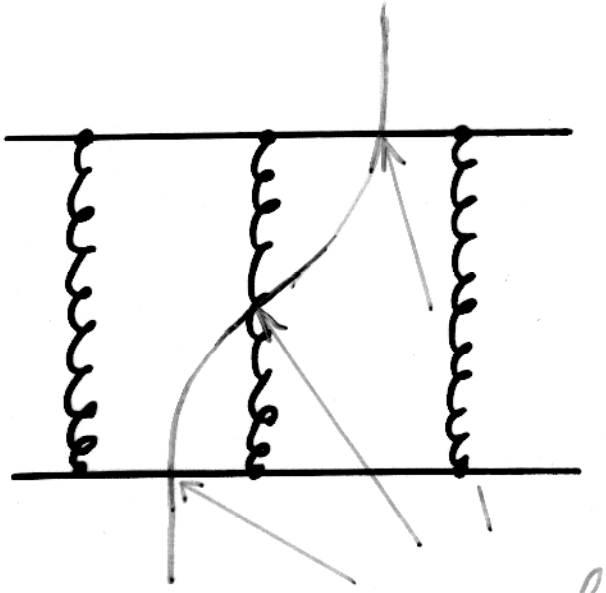
Im  $= \frac{1}{2}$ 

Im  $= 0$



Im  $\stackrel{?}{=} \frac{1}{2}$ 

# Cutting rules:



replace $\frac{1}{k^2-m^2}$

by $2\pi\,\delta_+(k^2-m^2)$

$\delta_+(k^2) \equiv \delta(k^2)\,\theta(k_0)$

e.g. $S = \frac{i\gamma.p + m}{p^2-m^2} \to 2\pi(\gamma.p+m)\ \delta^+(p^2-m^2)$

$$\mathrm{Im}\,A = \frac{1}{2}\sum_{\text{cuts}} A_c = \frac{1}{2}\sum_{\text{cuts}} a^*_{\text{left}}\, a_{\text{right}}$$

- much simpler
- sometimes finite although $\mathrm{Re}A \to \infty$
- imposes physical intermediate states

<u>physical gauges</u>: only asymptotic polarisations propagate

e.g. $n^2 = 0$

$$D_{\mu\nu} \doteq -g_{\mu\nu} + \frac{n_\mu p_\nu + n_\nu p_\mu}{n.p}$$

$$n \cdot D = 0$$

$$p \cdot D = \frac{p^2}{n.p} n \to 0$$

propagates 2 physical degrees of freedom

$\Rightarrow$ no problem with cut diagrams

<u>covariant gauges</u>

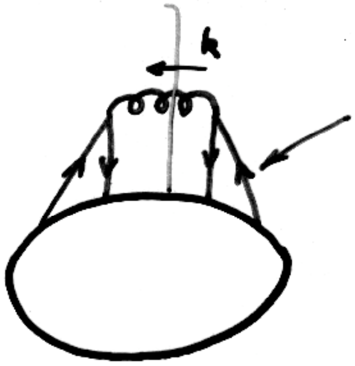
$$D_{\mu\nu} \doteq -g_{\mu\nu} + (1-\lambda)\frac{p_\mu p_\nu}{p^2}$$

$$\overset{n}{\cancel{p}} \cdot D \not\to 0$$

$$\Rightarrow \quad D_{\mu\nu} \doteq (\varepsilon_1^T)_\mu (\varepsilon_1^T)_\nu + (\varepsilon_2^T)_\mu (\varepsilon_2^T)_\nu$$

$$+ \; \varepsilon_\mu^L \varepsilon_\nu^L \; \left[ - \varepsilon_\mu^0 \varepsilon_\nu^0 \right]$$

need to be cancelled

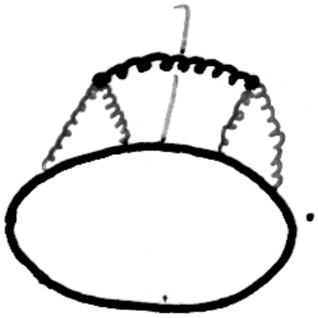



current conserved

$\partial_\mu j^\mu = 0$

$\Rightarrow$ cancellation

QED and quark-gluon OK

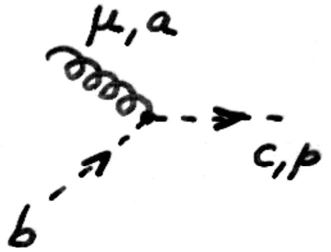


problem

In gluon loop diagrams, introduce an unphysical particle to cancel unphysical polarisations

"ghost" = spin 0 obeying Fermi-Dirac stat.!

$$\mathcal{L}_{ghost} = \Sigma (\partial_\mu \bar{\omega}_a)(\delta_{ab}\partial^\mu - g f_{abc} A_c^\mu)\omega_b$$

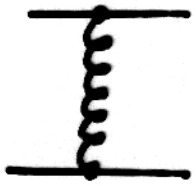



$$\boxed{-g f^{abc} p_\mu}$$

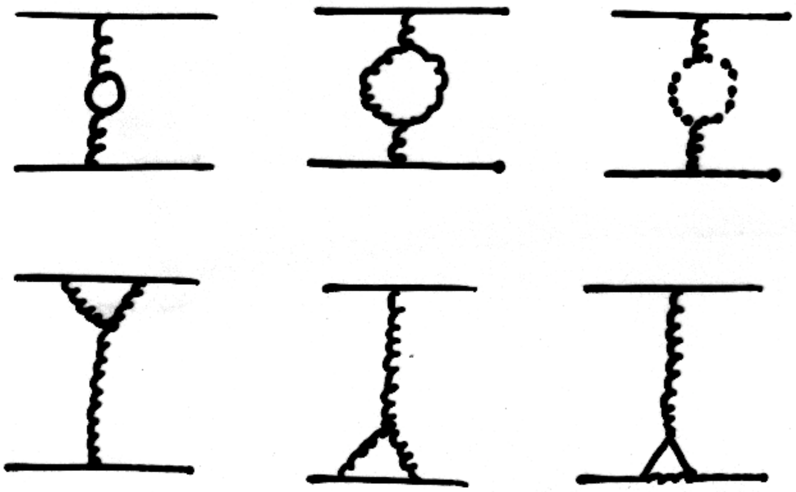
$$\boxed{\frac{i}{p^2}}$$

$+ \ldots$

A priori $\alpha_s \sim 1 \Rightarrow$ perturbation theory fails?

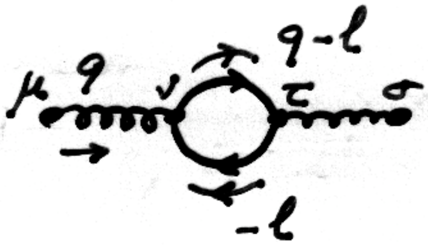


$O(g^2)$



$\Big\}\ O(g^4)$

fermion loop

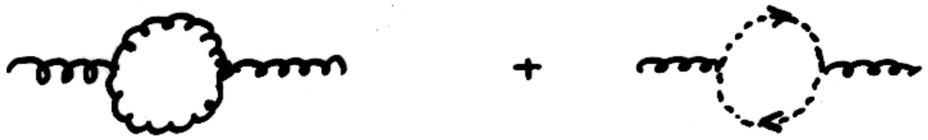



$$\sim g^4 \frac{g_{\mu\nu}}{q^2}\left[-\int \frac{d^4 l}{(2\pi)^4}\,\frac{\mathrm{Tr}\left(\gamma^\tau (q-l)\cdot\gamma\,\gamma^\nu\, l\cdot\gamma\right)}{(q-l)^2\, l^2}\right]\frac{g_{\tau\sigma}}{q^2}$$

$$\times\ \mathrm{Tr}\left(\frac{\lambda_a}{2}\,\frac{\lambda_b}{2}\right)$$

$$\sim \frac{g_{\mu\nu}}{q^2}\, g^4\, I^{\nu\tau}(q^2)\,\frac{g_{\tau\sigma}}{q^2}$$

$$I^{\nu\tau}(q^2) = -i g^{\nu\tau} q^2 g^4\left[\frac{\alpha_s}{3\pi}\underbrace{\int^\infty \frac{dl^2}{l^2}}_{\log\ \mathrm{div}} - \text{finite}\right]$$



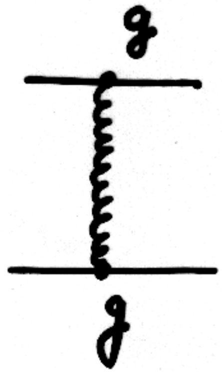
$$I^{\nu\tau} = + i g^{\mu\nu} \, q^2 \left[\frac{\alpha_s}{3\pi}\int^{\infty}\frac{dl^2}{l^2} + \text{finite}\right]$$

$$\alpha_s \, \frac{g_{\mu\nu}}{q^2}$$

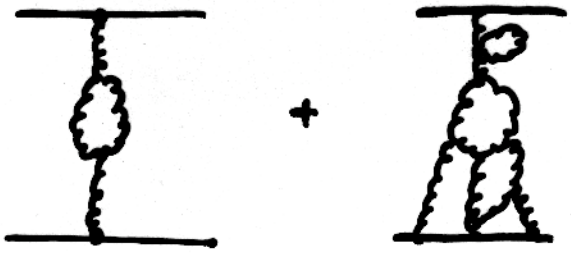
$$\sim \frac{\alpha_s \, g_{\mu\nu}}{q^2} \, \alpha_s \, I(q^2)$$

$$I(q^2) = \underbrace{\frac{33-2n_f}{12\pi}}_{4\beta_0} \log\frac{\infty}{q^2}$$

These infinities can be reabsorbed in the parameters of the theory



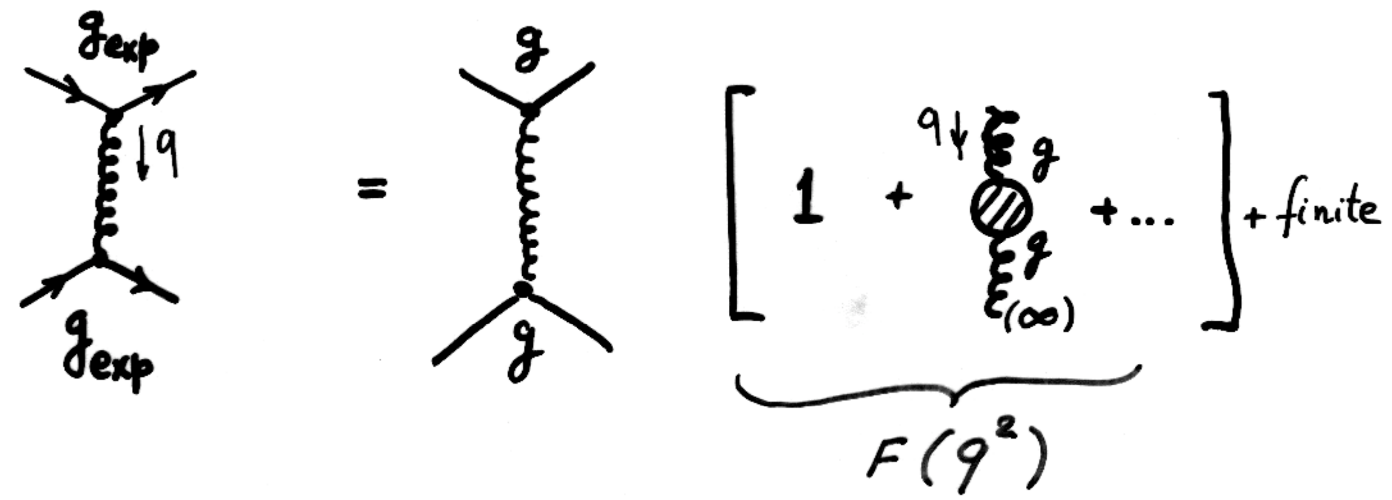

measure "$g^2$"



also there

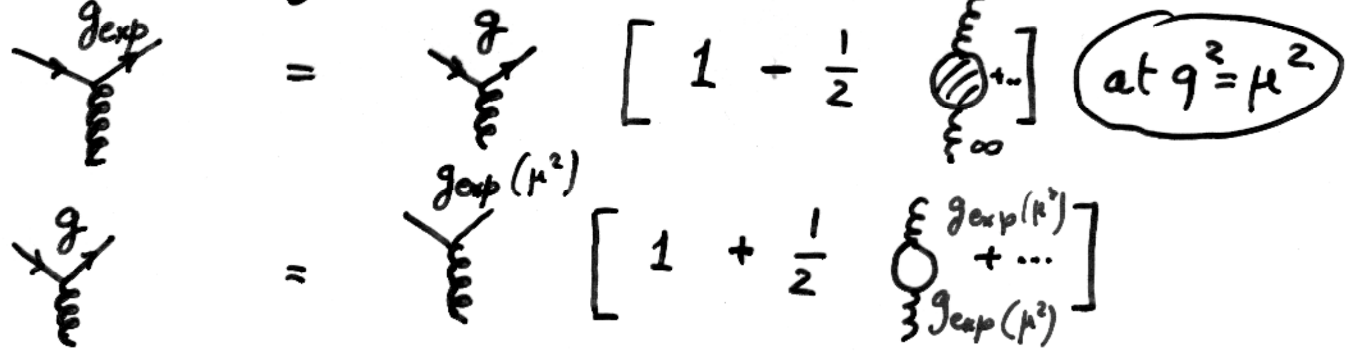
$\rightarrow$ not really $g^2$

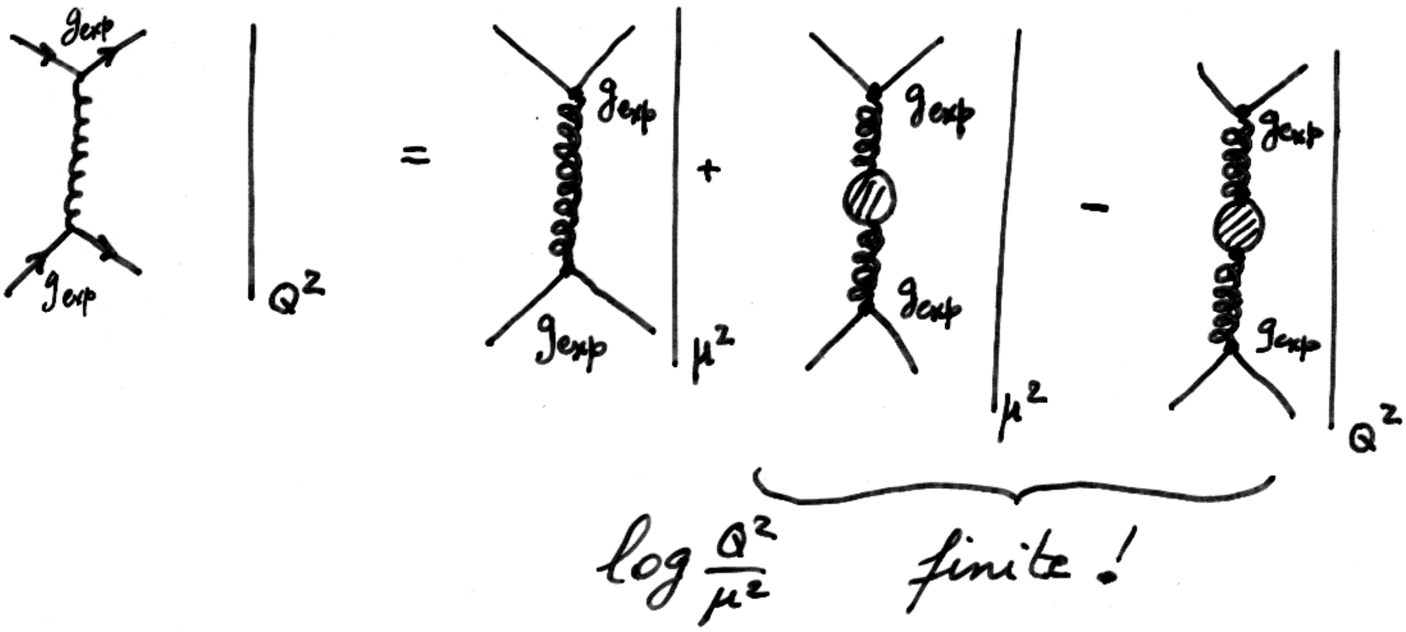
# Lowest-order renormalisation




$\Rightarrow$ can only measure $g_{exp}(\mu^2 = q^2)$

eliminate $g$:




$\Rightarrow$ infinities can be reabsorbed in the definition of $g$

## Renormalisability

all infinities can be reabsorbed in the parameters of the lagrangian

$$\begin{cases} m_i \\ g \\ \text{wave-function normalisation} \end{cases}$$

to all orders of perturbation theory

$$\Rightarrow \quad \begin{cases} m_i^R(q^2) \\ g_R(q^2) \\ Z_R(q^2) \end{cases}$$

Note: $\alpha_s^R$ from 

is the same as that from

 or  !

# Renormalisation scheme:

 $=$  $\left[ 1 - \frac{1}{2}\right.$  $\left. + \ldots \right]$

$\to \infty$

can choose to include finite constants

gauge invariance $\longrightarrow$ dimensional regularisation

$d = 4 - \varepsilon$

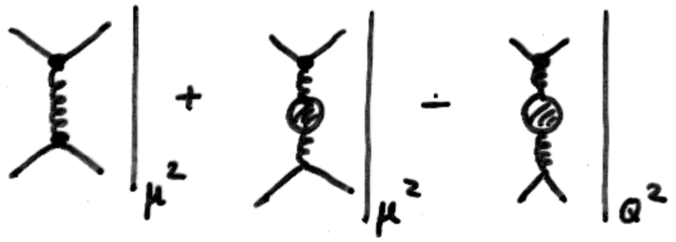
 $\sim \frac{2}{\varepsilon}$ MS

 $\sim -\gamma_E + \frac{2}{\varepsilon} + \log 4\pi$ $\overline{MS}$

 $\Big|_{Q^2} \longrightarrow$  $\Big|_{q^2} -$  $\Big|_{\mu^2}$ MOM

$\alpha_s(q^2)$ depends on the scheme

## Renormalisation group: physics does not depend on $\mu^2$



$$\equiv \mathcal{M}\left(g(\mu^2), \mu^2/Q^2\right)$$

$$\frac{d\mathcal{M}}{d\log \mu^2/Q^2} = \frac{\partial}{\partial \log \mu^2/Q^2} + \boxed{\frac{\partial \alpha_s}{\partial \log \mu^2/Q^2}}\,\frac{\partial \mathcal{M}}{\partial \alpha_s} = 0$$

$$\mathcal{M} = \sum \underline{c_n\left(\log \tfrac{\mu^2}{Q^2}\right)}\, \alpha_s^n(\mu^2) \qquad \beta(\alpha_s)$$

$$\beta(\alpha_s) = -\alpha_s \sum \beta_n \left(\frac{\alpha_s}{4\pi}\right)^{n+1}$$

$$\beta_0 = 11 - \frac{2}{3} n_f$$

$$\beta_1 = 102 - \frac{38}{3} n_f$$

$$\beta_2 = \frac{2857}{2} - \frac{5033}{18} n_f + \frac{325}{54} n_f^2$$

$$\left[-\frac{\partial}{\partial t} + \beta(\alpha_s)\frac{\partial}{\partial \alpha_s}\right]\mathcal{M}(\alpha_s, e^t) \qquad t = \log\frac{Q^2}{\mu^2}$$

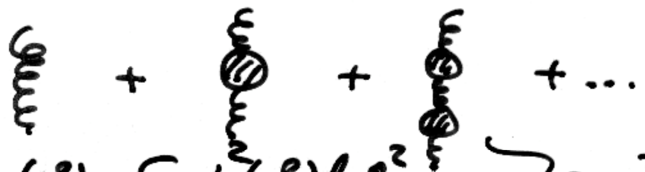
Solve by defining a new function:
the running coupling $\alpha_s(Q^2)$

$\Rightarrow$ $\mathcal{M}(\alpha_s(Q^2), 1)$ is solution

$$\begin{cases} \dfrac{\partial \alpha_s(Q^2)}{\partial t} = \beta\left(\alpha_s(Q^2)\right) \\ \alpha_s(\mu^2) = \alpha_s(Q^2 = \mu^2) \quad \text{initial condition} \end{cases}$$

$$\Rightarrow \qquad \alpha_s(Q^2) \approx \frac{\alpha_s(\mu^2)}{1 + \alpha_s(\mu^2)\left(\log \frac{Q^2}{\mu^2}\right)\left(\frac{33 - 2n_f}{12\pi}\right)}$$

can also be obtained from



$\alpha_s(\mu^2) \qquad C\alpha_s(\mu^2)\log\frac{Q^2}{\mu^2} \qquad C^2\alpha_s^3\log^2$

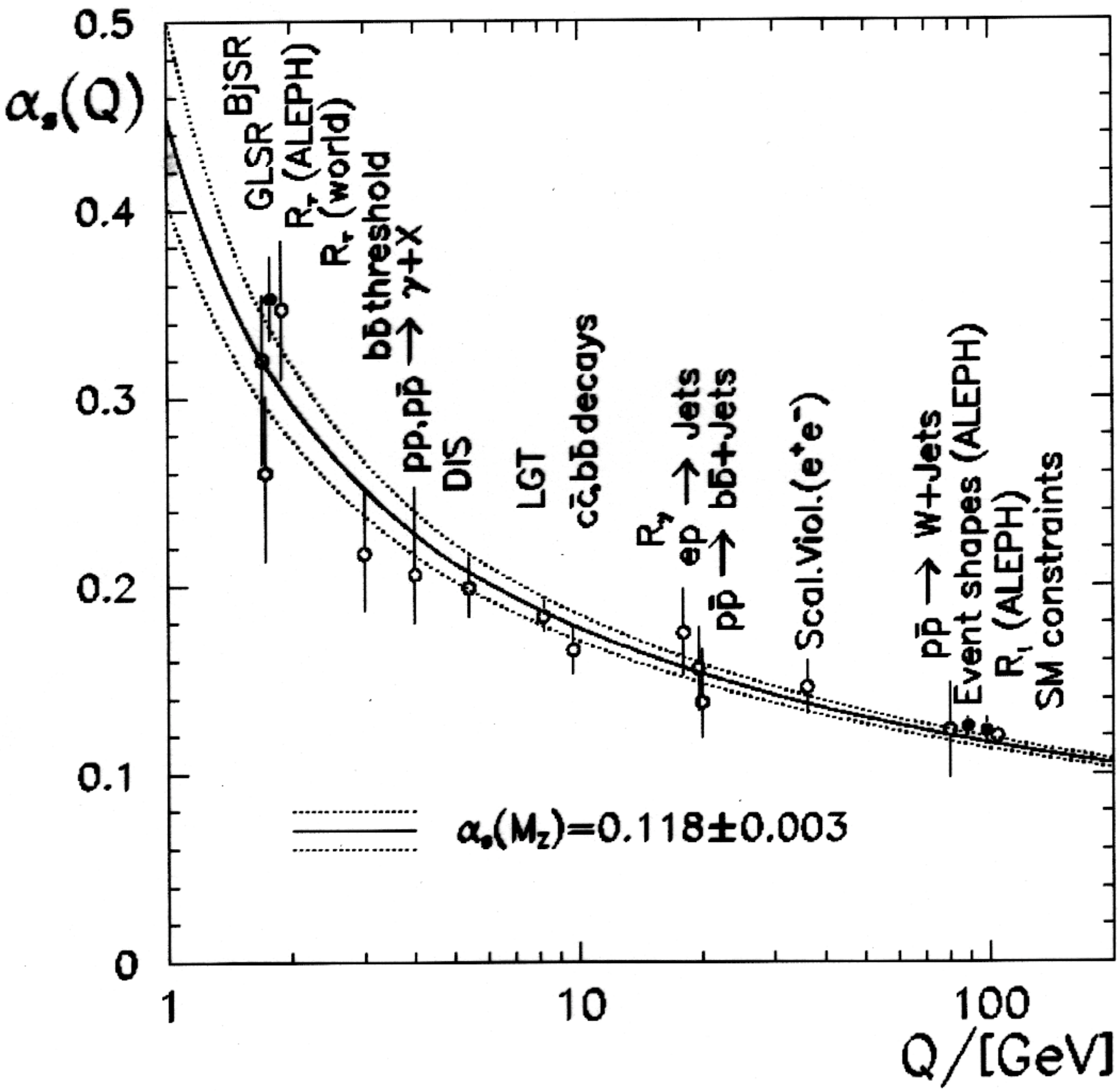
$$\alpha_s(Q^2) \longrightarrow 0 \quad \text{as } Q^2 \to \infty$$

$$\alpha_s(Q^2) \longrightarrow \infty \quad \text{for } Q^2 = \mu^2 e^{-\frac{12\pi}{(33-2n_f)\alpha_s(\mu^2)}}$$

Landau pole

$\Lambda_{QCD}$

$\Rightarrow$ perturbation theory possible at SHORT DISTANCE


0.5
0.4
0.3
0.2
0.1
0
$\alpha_s(Q)$
BjSR
GLSR
$R_\tau$ (ALEPH)
$R_\tau$ (world)
$b\bar{b}$ threshold
$pp, p\bar{p} \rightarrow \gamma + X$
DIS
LGT
$c\bar{c}, b\bar{b}$ decays
$R_\gamma$
$ep \rightarrow$ Jets
$p\bar{p} \rightarrow b\bar{b}$+Jets
Scal.Viol.($e^+e^-$)
$p\bar{p} \rightarrow$ W+Jets
Event shapes (ALEPH)
$R_l$ (ALEPH)
SM constraints
$\alpha_s(M_Z)=0.118\pm0.003$
1
10
100
Q/[GeV]

## Note:

because $\beta(\alpha_s)$ is not the complete function, but only the first few terms, we really have

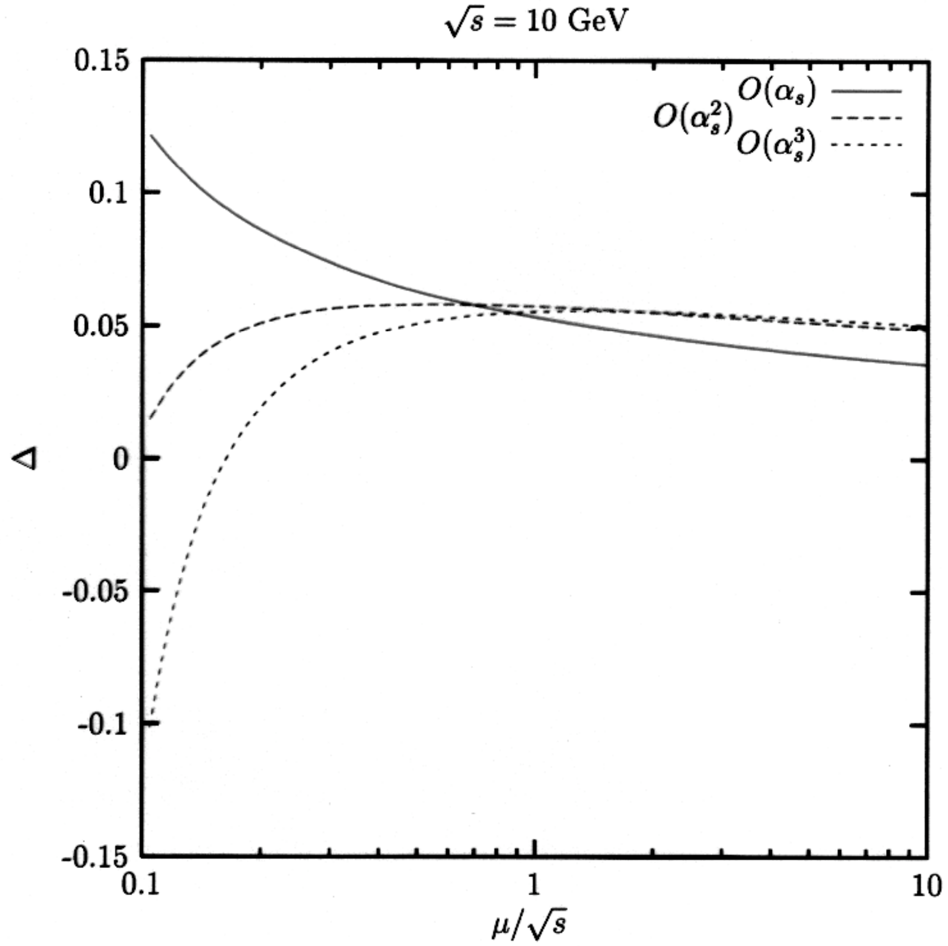
$$\frac{dR}{d\log\frac{Q^2}{\mu^2}} = \frac{\partial R}{\partial \log\frac{Q^2}{\mu^2}} + \underbrace{\left(\beta_0 \frac{\alpha_s}{4\pi} + \beta_1 \left(\frac{\alpha_s}{4\pi}\right)^2 + \dots\right)}_{n \text{ terms}} \frac{\partial R}{\partial \alpha_s}$$

$$= O(\alpha_s^{n+1})$$

$\Rightarrow$ residual $\mu$ dependence

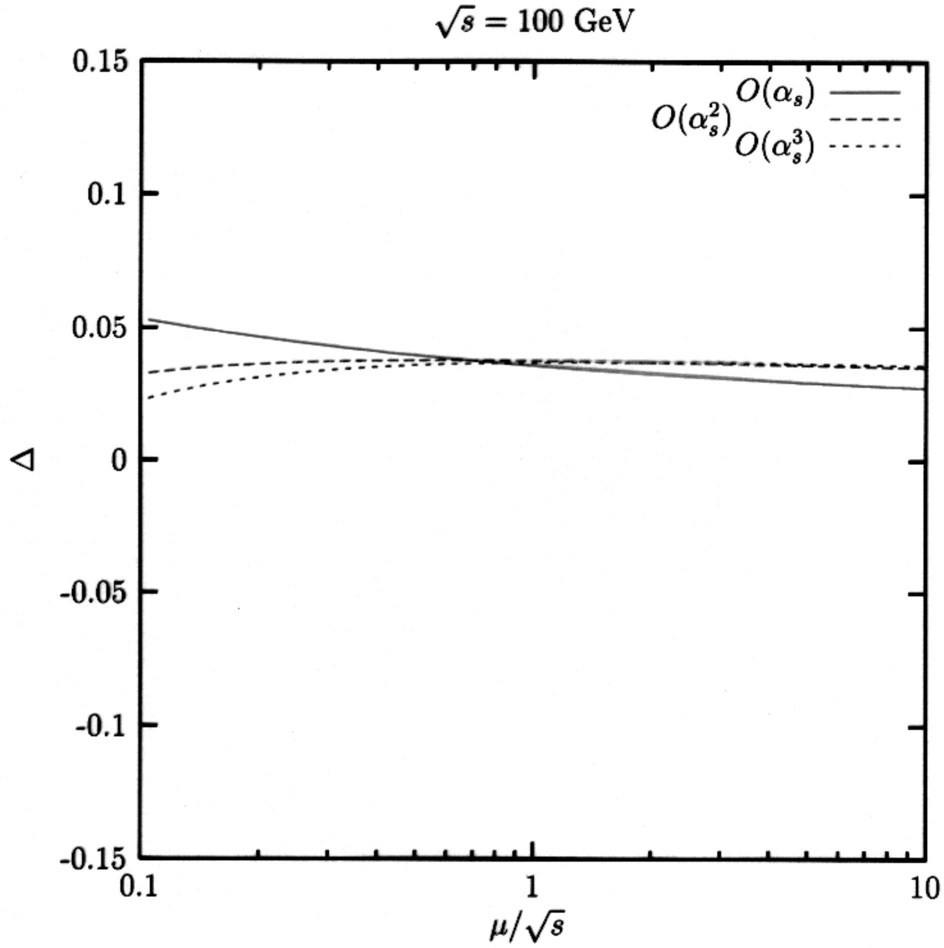
e.g. $$\frac{\sigma(ee\to h)}{\sigma(ee\to\mu\mu)} = \left(\sum_f Q_f^2\right)\left[1+\Delta\right]$$

$$\begin{aligned}\Delta_{\overline{MS}}(\mu) = & \frac{\alpha_s(\mu)}{\pi} \\ & + \left(\frac{\alpha_s(\mu)}{\pi}\right)^2 \left(1.4092 + 1.9167 \log\frac{\mu^2}{s}\right) \\ & + \left(\frac{\alpha_s(\mu)}{\pi}\right)^3 \left(-12.805 + 7.8186 \log\frac{\mu^2}{s} \right. \\ & \left. \qquad + 3.674 \log^2\frac{\mu^2}{s}\right) \\ & + \dots\end{aligned}$$



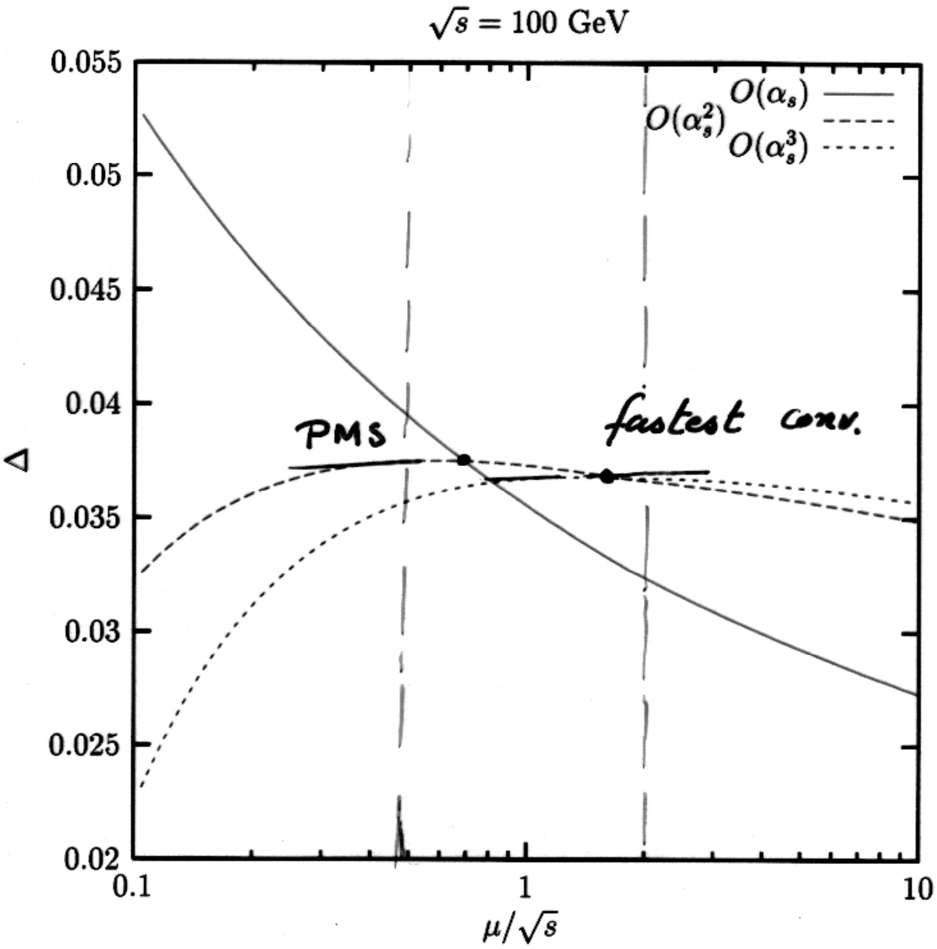

higher orders $\Rightarrow$ $\frac{d}{d\mu} R$ smaller

$\mu \approx \sqrt{s}$ most reasonable



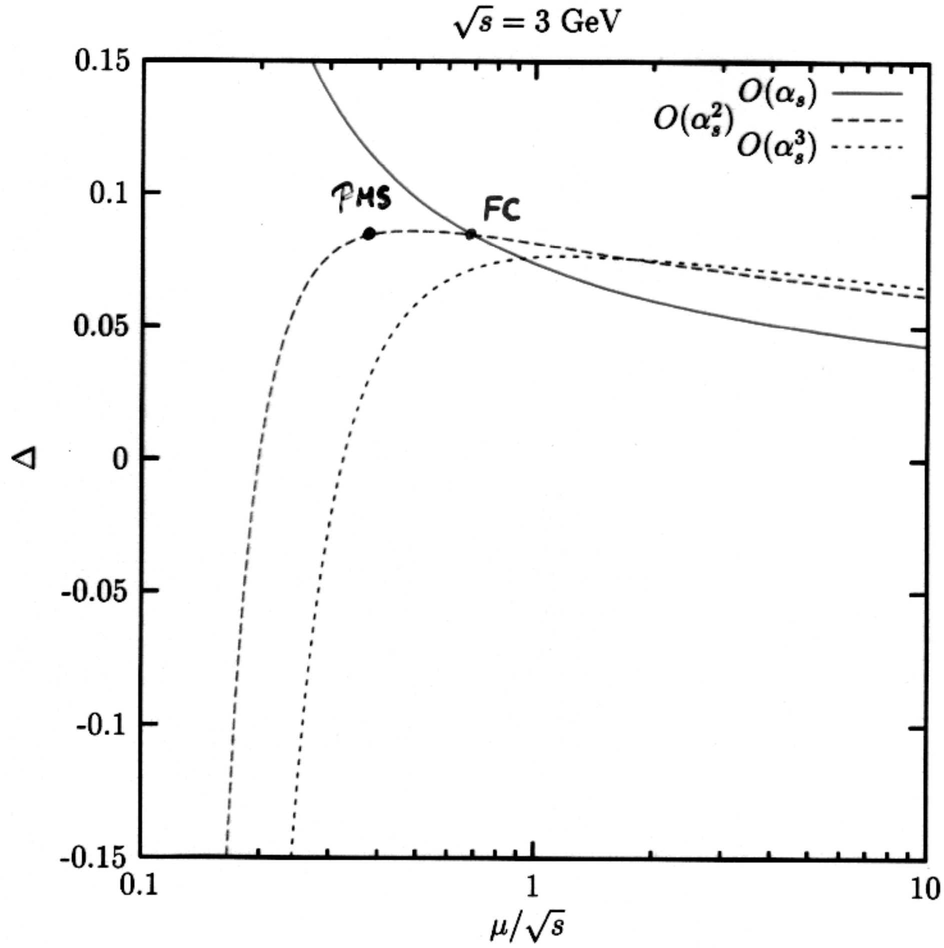

higher scale => smaller dependence

($\alpha_s$ smaller)




criteria to guess ?

$\frac{\mu}{\sqrt{s}}$ varied by 2 around 1




criteria fail if $\mu$ too small

# Evolution of quark masses:


$$\Rightarrow m(Q^2)$$

$$\left[\frac{\partial}{\partial \log\mu^2} + \beta(\alpha_s)\frac{\partial}{\partial\alpha_s} - \gamma_m(\alpha_s)\frac{\partial}{\partial \log m}\right] \mathcal{R}\left(\frac{Q^2}{\mu^2}, \alpha_s, \frac{m}{Q}\right) = 0$$

$$\gamma_m(\alpha_s) = \frac{\alpha_s}{\pi} + \frac{(303-10n_f)\alpha_s^2}{72\pi^2} + \dots$$

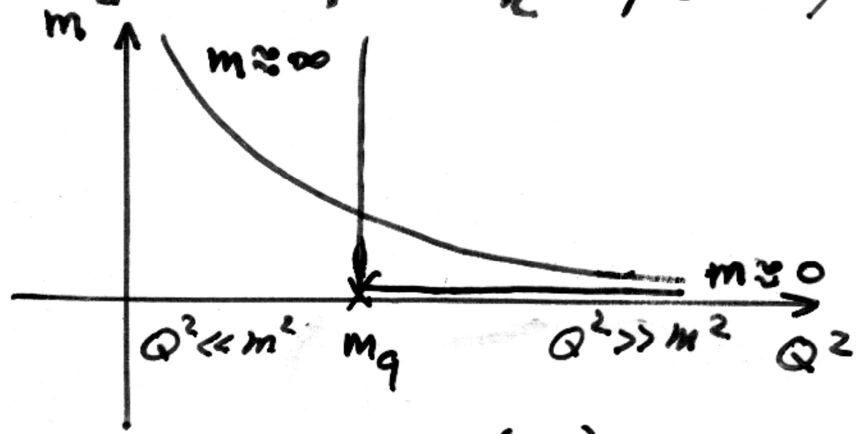
$$\gamma_0 = 1$$

## running mass:

$$\frac{\partial m}{\partial \log Q^2} = -\gamma_m(\alpha_s)\, m(Q^2)$$

$$m(Q^2) = m(\mu^2)\,\exp\left[-\int_{\mu^2}^{Q^2}\frac{dQ^2}{Q^2}\gamma_m(\alpha_s(Q^2))\right]$$

$$= \frac{m(\mu^2)}{\left(\log\frac{Q^2}{\mu^2}\,\frac{\alpha_s(\mu^2)}{\pi}\beta_0 + 1\right)^{\gamma_0/\beta_0}}$$




$$\Rightarrow \alpha_s^{(n_f)}$$

$$\alpha_s(Q^2) = \frac{12\pi}{(33-2n_f)\log\frac{Q^2}{\Lambda^2(n_f)}}$$

Define $\Lambda^{(2)}, \Lambda^{(3)}, \Lambda^{(4)} \ldots$ so that

$\alpha_s(Q^2)$ is continuous

$$\frac{12\pi}{(33-2n_f)\log\frac{Q^2}{\Lambda(n_f)^2}} = \frac{12\pi}{(33-2-2n_f)\log\frac{Q^2}{\Lambda(n_f+1)^2}}$$

$$\Lambda(n_f) = \Lambda(n_f+1)\left(\frac{Q}{\Lambda(n_f+1)}\right)^{\frac{2}{33-2n_f}}$$

$m_b \simeq 4.6$ GeV

$m_c \simeq 1.35$ GeV

$m_s \simeq 500$ MeV

$\Lambda^{(5)} \approx 200$ MeV      350 MeV

$\Rightarrow \Lambda^{(4)} \simeq 257$ MeV      430 MeV

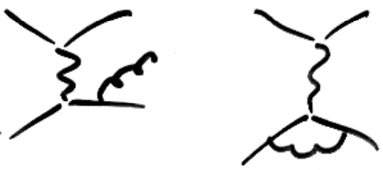
$\Lambda^{(3)} \simeq 291$ MeV      468 MeV

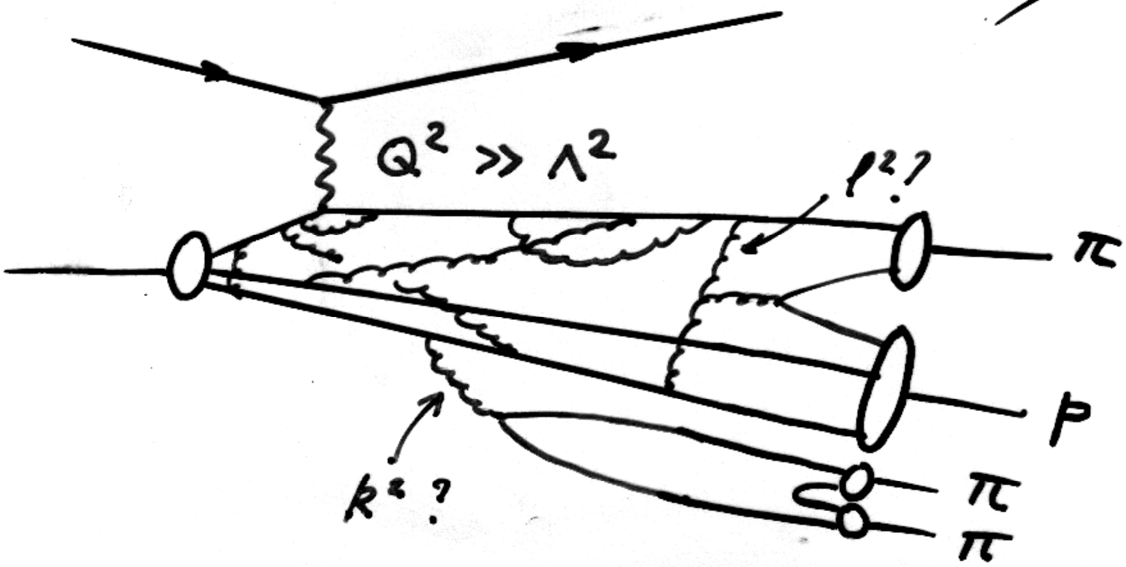
$\Lambda^{(2)} \simeq 302$ MeV      470 MeV

$\alpha_s = 1$ for $Q^2 = 3{,}7\,\Lambda^2 =$ 380 - 820 MeV

$Q \simeq 600-1000$ MeV

# How do we force short distances?





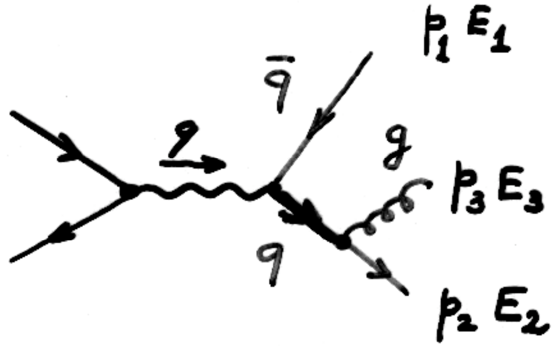

part of the physics is always long-distance

$\Rightarrow$ find observables for which it does not matter

e.g. what is the probability that the high-$Q^2$ virtual photon interacts with the proton

"I.R. finite" quantities

## long distances




$$\sqrt{s} = E_1 + E_2 + E_3$$

$$q^2 = s$$

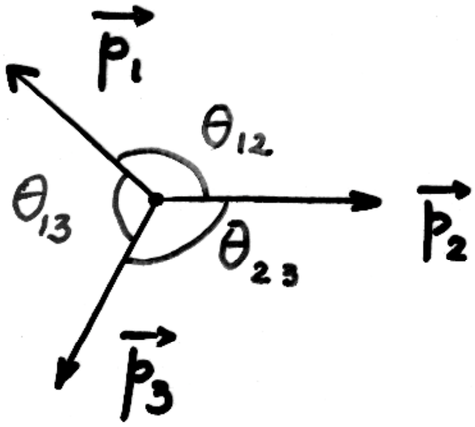
$$m \simeq 0$$

energy fractions $\quad x_i = \dfrac{2E_i^*}{\sqrt{s}} = \dfrac{2p_i \cdot q}{s}$

$$\begin{cases} \Sigma x_i = \dfrac{2(\Sigma p_i)\cdot q}{s} = 2 \\ x_i > 0 \end{cases}$$

$$(p_1 + p_2)^2 = (q - p_3)^2$$

$$2p_1 \cdot p_2 = s - 2q\cdot p_3 \Rightarrow 2E_1E_2(1 - \cos\theta_{12}) = s(1 - x_3)$$




(c.m. system)

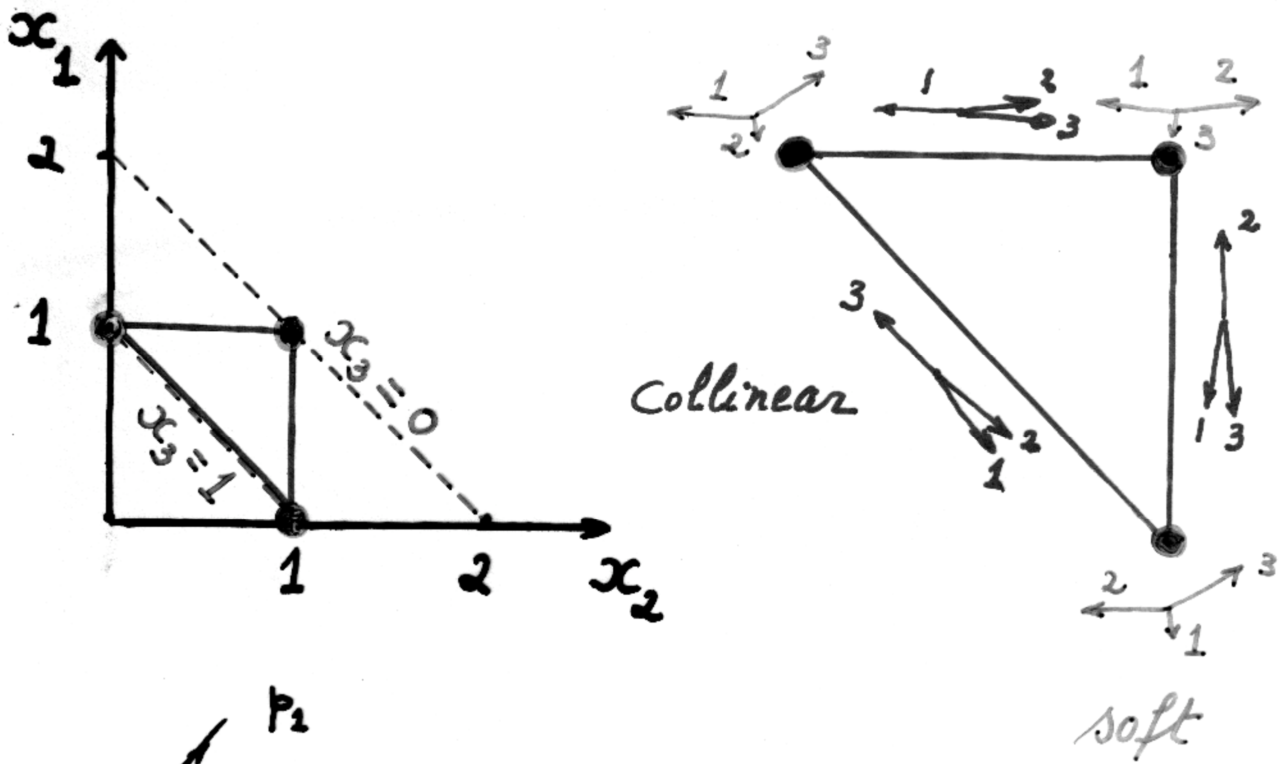
$$\begin{cases} x_1 x_2 (1 - \cos\theta_{12}) = 2(1 - x_3) \\ x_1 x_3 (1 - \cos\theta_{13}) = 2(1 - x_2) \\ x_2 x_3 (1 - \cos\theta_{23}) = 2(1 - x_1) \end{cases}$$

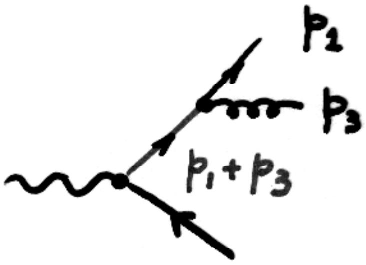
$$\Downarrow$$

$$x_i < 1$$

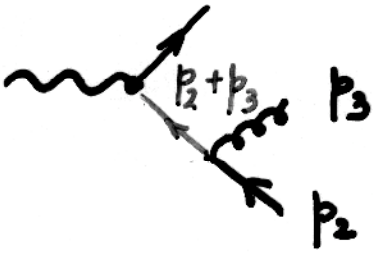
$x_i = 1 \Rightarrow \cos\theta_{jk} = 1$

$\Rightarrow \theta_{jk} = 0$




$$(p_1 + p_3)^2 = 2\, p_1 \cdot p_3$$
$$= 2 E_1 E_3 \left(1 - \cos\theta_{31}\right)$$



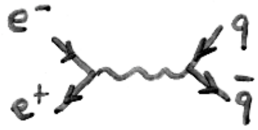

$$(p_2 + p_3)^2 = 2 E_2 E_3 \left(1 - \cos\theta_{23}\right)$$

**singularities** :

$$E_3 \to 0 \qquad \text{IR}$$

$$\left.\begin{array}{l} \theta_{31} \to 0 \\ \theta_{32} \to 0 \end{array}\right\} \text{collinear}$$

$$\frac{d\sigma}{dx_1\, dx_2} = \left[\frac{4\pi\alpha^2}{s} \sum Q^2\right] \frac{\alpha_s C_F}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$



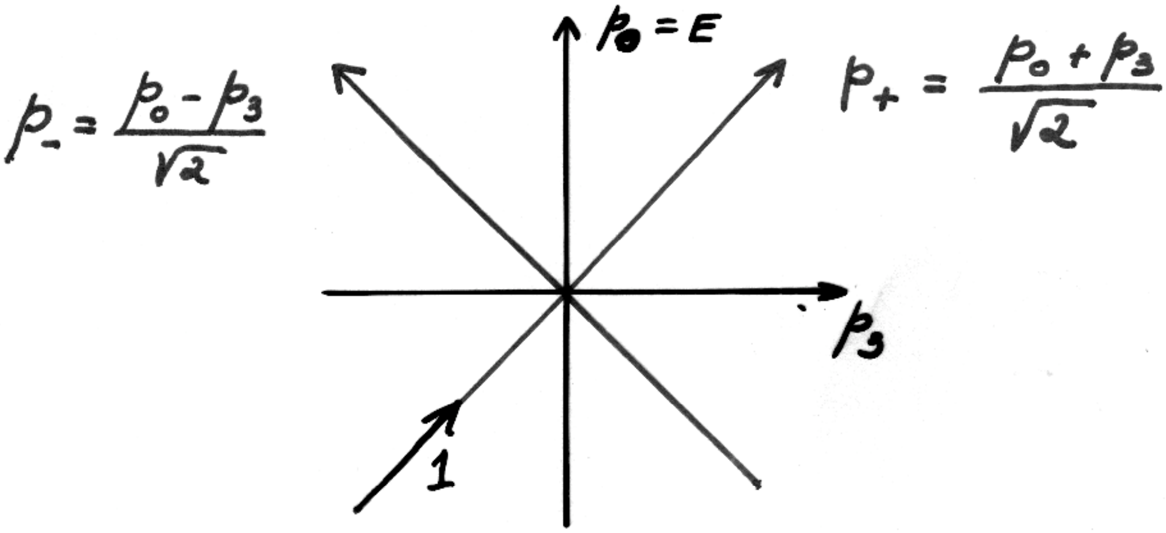
can also write

$$\frac{d\sigma}{dE_3\, d\cos\theta_{31}} \sim \frac{1}{E_3}\,\frac{1}{1-\cos\theta_{31}}$$

$$\Rightarrow \quad \log\frac{\Lambda_{IR}}{\sqrt{s}}\,, \quad \log\frac{m}{\sqrt{s}}$$

$\Rightarrow$ perturbation theory fails?

## Space-time structure:



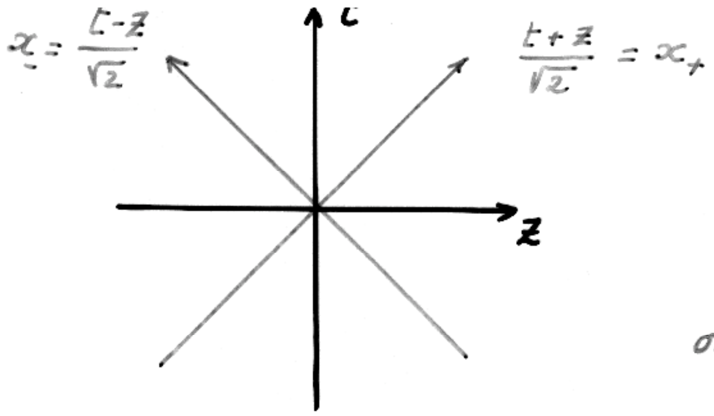

light-cone coordinates:

$$\left.\begin{array}{l} p_3 \text{ large} \\ \vec{p}_T \text{ finite} \end{array}\right\} \Rightarrow \quad p_+ \text{ large}, \; p_- \text{ small}$$

$$p\cdot p = 2p_+ p_- - \vec{p}_T^{\,2} = m^2$$

$$\Rightarrow \quad p_- = \frac{\vec{p}_T^{\,2} + m^2}{2p_+}$$

on-shell $\Rightarrow$ $p_+ > 0$, $p_- > 0$




$p_+ \to \infty$

$\Rightarrow p_- \to 0$

$\Rightarrow x_- \sim$ fixed

or: $p \cdot x = p^+ x_- + p^- x_+ - \vec{p}_T \cdot \vec{x}_T$

$x^\mp$ conjugate to $p^\pm$


choose axis $z$ so that

$\vec{k}_T = 0 \qquad k_3 > 0$

$k_+$ large

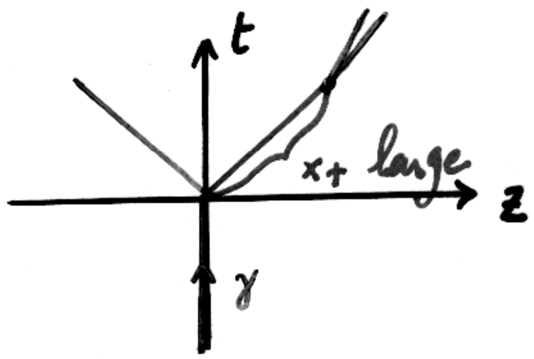
$$k^2 = 2 k_+ k_-$$

small if $\quad k_- = \frac{p_{3,T}^2}{2p_1^+} + \frac{p_{3,T}^2}{2p_3^+}$ small

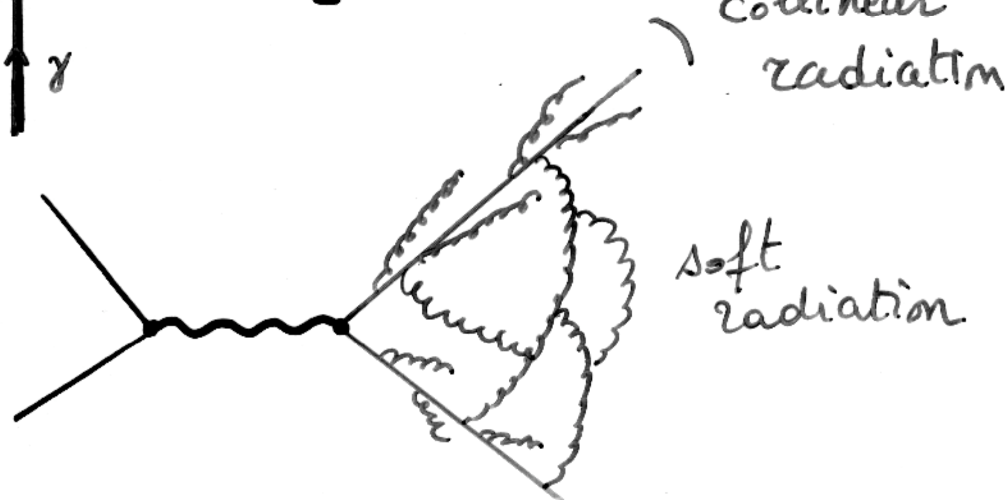
collinear: $\quad p_{3,T} \sim 0 \qquad p_1^+, p_3^+$ fixed

soft: $\quad p_{3T} \sim 0 \qquad p_3^+ \sim p_{3T}$

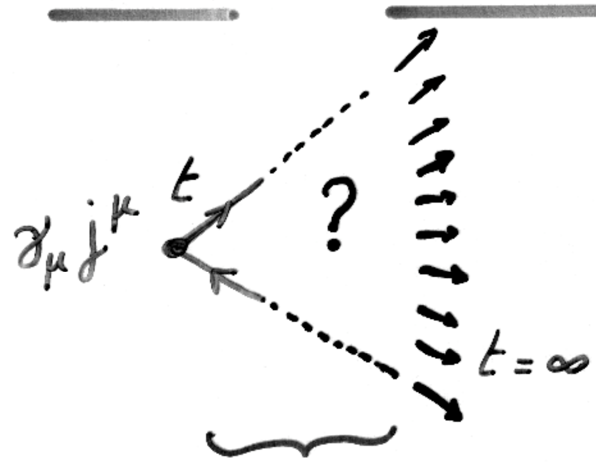
Small $k_- \Rightarrow$ large $x_+$




suggests:



# Trick : inclusive observables

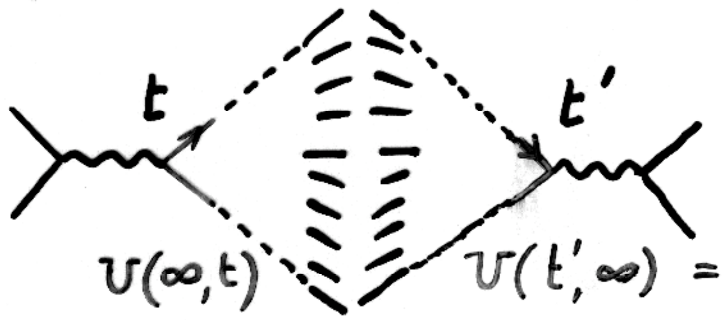



final state $|N\rangle$

evolution operator

$U(\infty,t) \equiv U(t\to\infty)$

$\sigma \sim$




$U(\infty,t)$ $\quad$ $U(t',\infty) = U^*(\infty,t')$

$\mathcal{M}$ $\quad$ $\mathcal{M}^*$

$$\sigma_{tot} \sim \sum_N \cdots \langle 0|\, j(t')\, U(t',\infty)\, |N\rangle\langle N|\, U(\infty,t)\, j(t)\, |0\rangle \cdots$$

$$\sim \langle 0|\, j(t')\, U(t',\infty)\, U(\infty,t)\, j(t)\, |0\rangle$$

$$\sim \langle 0|\, j(t')\, U(t',t)\, j(t)\, |0\rangle$$

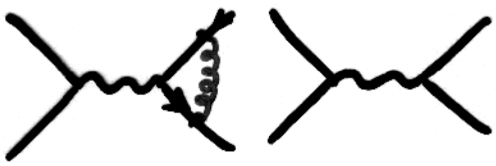
$e^{-i\sqrt{s}\,(t-t')} \Rightarrow t-t' \sim \frac{1}{\sqrt{s}}$ small

$\Rightarrow$ only short distance

$\Rightarrow \quad O(\alpha_s)$ *finite* :

$|\mathcal{M}|^2$ 

$+$ 

$+$ 

virtual corrections cancel the divergences of bremstrahlung

 $\Rightarrow \sigma_{tot}$ at $O(\alpha_s^0)$

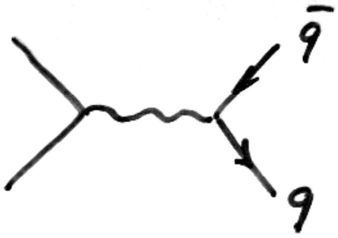
 $\Rightarrow \sigma_{tot}$ at $O(\alpha_s^1)$

NOT n-particle final state!

NB: $\langle q, ng \,|\, \hat{O} \,|\, q, mg \rangle = 0$

if definite $m, n$ : no naive S matrix

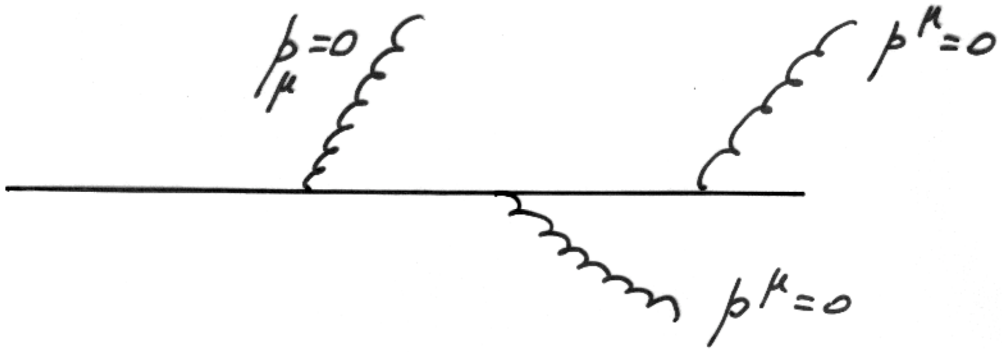
collective states?

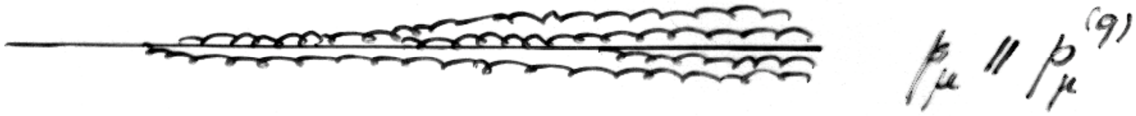



= inclusive jet cross section to $O(\alpha_s)$

$+ \ldots$ = inclusive cross section to $O(\underline{\underline{\alpha_s^2}})$

⋮




states with a fixed number of gluons do <u>not</u> exist