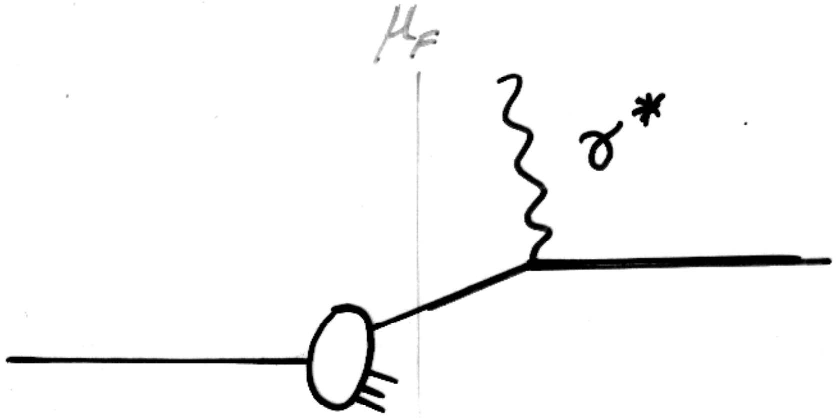


Same for fragmentation functions

large distance

short distance fluctuations

$\Downarrow$

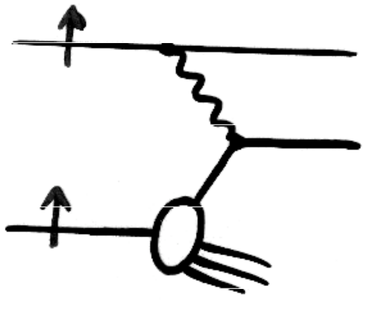
universal parton distibutions

evolution equations

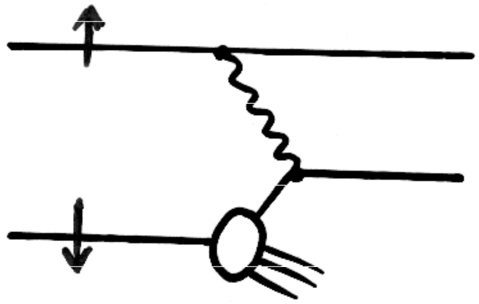
measured parton distibutions

Drell Yan
hadronic
jets

Spin-dependent structure functions



$\sigma^{\uparrow\uparrow}$



$\sigma^{\uparrow\downarrow}$

$$A = \frac{d\sigma^{\uparrow\uparrow} - d\sigma^{\uparrow\downarrow}}{d\sigma^{\uparrow\uparrow} + d\sigma^{\uparrow\downarrow}} \approx \frac{g_1(x,Q^2)}{F_1(x,Q^2)}$$

polarisation

unpolarised DIS

$$\frac{d\sigma^{\uparrow\uparrow}}{dx\,dy} - \frac{d\sigma^{\downarrow\uparrow}}{dx\,dy} = -\frac{8\pi\alpha^2 ME}{Q^2}\left[\left(2y - y^2 - \frac{xyM}{E}\right)2x\, g_1(x,Q^2) - \frac{4M}{E}x^2 y\, g_2(x,Q^2)\right]$$

$\Rightarrow$ g_1 and g_2

from antisymmetric part of $W_{\mu\nu}$

$$W^{\mu\nu} = \frac{i\epsilon^{\mu\nu\rho\sigma} q_\rho}{P\cdot q}\left[S_\sigma g_1 + \left(S_\sigma - \frac{S\cdot q}{P\cdot q}P_\sigma\right)g_2\right]$$

$S^2 = -1 \qquad S\cdot P = 0$

polarisation vector

Partonic interpretation

$$q(x) = q^{\uparrow}(x) + q^{\downarrow}(x)$$
$$\Delta q(x) = q^{\uparrow}(x) - q^{\downarrow}(x)$$

$$\Rightarrow \begin{cases} F_1 = \sum_q e_q^2 \dfrac{q(x) + \bar{q}(x)}{2} \\ g_1 = \sum_q e_q^2 \dfrac{\Delta q(x) + \Delta \bar{q}(x)}{2} \end{cases}$$

$SU(2)_F$ + current algebra

from β decay

$$\Rightarrow \int_0^1 dx\,(g_1^p - g_1^n) = \frac{1}{6}\left|\frac{g_A}{g_V}\right| = 0.209 \pm 0.001$$

Bjorken sum rule

$$\eta_q = \int_0^1 dx\,(\Delta u + \Delta \bar{u}) \qquad \eta_g = \int_0^1 dx\, \Delta g(x)$$

$\hookrightarrow g^{\uparrow} - g^{\downarrow}$

at $Q^2 = 10\,\mathrm{GeV}^2$ $\begin{cases} \eta_u \simeq 0.83 \\ \eta_d \approx -0.43 \\ \eta_s \approx -0.10 \end{cases}$

Gehrmann - Stirling
PRD 53 (1996) 6100

$\hookrightarrow$ Ellis - Jaffe sum rule wrong.

$$\frac{1}{2} = \frac{1}{2}\sum \eta_q + \eta_g + \langle L_z \rangle$$

spin sum rule

Evolution :

DGLAP equation for Δq_i, Δg

$$g_1 \sim \sum e_i^2 (\Delta q + \Delta \bar{q})(1 + O(\alpha_s))$$

$$+ \sum e_i^2 \int_x^1 \frac{d\xi}{\xi} \Delta g\left(\frac{x}{\xi}, Q^2\right) \frac{\alpha_s(Q^2)}{2\pi} \Delta C_g(\xi) + \dots$$

But evolution gives :

$$\frac{\partial}{\partial \log Q^2} \begin{pmatrix} \sum_q \eta_i \\ \eta_g \end{pmatrix} = \frac{\alpha_s(Q^2)}{2\pi} \begin{pmatrix} 0 & 0 \\ \frac{3}{2} C_F & \frac{11}{2} - \frac{1}{3} n_f \end{pmatrix} \begin{pmatrix} \sum_q \eta_i \\ \eta_g \end{pmatrix}$$

$$\Rightarrow \begin{cases} \dfrac{\partial}{\partial \log Q^2} \sum \eta_i = 0 & + O(\alpha_s^2) \\ \dfrac{\partial}{\partial \log Q^2} \alpha_s(Q^2)\, \eta_g = 0 & + O(\alpha_s^2) \end{cases}$$

The evolution of α_s cancels that of η_g

$\Rightarrow$ **gluons contribute as much as quarks to g_1**

$\eta_s = 0 \quad \Rightarrow \quad \eta_g \simeq 2$

Leader, Sidorov, Stamenov
hep-ph/0111267

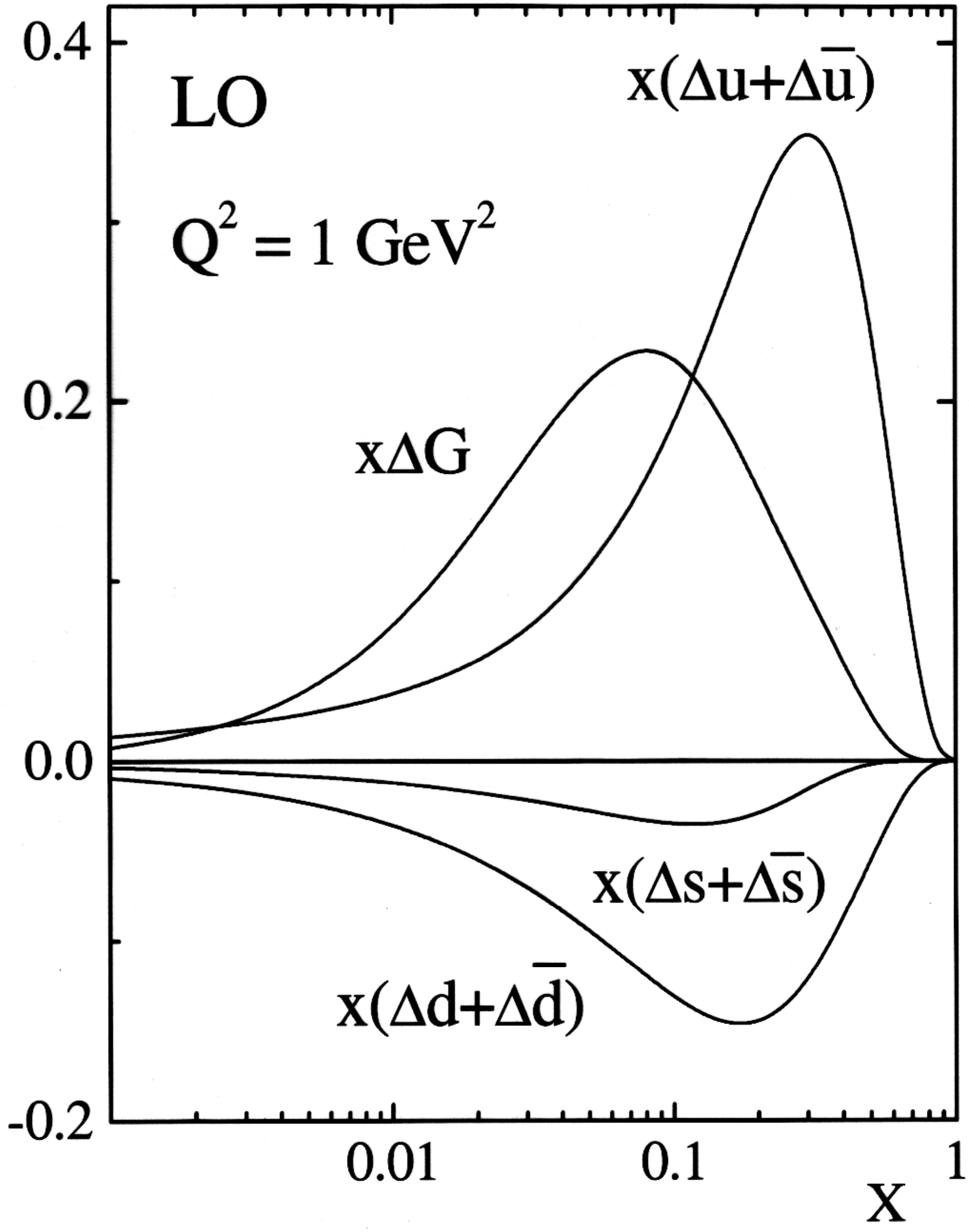


Fig. 2

Leader, Sidorov, Stamenov
hep-ph/0111267

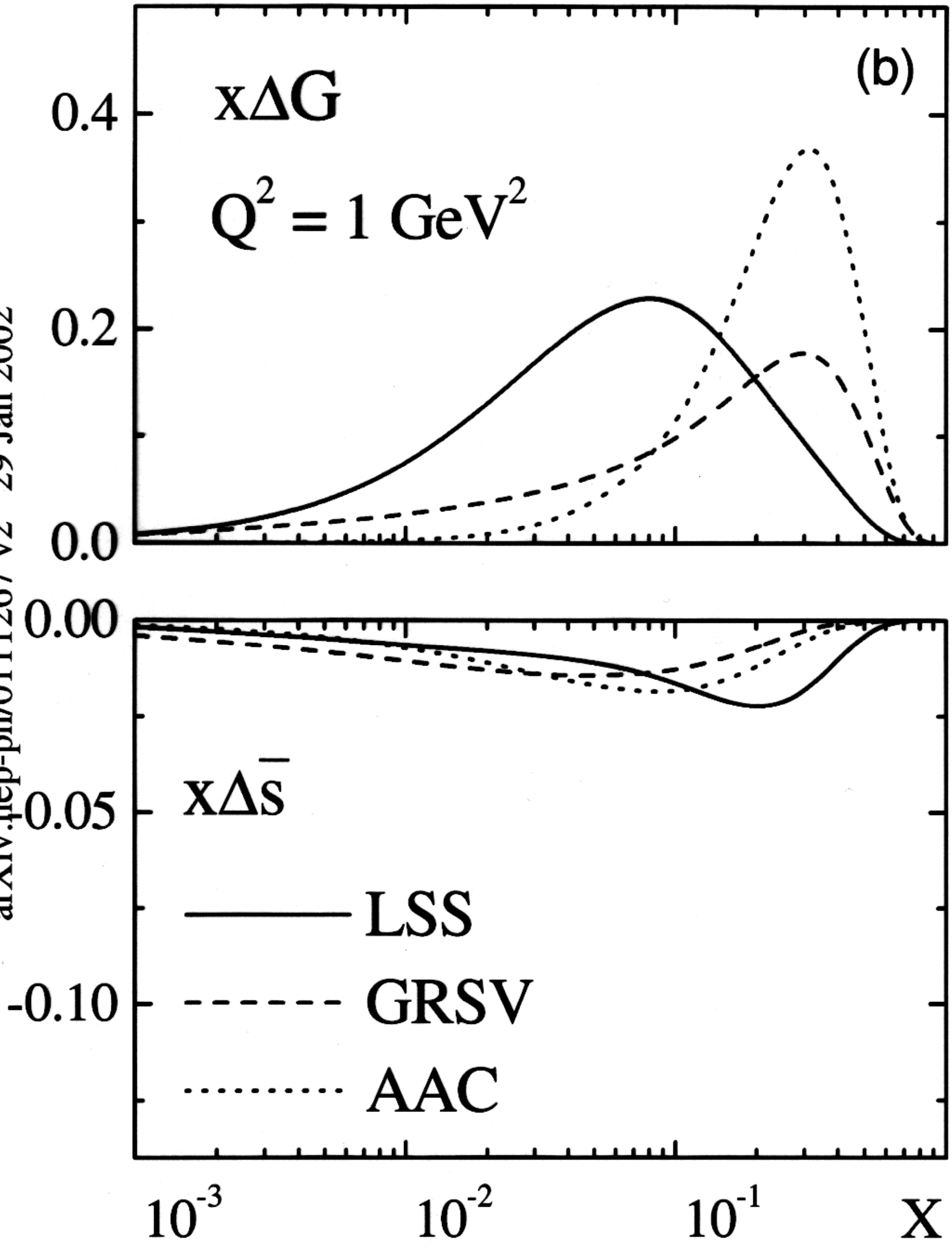


Fig. 8(b)

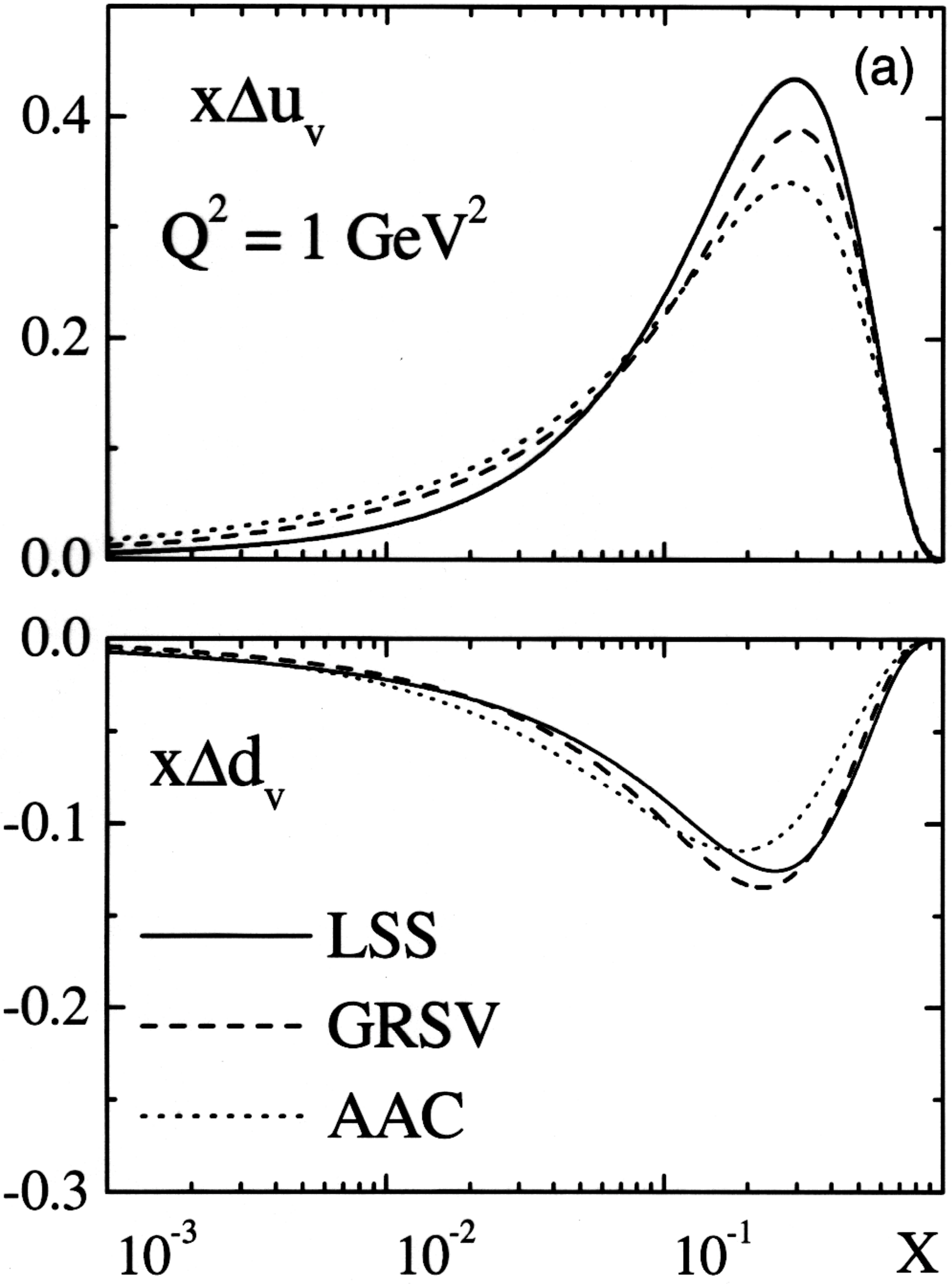
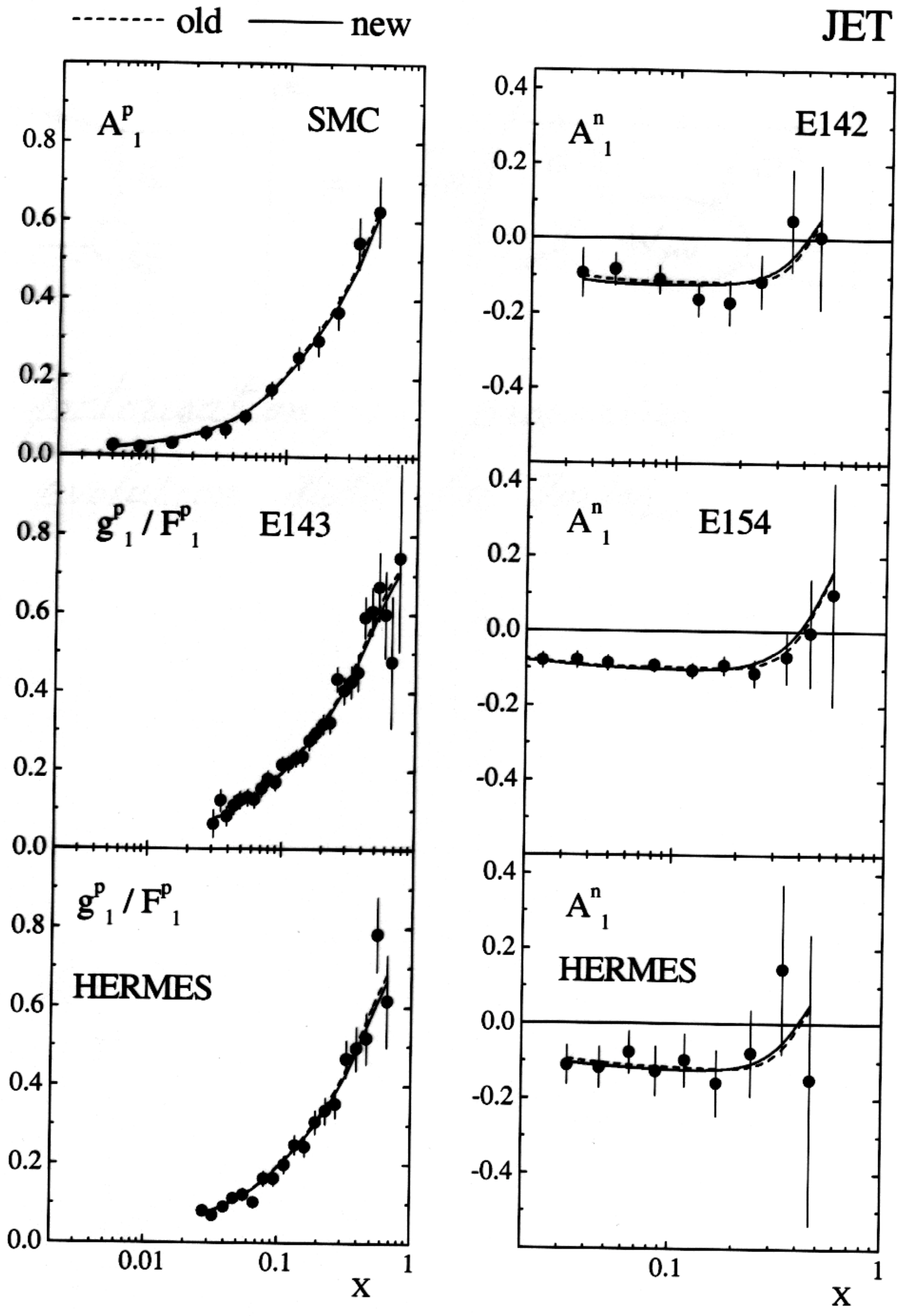

(a)
xΔu_v
Q² = 1 GeV²
xΔd_v
LSS
GRSV
AAC
0.4
0.2
0.0
0.0
-0.1
-0.2
-0.3
10⁻³
10⁻²
10⁻¹
X

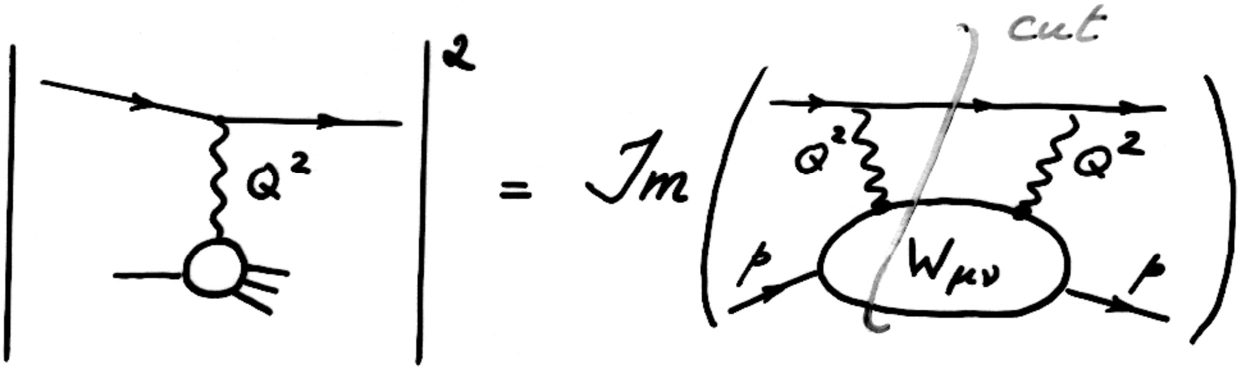
Fig. 8 (a)



old
new
JET
A^p_1
SMC
g^p_1 / F^p_1
E143
g^p_1 / F^p_1
HERMES
A^n_1
E142
A^n_1
E154
A^n_1
HERMES
0.0
0.2
0.4
0.6
0.8
-0.2
-0.4
0.01
0.1
1
x

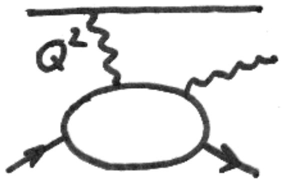
Fig. 3

Off-diagonal parton distributions

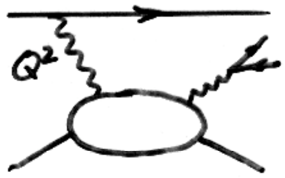


factorisation, I.R. properties, evolution hold for $\mathrm{Im}\, W_{\mu\nu}$

They hold for $W_{\mu\nu}$ itself and for its generalisations :



Deeply virtual Compton scattering

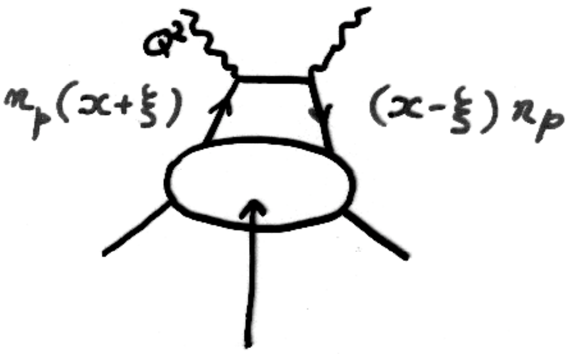


Elastic lepton pair production

Elastic vector meson production

Diffractive scattering

Partons :



Generalised
Parton
Distribution

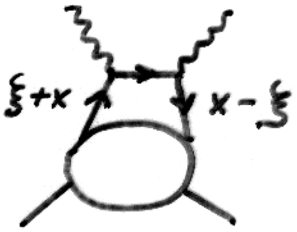
$Q^2 \to \infty$

x_B fixed

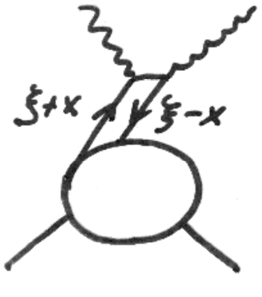
$\Delta^2 = t$ fixed

$\Rightarrow$ factorisation

2 kinematics :



1 parton emitted
and reabsorbed
$x > \xi$

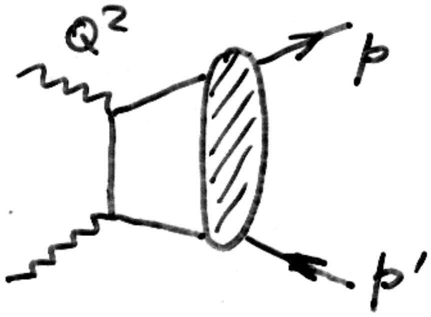


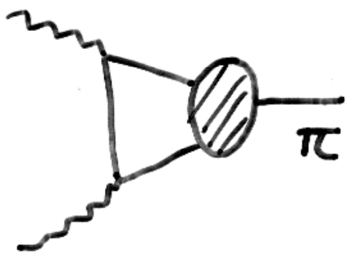
2 partons emitted : $q, \bar{q}$

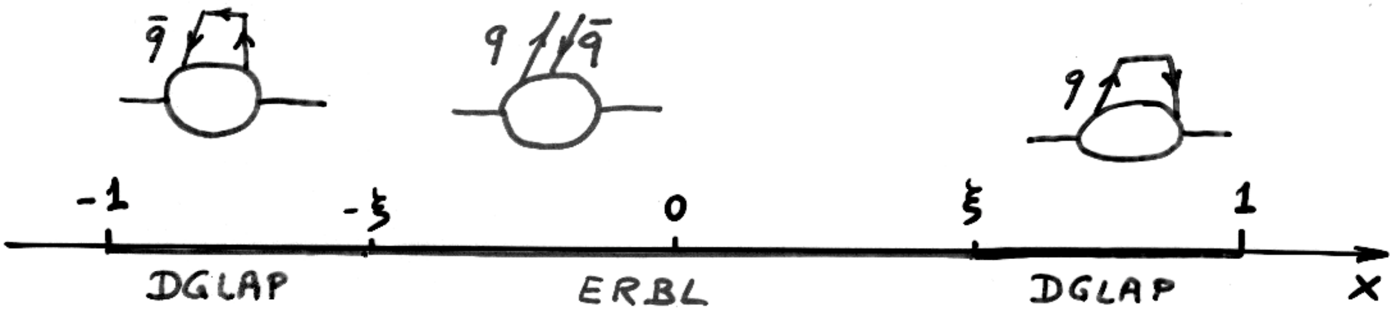
$\to$ new regime $\quad x < \xi$

$\Rightarrow$ probe of pairs of partons

Crossed processes :







Efremov - Radyushkin - Brodsky - Lepage

$\mathcal{H}_q(x,\xi,t)$ $\quad E_q$

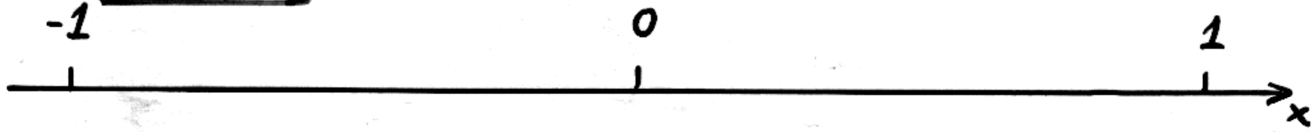
$\tilde{\mathcal{H}}_q(x,\xi,t)$ $\quad \tilde{E}_q$

$\mathcal{H}_g(x,\xi,t)$ $\quad E_g$

$\tilde{\mathcal{H}}_g(x,\xi,t)$ $\quad \tilde{E}_g$

helicity flip

$\boxed{p = p'}$



$\mathcal{H}_q(x,0,0) = -\bar{q}(-x)$ $\qquad \mathcal{H}_q(x,0,0) = q(x)$

$\tilde{\mathcal{H}}_q(x,0,0) = \Delta\bar{q}(-x)$ $\qquad \tilde{\mathcal{H}}_q(x,0,0) = \Delta q(x)$

$\mathcal{H}_g(x,0,0) = g(x)$

$\tilde{\mathcal{H}}_g(x,0,0) = \Delta g(x)$

$\boxed{\xi = 0\ ,\ t \neq 0}$

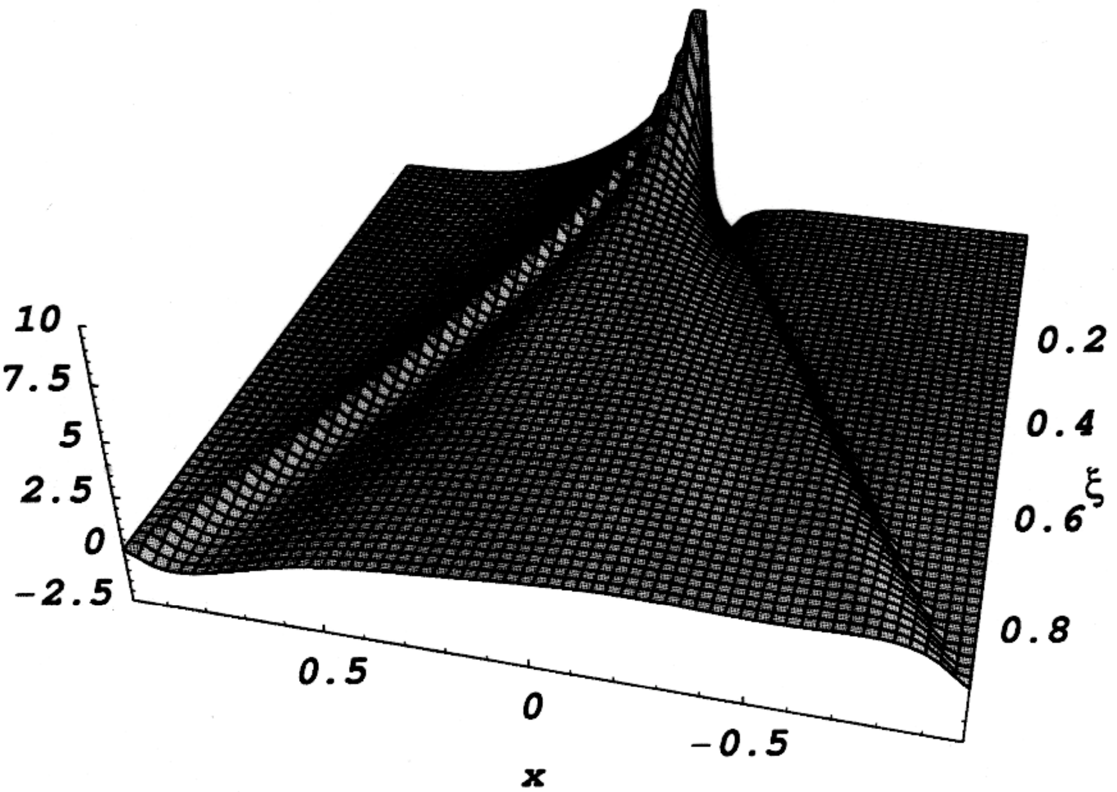
$$f(x,\vec{b}) = \int \frac{d^2\Delta}{(2\pi)^2}\, e^{-\vec{b}.\vec{\Delta}}\, \mathcal{H}(x, t = -\vec{\Delta}^2)$$

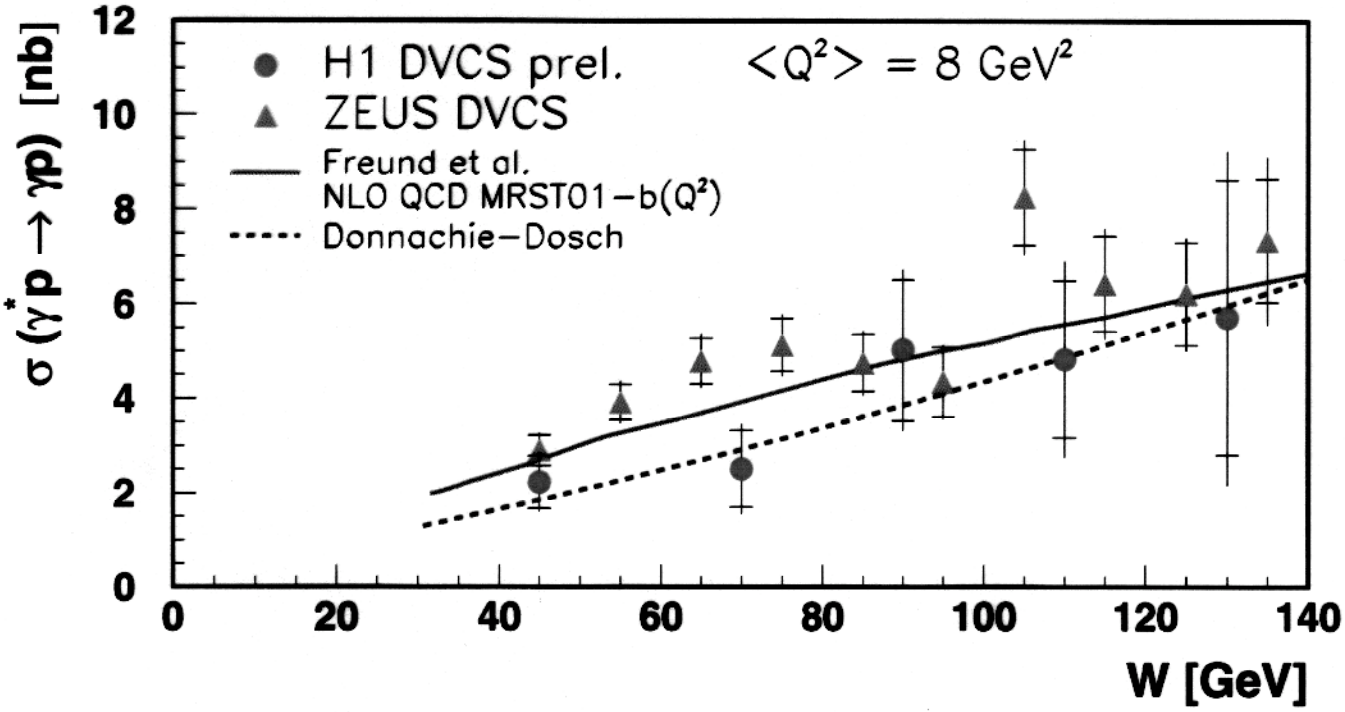
probability of parton with mom. fraction x

Models for GPD's exist

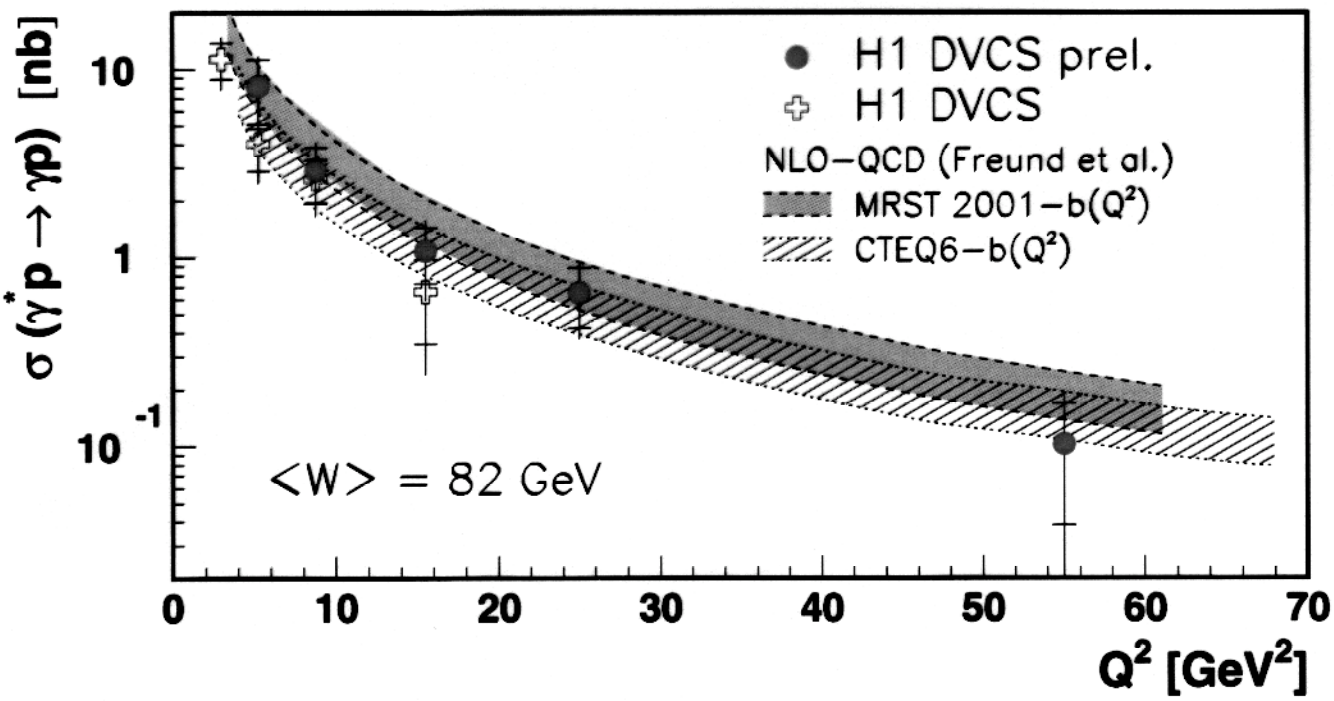
Goeke,
Polyakov,
Vanderhaeghen
Prog. Part. Phys.
47 (2001)

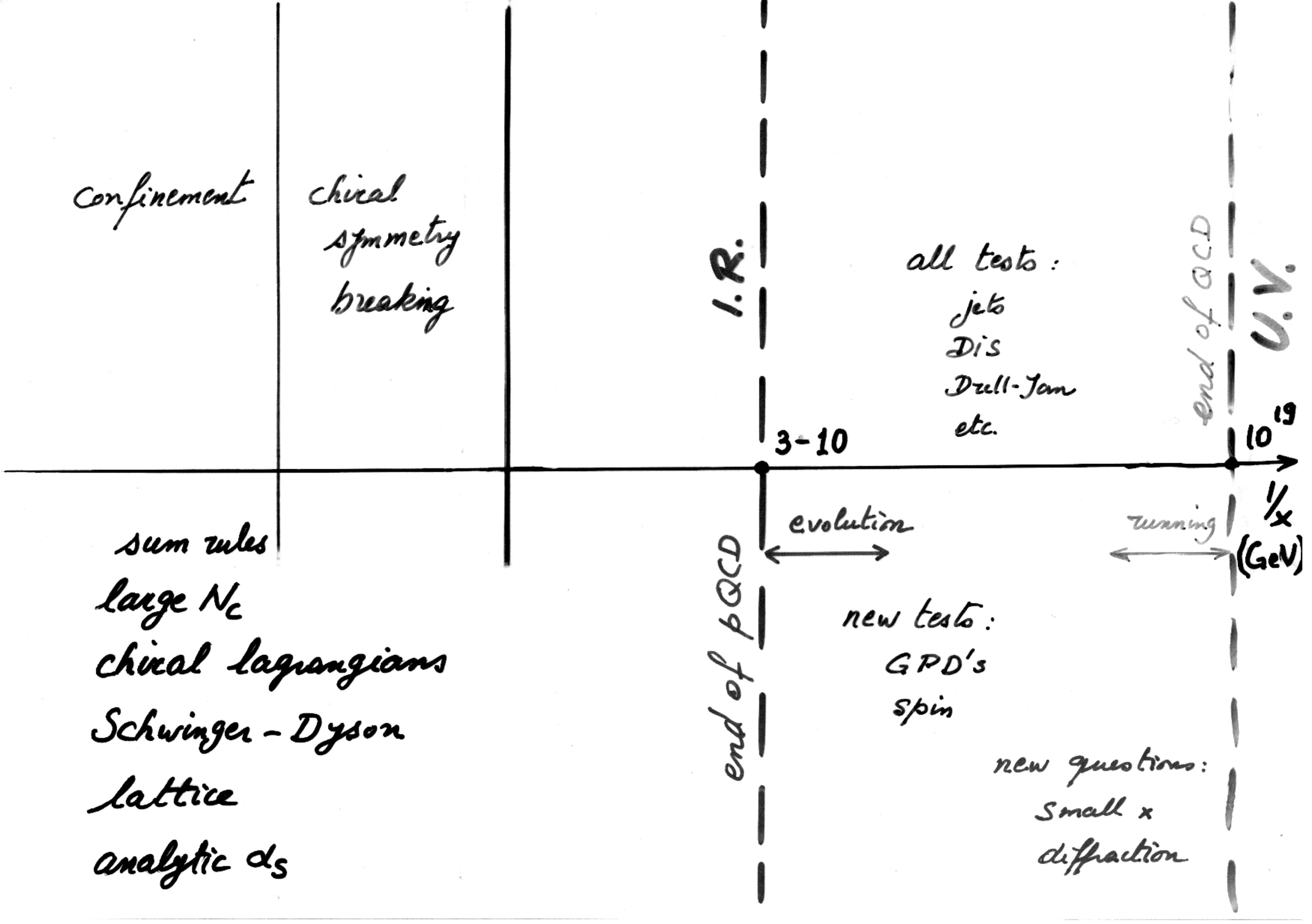
$H_q(x, \xi, t=0)$





H1 DVCS prel.
<Q²> = 8 GeV²
ZEUS DVCS
Freund et al.
NLO QCD MRST01−b(Q²)
Donnachie−Dosch
σ (γ*p → γp) [nb]
W [GeV]
0
20
40
60
80
100
120
140
0
2
4
6
8
10
12


H1 DVCS prel.
H1 DVCS
NLO−QCD (Freund et al.)
MRST 2001−b(Q^2)
CTEQ6−b(Q^2)
$\langle W \rangle$ = 82 GeV
$\sigma(\gamma^* p \rightarrow \gamma p)$ [nb]
Q^2 [GeV^2]


confinement
chiral symmetry breaking
I.R.
all tests:
jets
DIS
Drell-Yan
etc.
end of QCD
U.V.
3-10
10^{19}
$1/x$ (GeV)
evolution
running
end of pQCD
new tests:
GPD's
spin
new questions:
small x
diffraction
sum rules
large N_c
chiral lagrangians
Schwinger-Dyson
lattice
analytic α_s

Expansions:

α_s	pQCD
$\frac{1}{N_c}$	large-N_c
m_q	chiral lagrangians
$\frac{1}{m_q}$, v	HQET
N_f near 16.5	Stevenson

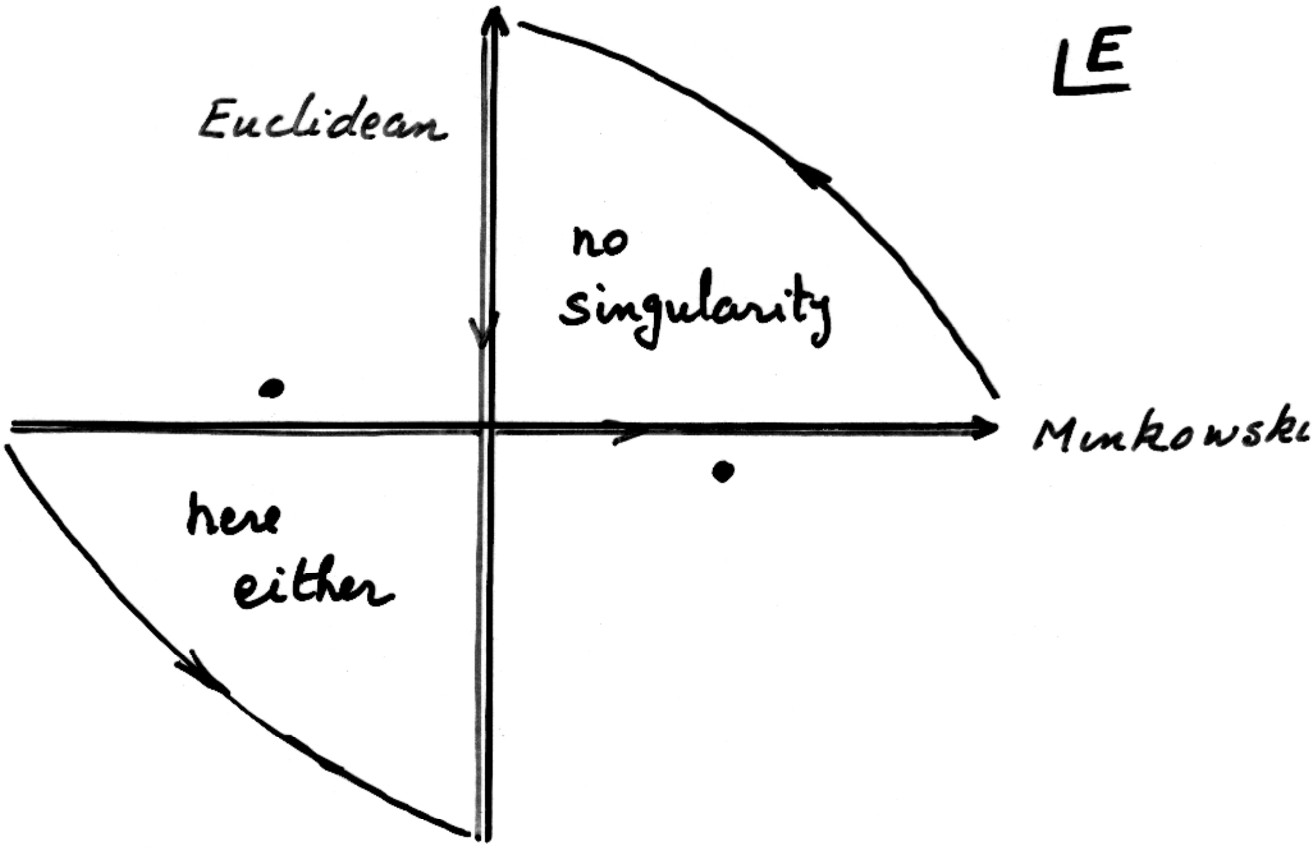
Analytic structure of Green functions

$$\bullet\!\!-\!\!-\!\!-\!\!-\!\!\bullet \;=\; \frac{\sum \psi^{\dagger}(k)\,\psi(k)}{k^2 - m^2 + i\varepsilon}$$

Sum over asymptotic states

real pole "mass shell"

particles to $t>0$
antiparticles to $t<0$
$\varepsilon \to 0$ if stable



$$\int_{-\infty}^{+\infty} dE\, d\vec{p} = \int_{-\infty}^{+\infty} dp_4\, d\vec{p} + \underbrace{\text{residues}}_{=0}$$

QCD

$\langle 0 | T\psi(x)\psi(x') | 0 \rangle$

$\Rightarrow$ pole at $k^2 = m$

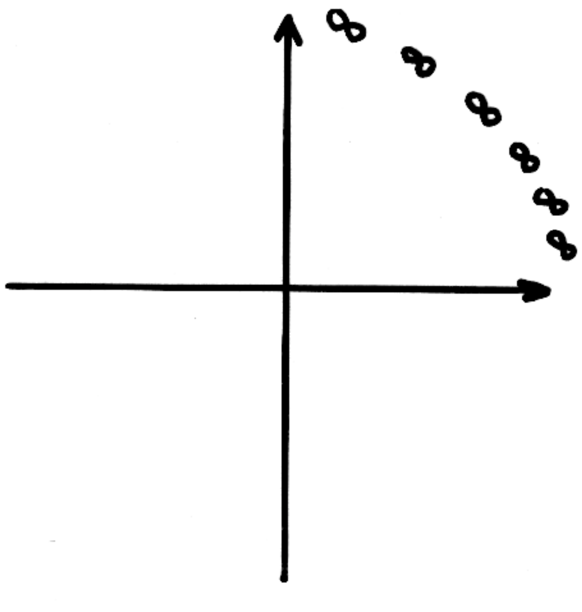
$\Rightarrow$ asymptotic states $\Rightarrow$ wrong

$|0\rangle$ = perturbative vacuum

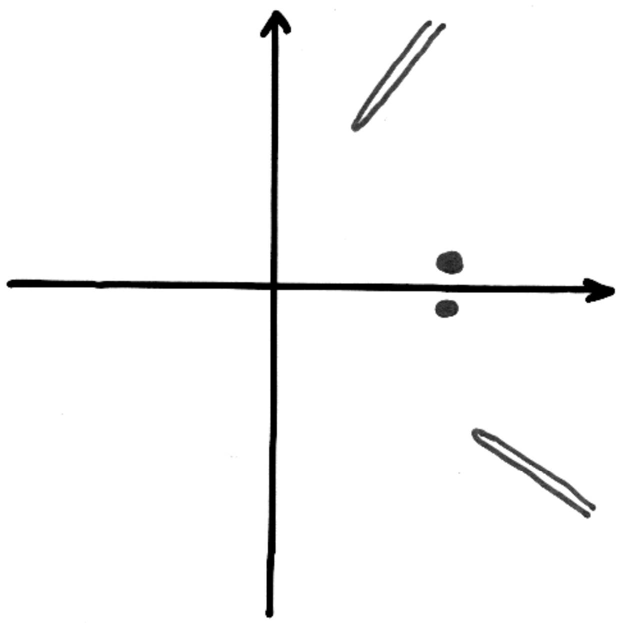
$|\Omega\rangle$ = true vacuum

$\langle \Omega | T\bar{\psi}(x)\psi(x') | \Omega \rangle$

- has no pole on E real
- $\sim$ perturbative if $x - x' \to 0$

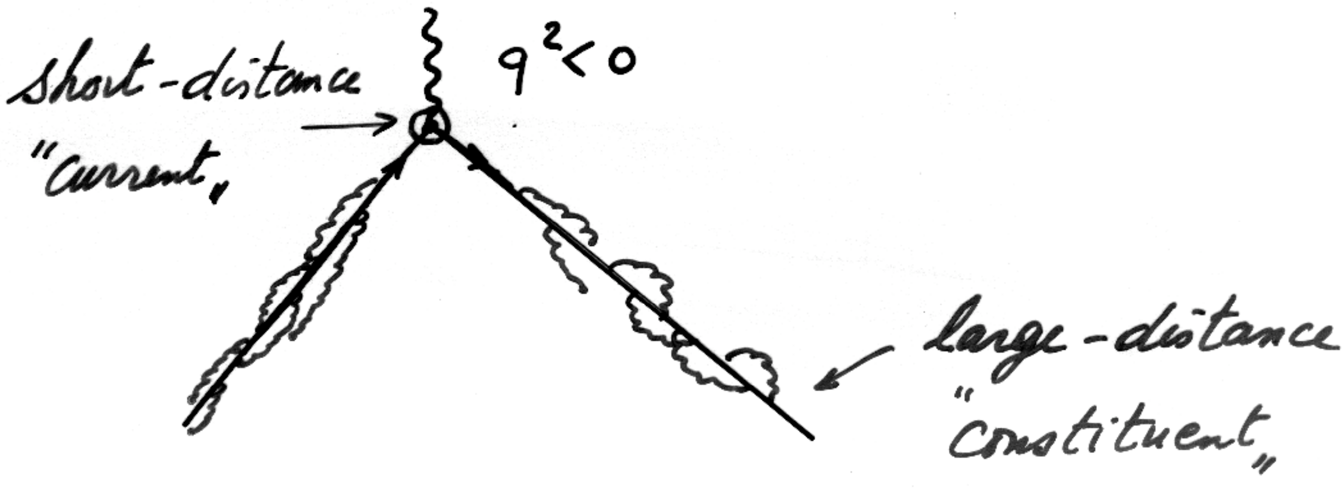


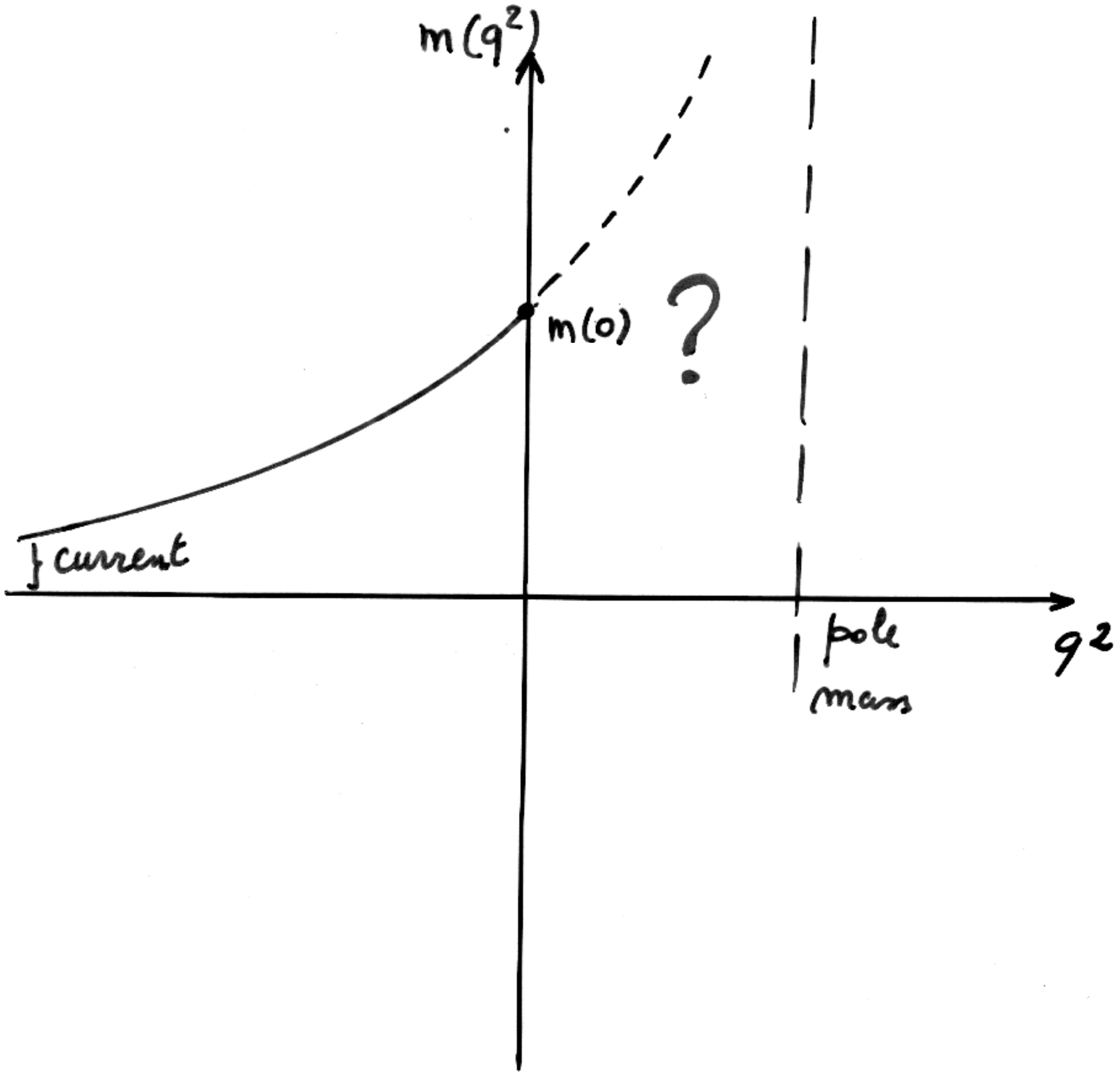
entire:
no poles
but $\to \infty$ at ∞



acausal:
funny poles/cuts
or non analytic!

Note on the mass of quarks





Imaginary time: Wick rotations

$$
\begin{aligned}
dx^\mu dx_\mu &= dt^2 - (d\vec{x})^2 && \text{Minkowski} \\
&= -d(i\tau)^2 - (d\vec{x})^2 && \text{Euclidean}
\end{aligned}
$$

Similarly $\quad E = p_0 \rightarrow i p_4$

⚠ light-cone mapped to 0

$\Rightarrow$ no scattering!

⚠ gives information about $p^\mu p_\mu < 0$. Need to continue to $p^\mu p_\mu > 0$

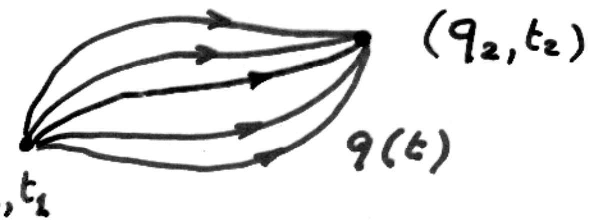
⚠ could be forbidden as gluons and quarks are not observed

Lattice QCD

path integrals :

$$\langle q_2 | e^{-i(t_2 - t_1)\hat{H}} | q_1 \rangle = \mathcal{N} \int \Pi \, dq(t) \; e^{iS}$$

$$S = \int_{q_1, t_1}^{q_2, t_2} L(q, \dot{q}) \, dt$$

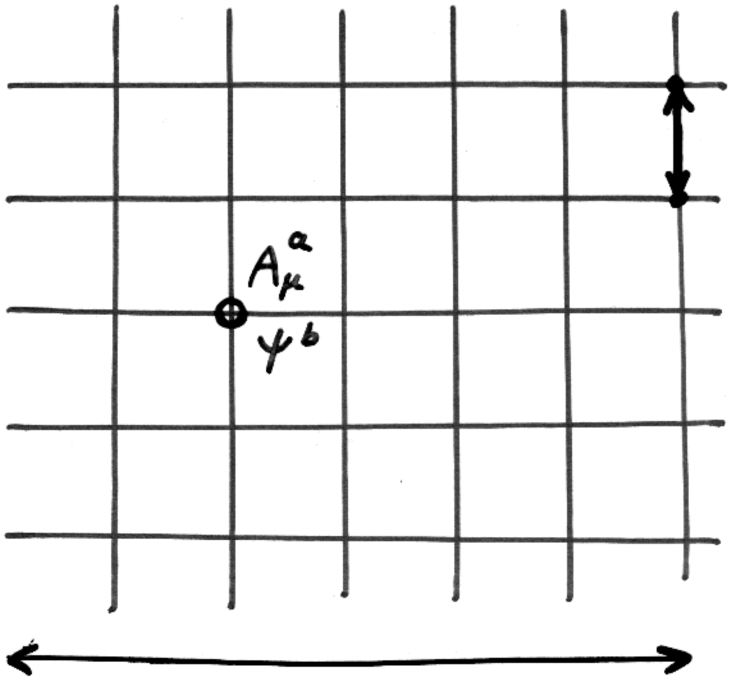


quantum mechanics

extend to QFT

Wick rotate : $iS \rightarrow -S_E = -\int d^4x_E \, \mathcal{L}_E$

discretize :



a : lattice spacing

UV cutoff

$n \times m \times k \times l$

hyperlattice

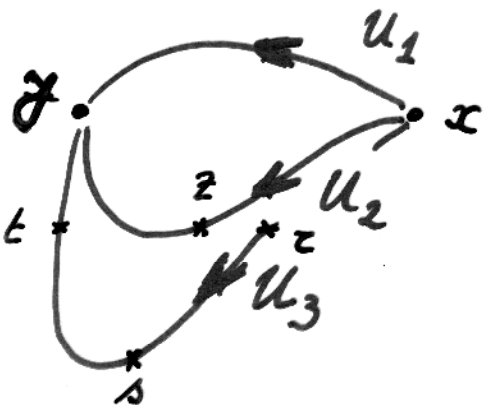
V : lattice volume

gauge invariant operators:

$$\bar{\psi}_j \overset{y}{\bullet} \qquad \overset{x}{\bullet} \psi_\ell \qquad O = \bar{\psi}_j(y)\, U(y,x)\, \psi_\ell(x)$$

$$U(y,x) = \left[\mathcal{P}\, e^{-ig\int_x^y dz_\mu A_a^\mu(z) \frac{\lambda_a}{2}} \right]_{j\ell}$$

$$\mathbb{A}_\mu = A_\mu^a \frac{\lambda^a}{2}$$



$$U(y,x) = 1 - ig\int_x^y \mathbb{A}\cdot dz$$
$$\underbrace{- \frac{1}{2} g^2 \int_z^y \mathbb{A}\cdot dz \int_x^z \mathbb{A}\, dz}_{\mathcal{P}:\ \text{path ordering}} + \dots$$

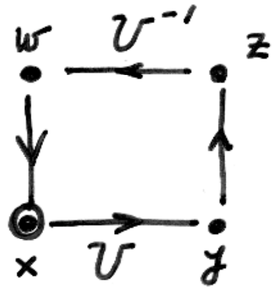
gauge transformation:

$$\begin{cases} \psi_a(x) \to \psi'_a(x) = \underbrace{\left(e^{i\frac{\lambda}{2}\cdot\theta(x)}\right)_{ab}}\psi_b(x) \\ \qquad\qquad\qquad\qquad \Omega(x)_{ab}\, \psi_b(x) \\ D_\alpha \psi \to D'_\alpha \psi' = \Omega(x)\, D_\alpha \psi \\ \mathbb{A}_\alpha \to \Omega\, \mathbb{A}_\alpha \Omega^{-1} + \frac{i}{g}(\partial_\alpha \Omega)\,\Omega^{-1} \end{cases}$$

$$U(y,x) \to \Omega(y)\, U(y,x)\, \Omega^{-1}(x)$$

matrix on the lattice.

Bosons: plaquette



$$U_{\square}(x)$$

$$= \underbrace{U(x,w)}_{U^{\dagger}(z,y)}\,\underbrace{U(w,z)}_{U^{\dagger}(y,x)}\,U(z,y)\,U(y,x)$$

$$\lim_{a\to 0} \mathrm{Tr}_{\mathrm{colour}}\, U_{\square}(x) = 3 - \frac{1}{2} a^4 g_L\, \mathrm{Tr}\left[\mathbb{F}_{\mu\nu}(x)\mathbb{F}_{\mu\nu}(x)\right]$$

$$\mathbb{F}_{\mu\nu} \equiv F^a_{\mu\nu}\frac{\lambda_a}{2}$$

One can write the Euclidean action as:

$$S_G = \beta \sum_{\square}\left(1 - \frac{1}{3}\,\mathrm{Tr}\, U_{\square}\right)$$

$$\xrightarrow{a\to 0} \underbrace{a^4 \sum_x}_{\int d^4x} \frac{\beta g_L^2}{12} \sum \mathrm{Tr}_{\mathrm{col.}}\left(\mathbb{F}_{\mu\nu}\,\mathbb{F}_{\mu\nu}\right)$$

$$\beta = \frac{6}{g_L^2}$$

Fermions:

$$S_F = a^4 \sum_x \left\{ \bar{\psi}_x \psi_x - \kappa\left(\sum \bar{\psi}_x (1-\gamma_\mu)\, U(x,y)\, \psi_y + c.c\right)\right\}$$

$$\kappa = \frac{1}{2(ma+4)}$$

Wilson fermions
"hopping parameter"

(also Kogut-Susskind)

Physical predictions

$$\Rightarrow \underbrace{\int dU \, d\psi \, d\bar{\psi}}_{\text{all sites}} \; e^{-[S_G + \bar{q} M q]}$$

$$= \int dU \; \underbrace{\det(M(U))}_{\text{all sites!}} \; e^{-S_G(U)}$$

quenched approximation:

$\det M(U) = 1$

no dynamical fermions

Continuum limit

$$\langle \hat{O} \rangle \sim \frac{\int dU \; O(U) e^{-S}}{\int dU \; e^{-S}}$$

2 parameters: g_L , a

$\uparrow$ bare

$$m = \frac{1}{a} f(g_L)$$

$$\Rightarrow \lim_{a \to 0} m = \text{finite}$$

$$\Rightarrow g_L = g_0(x=0) = 0$$

behaviour of $g_L(a)$ known for small a from perturbation theory

$\Rightarrow$ can take the limit

$g_L \simeq \log 1/a$

Higher orders $\Rightarrow$ $O(a)$ improved actions

$\Rightarrow$ 1) fix a in GeV^{-1} using an existing mass (m_ρ)

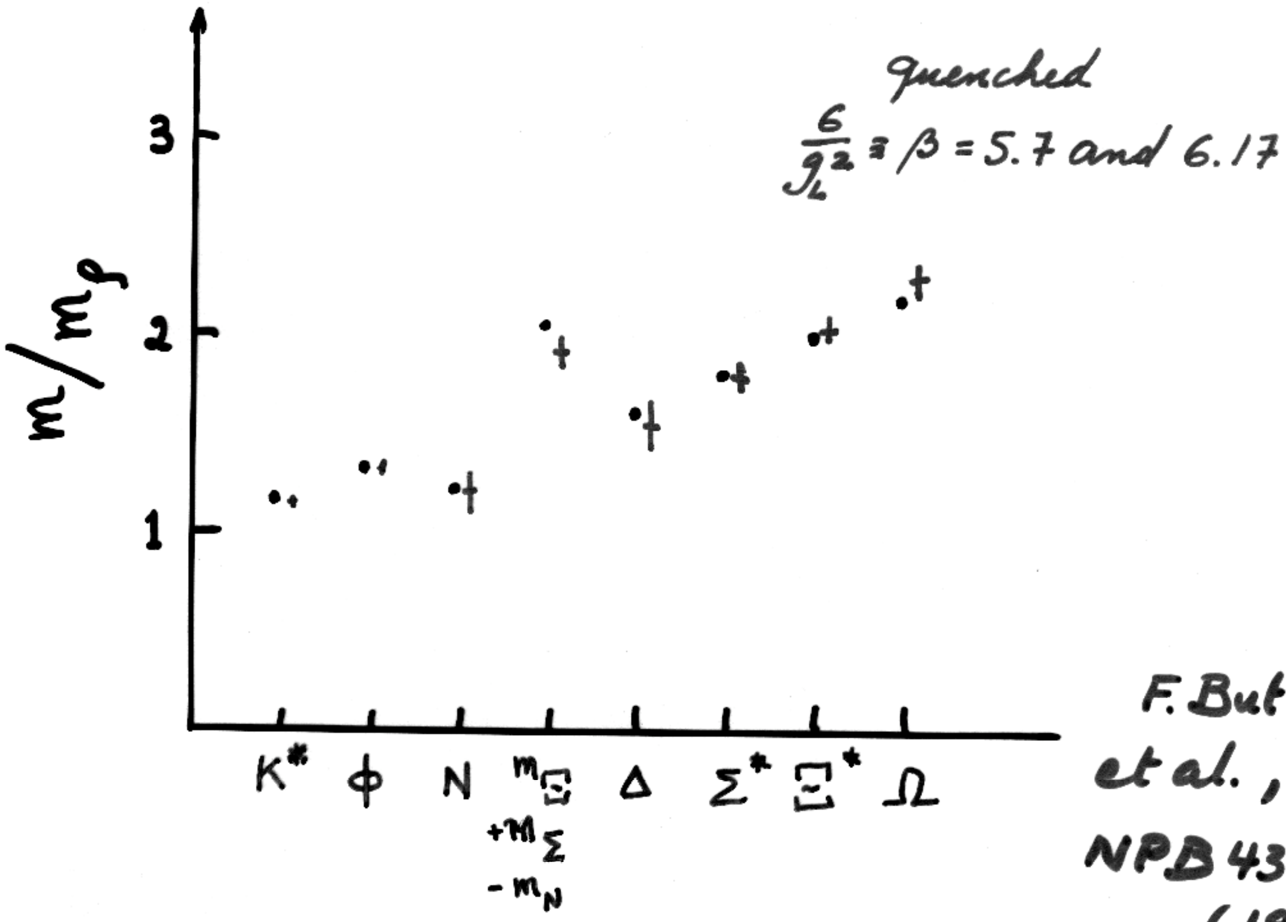
$g_L \simeq 1 \iff a \approx 1/2\ \mathrm{GeV}^{-1}$

$(1/10\ \mathrm{fm})$

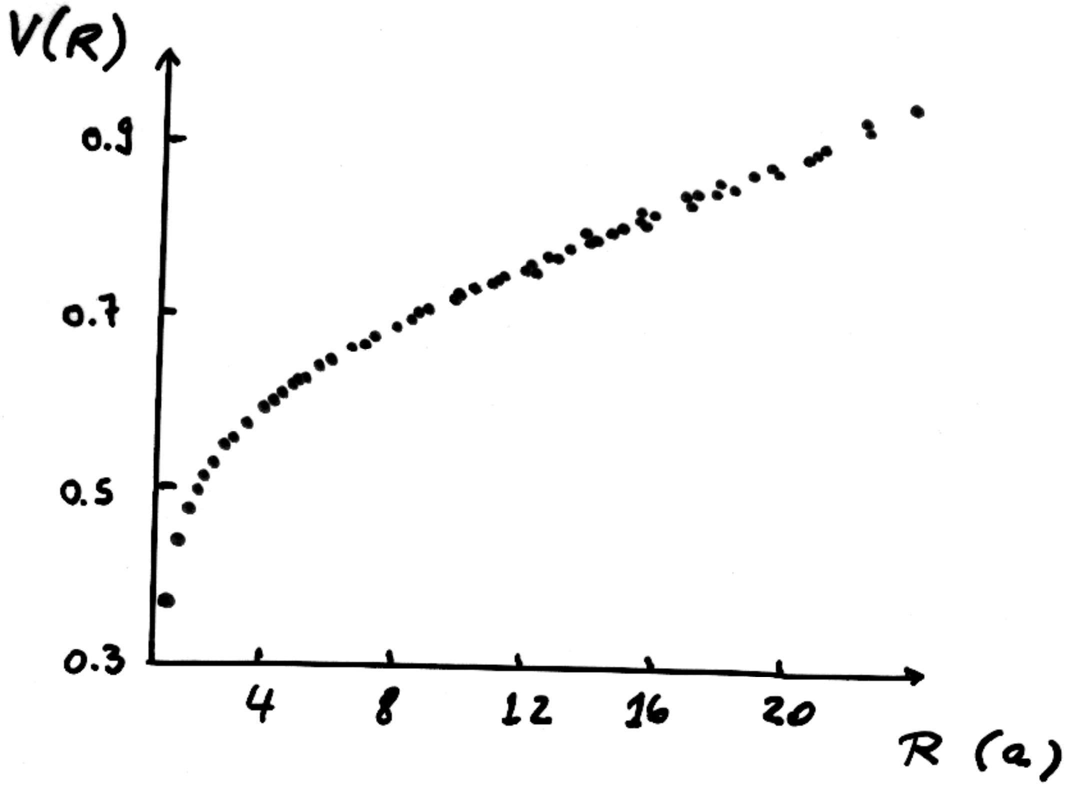
2) Calculate other quantities

⚠ hadrons, etc must fit on the lattice

⚠ quenched approximation can be bad for some observables and good for others



F. Butler et al., NPB 430, 179 (1994)



G. Bali - K. Schilling PRD 47, 661 (1993)

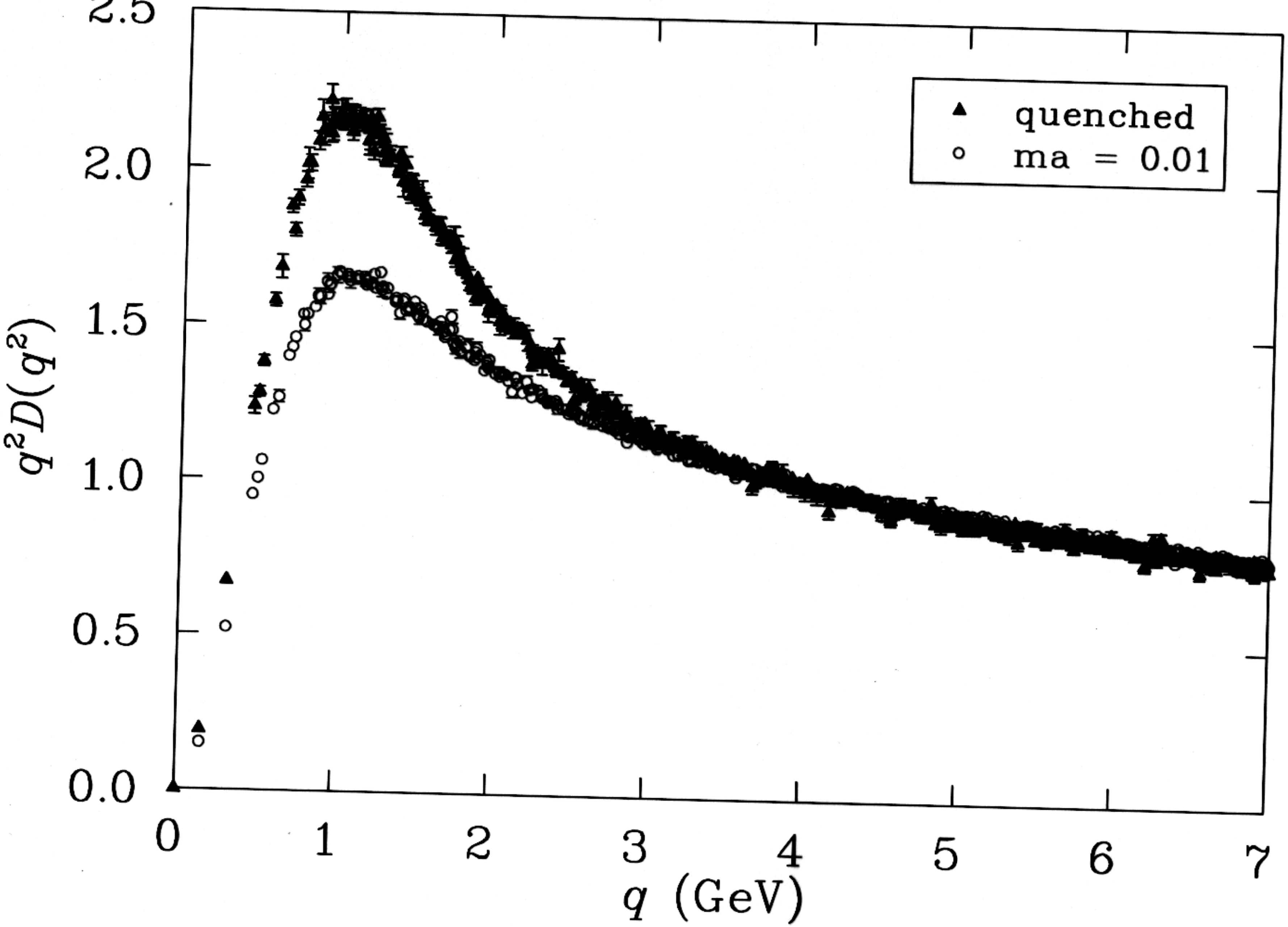
New: fix the gauge

→ study n-point functions

⇒ • gluon propagator suppressed at low k^2

• Goldstone boson in quark propagator

Gluon propagator



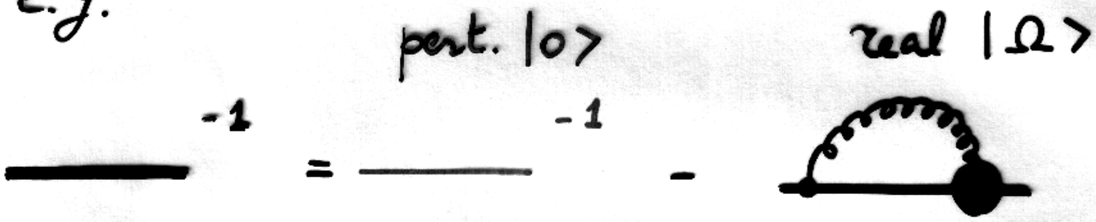
Other approach:

field equations of motion

$\Rightarrow$ infinite set of non-perturbative integro-differential equations

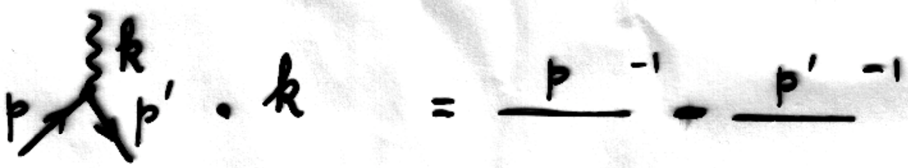
"Schwinger - Dyson"

e.g.



gauge invariance

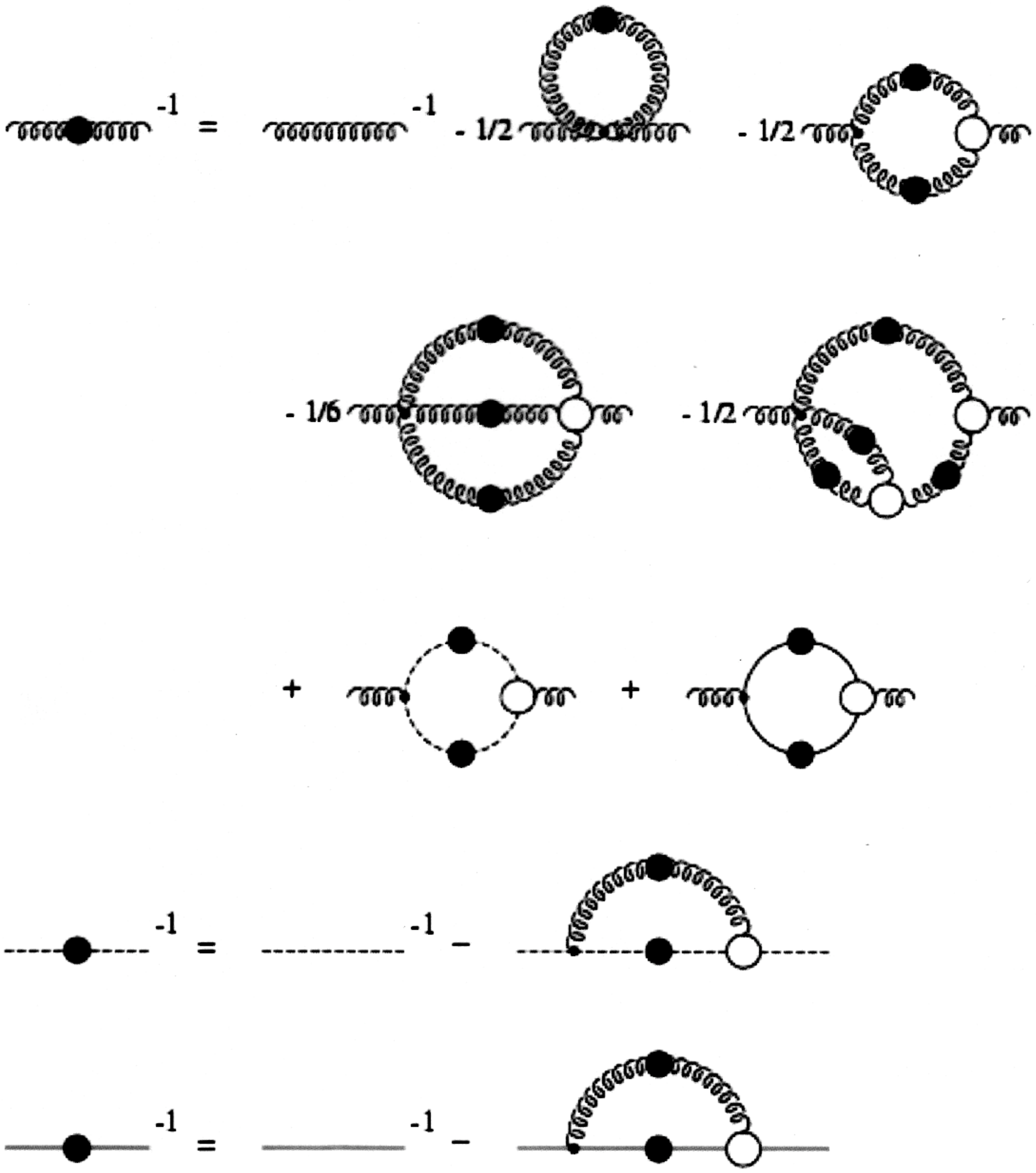
$\Rightarrow$ Slavnov - Taylor identities

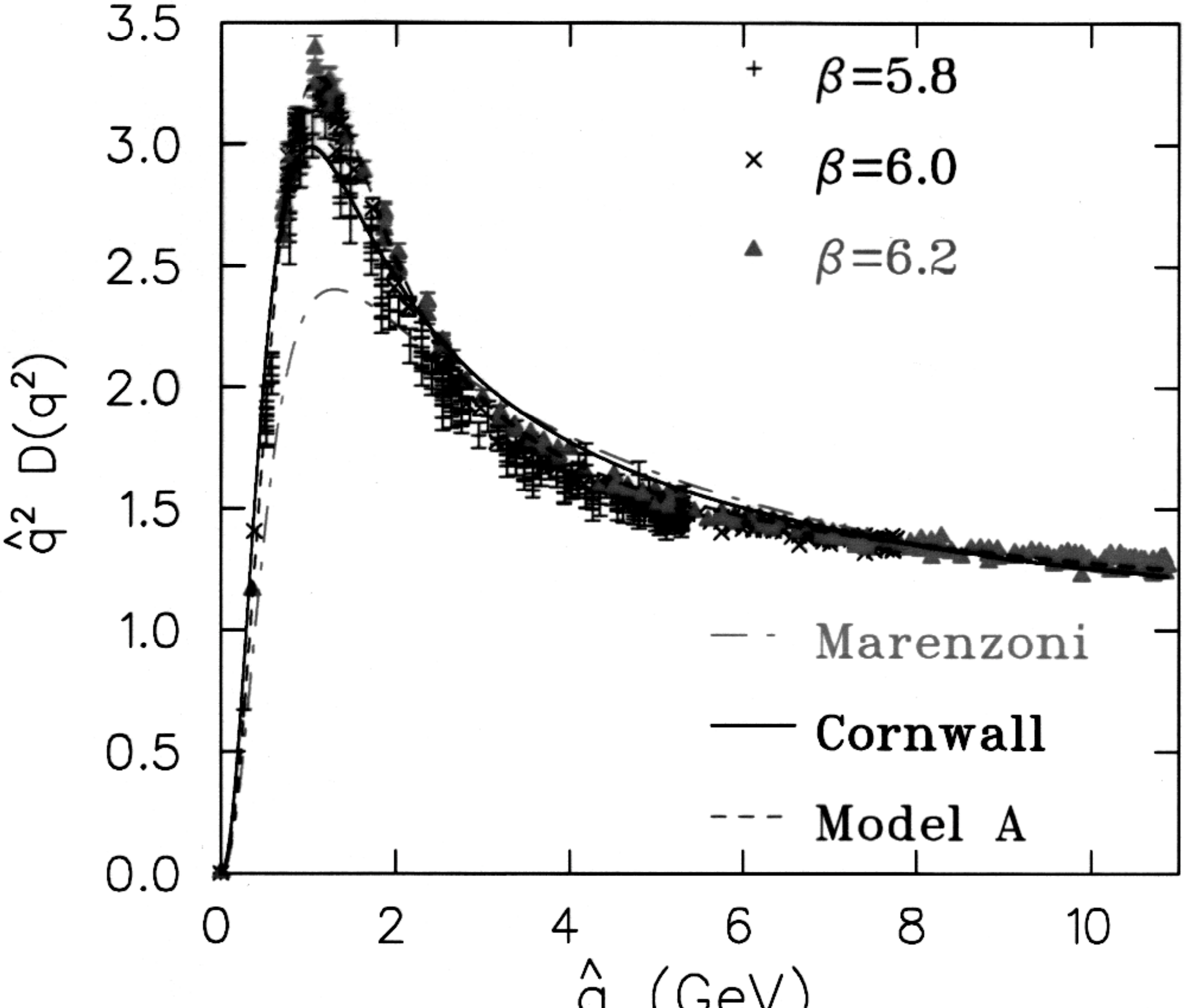


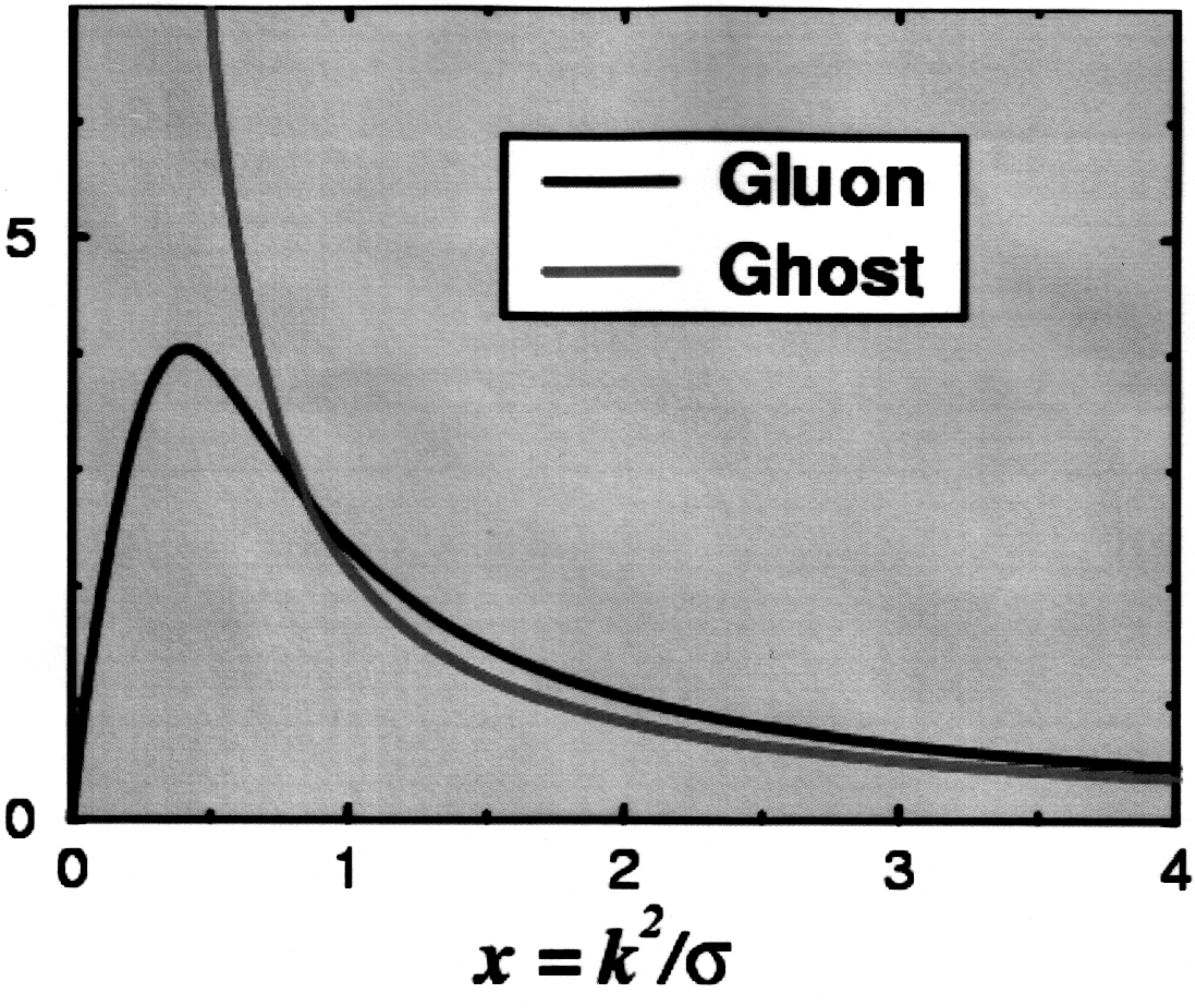
put all this together

$\Rightarrow$ truncate and solve

(and renormalise!)


-1 = -1 - 1/2 - 1/2
- 1/6 - 1/2
+ +
-1 = -1 −
-1 = -1 −


3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
$\hat{q}^2\ D(q^2)$
0
2
4
6
8
10
$\hat{q}$ (GeV)
β=5.8
β=6.0
β=6.2
Marenzoni
Cornwall
Model A


Gluon
Ghost
5
0
0
1
2
3
4
$x = k^2/\sigma$

Usual assumptions :

1) No hidden observables

2) Unique ground state (vacuum)

3) Asymptotic states

4) S matrix

5) Start with large distances
to understand small distances

6) zero-mass particles give $\frac{1}{r}$ potentials

~~Usual assumptions~~ : QCD

1) No ~~hidden observables~~

hidden FIELDS!

2) Unique ~~ground state~~ (vacuum)

many vacua (∞ number)!

3) ~~Asymptotic states~~

asymptotic states not there

4) ~~S matrix~~

not for $m=0$ bosons

5) Start with ~~large distances~~

to ~~understand small~~ distances

understand small distances only

6) zero-mass ~~particles give~~ $\frac{1}{r}$ potentials

potential $\sim r$
constant force

Conclusion

- excellent tests

  $\Rightarrow$ we understand $Q^2 > 5\ \text{GeV}^2$ (at moderate x)

- new unifying tools : GPD's
- convergence of several theoretical pictures of confinement
- prediction of :
  - * new unconfined state of matter
  - * new regime in scattering
  - * new states in spectrum

$\Rightarrow$ we may finally understand Yang-Mills theories !

$$\begin{tikzcd}[column sep=small] & \text{QCD} \arrow[dl] \arrow[dr, bend left] & \\ \text{partially understood} \arrow[rr, leftrightarrow, "\text{tests}"'] \arrow[d] & & \text{most events experimentally} \arrow[d] \\ \text{a lot of theoretical activity} & & \text{a must for experimentalists} \end{tikzcd}$$

$$\mathcal{L} = i\bar{\psi}\,\mathbb{D}\cdot\gamma\psi - \frac{1}{2}\,\mathrm{Tr}\{\mathbb{F}.\mathbb{F}\}$$

$$+ \mathcal{L}_{ghost} + \mathcal{L}_{gauge\ fixing}$$

"The Lord is subtle"