QCD at high-energy

statistical physics and beyond

Gregory Soyez

Based on : **G.S.**, **hep-ph/0504129, Phys. Rev. D72 (2005) 016007**

E. Iancu, G.S., D. Triantafyllopoulos, **hep-ph/0510094, Nucl. Phys. A768 (2006) 194**

C. Marquet, Y. Hatta, E. Iancu, G.S., D. Triantafyllopoulos, **hep-ph/0601150**

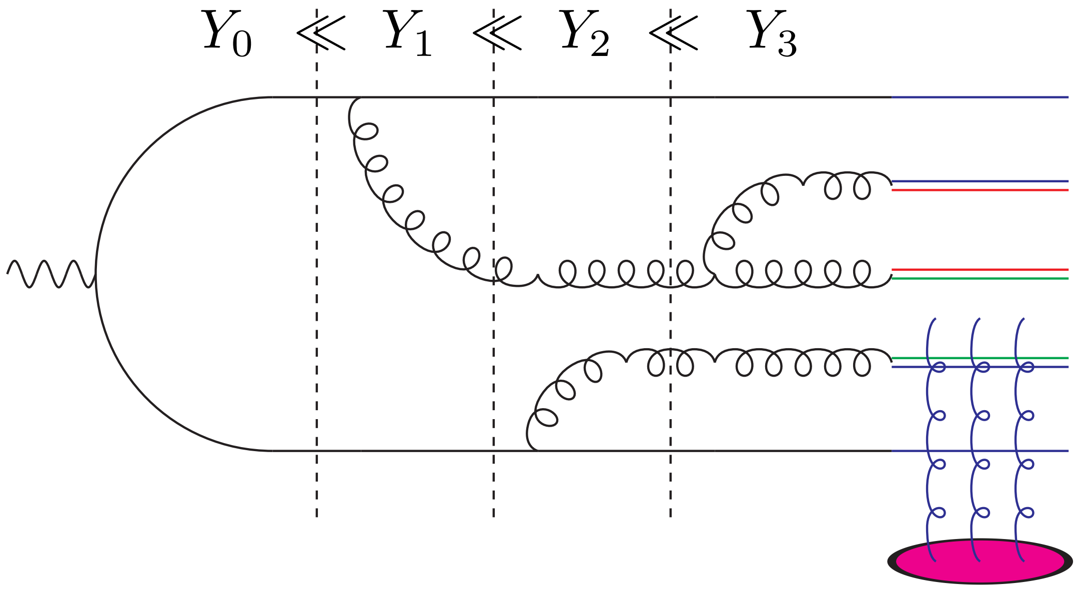
C. Marquet, R. Peschanski, G.S., **hep-ph/0512186**

Outline

- High-energy evolution equations
  - Unitarity and saturation
  - Dilute regime and fluctuations
    Evolution as a reaction-diffusion process
- Consequences
  - Geometric scaling in the mean field
  - Diffusive scaling with fluctuations
- Predictions vs. DIS data

Motivation

Photon-target collision



$n(r, Y)$ dipoles

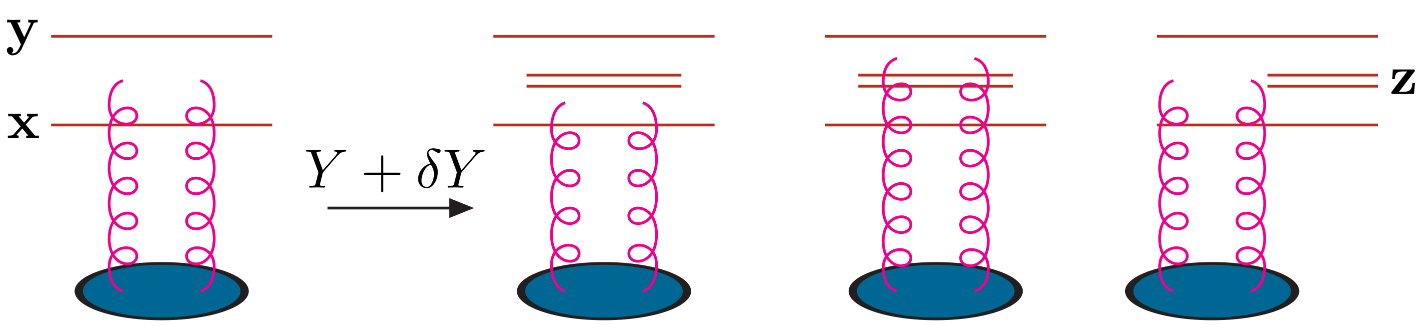
of size r

- High-energy: Bremsstrahlung of soft gluons
- degrees of freedom: energy s (rapidity $Y = \log(s) = \log(1/x)$)
  and transverse coordinates
- Large-N_c: gluon at $\mathbf{z}$ = $q\bar{q}$ pair at $\mathbf{z}$
  $\Rightarrow$ gluon emission = dipole splitting

BFKL and BK evolution

Consider a $q\bar{q}$ dipole at large rapidity $Y = \log(s) = \log(1/x)$

Rapidity increase $\Rightarrow$ Splitting into 2 dipoles



$$\partial_Y \langle T_{\mathbf{xy}} \rangle = \bar{\alpha} \int_z \mathcal{M}_{\mathbf{xyz}} \underbrace{[\langle T_{\mathbf{xz}} \rangle + \langle T_{\mathbf{zy}} \rangle - \langle T_{\mathbf{xy}} \rangle]}_{\text{Linear BFKL}}$$

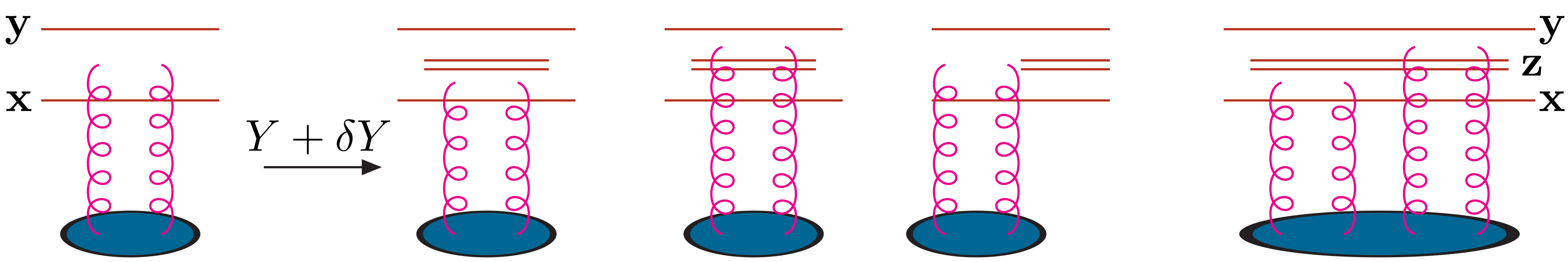
[Balitsky, Fadin, Kuraev, Lipatov, 78]

Solution: $e^{\omega Y}$ but violates unitarity

BFKL and BK evolution

Consider a $q\bar{q}$ dipole at large rapidity $Y = \log(s) = \log(1/x)$

Rapidity increase $\Rightarrow$ Splitting into 2 dipoles



$$\partial_Y \langle T_{\mathbf{xy}} \rangle = \bar{\alpha} \int_z \mathcal{M}_{\mathbf{xyz}} [\underbrace{\langle T_{\mathbf{xz}} \rangle + \langle T_{\mathbf{zy}} \rangle - \langle T_{\mathbf{xy}} \rangle}_{\text{Linear BFKL}} \underbrace{- \langle T_{\mathbf{xz}} T_{\mathbf{zy}} \rangle}_{\text{Unitarity}}]$$

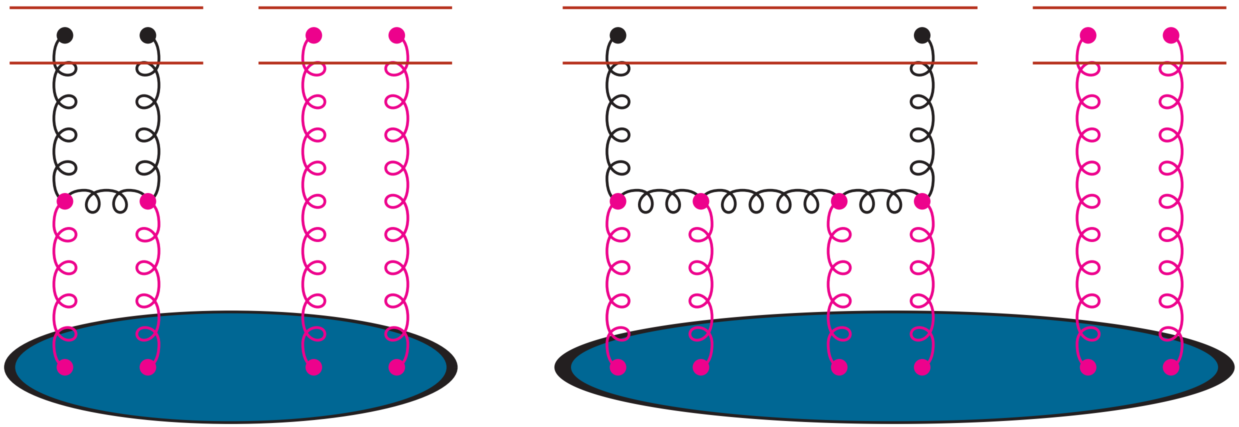
- $\langle T \rangle$, $\langle T^2 \rangle$, ...: JIMWLK/Balitsky equations (at large N_c)
- Mean-field approximation: $\langle T^2 \rangle = \langle T \rangle^2$ (BK equation)

[Balitsky 96, Kovchegov 99]

Fluctuations

Consider evolution of $\langle T^{(2)} \rangle$

[E. Iancu, D. Triantafyllopoulos]
Also A. Mueller, S. Munier, A. Shoshi, S. Wong



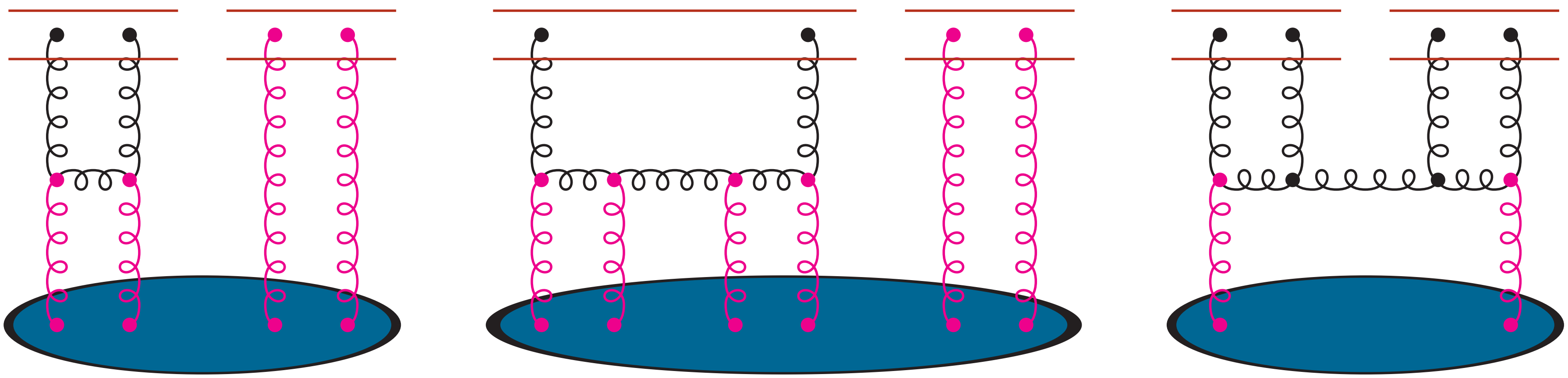
$$\partial_Y \left\langle T^{(2)} \right\rangle = \underbrace{\bar{\alpha}\mathcal{M} \otimes \left\langle T^{(2)} \right\rangle}_{\text{BFKL}} - \underbrace{\bar{\alpha}\mathcal{M} \otimes \left\langle T^{(3)} \right\rangle}_{\text{saturation}}$$

- saturation $\longrightarrow T \sim 1$ dense regime

Fluctuations

Consider evolution of $\langle T^{(2)} \rangle$

[E. Iancu, D. Triantafyllopoulos]
Also A. Mueller, S. Munier, A. Shoshi, S. Wong



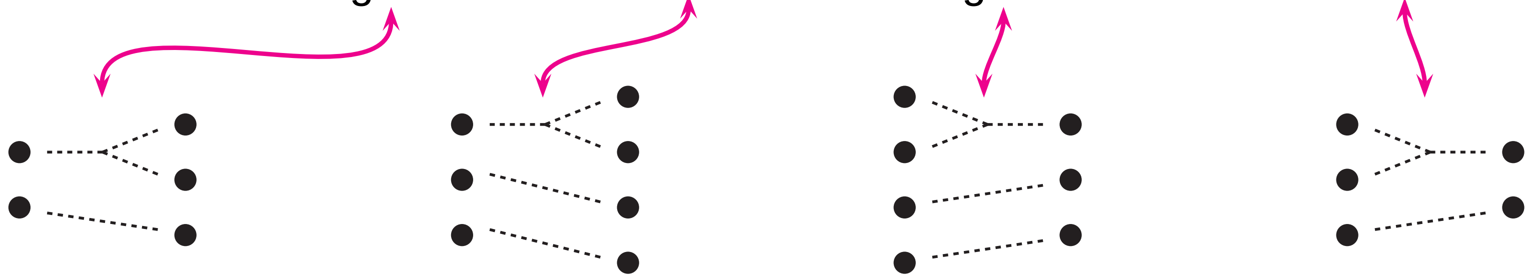
$$\partial_Y \left\langle T^{(2)} \right\rangle = \underbrace{\bar{\alpha}\mathcal{M} \otimes \left\langle T^{(2)} \right\rangle}_{\text{BFKL}} - \underbrace{\bar{\alpha}\mathcal{M} \otimes \left\langle T^{(3)} \right\rangle}_{\text{saturation}} + \underbrace{\bar{\alpha}\alpha_s^2\,\mathcal{K} \otimes \langle T \rangle}_{\text{fluctuations}}$$

- saturation $\longrightarrow T \sim 1$ dense regime
- fluctuations $\longrightarrow T \sim \alpha_s^2$ dilute regime

Reaction-diffusion process $A \underset{\sigma}{\overset{\gamma}{\rightleftharpoons}} A + A$

Master equation: $P_n \equiv$ proba to have n particles

$$\partial_t P_n = \underbrace{\gamma\,(n-1)P_{n-1}}_{\text{gain}} - \underbrace{\gamma\,nP_n}_{\text{loss}} + \underbrace{\sigma\,n(n+1)P_{n+1}}_{\text{gain}} - \underbrace{\sigma\,n(n-1)P_n}_{\text{loss}}$$



$\Rightarrow$ evolution of particle densities $\langle n \rangle$ and correlators $\langle n^k \rangle$:

$$\partial_t \left\langle n^k \right\rangle = \gamma\,k \left\langle n^k \right\rangle + \gamma\,k(k-1)\left\langle n^{k-1} \right\rangle - \sigma\,k(k+1)\left\langle n^{k+1} \right\rangle$$

Scattering amplitude for this system off a target (from boost invariance)

$$\mathcal{A}(t) = \sum_{k=0}^{\infty} (-)^k \left\langle n^k \right\rangle_{t_0} \left\langle T^k \right\rangle_{t-t_0} \Rightarrow$$

$$\partial_t \left\langle T^k \right\rangle = \underbrace{\gamma \left\langle T^k \right\rangle}_{\text{BFKL}} - \underbrace{\gamma \left\langle T^{k+1} \right\rangle}_{\text{sat.}} + \underbrace{\sigma \left\langle T^{k-1} \right\rangle}_{\text{fluct.}}$$

Reaction-diffusion & QCD

For QCD particle = (effective) dipoles

Dipole plitting $\equiv$ BFKL kernel

$$\gamma \sim \bar{\alpha} \frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{z}-\mathbf{y})^2}$$

Effective dipole merging

$$\sigma(\mathbf{x}_1\mathbf{y}_1, \mathbf{x}_2\mathbf{y}_2 \to \mathbf{uv}) \sim \bar{\alpha}\alpha_s^2 \nabla_{\mathbf{u}}^2 \nabla_{\mathbf{v}}^2 \left\{ \mathcal{M}_{\mathbf{uvz}} \log^2\left[\frac{(\mathbf{x}_1-\mathbf{u})^2(\mathbf{y}_1-\mathbf{z})^2}{(\mathbf{x}_1-\mathbf{z})^2(\mathbf{y}_1-\mathbf{u})^2}\right] \log^2\left[\frac{(\mathbf{x}_2-\mathbf{v})^2(\mathbf{y}_2-\mathbf{z})^2}{(\mathbf{x}_2-\mathbf{z})^2(\mathbf{y}_2-\mathbf{v})^2}\right]\right\}$$

Remarks:

- merging not always positive
- fluctuations = gluon-number fluctuations
- Can be obtained from projectile or target point of view
- Known at large N_c.

Consequences

Lessons from statistical physics and beyond

BK vs. F-KPP equation

[S. Munier, R. Peschanski]

b-independent BK in momentum space

$$\partial_{\bar\alpha Y} T(k) = \chi_{\text{BFKL}}(-\partial_{\log(k^2)}) T(k) - T^2(k)$$

Diffusive approximation:

$\chi_{\text{BFKL}}(-\partial_{\log(k^2)})$ up to $\partial^2_{\log(k^2)}$

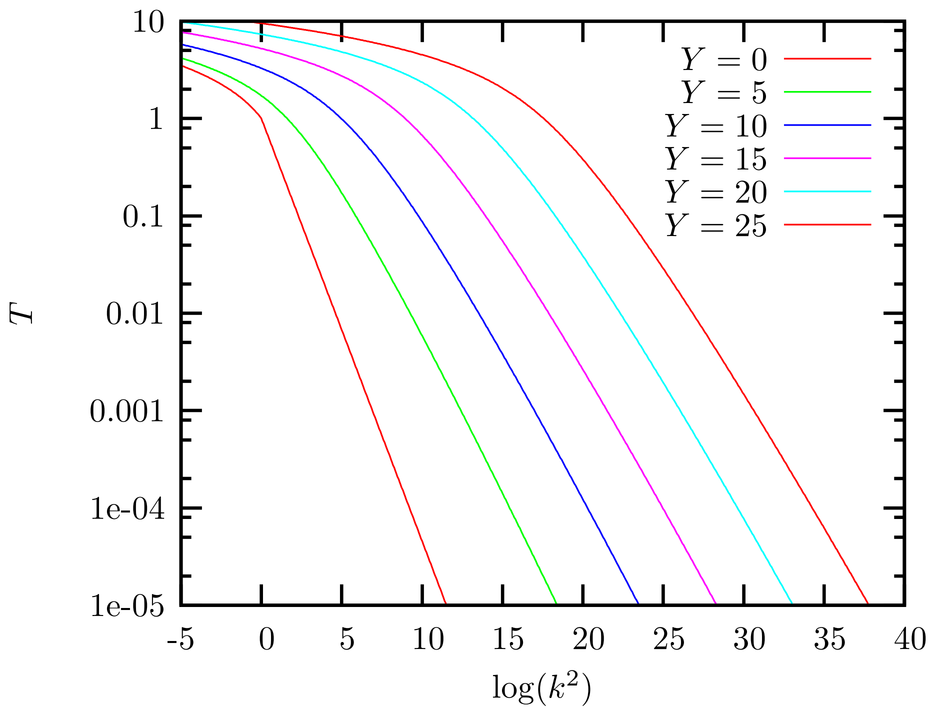
Time $t = \bar\alpha Y$, Space $x \approx \log(k^2)$, $u \propto T$

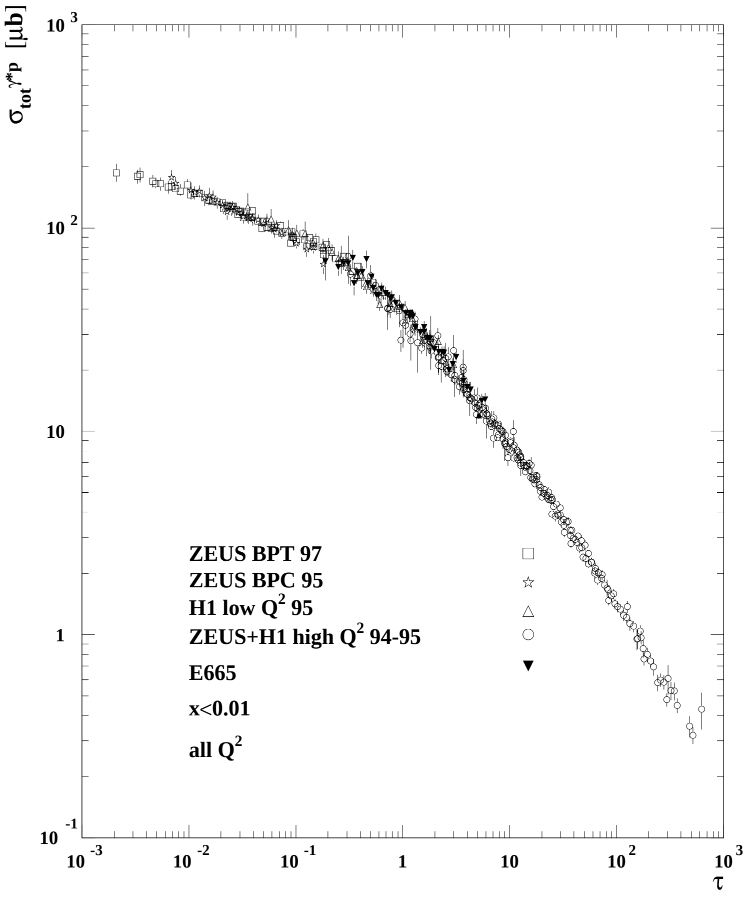
$$\partial_t u(x,t) = \partial_x^2 u(x,t) + u(x,t) - u^2(x,t)$$

Fisher-Kolmogorov-Petrovsky-Piscounov (F-KPP)

Prediction:

formation of a traveling-wave pattern





$$T(k,Y) = T(\log(k^2) - v_c Y) = T\left(\frac{k^2}{Q_s^2(Y)}\right) \qquad \text{with } Q_s^2 \sim \exp(v_c Y)$$

Geometric scaling (speed of the wave $\rightarrow$ energy dependence of Q_s^2)

With fluctuations

no b-dependence + local approximation for fluctuations (introduces a factor κ)

$\longrightarrow$ Langevin equation

$$\partial_Y T(k,Y) = \bar\alpha K_{\text{BFKL}} \otimes T(k,Y) - \bar\alpha T^2(k,Y) + \bar\alpha\sqrt{\kappa\alpha_s^2 T(k,Y)}\,\nu(k,Y)$$

with $\langle \nu(k,Y)\rangle = 0$

$$\langle \nu(k,Y)\nu(k',Y')\rangle = \frac{1}{\bar\alpha}\delta(Y-Y')\,k\delta(k-k')$$

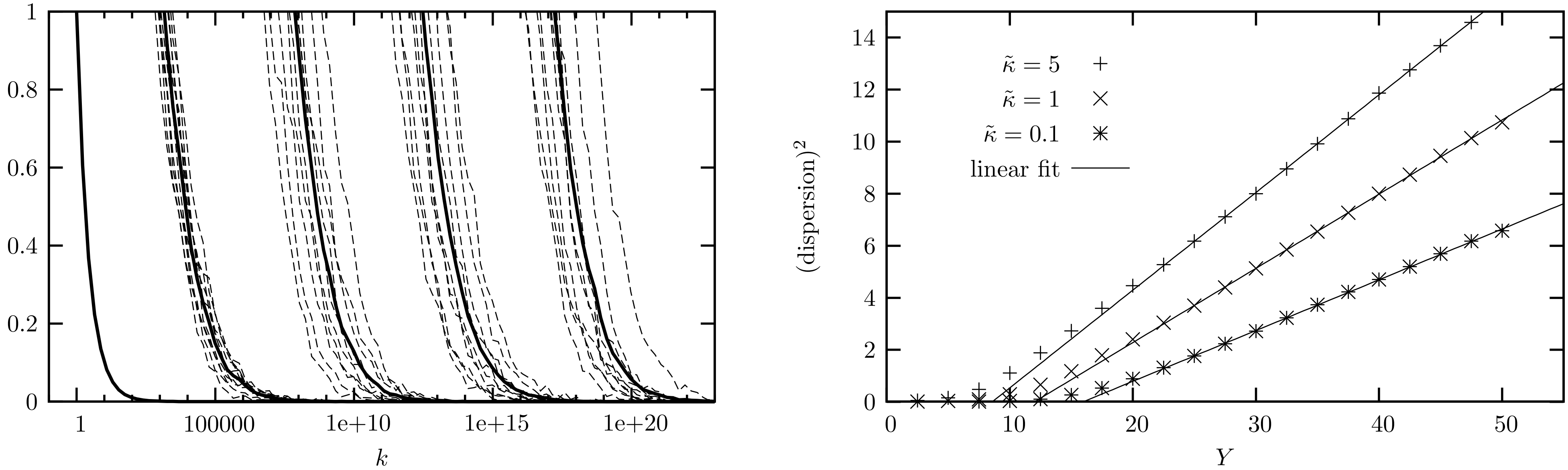
Diffusive approximation

$$\partial_t u(x,t) = \partial_x^2 u + u - u^2 + \sqrt{2\kappa u}\,\nu(x,t)$$

stochastic F-KPP

Numerical analysis

[G.S. 05]



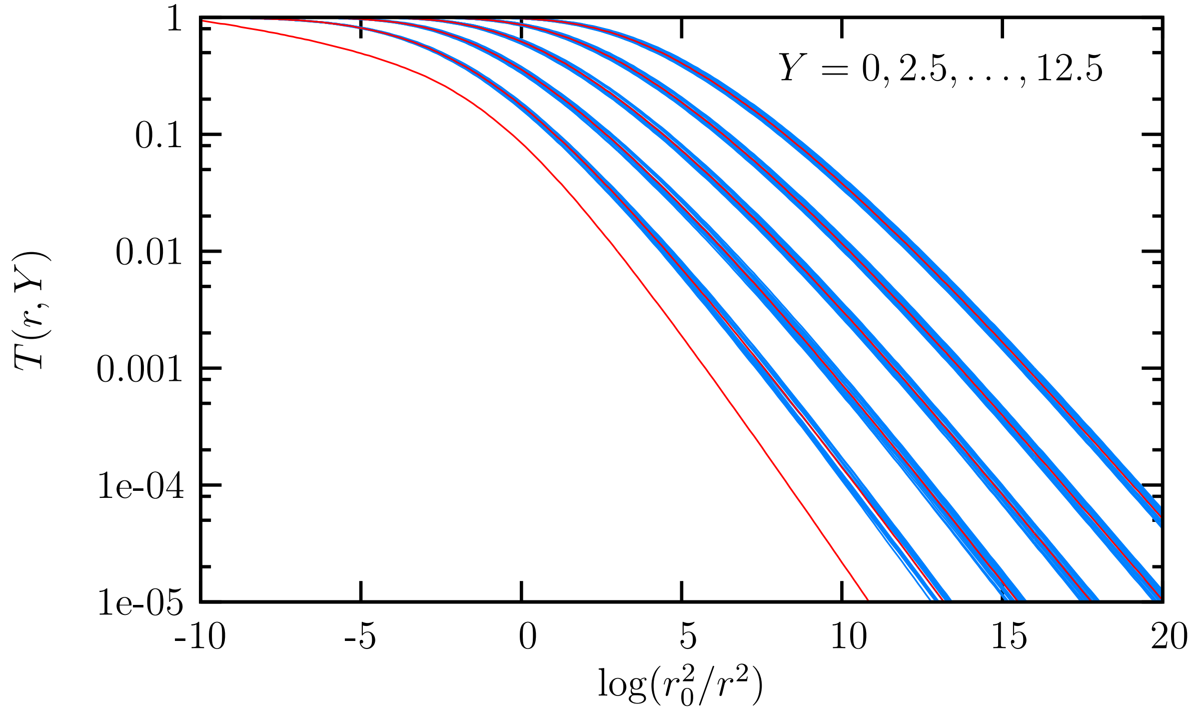
- Dispersion of the events $\Rightarrow$ geometric scaling violations

$$\Delta \log[Q_s^2(Y)] \approx \sqrt{D_{\text{diff}}\bar\alpha Y}$$

- No important dispersion in early stages of the evolution !

[E. Iancu, A. Mueller, S. Munier, G.S., in preparation]

Beyond local approximation for the fluctuations:



Idea: matching between

- mean field (BK) for saturation
- (random) dipole splitting in the dilute regime

High-energy behaviour

Evolution with saturation & fluctuations $\equiv$

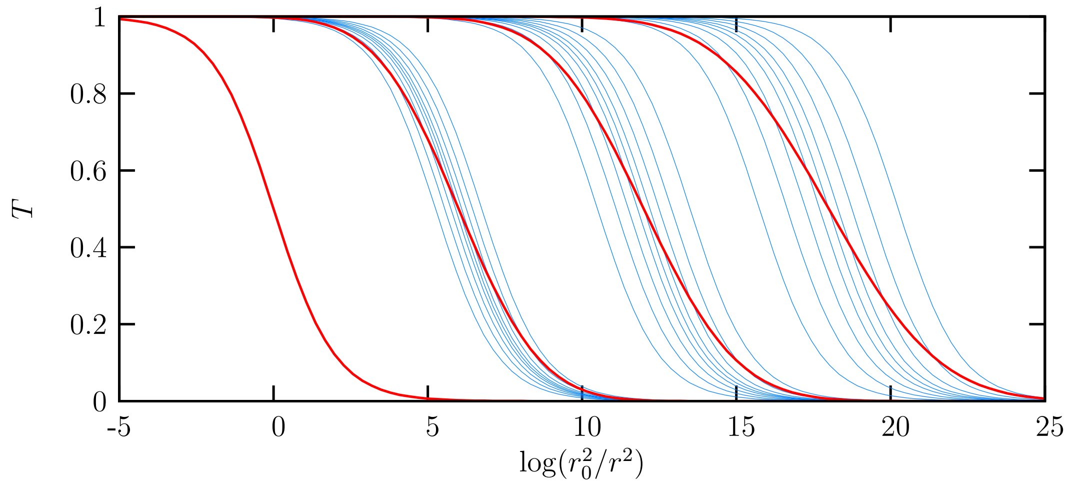
- superposition of unitary front (with geometric scaling)
- with a dispersion (yielding geometric scaling violations)

$$\langle T(r,Y)\rangle = \int d\rho_s\, T_{\text{event}}(\rho-\rho_s)\,\frac{1}{\sqrt{\pi}\sigma}\exp\left(-\frac{(\rho_s-\bar{\rho}_s)^2}{\sigma^2}\right)$$

with $\rho = \log(1/r^2)$, $\rho_s = \log(Q_s^2)$

$$T_{\text{event}}(\rho-\rho_s) = \begin{cases} 1 & r > 1/Q_s \quad \text{saturation} \\ r^2 Q_s^2 & r < 1/Q_s \quad \text{colour transparency} \end{cases}$$

High-energy behaviour



dispersion $\sim DY$

Energy:	Intermediate	High energy
Physics:	Mean field (BK)	Fluctuations
Amplitude:	Geometric scaling $\langle T\rangle = f\left[\log(k^2/Q_s^2)\right]$	Diffusive scaling $\langle T\rangle = f\left[\log(k^2/Q_s^2)/\sqrt{DY}\right]$

At high-energy, amplitudes are dominated by black-spots i.e. rare fluctuations at saturation: $T = 1$ or 0

Describing F_2

Following fits to the F_2^p data:

Saturation fit: [Iancu, Itakura, Munier]

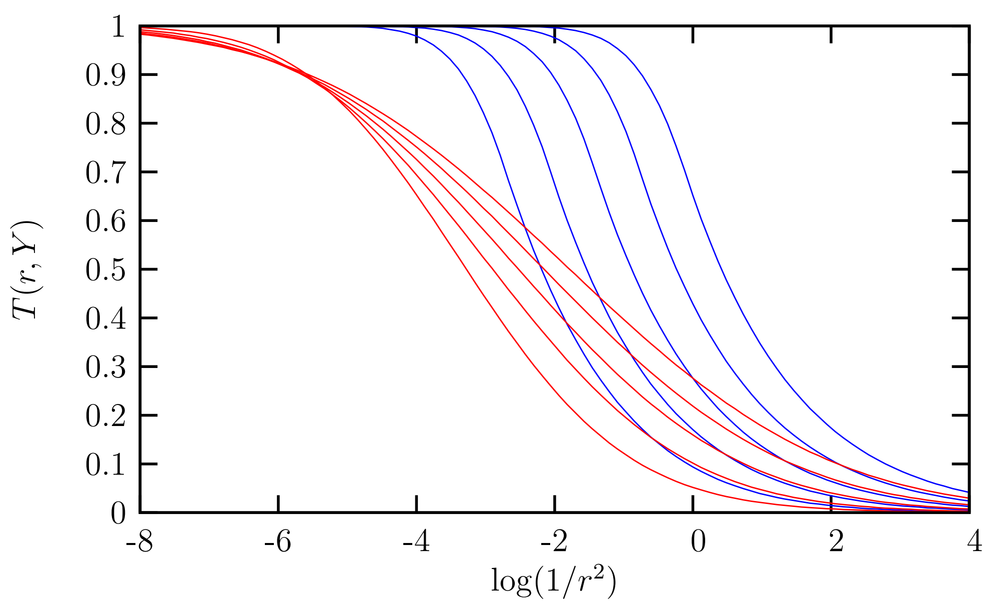
$$\langle T(r,Y)\rangle = \begin{cases} (r^2Q_s^2)^{\gamma_c} e^{-\frac{2\log^2(rQ_s)}{CY}} & r < 1/Q_s \\ 1 - e^{-a-b\log^2(rQ_s)} & r > 1/Q_s \end{cases}$$

$$Q_s^2(Y) = \lambda Y,\ \rho_s = \log(Q_s^2)$$

Saturation+fluctuations fit: [in preparation]

$$\langle T(r,Y)\rangle = \int d\rho_s\, T(r,\rho_s) \frac{1}{\sqrt{\pi}\sigma} e^{-\frac{(\rho_s-\bar{\rho}_s)^2}{\sigma^2}}$$

$$T(r,\rho_s) = \begin{cases} r^2Q_s^2 & r < 1/Q_s \\ 1 & r > 1/Q_s \end{cases}$$



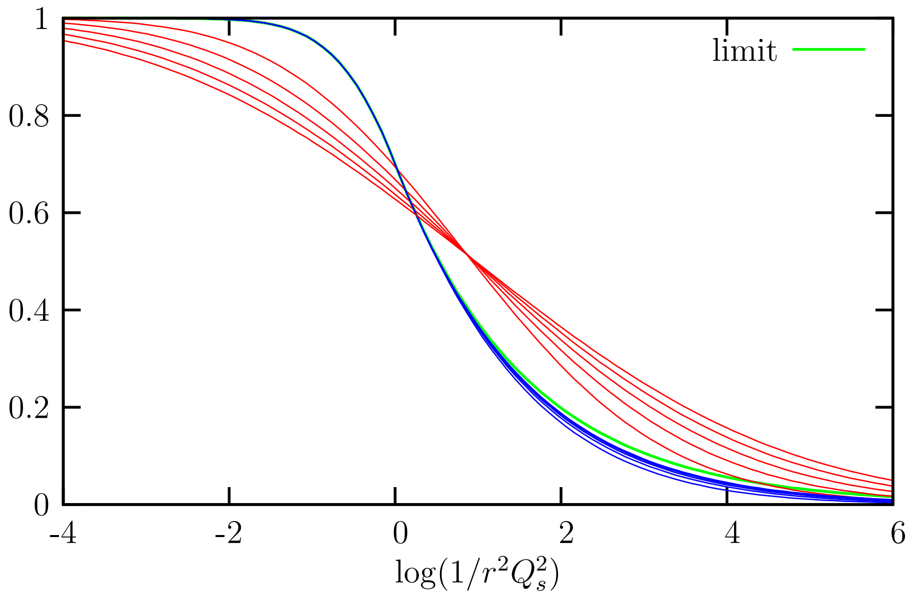
Following fits to the F_2^p data:

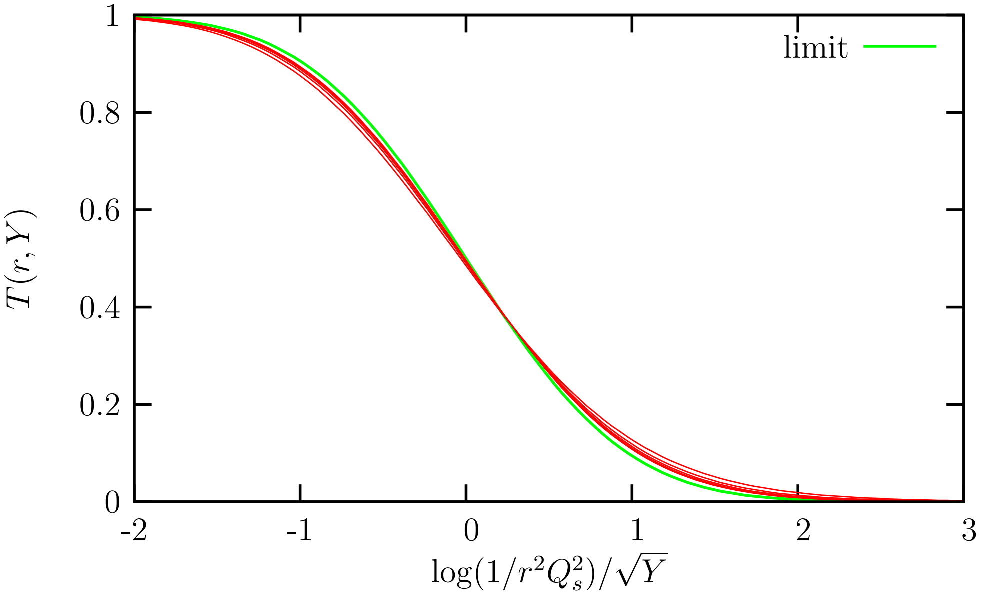
Saturation fit: [Iancu, Itakura, Munier]

$$\langle T(r,Y)\rangle = (r^2Q_s^2)^{\gamma_c} e^{-\frac{2\log^2(rQ_s)}{CY}} \qquad \to (r^2Q_s^2)^{\gamma_c}$$

Saturation+fluctuations fit: [in preparation]

$$\langle T(r,Y)\rangle = \int d\rho_s\, T(r,\rho_s) \frac{1}{\sqrt{\pi}\sigma} e^{-\frac{(\rho_s-\bar{\rho}_s)^2}{\sigma^2}} \qquad \to \frac{1}{2}\mathrm{erfc}\left(\frac{-\log(r^2\bar{Q}_s^2)}{\sqrt{2DY}}\right)$$

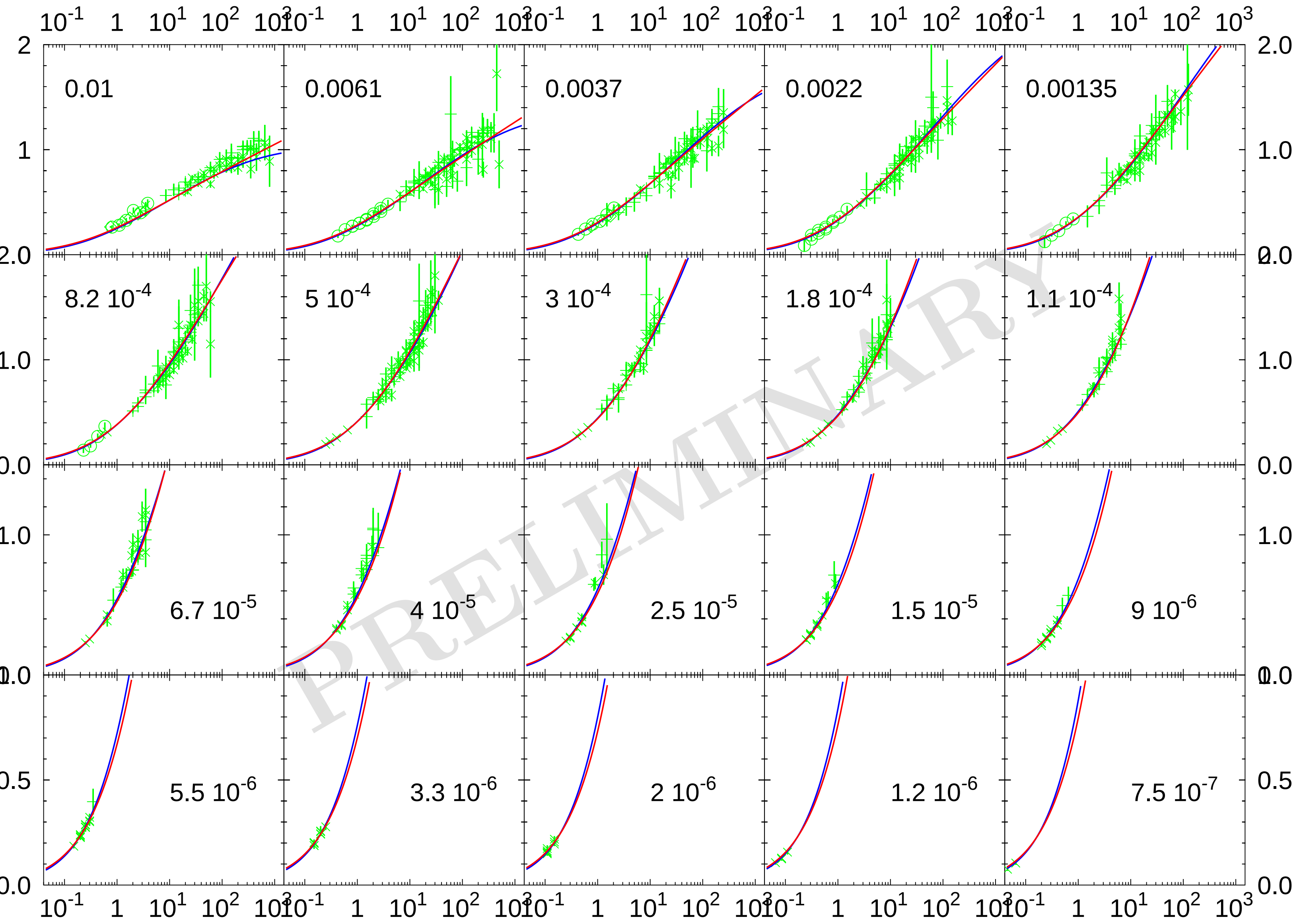




$\xrightarrow{Y\to\infty}$ Geometric scaling

$\xrightarrow{Y\to\infty}$ Diffusive scaling

Describing F_2



Both fits can describe the data for $x \le 0.01$

Conclusion

	High-energy QCD		Statistical Physics		
1974	Linear: BFKL	$\leftrightarrow$	Dipole splitting		
1999	Saturation:				Geometric
	JIMWLK/BK	$\overset{2003}{\leftrightarrow}$	F-KPP	$\leftrightarrow$	Scaling
2005	Fluctuations				
	b-indep.	$\leftrightarrow$	sF-KPP	$\leftrightarrow$	Diffusive
	full	$\leftrightarrow$	reaction-diffusion		Scaling

Perspectives

- phenomenological tests:
  - applications for diffraction (see talks by E. Iancu and C. Marquet)
  - Non-zero momentum transfer (DVCS, ρ mesons)
  - Predictions for LHC (under study)
- theoretical extensions:
  - include running coupling effects
  - include b-dependent fluctuations (under study)
  - better analytic understanding (under study)
  - beyond large-N_c (see next talk by Y. Hatta)