Saturation in High-Energy QCD

Scaling laws and phenomenological applications

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Based on : G.S., hep-ph/0504129, Phys. Rev. D72 (2005) 016007

Y. Hatta, E. Iancu, C. Marquet, G.S., D. Triantafyllopoulos, hep-ph/0601150, Nucl. Phys. A773 (2006) 95

E. Iancu, C. Marquet, G.S., hep-ph/0605174, to appear in Nucl. Phys. A

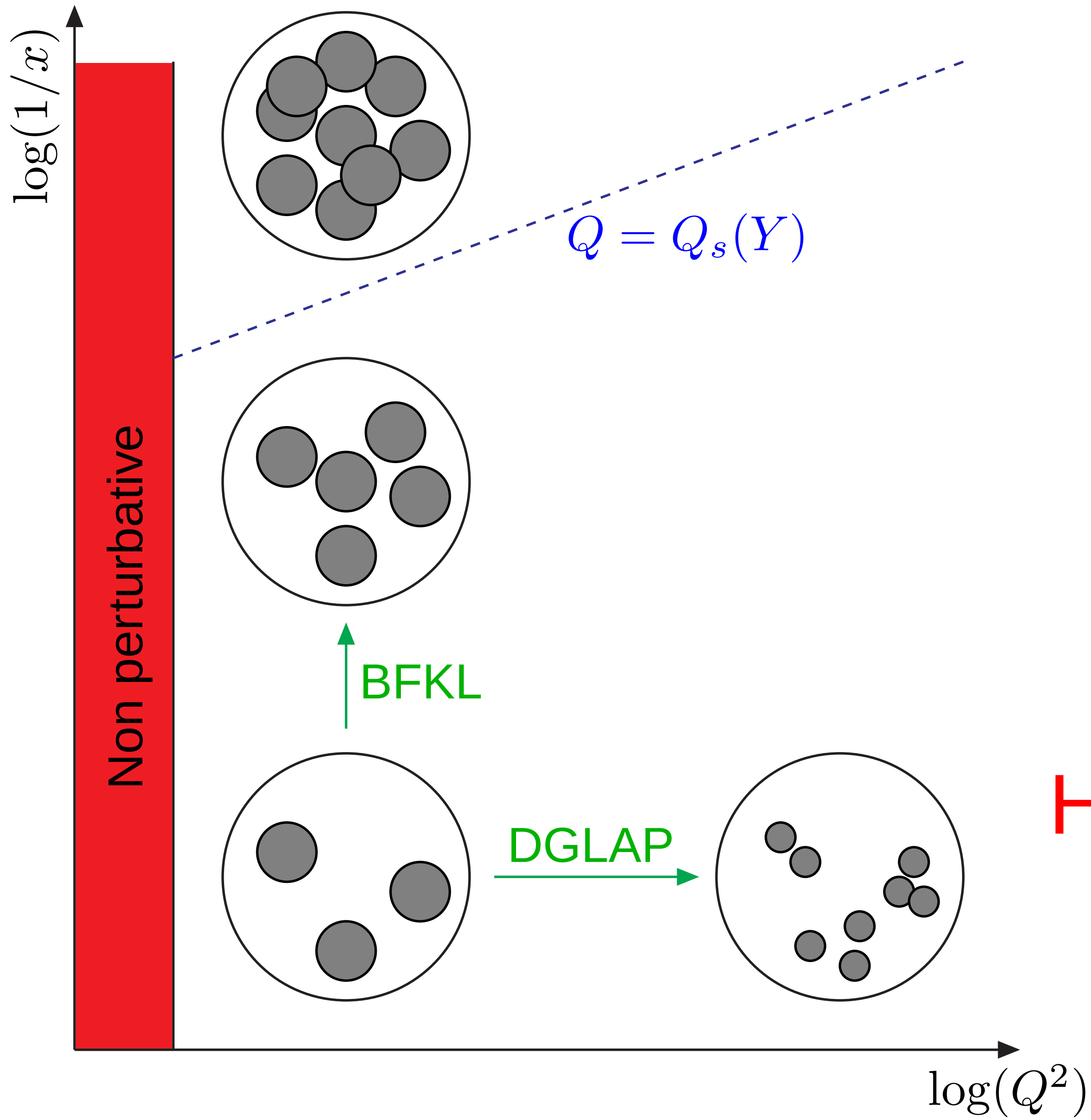
C. Marquet, G.S., B-W. Xiao, hep-ph/0606233, Phys. Lett. B639 (2006) 635

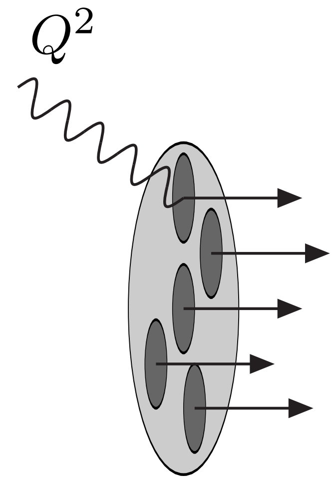
C. Marquet, R. Peschanski, G.S., in preparation

Outline

- Perturbative evolution in high-energy QCD:
  - Leading log approx.: BFKL equation
  - Unitarity/Saturation effects: Balitsky/JIMWLK and BK equation
  - Fluctuation effects: towards a new evolution
- Asymptotic solutions:
  - saturation $\Rightarrow$ geometric scaling
  - fluctuation $\Rightarrow$ Stochastic evolution $\Rightarrow$ Diffusive scaling
- Phenomenological consequences
  - Geometric scaling for F_2 and in vector meson production
  - Diffusive scaling in DIS, diffractive DIS and forward gluon production

Motivation: why saturation ?



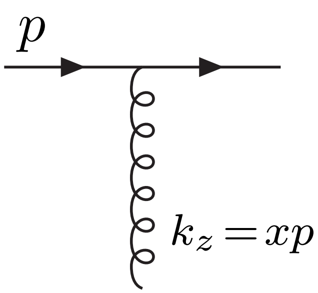


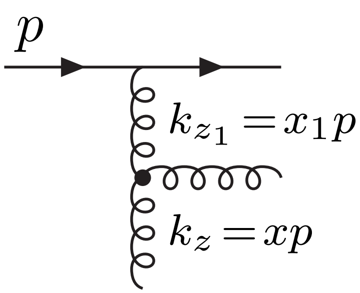
Size $\sim 1/Q$

Energy $\sim Q^2/x$

How to describe this in QCD ?

Bremsstrahlung:





$x \ll 1$ $\quad$ $x \ll x_1 \ll 1$

Probability of emission

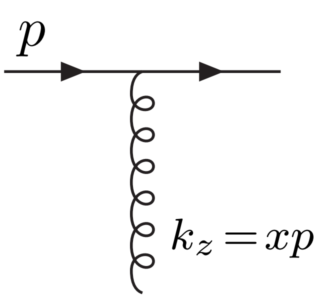
$$dP \sim \alpha_s \frac{dk_\perp^2}{k_\perp^2} \frac{dx}{x}$$

In the small-x limit

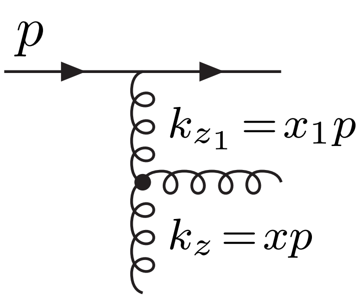
$$\int_x^1 \frac{dx_1}{x_1} \sim \alpha_s \log(1/x)$$

Motivation: why resummation ?

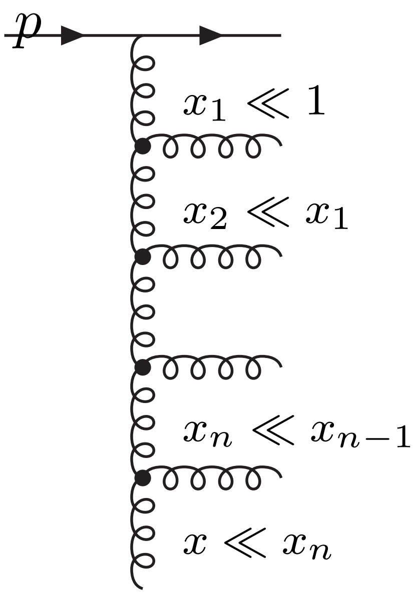
Bremsstrahlung:



$x \ll 1$



$x \ll x_1 \ll 1$



Probability of emission

$$dP \sim \alpha_s \frac{dk_\perp^2}{k_\perp^2} \frac{dx}{x}$$

In the small-x limit

$$\int_x^1 \frac{dx_n}{x_n} \dots \int_{x_2}^1 \frac{dx_1}{x_1} \sim \frac{1}{n!} \alpha_s^n \log^n(1/x)$$

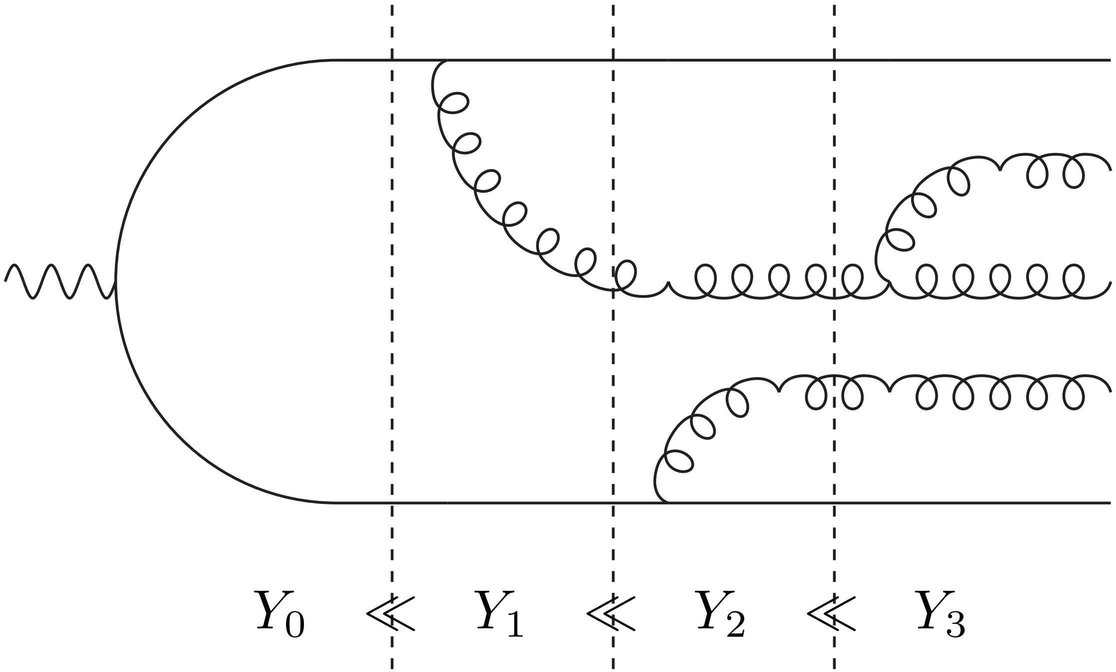
Same order when $\alpha_s \log(1/x) \sim 1$

Perturbative evolution in high-energy QCD

Dipole picture

Consider a fast-moving $q\bar{q}$ dipole (Rapidity: $Y = \log(s)$)

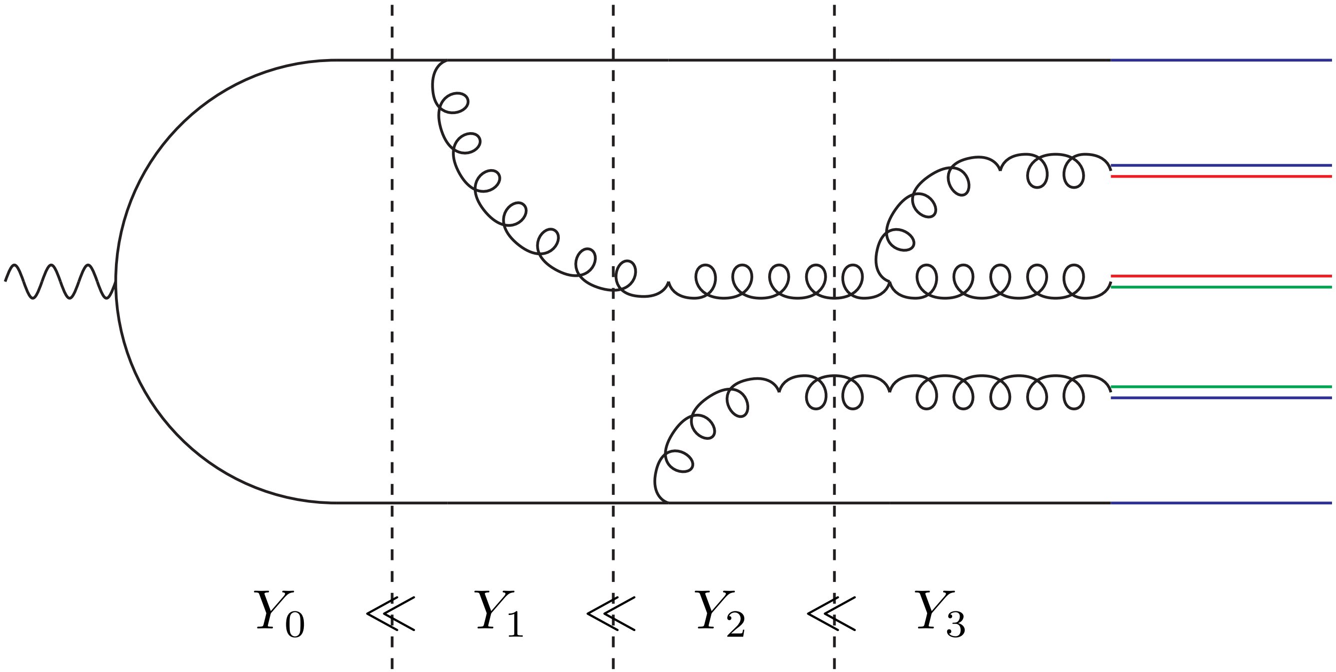
[Mueller,93]



- Probability $\bar{\alpha}K$ of emission
- Independent emissions in coordinate space (transverse plane)

Dipole picture

Consider a fast-moving $q\bar{q}$ dipole (Rapidity: $Y = \log(s)$)

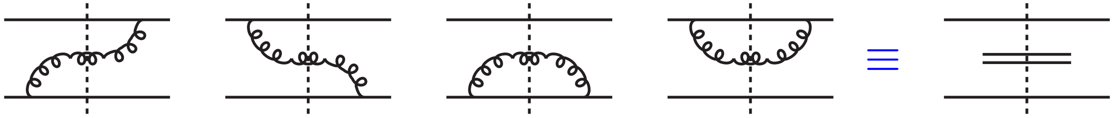


[Mueller,93]

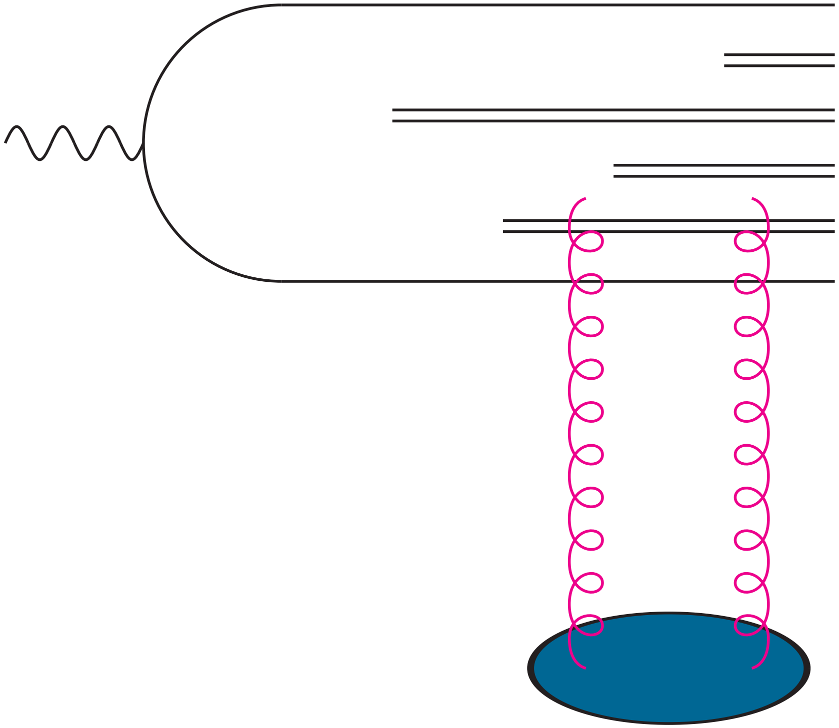
$n(r, Y)$ dipoles

of size r

- Probability $\bar{\alpha}K$ of emission
- Independent emissions in coordinate space (transverse plane)
- Large-N_c approximation



How to observe this system ?

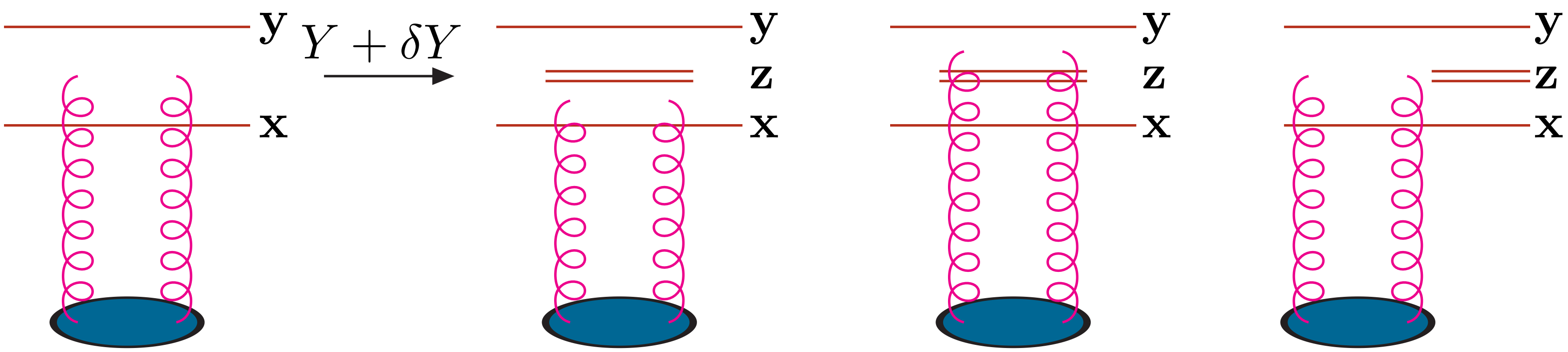


$$T(r,Y) \approx \alpha_s^2 n(r,Y)$$

Count the number of dipoles of a given size

BFKL evolution (2/4)

Consider a small increase in rapidity ⇒ splitting



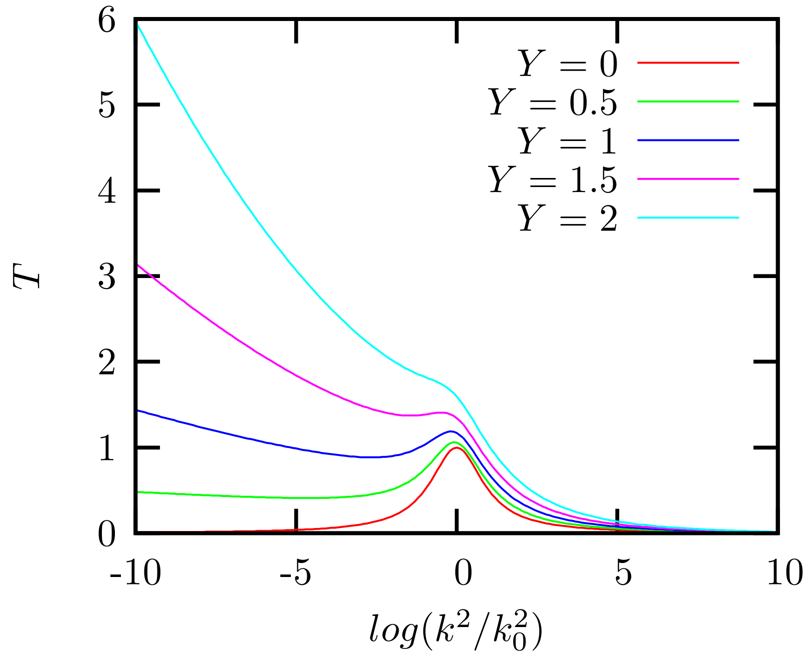
$$\partial_Y T(\mathbf{x},\mathbf{y};Y) = \bar{\alpha} \int \underbrace{d^2z \frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{z}-\mathbf{y})^2}}_{\text{Emission proba from pQCD}} \underbrace{[T(\mathbf{x},\mathbf{z};Y) + T(\mathbf{z},\mathbf{y};Y) - T(\mathbf{x},\mathbf{y};Y)]}_{\text{all possible interactions}}$$

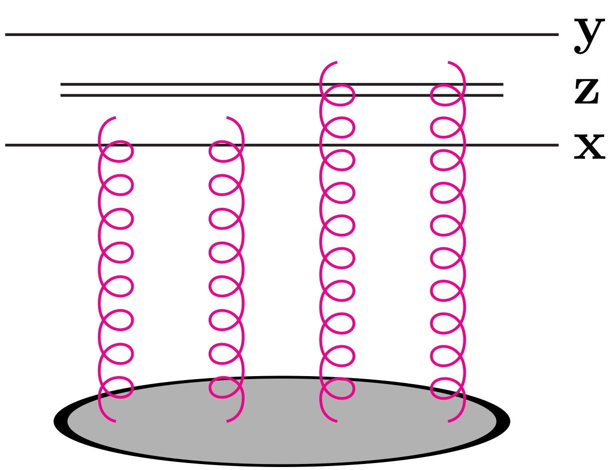
[Balitsky,Fadin,Kuraev,Lipatov,78]

The solution goes like

$$T(Y) \sim e^{\omega Y} \qquad \text{with} \quad \omega = 4\bar{\alpha}\log(2) \approx 0.5$$

- Fast growth of the amplitude
- Intercept value too large
- Violation of the Froissart unitarity:
  $T(Y) \le C \log^2(s) \qquad T(r,b) \le 1$
- problem of diffusion in the infrared





Multiple scattering

- ⋆ Proportional to T^2
- ⋆ important when $T \approx 1$

$\langle \cdot \rangle \equiv$ average over target field

$$\partial_Y \langle T(\mathbf{x},\mathbf{y};Y)\rangle = \bar{\alpha} \int d^2z \, \frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{z}-\mathbf{y})^2} \left[\langle T(\mathbf{x},\mathbf{z};Y)\rangle + \langle T(\mathbf{z},\mathbf{y};Y)\rangle - \langle T(\mathbf{x},\mathbf{y};Y)\rangle - \langle T(\mathbf{x},\mathbf{z};Y) T(\mathbf{z},\mathbf{y};Y)\rangle\right]$$

But

$\partial_Y \langle T(\mathbf{x},\mathbf{y};Y)\rangle$ contains a new object: $\langle T(\mathbf{x},\mathbf{z};Y) T(\mathbf{z},\mathbf{y};Y)\rangle$

In general: complete hierarchy [Balitsky, 96]

$$\partial_Y \langle T^k\rangle \longrightarrow \underbrace{\langle T^k\rangle}_{\text{BFKL}}, \underbrace{\langle T^{k+1}\rangle}_{\text{saturation}}$$

- Beyond large-N_c: the hierarchy involves quadrupoles, sextupoles, ...
- Balitsky hierarchy $\equiv$ JIMWKL eq. (Colour Glass Condensate formalism)

Mean field approx.: $\langle T_{\mathbf{xz}} T_{\mathbf{zy}}\rangle = \langle T_{\mathbf{xz}}\rangle\langle T_{\mathbf{zy}}\rangle$

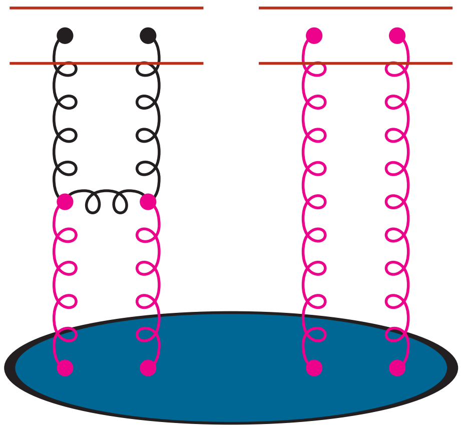
$$\partial_Y \langle T_{\mathbf{xy}}\rangle = \frac{\bar\alpha}{2\pi}\int d^2z\, \frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{z}-\mathbf{y})^2}\left[\langle T_{\mathbf{xz}}\rangle + \langle T_{\mathbf{zy}}\rangle - \langle T_{\mathbf{xy}}\rangle - \langle T_{\mathbf{xz}}\rangle\langle T_{\mathbf{zy}}\rangle\right]$$

[Balitsky 96,Kovchegov 99]

Simplest perturbative evolution equation satisfying unitarity constraint

Consider evolution of $\langle T^{(2)} \rangle$

[E. Iancu, D. Triantafyllopoulos, 05]
Also A. Mueller, S. Munier, A. Shoshi, S. Wong



- Usual BFKL ladder

$$\partial_Y \langle T^{(2)} \rangle \propto \langle T^{(2)} \rangle$$

Fluctuations

Consider evolution of $\langle T^{(2)} \rangle$

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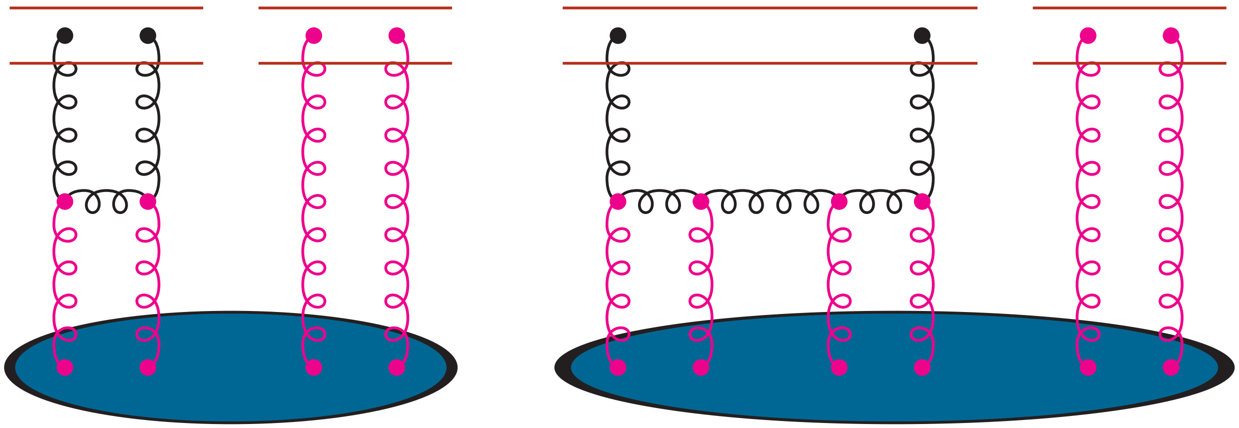

- Usual BFKL ladder $\partial_Y \langle T^{(2)} \rangle \propto \langle T^{(2)} \rangle$
- fan diagram $\longrightarrow$ saturation effects $\partial_Y \langle T^{(2)} \rangle \propto \langle T^{(3)} \rangle$

Consider evolution of $\langle T^{(2)} \rangle$ [E. Iancu, D. Triantafyllopoulos, 05]
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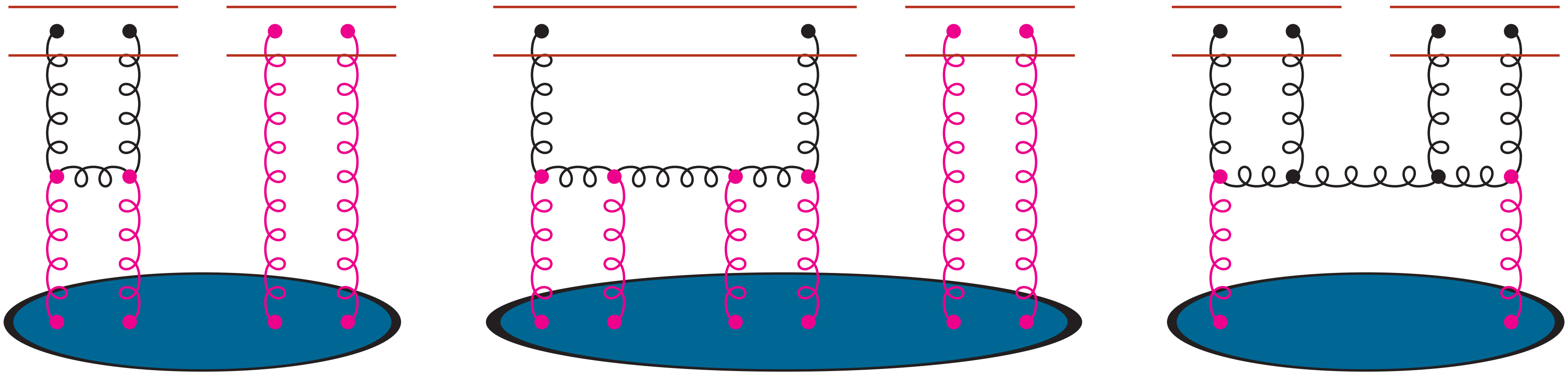
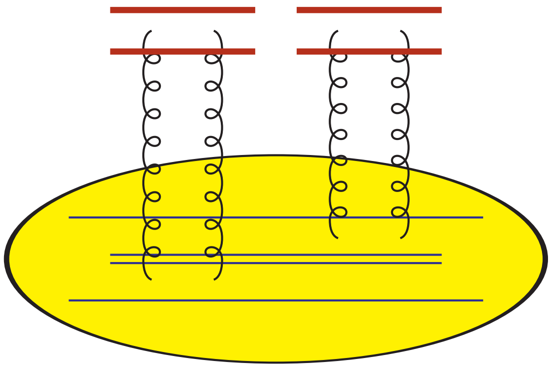


- Usual BFKL ladder $\partial_Y \langle T^{(2)} \rangle \propto \langle T^{(2)} \rangle$
- fan diagram $\longrightarrow$ saturation effects $\partial_Y \langle T^{(2)} \rangle \propto \langle T^{(3)} \rangle$
- splitting $\longrightarrow$ gluon-number fluctuations $\partial_Y \langle T^{(2)} \rangle \propto \langle T \rangle$
  $\longrightarrow$ pomeron loops



$\Rightarrow$ complicated hierarchy

$$\partial_Y \langle T^{(2)}(\mathbf{x}_1,\mathbf{y}_1;\mathbf{x}_2,\mathbf{y}_2)\rangle$$

$$= \frac{\bar\alpha}{2\pi}\int d^2z\, \frac{(\mathbf{x}_2-\mathbf{y}_2)^2}{(\mathbf{x}_2-\mathbf{z})^2(\mathbf{z}-\mathbf{y}_2)^2}\Big[\langle T^{(2)}(\mathbf{x}_1,\mathbf{y}_1;\mathbf{x}_2,\mathbf{z})\rangle + \langle T^{(2)}(\mathbf{x}_1,\mathbf{y}_1;\mathbf{z},\mathbf{y}_2)\rangle$$
$$-\langle T^{(2)}(\mathbf{x}_1,\mathbf{y}_1;\mathbf{x}_2,\mathbf{y}_2)\rangle - \langle T^{(3)}(\mathbf{x}_1,\mathbf{y}_1;\mathbf{x}_2,\mathbf{z};\mathbf{z},\mathbf{y}_2)\rangle + (1\leftrightarrow 2)\Big]$$
$$+ \frac{\bar\alpha}{2\pi}\left(\frac{\alpha_s}{2\pi}\right)^2\int_{\mathbf{uvz}} \mathcal{M}_{\mathbf{uvz}}\mathcal{A}_0(\mathbf{x}_1\mathbf{y}_1|\mathbf{uz})\mathcal{A}_0(\mathbf{x}_2\mathbf{y}_2|\mathbf{zv})\,\nabla^2_{\mathbf{u}}\nabla^2_{\mathbf{v}}\langle T^{(1)}(\mathbf{u},\mathbf{v})\rangle$$

- Saturation: important when $T^{(2)}\sim T^{(1)}\sim 1$ i.e. near unitarity
- Fluctuations: important when $T^{(2)}\sim \alpha_s^2 T^{(1)}$ or $T\sim\alpha_s^2$ i.e. dilute regime
- Langevin formulation: fluctuation = noise

Solutions

The BK equation

Case 1: no impact parameter dependence

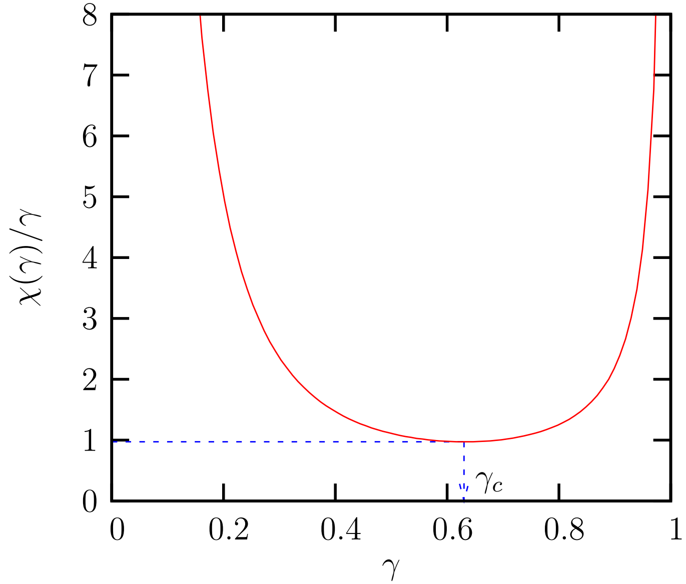
$$T_{\mathbf{xy}} \to T\left(\mathbf{r} = \mathbf{x} - \mathbf{y}, \mathbf{b} = \frac{\mathbf{x}+\mathbf{y}}{2}\right) \to T(r)$$

Note:

- all arguments work for $T(r)$ or its Fourier transform $\tilde{T}(k)$
- for $\tilde{T}$, the non-linear term is simply $-\tilde{T}^2(k)$

BK equation: $\partial_Y T = \underbrace{\chi(-\partial_L)T}_{\text{BFKL}} - T^2$

[S.Munier,R.Peschanski,03]



When $T \ll 1$ BFKL works: $\partial_Y T = \chi(-\partial_L)T$

Solution known:

$$\begin{aligned} T(k) &= \int \frac{d\gamma}{2i\pi} T_0(\gamma) \exp\left[\chi(\gamma)\bar{\alpha}Y - \gamma L\right] \\ &= \int \frac{d\gamma}{2i\pi} T_0(\gamma) \exp\left[-\gamma\left(L - \frac{\chi(\gamma)}{\gamma}\bar{\alpha}Y\right)\right] \end{aligned}$$

$\frac{\chi(\gamma)}{\gamma}$ min. when $\gamma = \gamma_c$

$\Rightarrow$ Wave of slope γ travels at speed $v = \chi(\gamma)/\gamma$

$Y = Y_0$

Evolution mechanism

[S.Munier,R.Peschanski,03]

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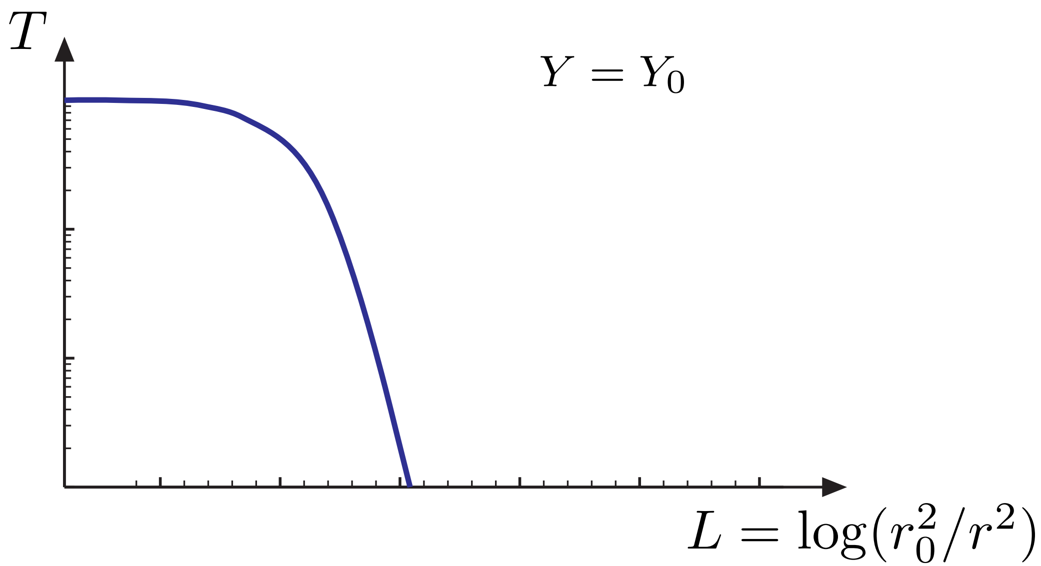

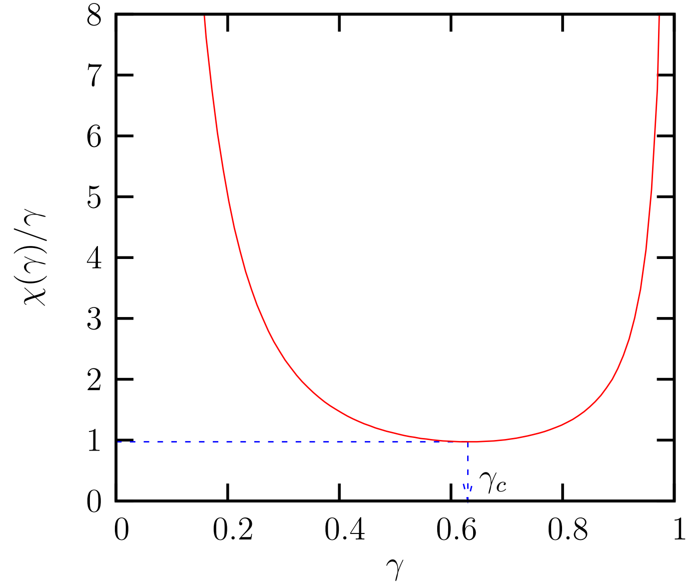

$\frac{\chi(\gamma)}{\gamma}$ min. when $\gamma = \gamma_c$

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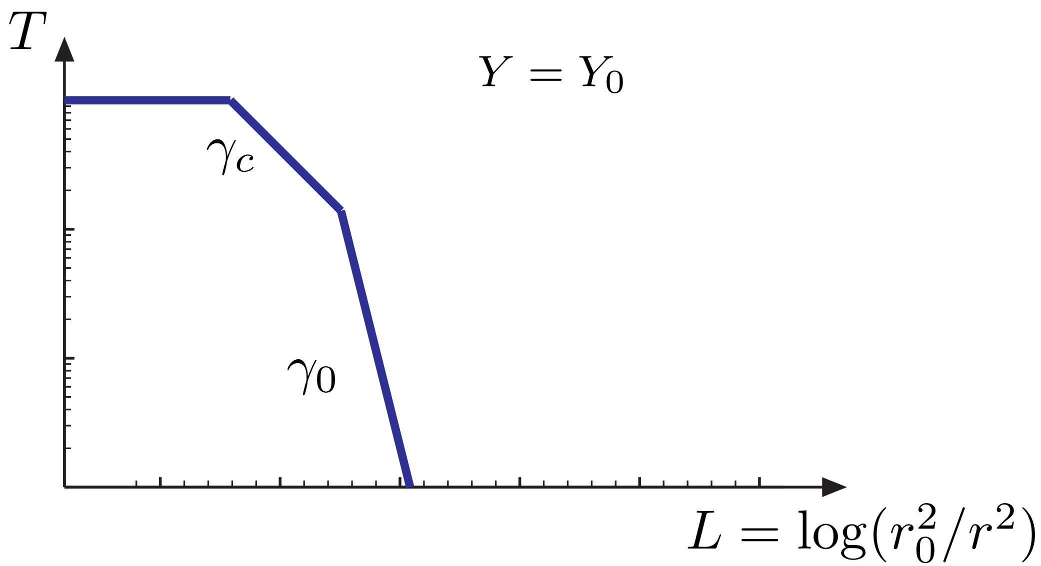

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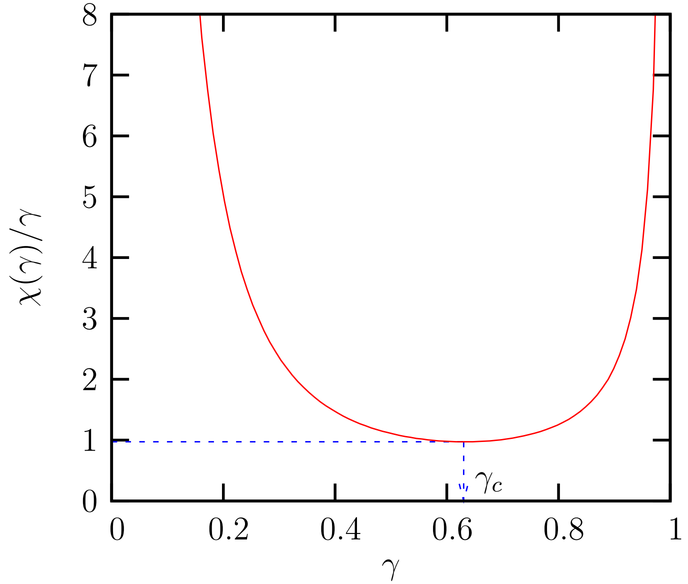

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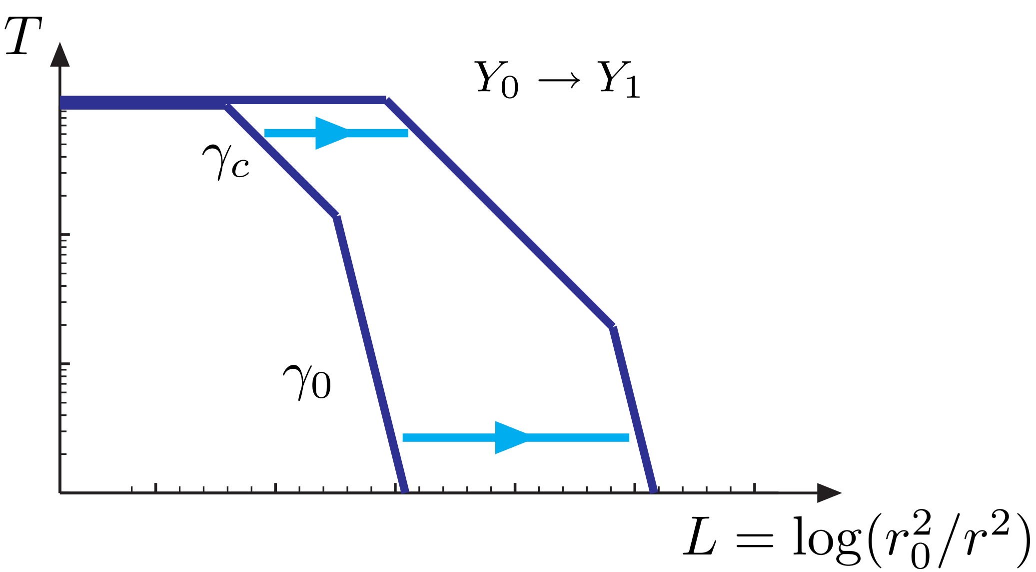

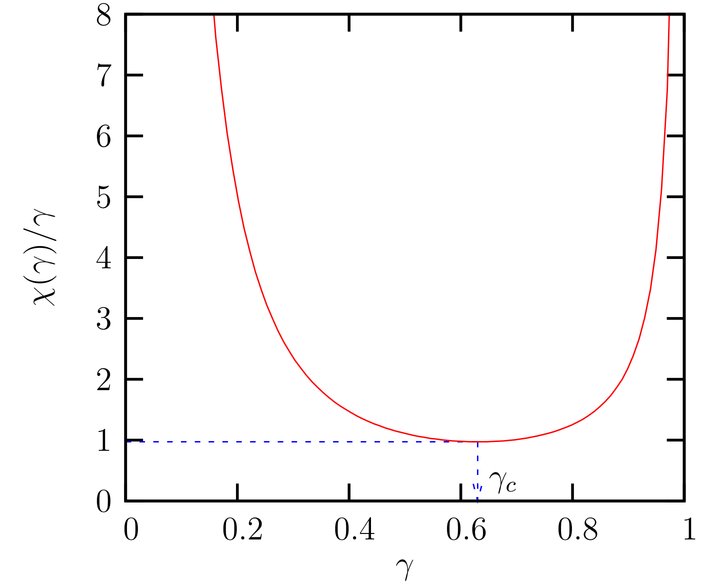

$\frac{\chi(\gamma)}{\gamma}$ min. when $\gamma = \gamma_c$

The minimal speed is selected during evoution

Geometric scaling

Consequence: geometric scaling ($Q_s \equiv$ saturation scale $\equiv$ front position)

$$T(r,Y) = T\left[rQ_s(Y)\right] \qquad \text{with} \qquad Q_s^2(Y) = v_c\bar{\alpha}Y$$

$$\overset{rQ_s \ll 1}{=} \underbrace{\left[r^2Q_s^2(Y)\right]^{\gamma_c}}_{\text{slope } \gamma_c} \quad \underbrace{\exp\left[\frac{\log^2(r^2Q_s^2)}{2\chi''(\gamma_c)\bar{\alpha}Y}\right]}_{\text{scaling window}}$$

scaling window

$|\log(r^2Q_s^2)| \lesssim \sqrt{\chi''(\gamma_c)\bar{\alpha}Y}$

Geometric scaling

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$$|\log(r^2Q_s^2)| \lesssim \sqrt{\chi''(\gamma_c)\bar{\alpha}Y}$$

- Generic arguments: exponential rise + saturation $\Rightarrow$ select γ_c
- Parameters fixed by linear kernel only
- Saturation effects even though $T \ll 1$

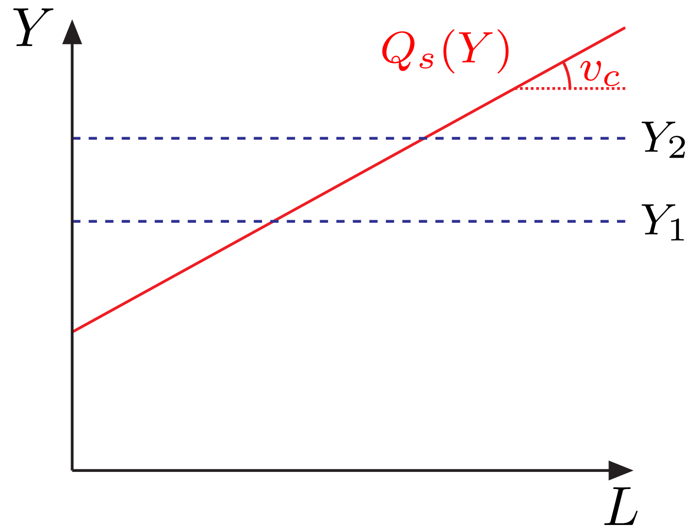
Consequence: geometric scaling ($Q_s \equiv$ saturation scale $\equiv$ front position)

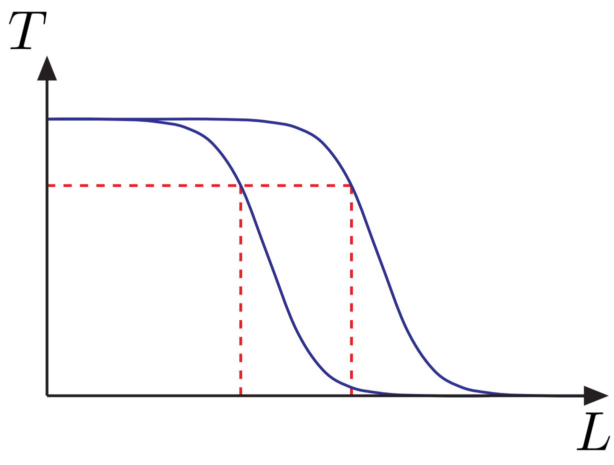
$$T(r,Y) = T\left[rQ_s(Y)\right] \quad \text{with} \quad Q_s^2(Y) = v_c\bar\alpha Y$$

$$\overset{rQ_s\ll 1}{=} \underbrace{\left[r^2Q_s^2(Y)\right]^{\gamma_c}}_{\text{slope } \gamma_c} \underbrace{\exp\left[\frac{\log^2(r^2Q_s^2)}{2\chi''(\gamma_c)\bar\alpha Y}\right]}_{\text{scaling window}}$$

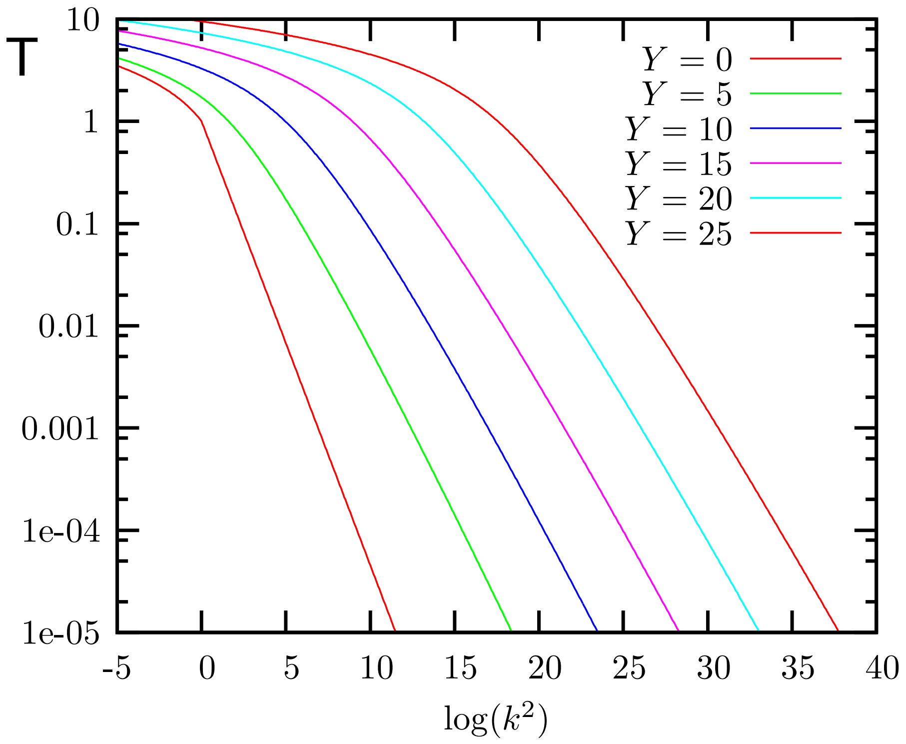
$$|\log(r^2Q_s^2)| \lesssim \sqrt{\chi''(\gamma_c)\bar\alpha Y}$$

Interpretation: invariance along the saturation line





Numerical simulations:



Geometric scaling

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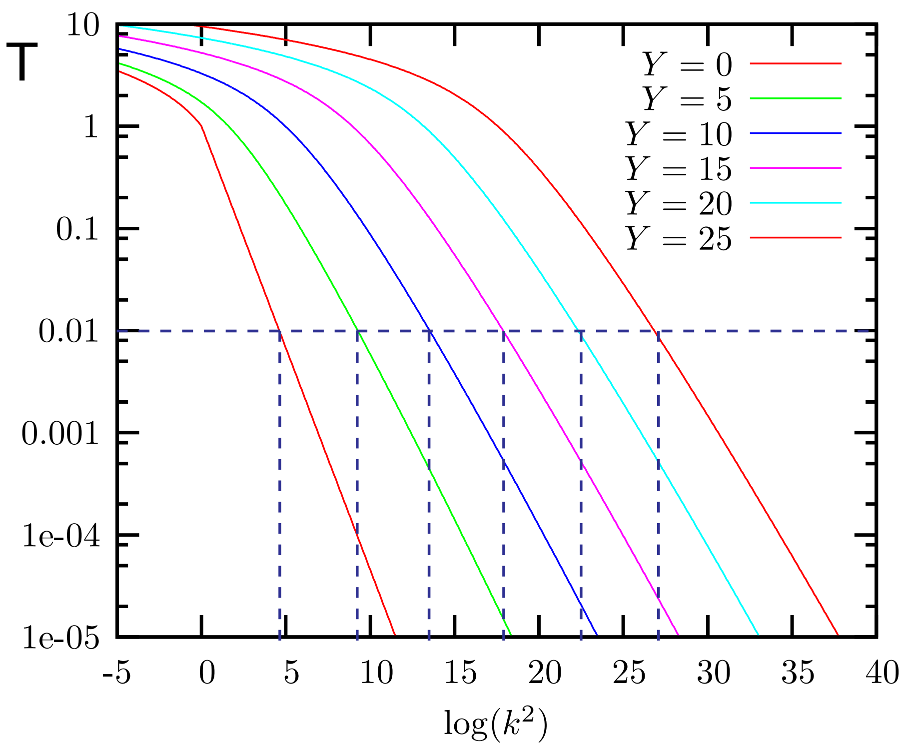
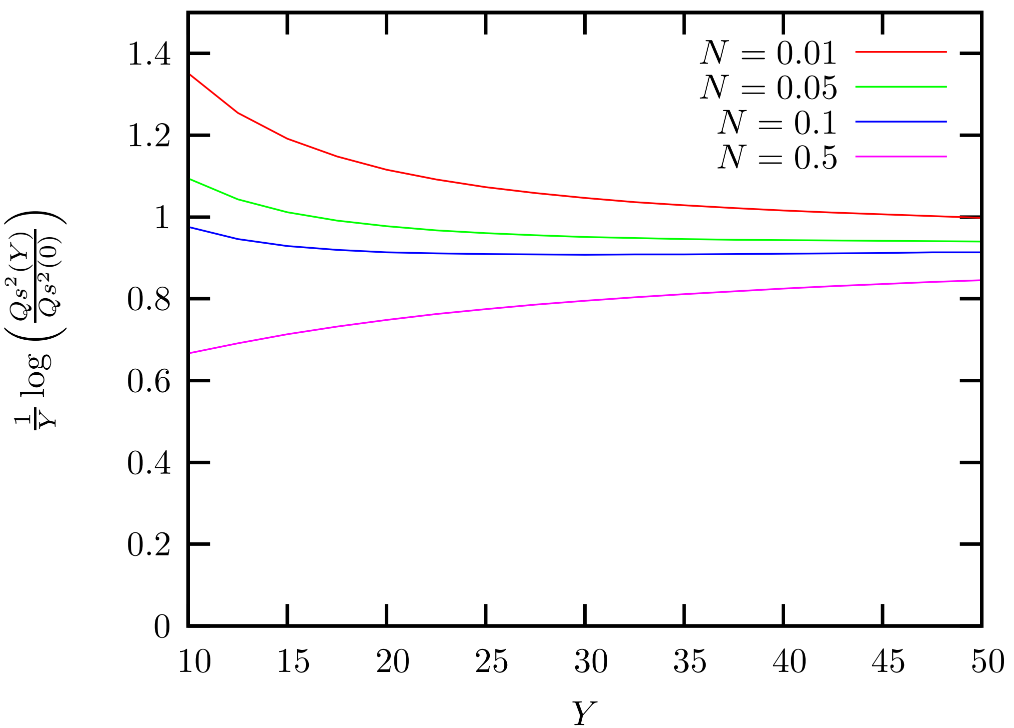




$$T(k,Y) = T\left(\frac{k^2}{Q_s^2(Y)}\right) \approx \left(\frac{k^2}{Q_s^2(Y)}\right)^{-\gamma_c} \qquad Q_s^2(Y) \propto \exp(v_c Y)$$

The full BK equation

Case 2: including impact parameter

Go to momentum space: use momentum transfer $\mathbf{q}$

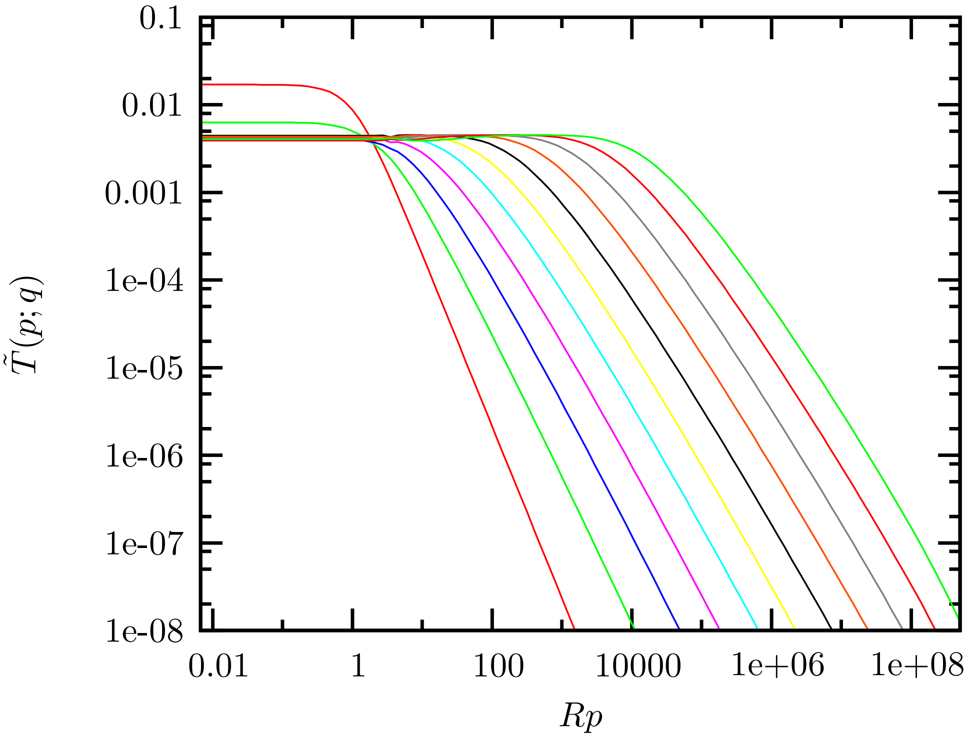
$$\tilde{T}(\mathbf{k},\mathbf{q}) = \int d^2x\, d^2y\, e^{i\mathbf{k}.\mathbf{x}} e^{i(\mathbf{q}-\mathbf{k}).\mathbf{y}} \frac{T(\mathbf{x},\mathbf{y})}{(\mathbf{x}-\mathbf{y})^2}$$

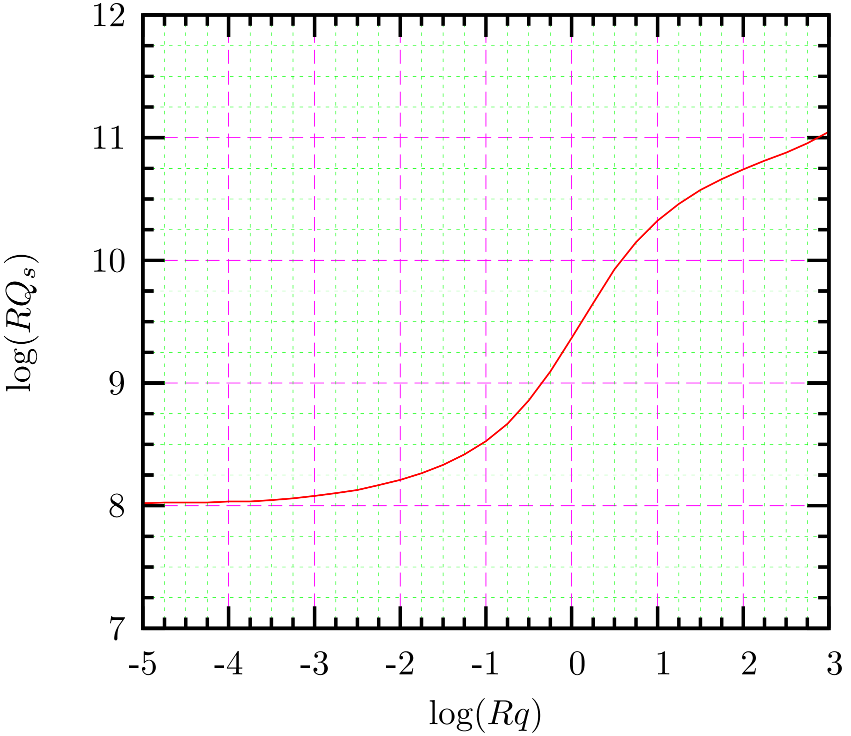
new form of the BK equation

$$\begin{aligned}\partial_Y \tilde{T}(\mathbf{k},\mathbf{q}) &= \frac{\bar{\alpha}}{\pi} \int \frac{d^2k'}{(k-k')^2} \left\{ \tilde{T}(\mathbf{k}',\mathbf{q}) - \frac{1}{4}\left[\frac{k^2}{k'^2} + \frac{(q-k)^2}{(q-k')^2}\right] \tilde{T}(\mathbf{k},\mathbf{q}) \right\} \\ &- \frac{\bar{\alpha}}{2\pi} \int d^2k'\, \tilde{T}(\mathbf{k},\mathbf{k}')\tilde{T}(\mathbf{k}-\mathbf{k}',\mathbf{q}-\mathbf{k}')\end{aligned}$$

[C.Marquet, R.Peschanski, G.S., 05]

Numerical simulations





One can prove analytically that:

- traveling wave at large k: BFKL $\Rightarrow$ same γ_c, v_c
- q dependence: $Q_s^2(q,Y) = \min(k_0^2, q^2)\exp(v_c Y)$

Predicts geometric scaling for t-dependent processes

Solutions

Fluctuation effects

Event evolution

no b-dependence + coarse-graining (local fluctutions) $\longrightarrow$ Langevin equation

$$\partial_Y T(k,Y) = \bar\alpha K_{\text{BFKL}} \otimes T(k,Y) - \bar\alpha T^2(k,Y) + \bar\alpha\sqrt{\kappa\alpha_s^2 T(k,Y)}\,\nu(k,Y)$$

with a Gaussian white noise $\langle \nu(k,Y)\rangle = 0$

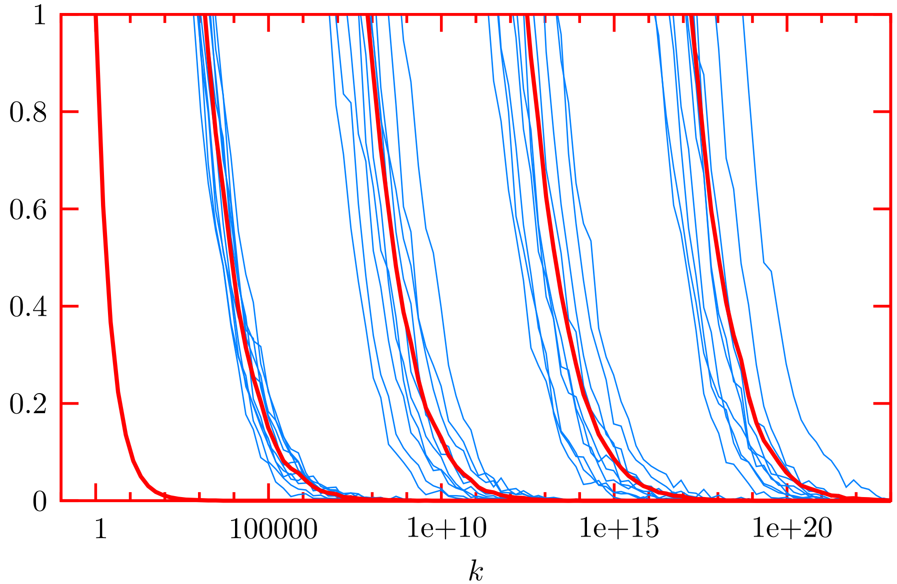
$$\langle \nu(k,Y)\nu(k',Y')\rangle = \frac{1}{\bar\alpha}\delta(Y-Y')\,k\delta(k-k')$$

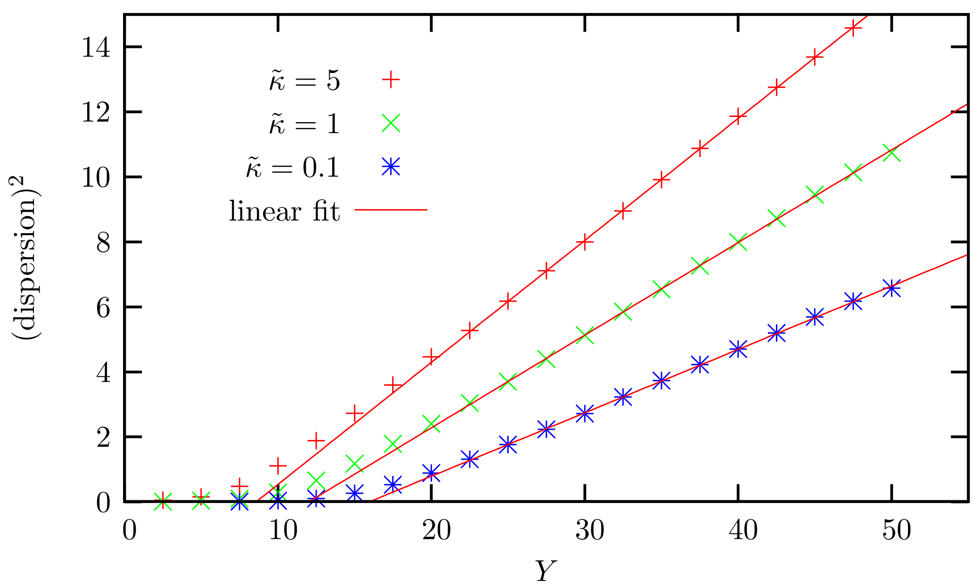
Remarks:

- noise $\equiv$ fluct. target field $\Rightarrow$ Different events $\equiv$ different target fields
- stochasticity as seen in detectors
- observables obtained by averaging over events

Numerical simulations

[G.S., 05]

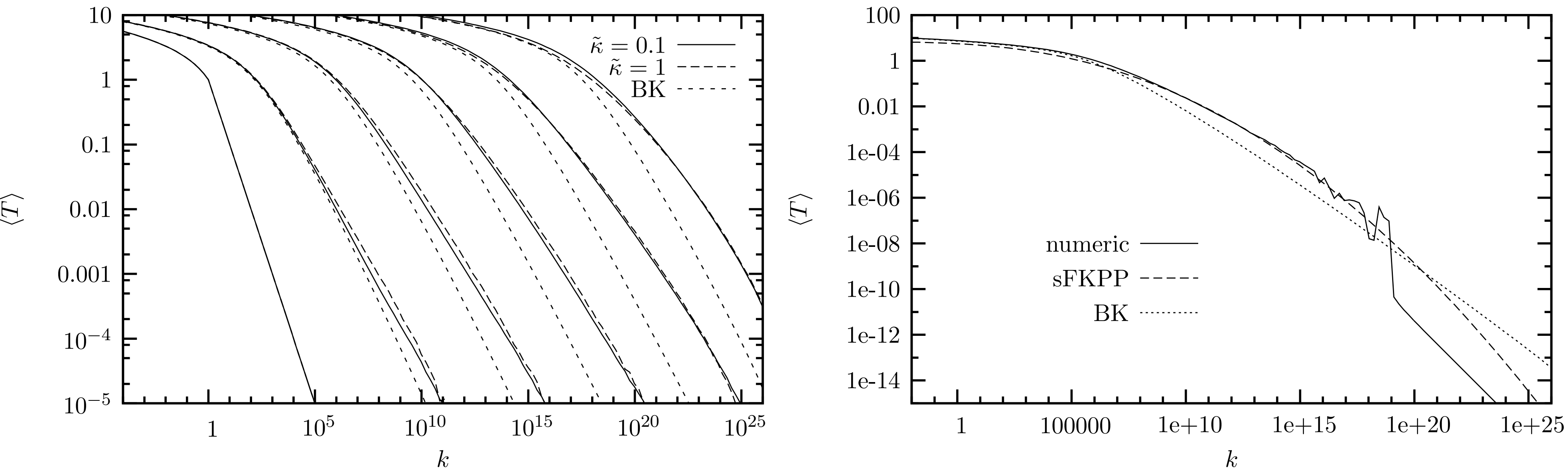




- Traveling wave/Geometric scaling for each event
- Dispersion of the events

$$\Delta \log[Q_s^2(Y)] \approx \sqrt{D_{\text{diff}}\bar{\alpha}Y} \quad \text{with} \quad D_{\text{diff}} \underset{\alpha_s^2\kappa\to 0}{\sim} \frac{1}{|\log^3(\alpha_s^2\kappa)|}.$$

[G.S., 05]



- Clear effect of fluctuations: dispersion $\Rightarrow$ spreading
- Violations of geometric scaling
- Agrees with predictions from statistical mechanics (sFKPP)

High-energy behaviour

Evolution with saturation & fluctuations $\equiv$

- superposition of unitary front (with geometric scaling)
- + dispersion of their position i.e. stochastic Q_s (geom. scaling violations)

$$\langle T(r,Y)\rangle = \int d\rho_s\, T_{\text{event}}(\rho-\rho_s)\, P(\rho_s)$$

with $\rho = \log(1/r^2)$, $\rho_s = \log(Q_s^2)$

Evolution with saturation & fluctuations $\equiv$

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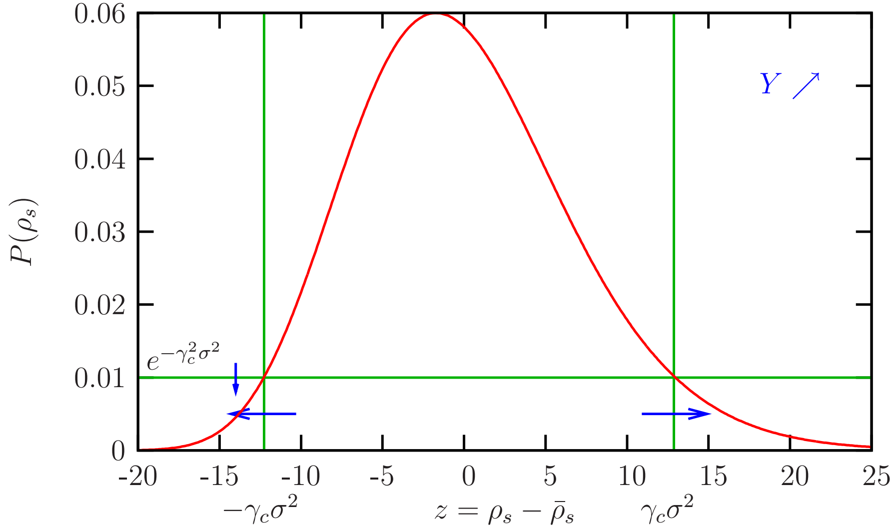
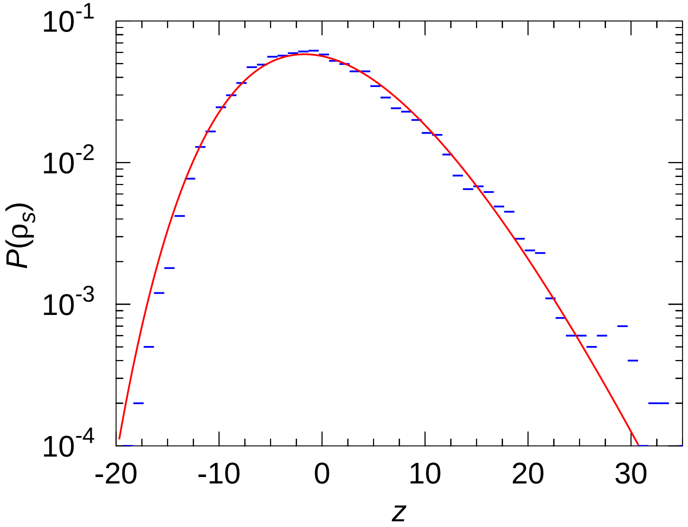
$$\langle T(r,Y)\rangle = \int d\rho_s\, T_{\text{event}}(\rho-\rho_s)\,\frac{1}{\sqrt{\pi}\sigma}\exp\left(-\frac{(\rho_s-\bar{\rho}_s)^2}{\sigma^2}\right)$$

with $\rho = \log(1/r^2)$, $\rho_s = \log(Q_s^2)$ [C.Marquet, G.S., B.Xiao, 06]

$P(\rho_s)$ can be taken as Gaussian: mean $\bar{\rho}_s \sim \lambda Y$, dispersion $\sigma^2 \sim DY$





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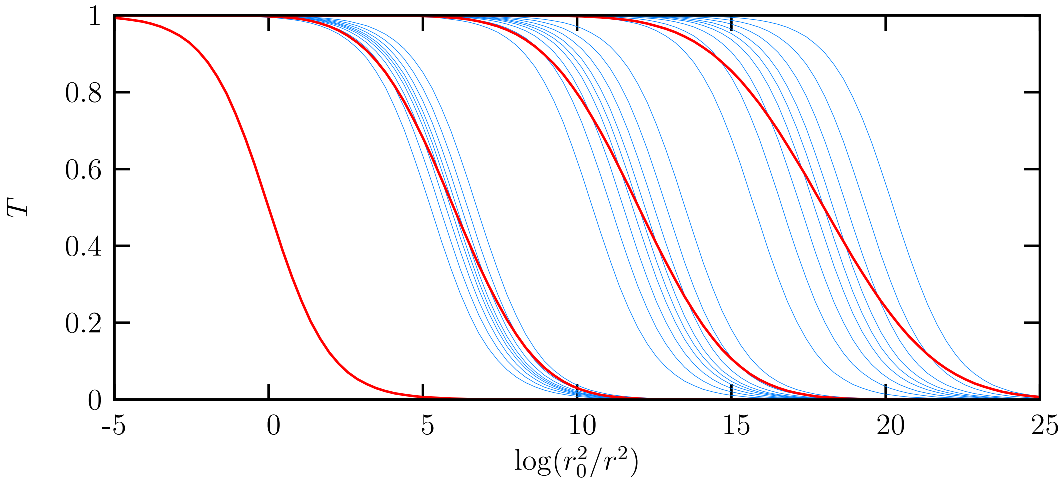
$$T_{\text{event}}(\rho-\rho_s) = \begin{cases} 1 & r > Q_s \quad \text{saturation} \\ (r^2Q_s^2)^\gamma & r < Q_s \quad \text{geometric scaling} \end{cases}$$

High-energy behaviour

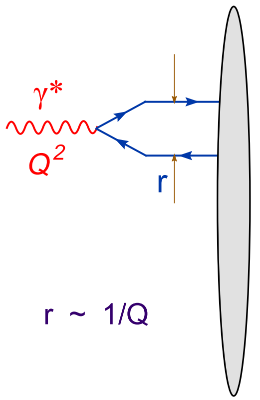
[E. Iancu, Y. Hatta, C. Marquet, G.S., D. Triantafyllopoulos, 06]

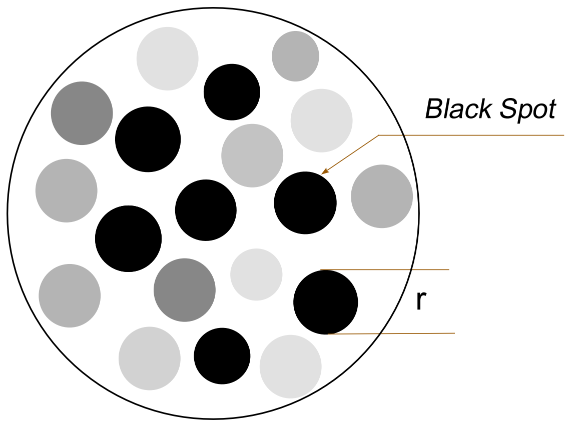


dispersion $\sim DY$

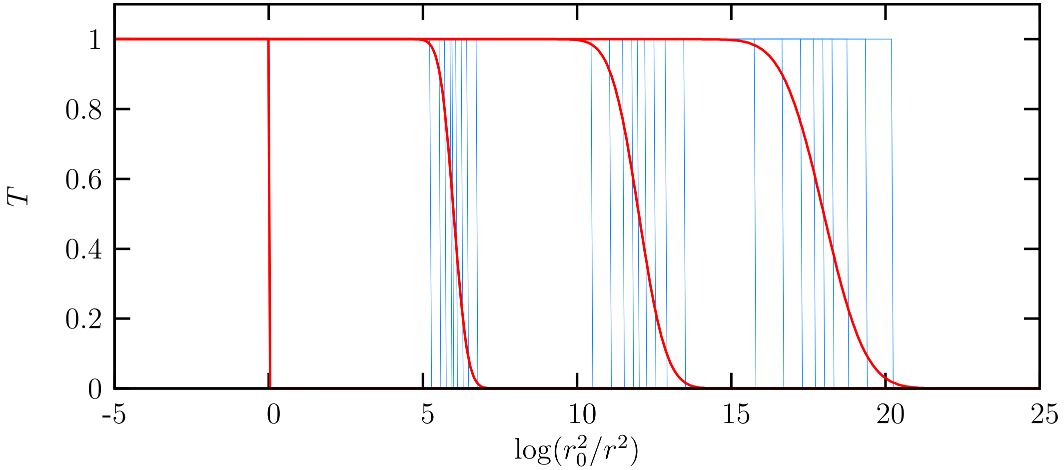
Case 1: Y not too large $\Rightarrow$ small dispersion $\Rightarrow$ Mean field picture $\langle T\rangle \approx T_{\text{event}}$
$\Rightarrow$ geometric scaling:

$$\langle T\rangle = f\left[\log(k^2/Q_s^2)\right]$$
$$\langle T^{(k)}\rangle = \langle T\rangle^k$$





[E. Iancu, Y. Hatta, C. Marquet, G.S., D. Triantafyllopoulos, 06]



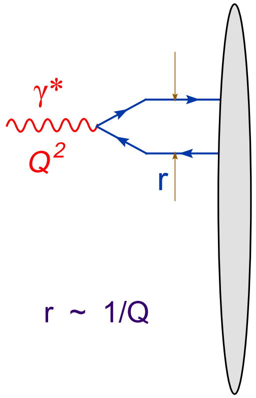
dispersion $\sim DY$

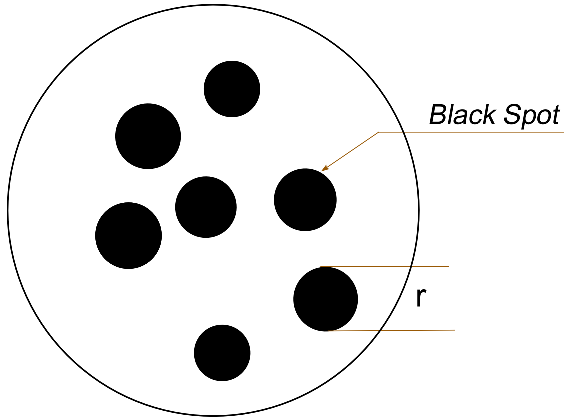
Case 2: Y higher energy $\Rightarrow$ dominated by disperion $\Rightarrow T = 0$ or $T = 1$

$\Rightarrow$ diffusive scaling:

$$\langle T \rangle = f\left[\log(k^2/Q_s^2)/\sqrt{DY}\right]$$

$$\langle T^{(k)} \rangle = \langle T \rangle$$





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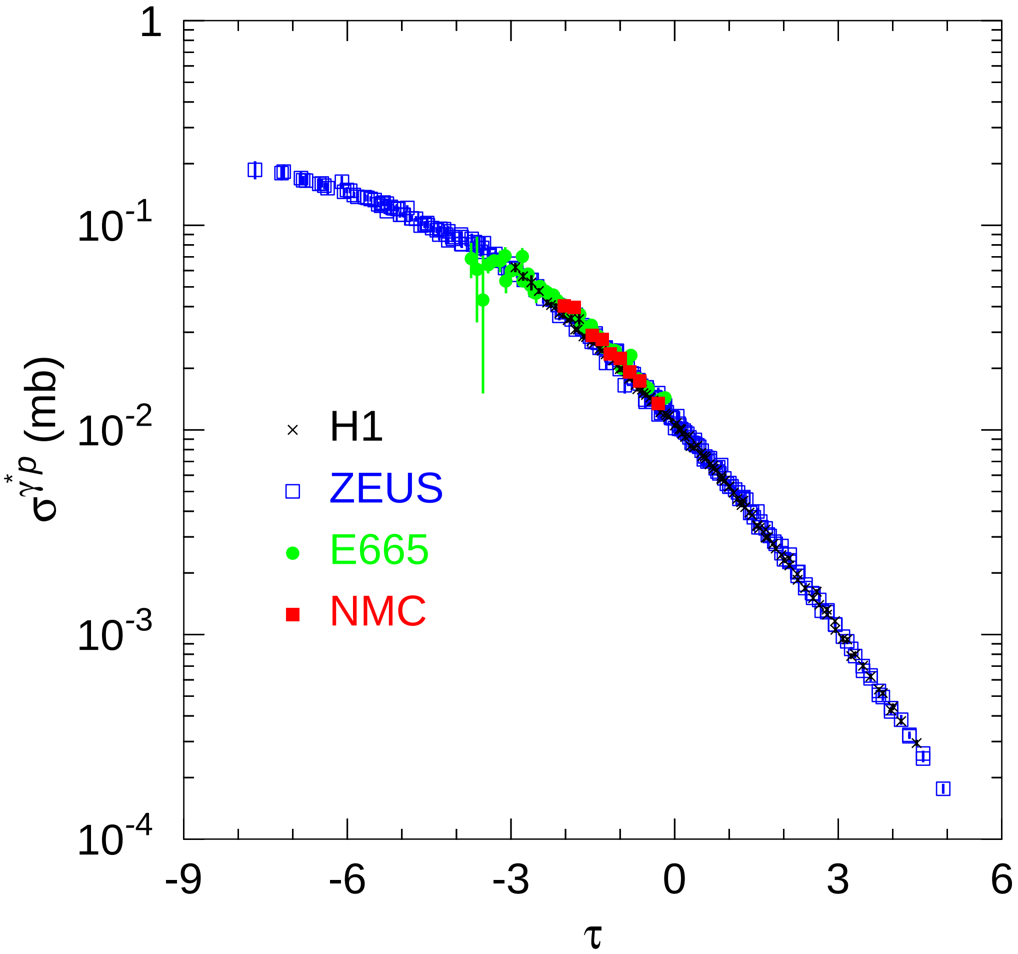
Phenomenology

Geometric scaling in F_2

Observation



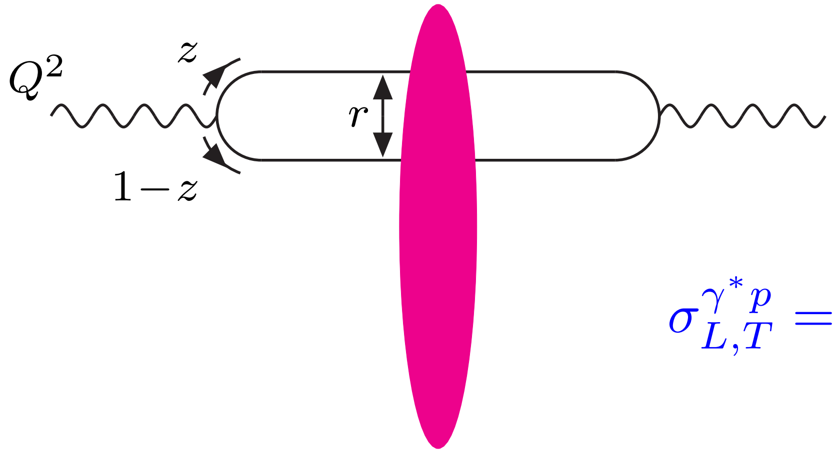
[A.Stasto, K.Golec-Biernat, 01]

$$\sigma^{\gamma^* p}(Q^2, x) = \sigma(\tau)$$
$$\tau = \log(Q^2) - \lambda Y$$
$$\lambda \approx 0.32$$

[F.Gelis, R.Peschanski, L.Shoeffel, G.S., hep-ph/0610436]

$\tau = \log(Q^2) - \lambda Y$ or $\tau = \log(Q^2) - \lambda\sqrt{Y}$

Factorisation formula: [E.Iancu, K.Itakura, S.Munier, 03]



$$\sigma_{L,T}^{\gamma^* p} = \int d^2r \int_0^1 dz\, \left|\Psi_{L,T}(z,r;Q^2)\right|^2\, 2\pi R_p^2 T(\mathbf{r};Y)$$

- $\Psi_{L,T} \equiv$ photon wavefunction from QED
- dipole amplitude: scaling variable $\tau = \log(r^2 Q_s^2/4)$

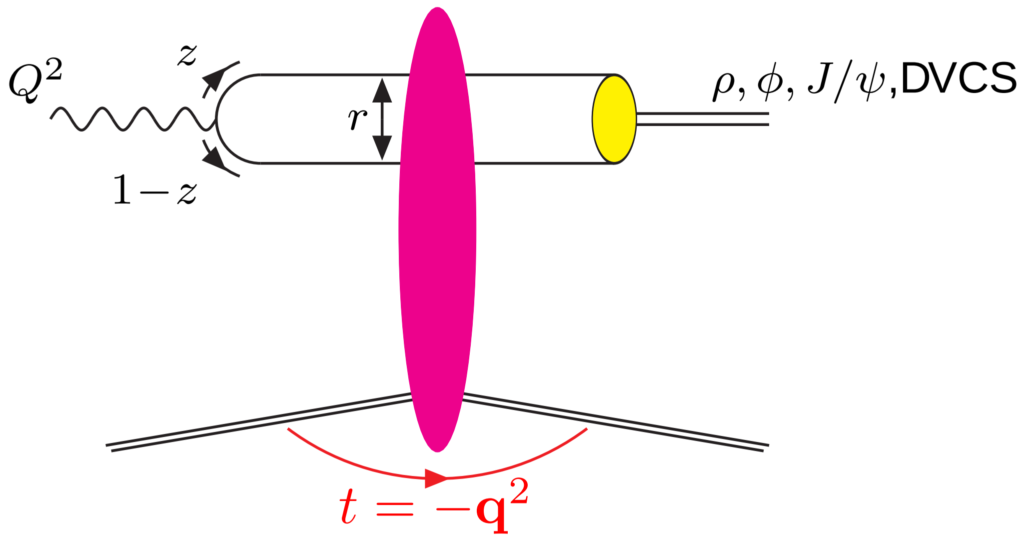
$$T(r;Y) = \begin{cases} T_0 \exp\left(\gamma_c \tau - \frac{\tau^2}{2\bar{\alpha}\chi_c'' Y}\right) & \text{if } rQ_s < 2 \quad \text{(travelling wave)} \\ 1 - \exp[-a(\tau+b)^2] & \text{if } rQ_s > 2 \quad \text{(McLerran-Venugopalan)} \end{cases}$$

$Q_s^2(Y) = k_0^2 \exp(\lambda Y) \quad \Rightarrow \lambda \approx 0.25$, in agreement with NLO BFKL predictions.

Phenomenology

Geometric scaling in vector-meson production

[C.Marquet, R.Peschanski, G.S., to appear]



Factorisation formula:

$$\mathcal{A}_{L,T}^{\gamma^* p \to Vp} = i \int d^2r \int_0^1 dz\, \Psi_{L,T}(z,r;Q^2) \Psi_{L,T}^*(z,r,q:M_V^2)\, e^{iz\mathbf{q}.\mathbf{r}}\, \sigma_{\text{dip}}(\mathbf{r},\mathbf{q};Y)$$

$\rightarrow \frac{d\sigma}{dt}$, σ_{el} for $\rho, \phi, J/\psi$,DVCS

- photon wavefunction: from QED
  Vector-mesons wavefunction: Boosted-Gaussian model
- dipole amplitude:

$$\sigma_{\text{dip}}(r, q; Y) = 2\pi R_p^2 e^{-b|t|}\, T_{\text{IIM}}\left(r, Q_s^2(q, Y)\right)$$

  - Normalisation: only one slope b (no Q^2 dependence)
  - T-matrix: t-dependent saturation scale from theoretical predictions:

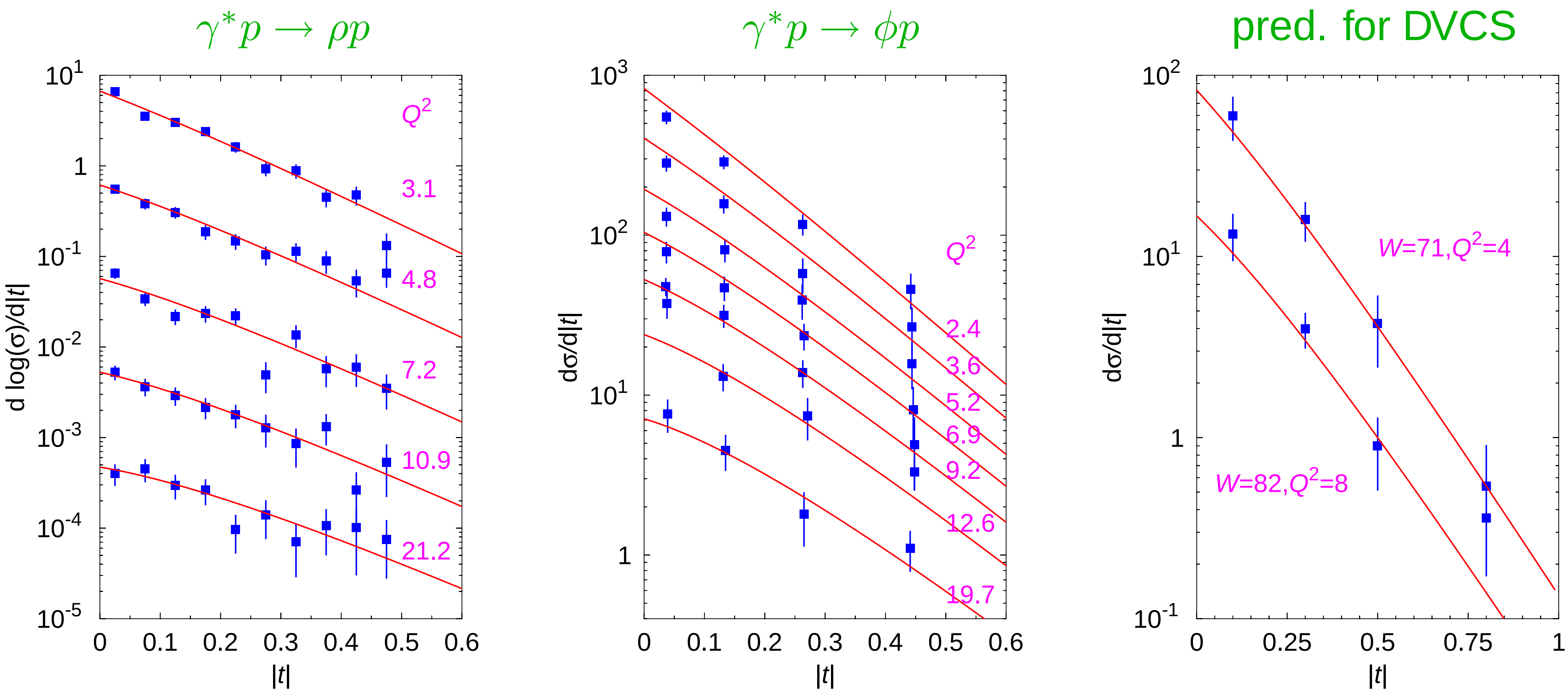
$$Q_s^2 = k_0^2\, e^{\lambda Y} \qquad \longrightarrow \qquad Q_s^2 = k_0^2\, (1 + c|t|)\, e^{\lambda Y}$$

Hence:

$$b,\ c \qquad \longrightarrow \qquad \left.\frac{d\sigma}{dt},\ \sigma_{\text{el}}\right|_{\rho,\phi}$$

(201 data)

Example: differential cross-section:



Future phenomenology

Diffusive scaling at HERA and LHC

Motivations

[Y.Hatta,E.Iancu,C.Marquet,G.S.,D.Triantafyllopoulos,06]

We have seen that, at high-energy,

$$\langle T(r,Y)\rangle = T\left(\frac{\log(r\bar{Q}_s)}{\sqrt{Y}}\right) = \frac{1}{2}\,\mathrm{erfc}\left(\frac{\log^2(r^2\bar{Q}_s^2)}{\sigma^2}\right)$$

with

$$\bar{Q}_s^2(Y) = k_0^2\, e^{\lambda Y} \qquad \text{and} \qquad \sigma^2 = DY$$

Note: λ and D (or Q_s and σ^2) taken as parameters

Consequences on

- DIS and diffractive DIS (DDIS)
- gluon/forward jet production

DIS and DDIS

- Total cross-section

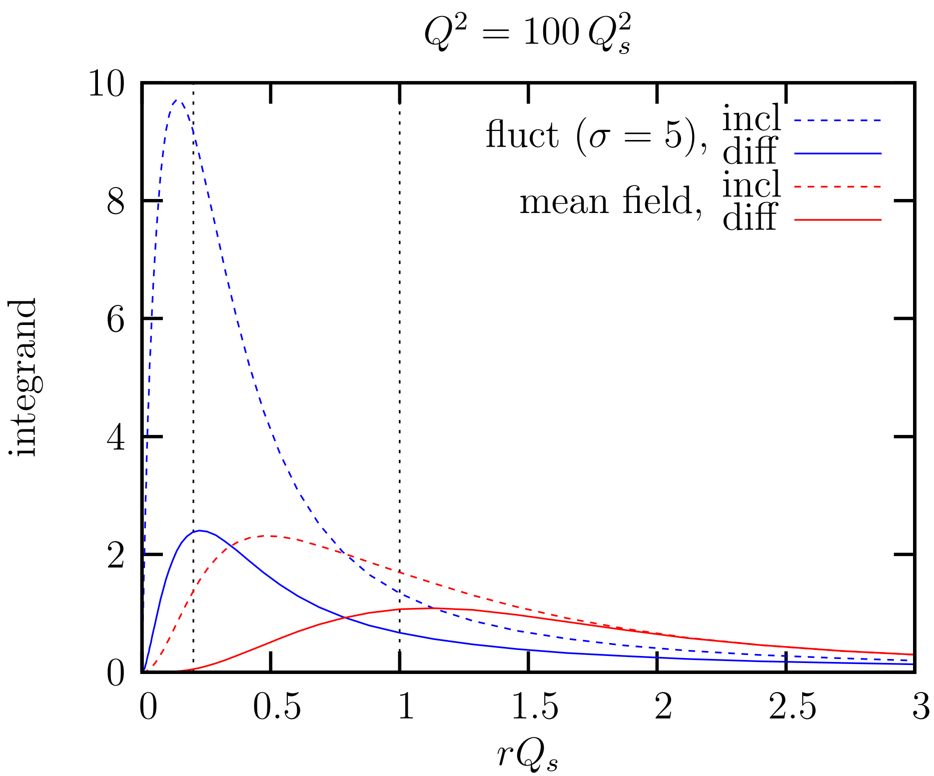
$$\sigma_{\text{DIS}} = \int dr\, \left|\Psi(r,Y;Q^2)\right|^2 \langle T(r,Y)\rangle$$
$$\to \text{cst.}\, \sigma\, \Phi_1\left(\frac{\log(r^2\bar{Q}_s^2)}{\sigma}\right)$$

- Diffractive cross-section

$$\sigma_{\text{DDIS}} = \int dr\, \left|\Psi(r,Y;Q^2)\right|^2 \langle T(r,Y)\rangle^2 + (q\bar{q}g) + \dots$$
$$\to \text{cst.}\, \sigma\, \Phi_2\left(\frac{\log(r^2\bar{Q}_s^2)}{\sigma}\right)$$

$\Rightarrow$ diffusive scaling for $\frac{1}{\sqrt{Y}}\sigma_{\text{DIS}}$ and $\frac{1}{\sqrt{Y}}\sigma_{\text{DDIS}}$

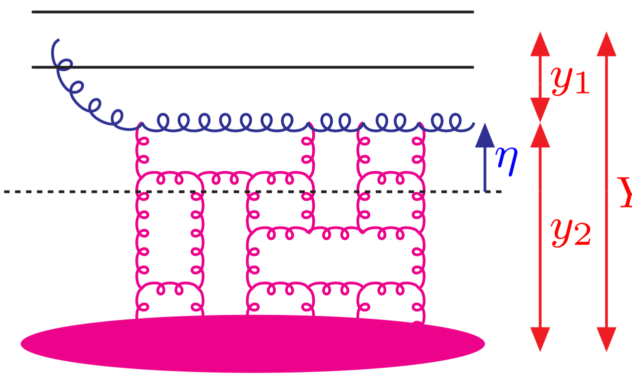
- Typical dipole scales in $|\Psi|^2 \otimes \langle T \rangle^{(1,2)}$:



	DIS	DDIS
saturation	$r \sim 1/Q$	$r \sim 1/Q_s$
fluctuations	$r \sim 1/Q$	$r \sim 1/Q$

- Diffraction dominated by elastic amplitudes dense-dilute scattering:

- dA or pp at forward rapidities
- dilute projectile $\to$ dipoles
- gluon at rapidity η

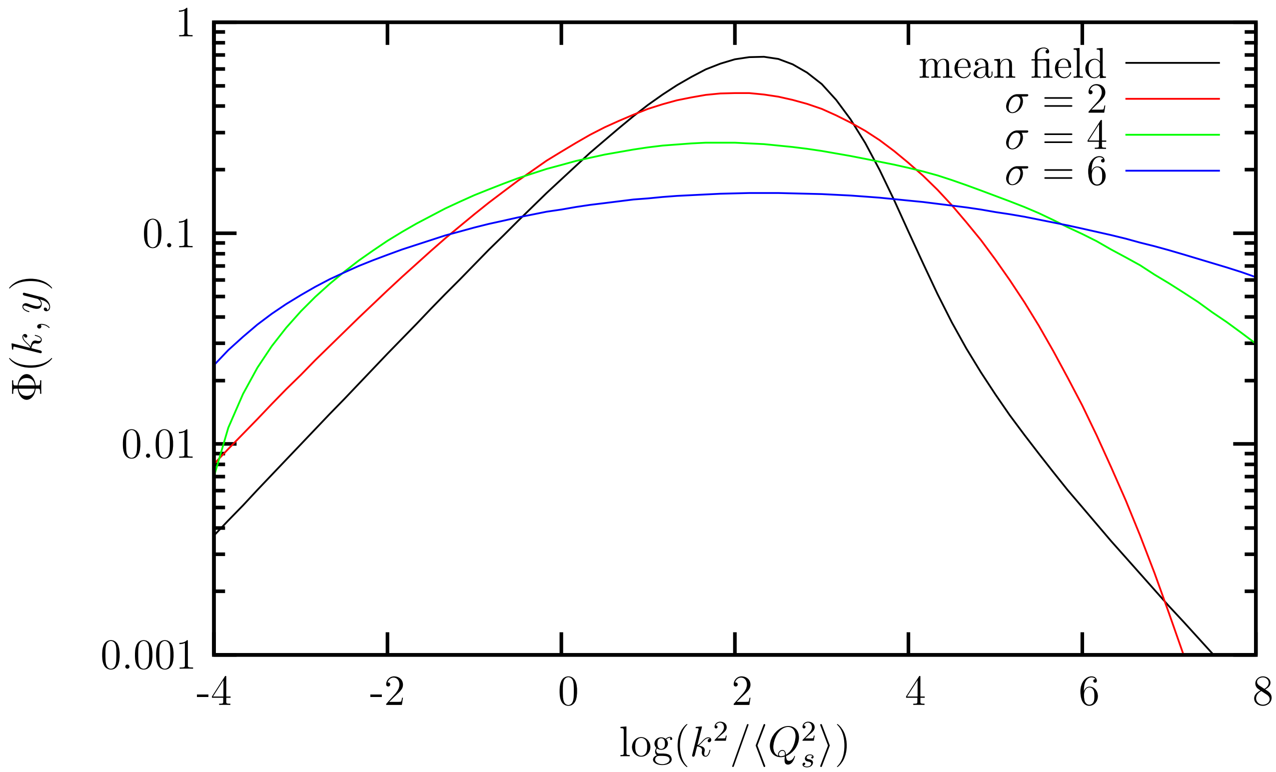


$$\frac{d\sigma}{d\eta\, d^2k\, d^2b} = \frac{\bar\alpha}{k^2}\int \frac{d^2p}{(2\pi)^2}\,\phi(\mathbf{p},y_1)\,\Phi(\mathbf{k}-\mathbf{p},y_2)$$

Projectile unintegrated gluon density $\phi(\mathbf{p},y_1) = \int \frac{d^2r}{2\pi}\, e^{i\mathbf{p}.\mathbf{r}}\, n(\mathbf{r},y_1)$

Target contribution $\Phi(\mathbf{k},y_2) = \int d^2r\, e^{i\mathbf{k}.\mathbf{r}}\, \nabla^2_{\mathbf{r}}\langle 2T(r,y_2) - T^2(r,y_2)\rangle$

[E.Iancu,C.Marquet,G.S.,06]



$$\Phi(k,Y) \to \frac{1}{\sigma} \exp\left[\frac{\log^2(k^2/\bar{Q}_s^2)}{\sigma^2}\right] \qquad \Rightarrow \text{ diffusive scaling for } \sqrt{Y}\,\Phi(k,Y)$$

Conclusion

Part 1: Evolution equations towards high-energy

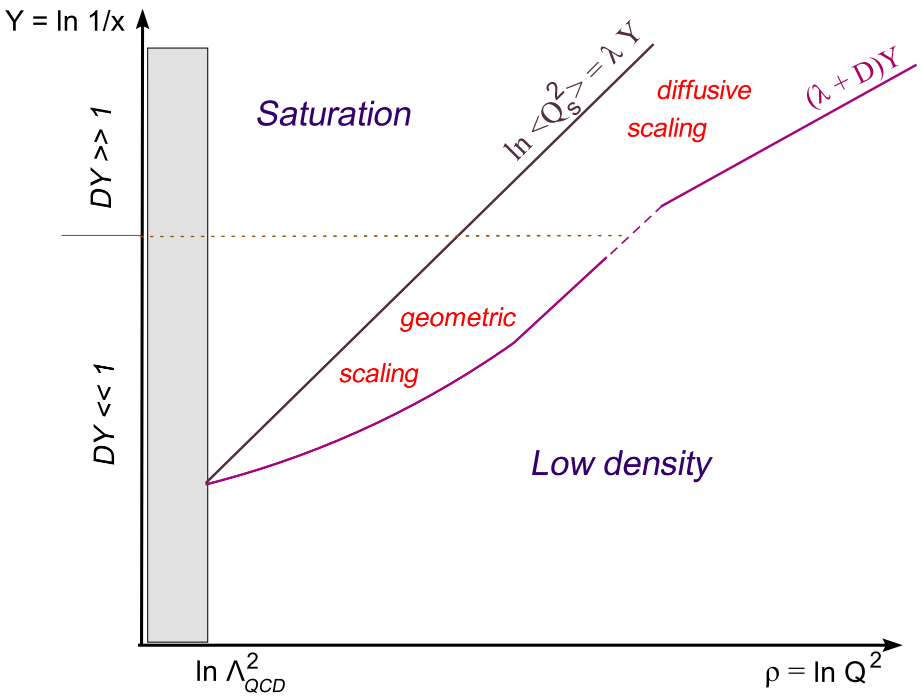
Infinite hierarchy:

contribution	$\partial_Y \langle T^k \rangle$	importance	diagrams
BFKL	$\langle T^k \rangle$	resums $\alpha_s^n \log^n(s)$	ladders
saturation	$\langle T^{k+1} \rangle$	near unitarity: $T \lesssim 1$	fan
fluctuations	$\langle T^{k-1} \rangle$	dilute tail: $T \gtrsim \alpha_s^2$	splittings & loops

Perspectives:

- beyond 2 gluon-exchange approximation
  ([J.T.Amaral, E.Iancu, G.S., D.Triantfyllopoulos, hep-ph/0611105])
- beyong large-N_c approximation

Part 2: Solutions for scattering amplitudes



Geometric scaling

$$T = T(rQ_s)$$

$$Q_s = \exp(\lambda Y)$$

Diffusive scaling

$$T = T[\log(rQ_s)/\sigma]$$

$$Q_s = \exp(\lambda Y),\ \sigma^2 = DY$$

General predictions of saturation

even when $T \ll 1$

Note: Knowledge of preasympt.

Perspectives:

- Better analytic control of the fluctuation effects
- include impact-parameter dependence

Conclusion

Part 3: Phenomenological consequences

- HERA:
  - geometric scaling for F_2, DVCS and VM-production
    $\Rightarrow$ indications for saturation
  - diffusive scaling for F_2 and F_2^D at higher energy
- LHC:
  - diffusive scaling predicted for dense-dilute collisions (dA or forward pp)

Perspectives:

- Control of the interplay between geometric and diffusive scaling (HERA ?)
- More predictions for LHC
- Applications to dense-dense collisions

Conclusion

- generic scaling laws from high-energy QCD
- interesting links with statistical physics
- hints from HERA and TEVATRON

→ LHC