Jet finding at the LHC era

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arXiv:0704:0292, arXiv:0802:1188, arXiv:0802:1189,

arXiv:0803.0678 + works in preparation

Plan

- Foreword: why jets? what are they?

  introducing the basic concepts

- Part 0: recent progress: building a solid toolkit

  jet definitions meeting the fundamental requirements

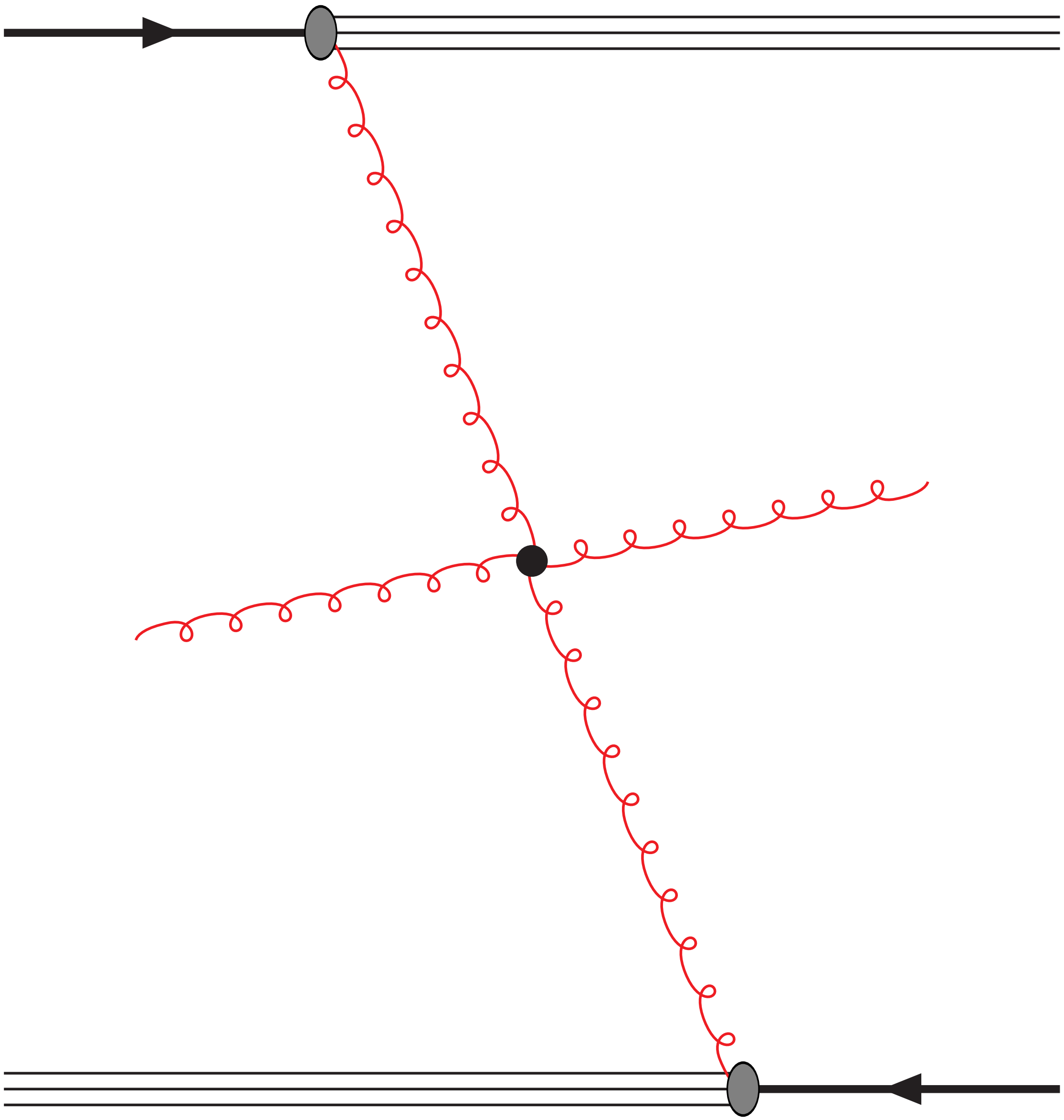
- Part 1: jets in pp collisions
  - Choosing the adapted jet definition

    which jet algorithm is best suited?
  - Subtracting pileup background using jet areas
    - defining areas
    - analytic control
    - using them for pileup subtraction

- Part 2: jets in heavy-ion collisions
  - subtraction subtleties
  - preliminary results

Foreword: why jets? what are they?

General (over)simplified picture



Hard scattering $(2 \to n)$

computed exactly at $\mathcal{O}(\alpha_s^p)$

$gg \to gg$, $gg \to ggg$,

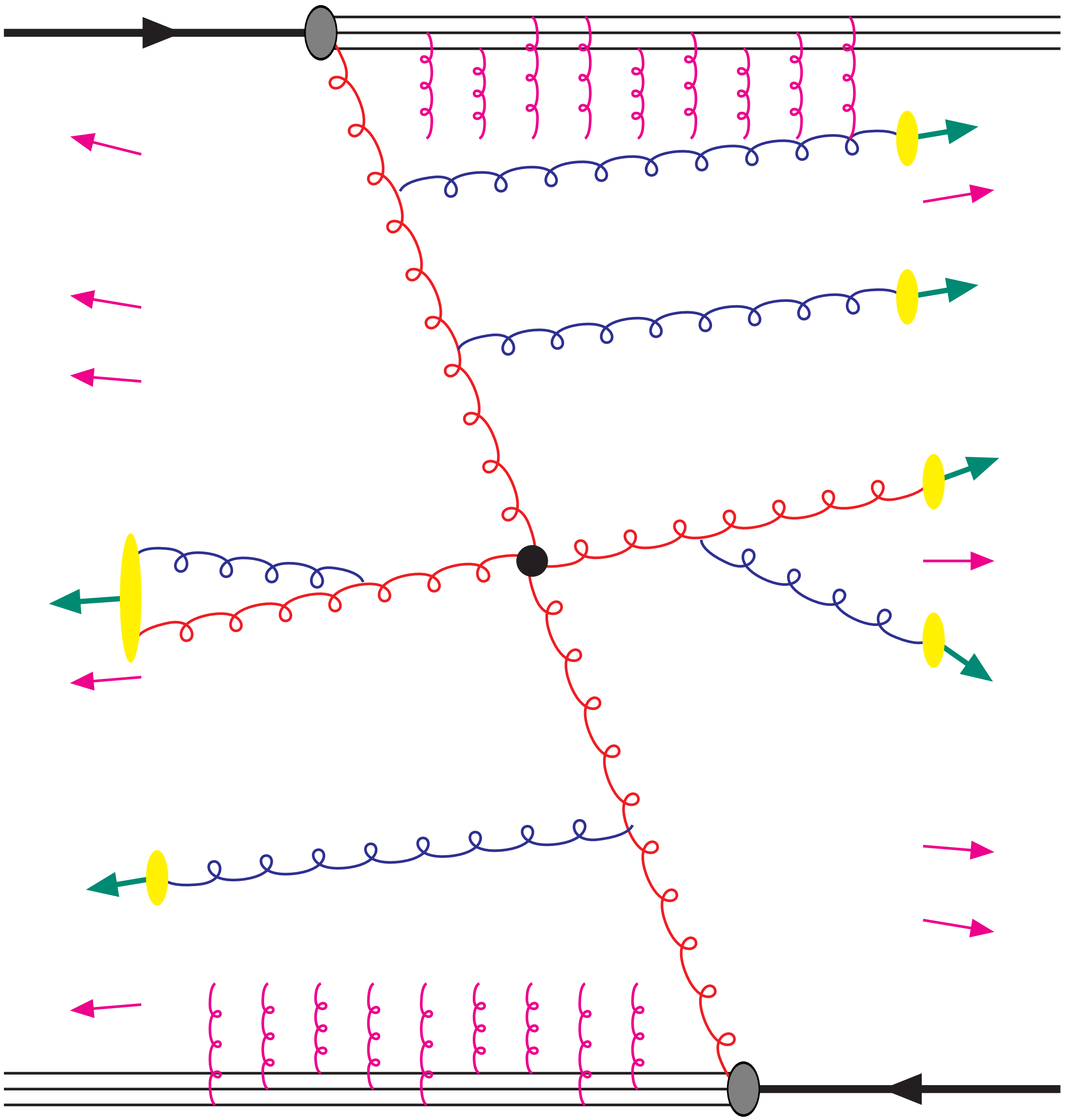
$gg \to gggg$,

$gg \to H \to b\bar{b}$,

$gg \to t\bar{t} \to \mu\nu_\mu b\bar{b}q\bar{q}$,

$gg \to Z' \to q\bar{q}$, ...

General (over)simplified picture



Hard scattering ($2 \to n$)

Parton level

$\approx$ resummed collinear div.

$\sum_i \alpha_s^i \log^i(p_t^2/\mu^2)$

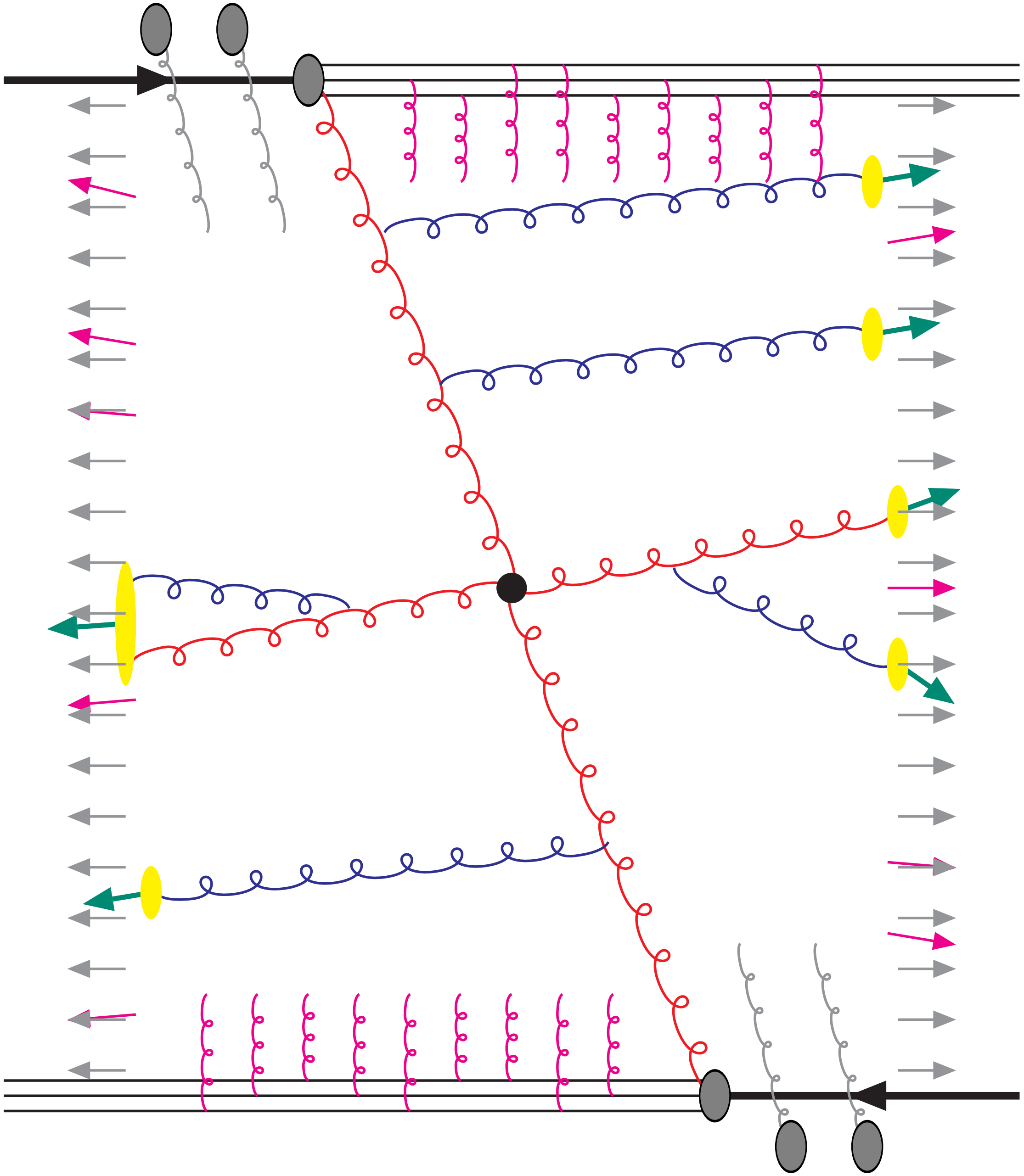
Hadron level: hadronisation

Underlying event

beam remnants interactions

$\Rightarrow$ soft background

General (over)simplified picture



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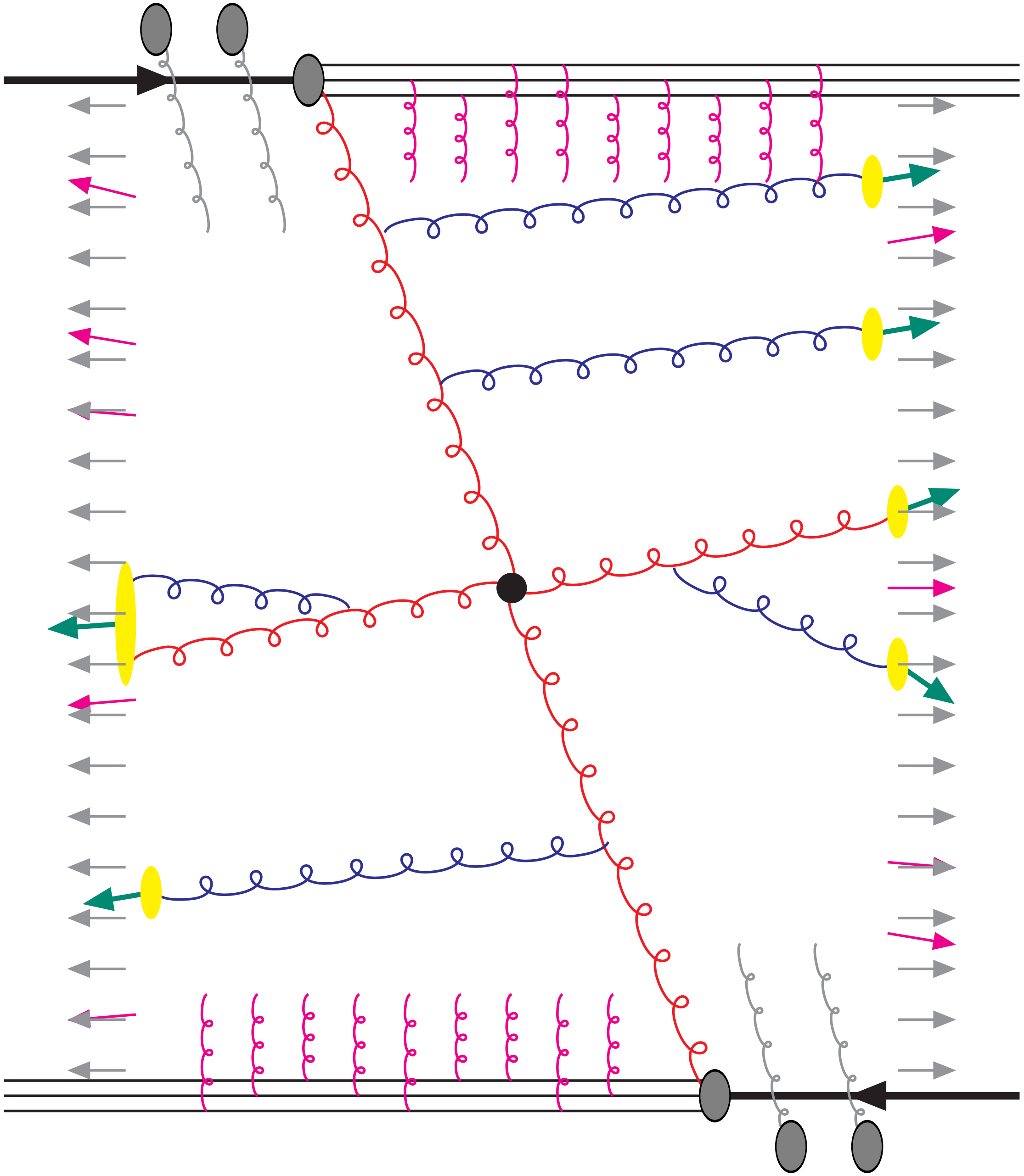

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"Jets" $\equiv$ hard partons

Parton ambiguous

$\Rightarrow$ multiple jet definitions

Two classes of algorithms

Class 1: recombination	Cass 2: cone
Successive recombinations of the "closest"$^{(a)}$ pair of particle	find directions of energy flow $\equiv$ stable cones$^{(b)}$
Nice perturbative behaviour	Small sensitivity to soft radiation (UE,PU)
Often used in $e^\pm e^\pm$, $e^\pm p$	Often used in pp

$^{(a)}$ Distance: (stop when $d_{\min} > R$)

$$k_t: \quad d_{i,j} = \text{min}(k_{t,i}^2, k_{t,j}^2)(\Delta\phi_{i,j}^2 + \Delta y_{i,j}^2)$$

$$\text{Aachen/Cam.:} \quad d_{i,j} = \Delta\phi_{i,j}^2 + \Delta y_{i,j}^2$$

$^{(b)}$ **stable cones** (radius R) such that:
the total momentum of its contents points in the direction of its centre

How the cone works...

- Seeded (iterative) approaches: iterate from an initial position until stable
  - seed = initial particle
  - seed = midpoint between stable cones found at first step
- One has to deal with overlapping stable cones: 2 subclasses

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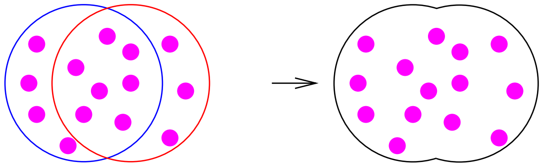
Class 2(a): cone with split-merge (ex.: JetClu, Atlas, MidPoint):

$\tilde{p}_{t,\text{shared}} > f\,\tilde{p}_{t,\text{min}}$



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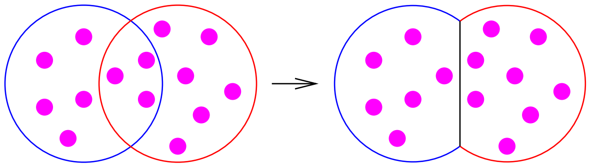

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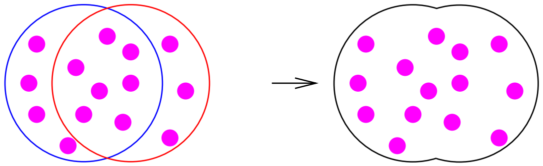

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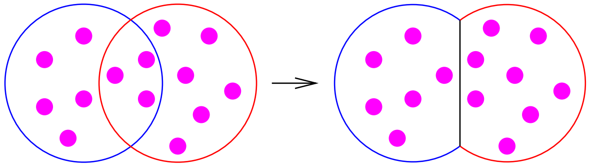


Class 2(b): cone with progressive removal (ex.: Iterative Cone)

- iterate from the hardest seed
- remove the stable cone as a jet and start again

Idea: “regular/circular” jets

20th century jet finders

Recombination:

- k_t algorithm
- Cambridge/Aachen alg.

Cone:

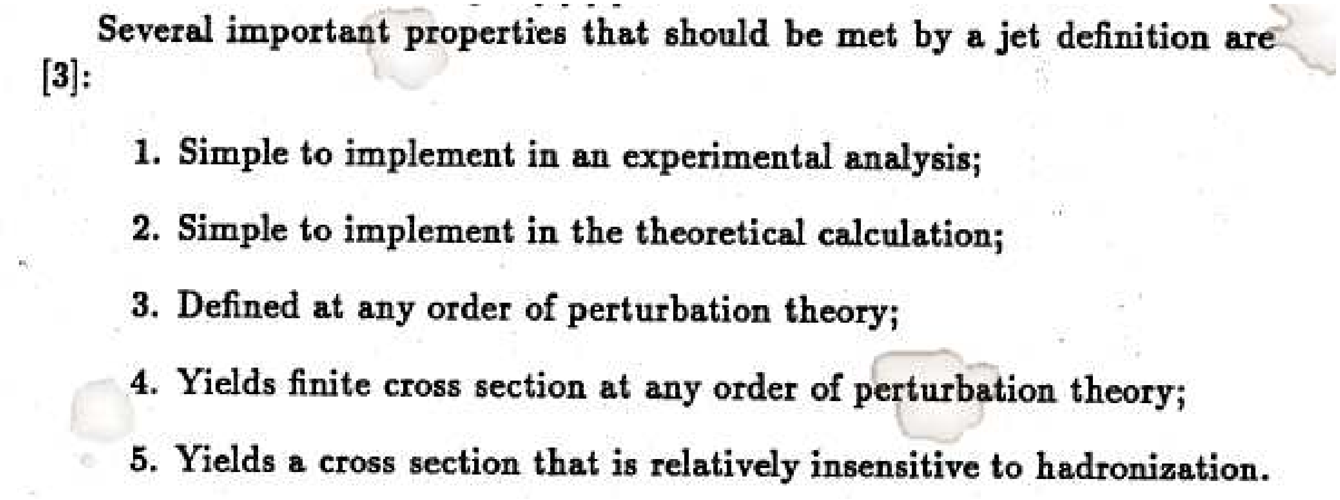
- CDF JetClu
- CDF MidPoint
- D0 (run II) Cone
- PxCone
- ATLAS Cone
- CMS Iterative Cone
- PyCell/CellJet
- GetJet

Part 0

21st century: towards a solid toolkit

1990: fixing the rules

SNOWMASS accords, Tevatron 1990 (i.e. old!):



Several important properties that should be met by a jet definition are [3]:

1. Simple to implement in an experimental analysis;
2. Simple to implement in the theoretical calculation;
3. Defined at any order of perturbation theory;
4. Yields finite cross section at any order of perturbation theory;
5. Yields a cross section that is relatively insensitive to hadronization.

i.e. usable by theoreticians (*e.g.* finite perturbative results)

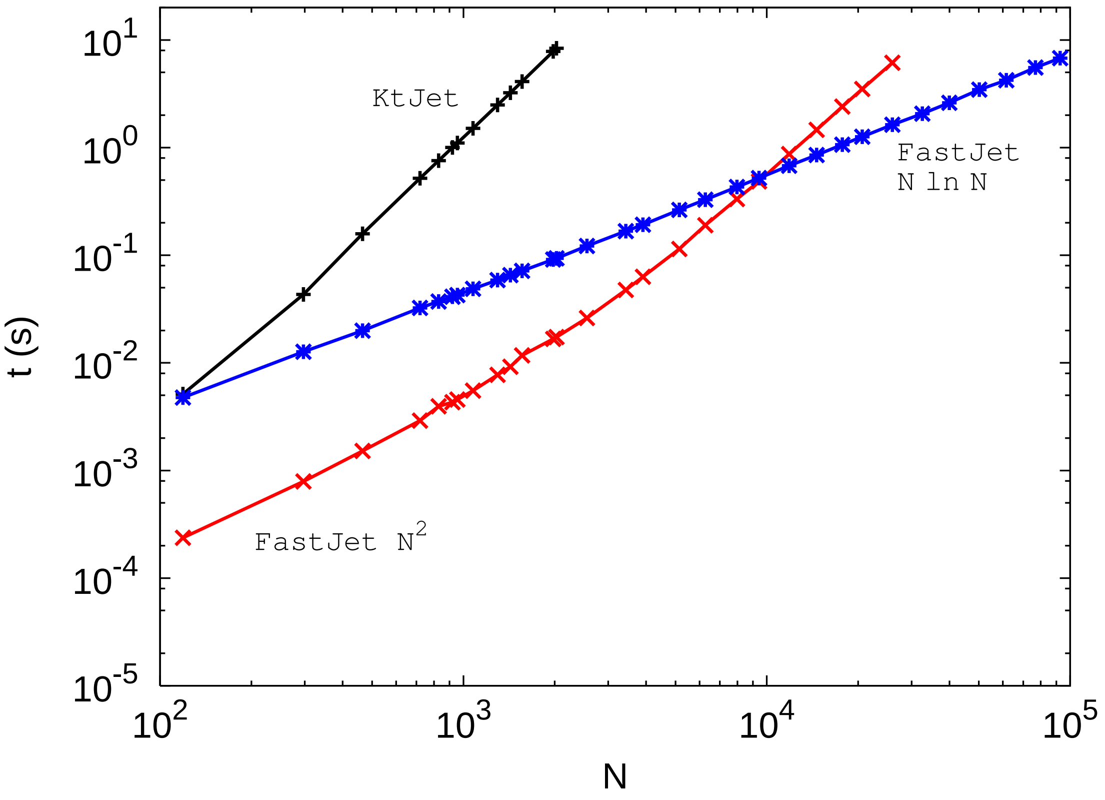
and experimentalists (*e.g.* fast enough, not much UE sensitivity)

Speed improvement

[M. Cacciari, G. Salam, 06]

Speeding up the k_t and Cam/Aachen algorithms

- using computational-geometry techniques: $\mathcal{O}\left(N^3\right) \rightarrow \mathcal{O}\left(N \log N\right)$
- `C++` implementation in `FastJet`

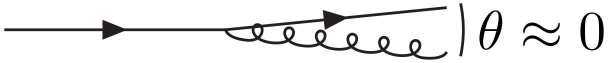


QCD divergences

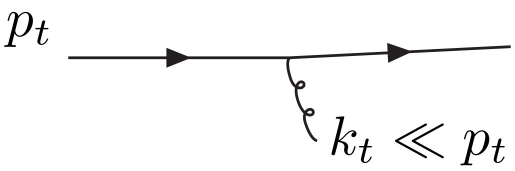
QCD probability for gluon bremsstrahlung at angle θ and $\perp$-mom. k_t:

$$dP \propto \alpha_s \frac{d\theta}{\theta} \frac{dk_t}{k_t}$$

Two divergences:



Collinear



Soft

QCD divergences

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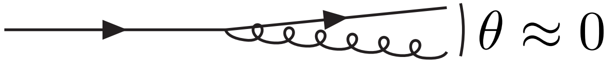

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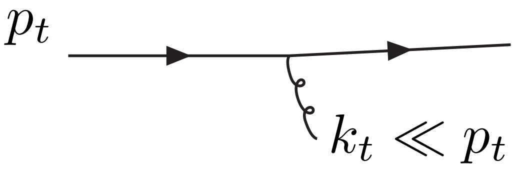
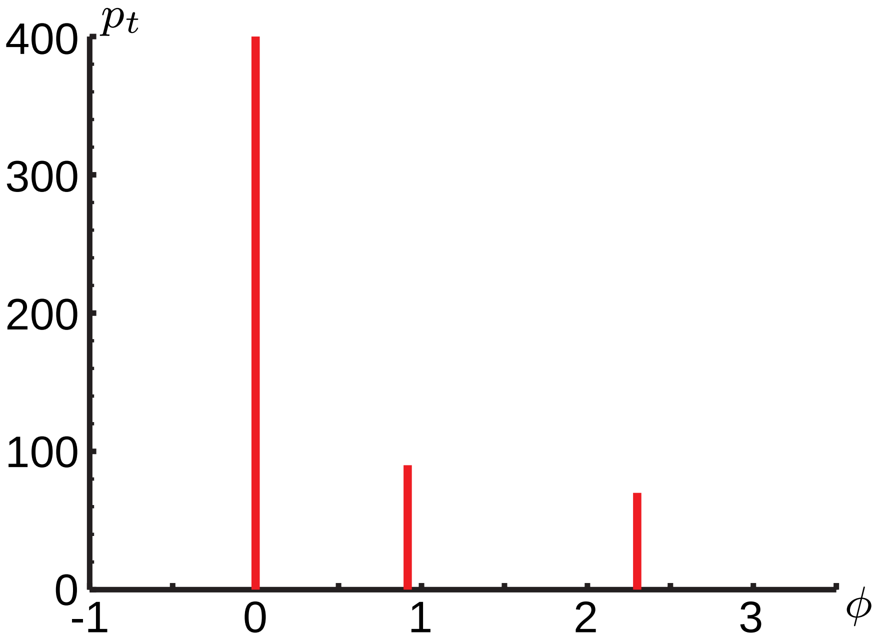


Soft

For pQCD to make sense, the (hard) jets should not change when

- one has a collinear splitting
  i.e. replaces one parton by two at the same place (η, ϕ)
- one has a soft emission *i.e.* adds a very soft gluon

IR unsafety of the Midpoint alg



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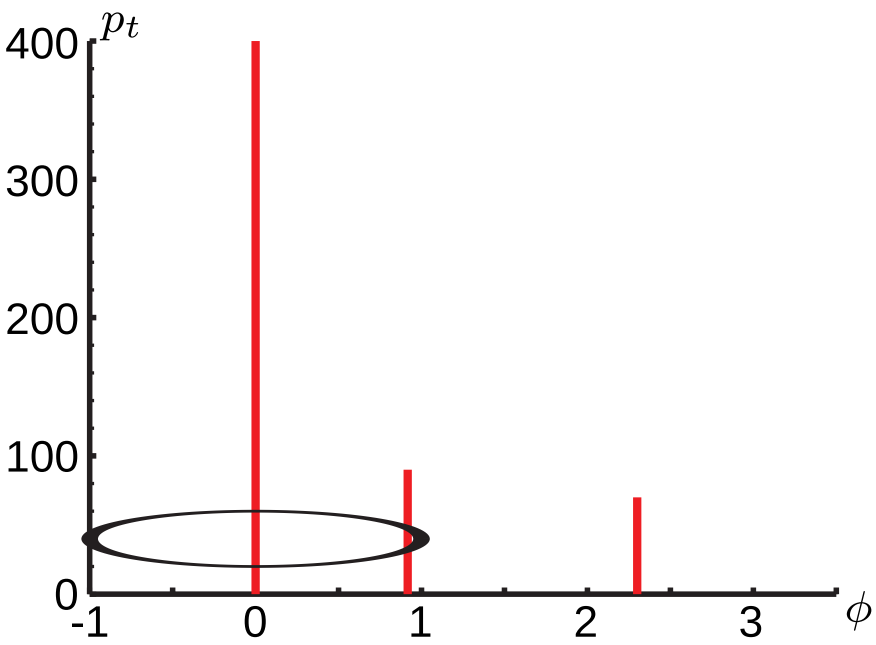

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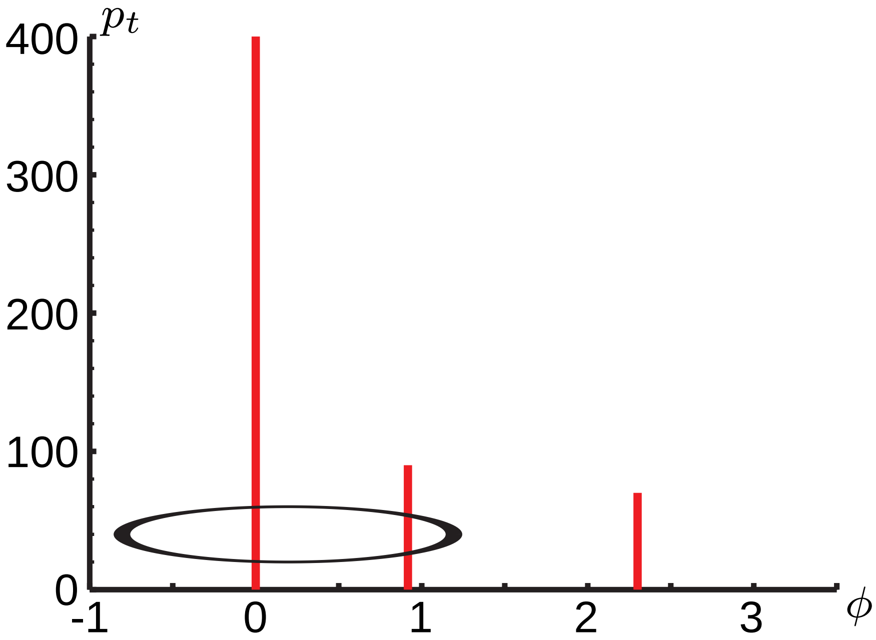

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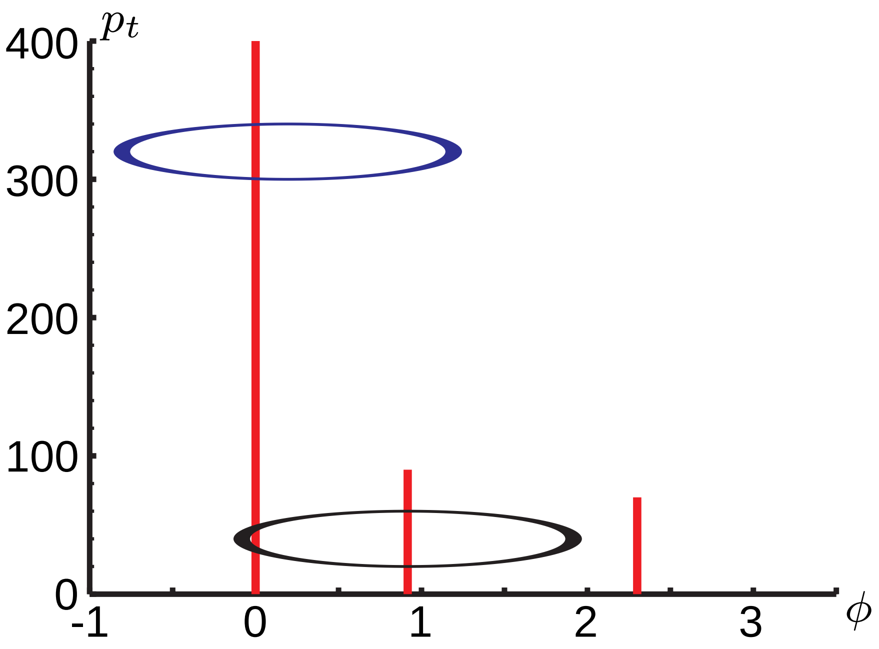

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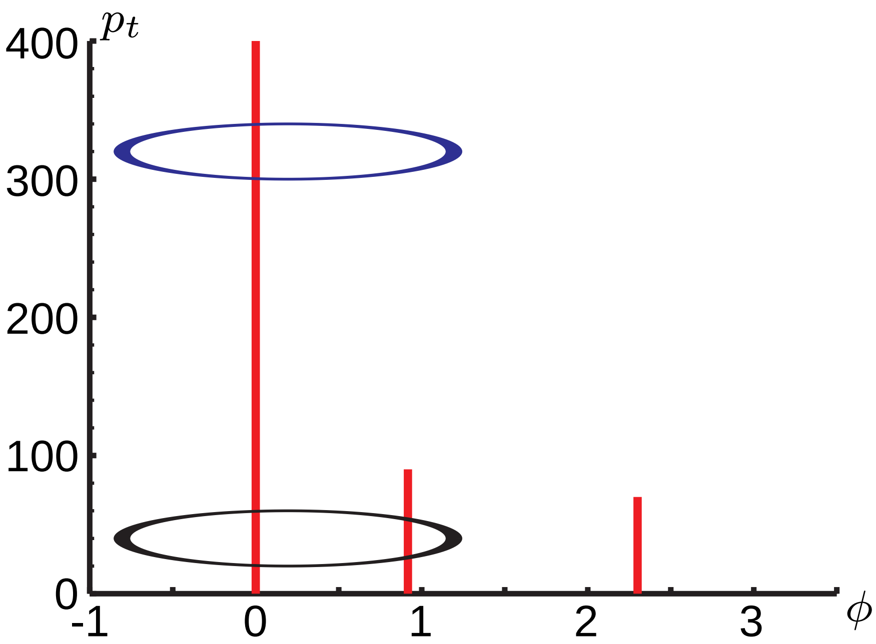

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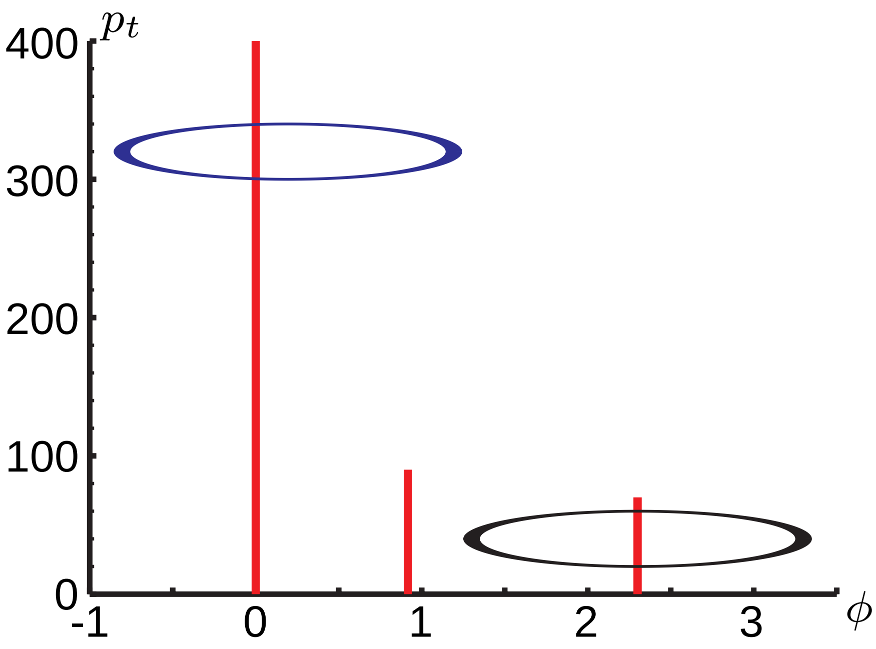

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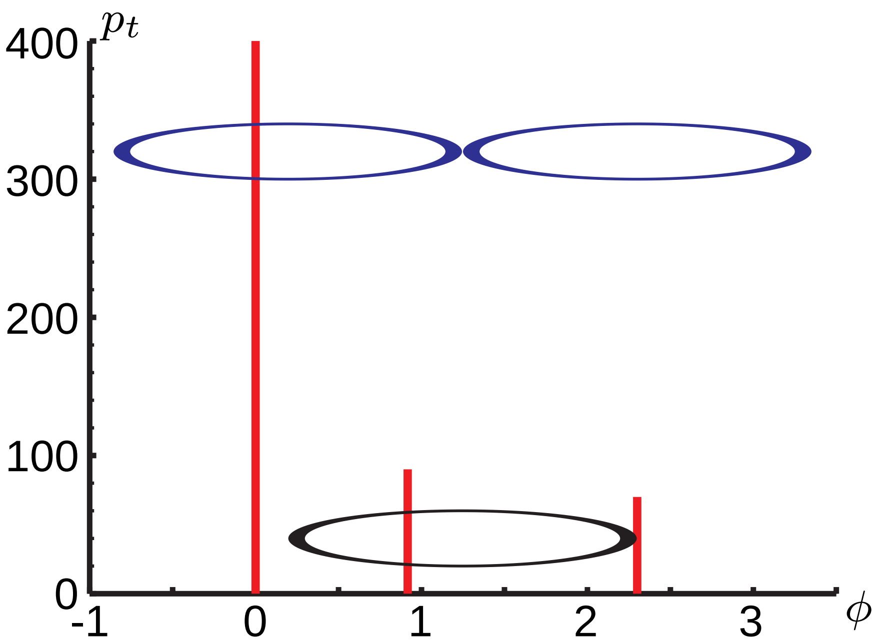

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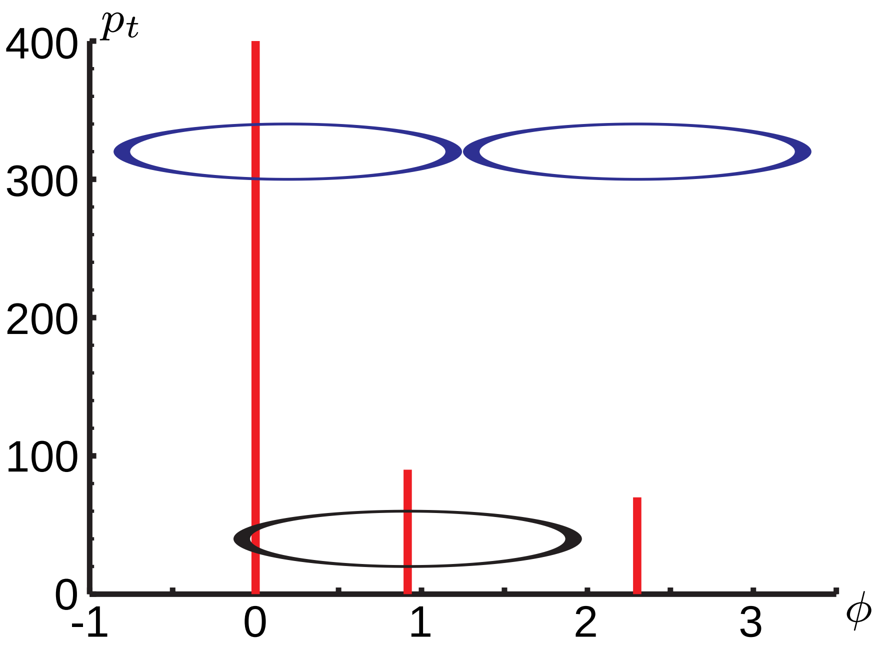

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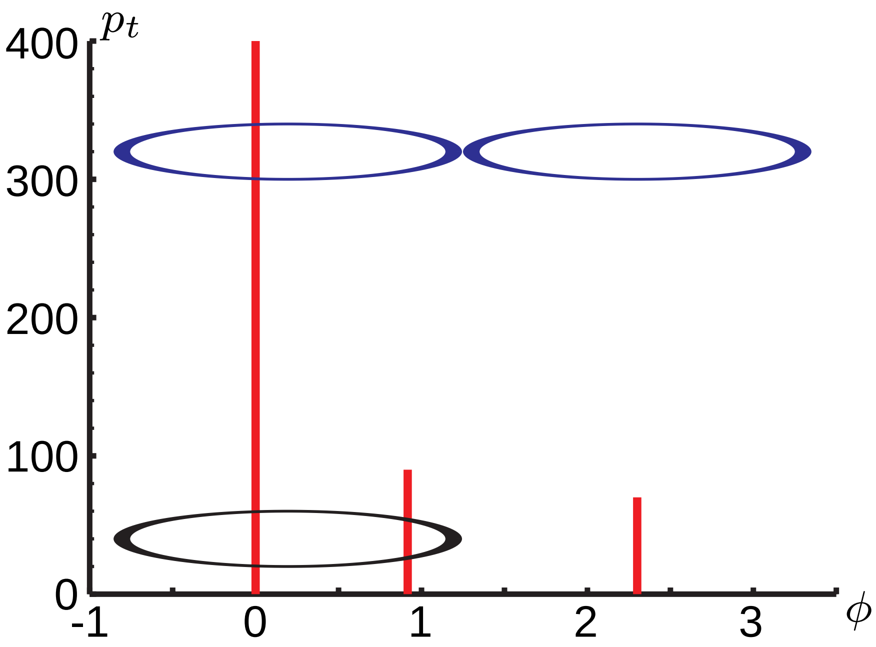

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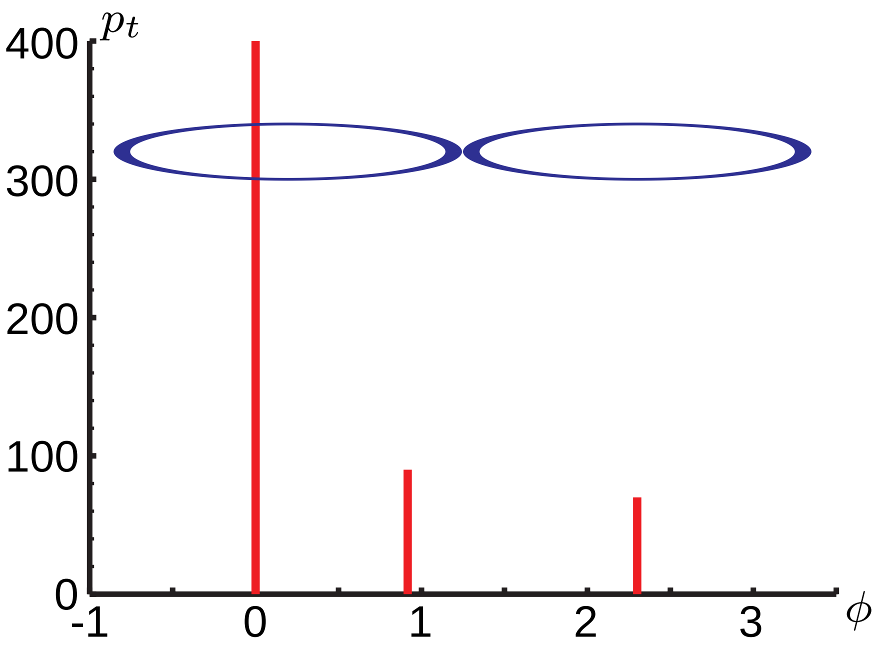

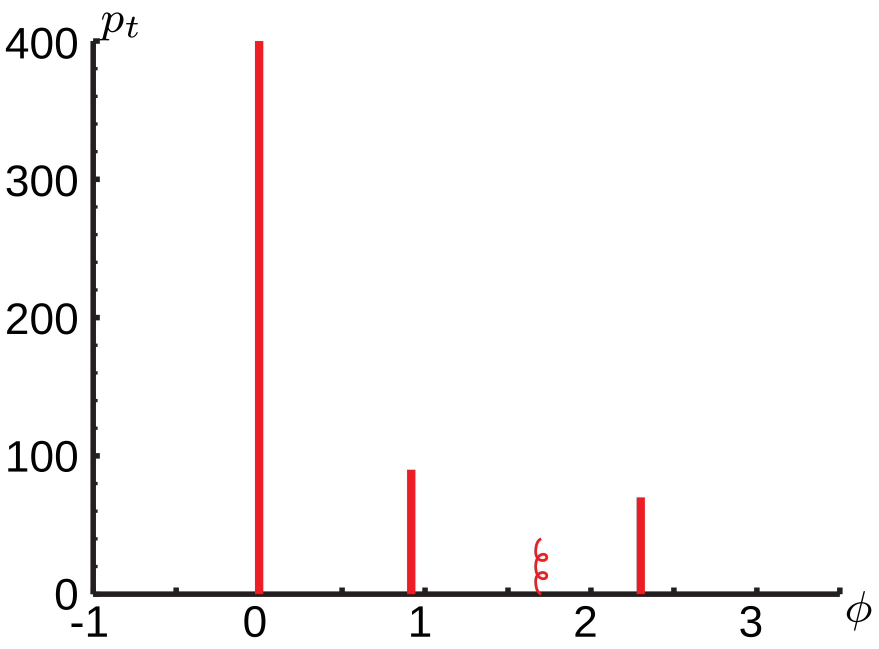

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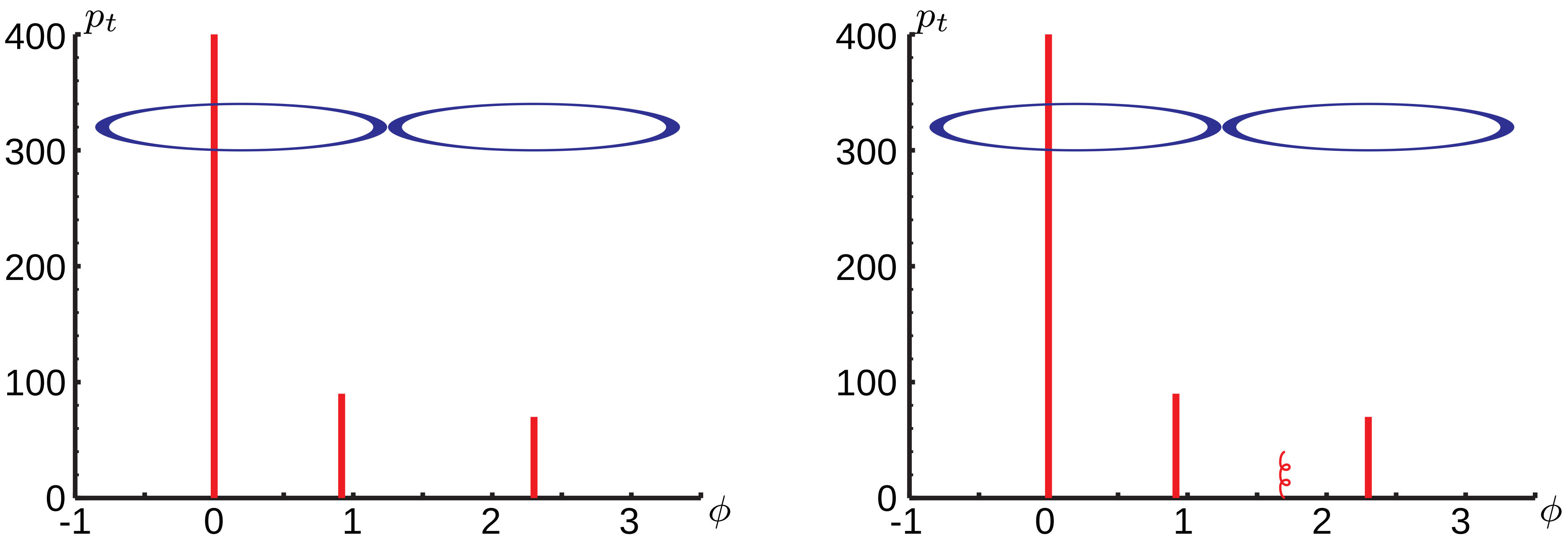

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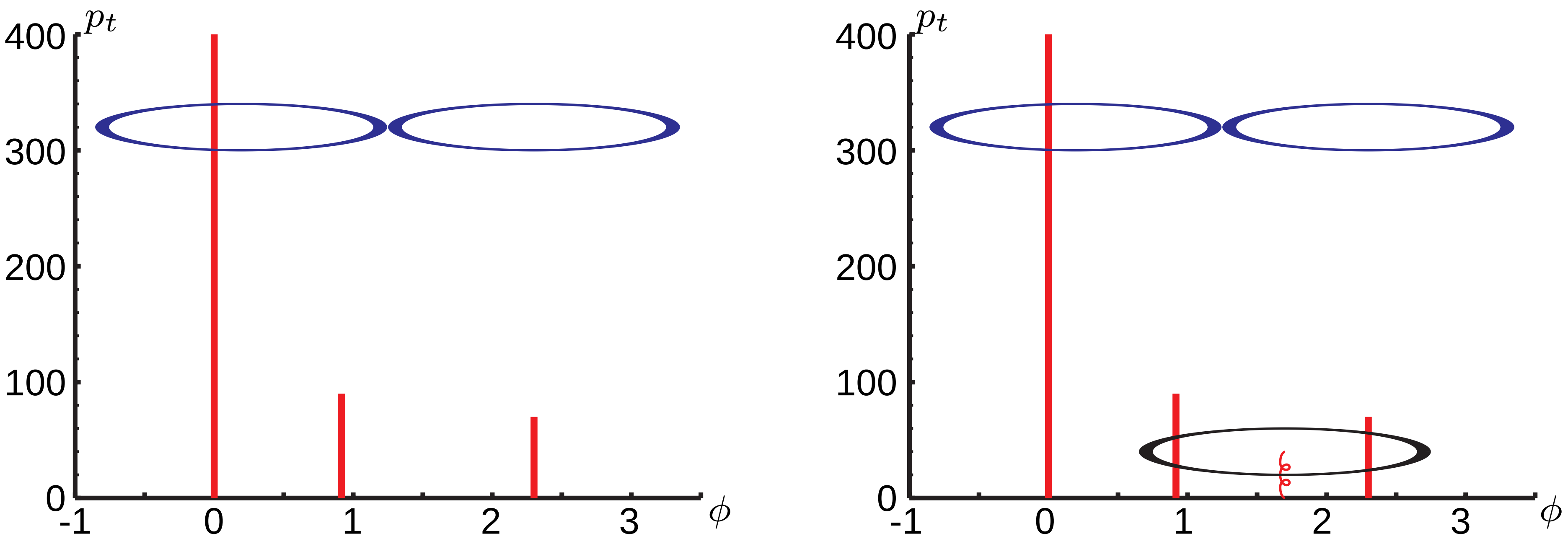

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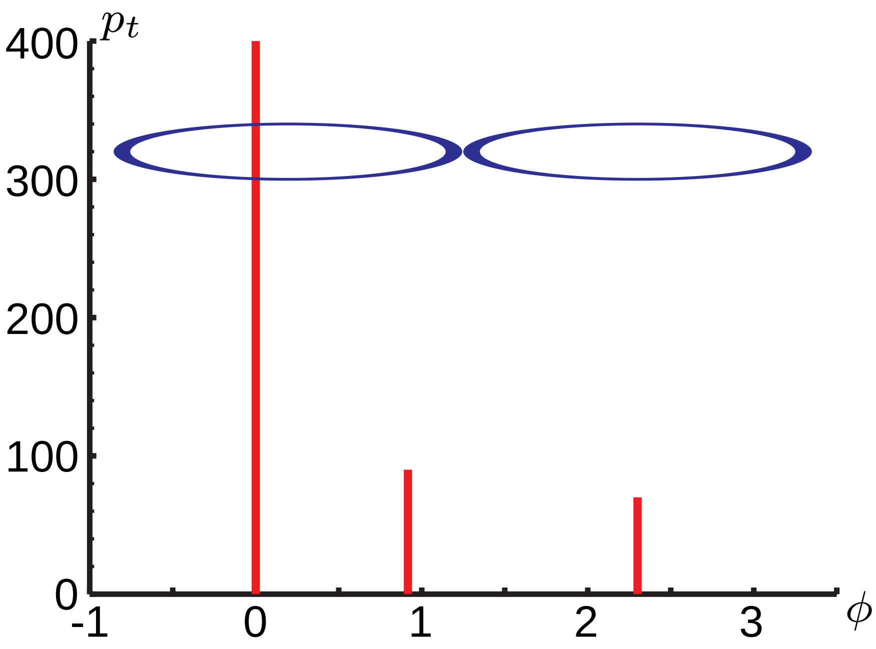

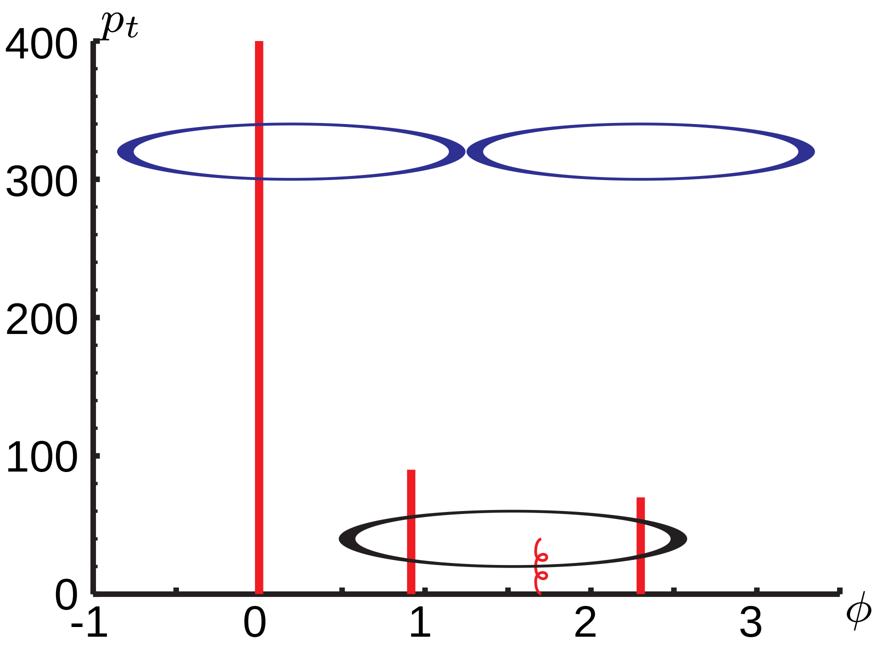

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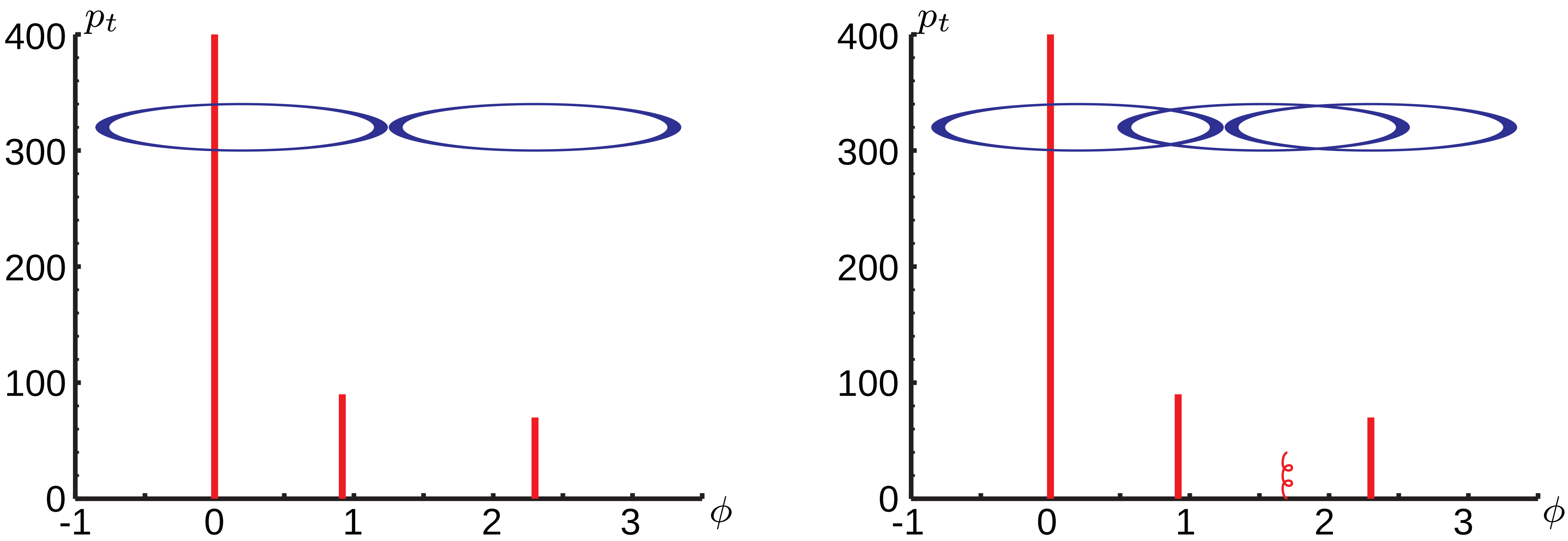

Stable cones:

Midpoint: {1,2} & {3} {1,2} & {3} & {2,3}

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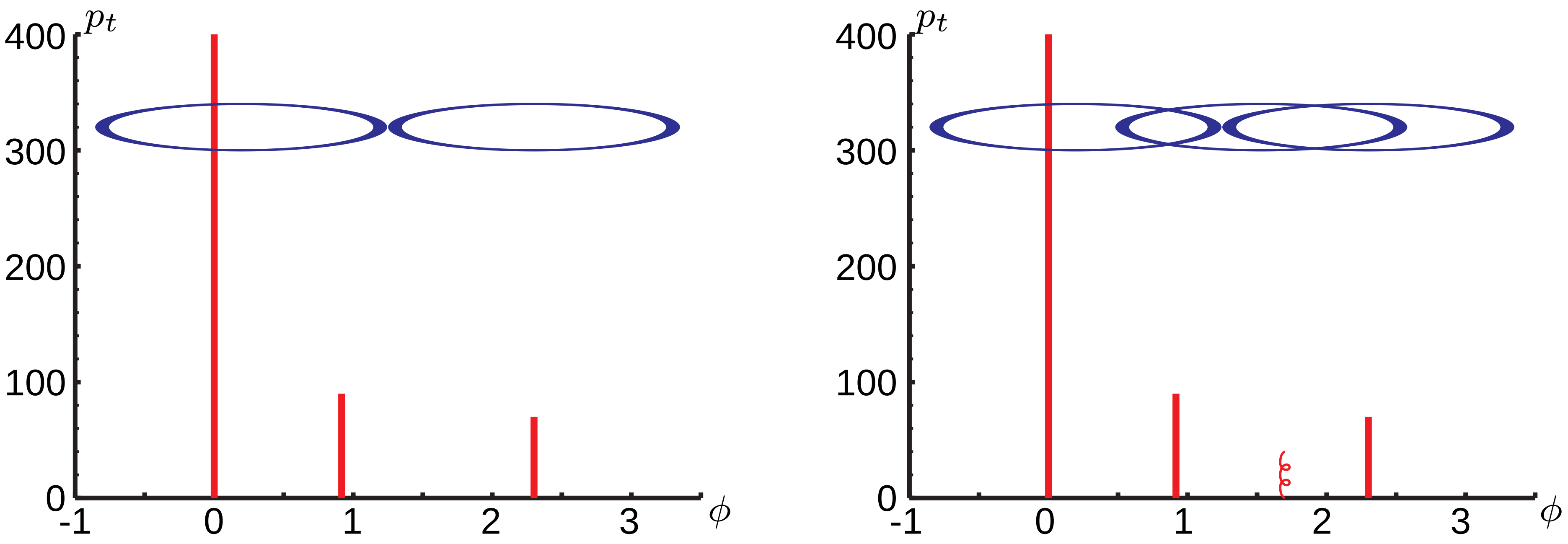

Stable cones:

Midpoint:	{1,2} & {3}	{1,2} & {3} & {2,3}

Jets: $(f = 0.5)$

Midpoint:	{1,2} & {3}	{1,2,3}

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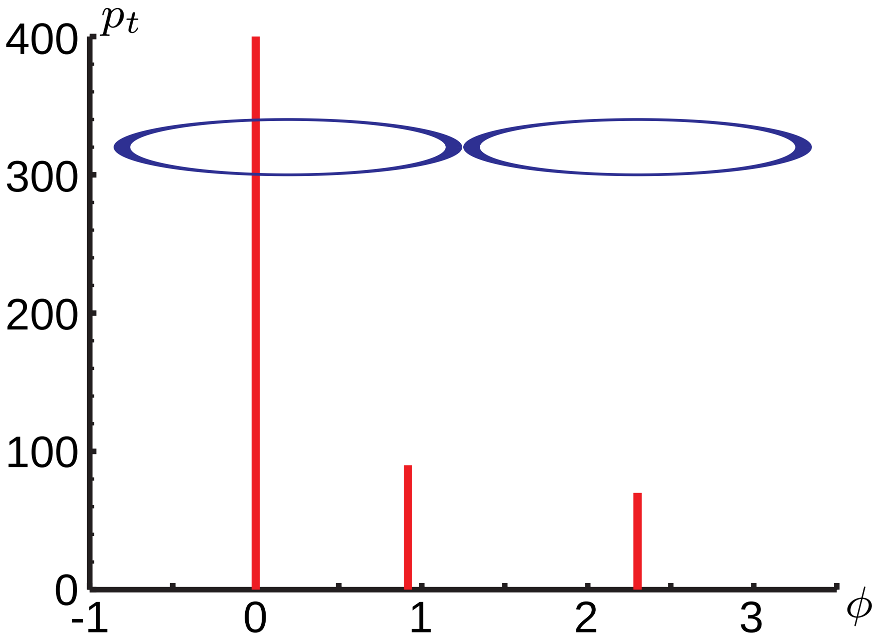

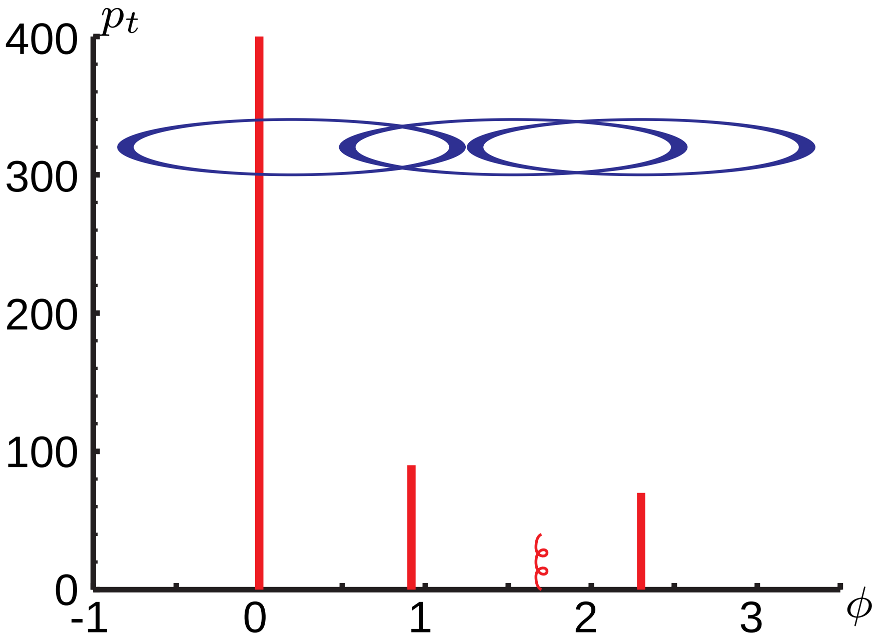

Stable cones:		
Midpoint:	{1,2} & {3}	{1,2} & {3} & {2,3}
Seedless:	{1,2} & {3} & {2,3}	{1,2} & {3} & {2,3}
Jets: ($f = 0.5$)		
Midpoint:	{1,2} & {3}	{1,2,3}
Seedless:	{1,2,3}	{1,2,3}

Stable cone missed ⟶ IR unsafety of the midpoint algorithm

Solution: SISCone

- Solution: use a seedless approach, find ALL stable cones
- Naive approach: check stability of each subset of particle

Solution: SISCone

- Solution: use a seedless approach, find ALL stable cones
- Naive approach: check stability of each subset of particle
  Complexity is $\mathcal{O}\left(N2^N\right)$
  $\Rightarrow$ definitely unrealistic: 10^{17} years for $N = 100$
- Midpoint complexity: $\mathcal{O}\left(N^3\right)$

Solution: SISCone

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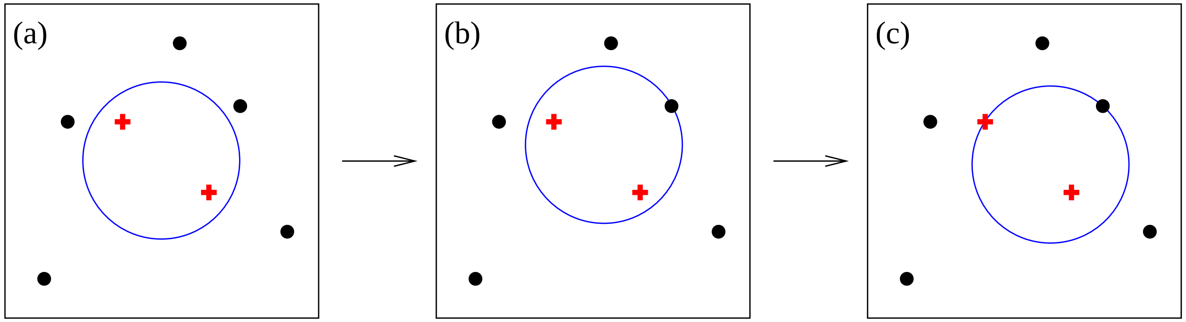
- Midpoint complexity: $\mathcal{O}\left(N^3\right)$

Idea: use geometric arguments



- Each enclosure can be moved (in any direction) until it touches a point
- ... then rotated until it touches a second one

Solution: SISCone

- Solution: use a seedless approach, find ALL stable cones
- Midpoint complexity: $\mathcal{O}\left(N^3\right)$

Idea: use geometric arguments

$\Rightarrow$ Enumerate all pairs of particles
with 2 circle orientations and 4 possible inclusion/exclusion
$\longrightarrow$ find all enclosures

- Complexity: $\mathcal{O}\left(N^3\right)$, with improvements: $\mathcal{O}\left(N^2\log(N)\right)$

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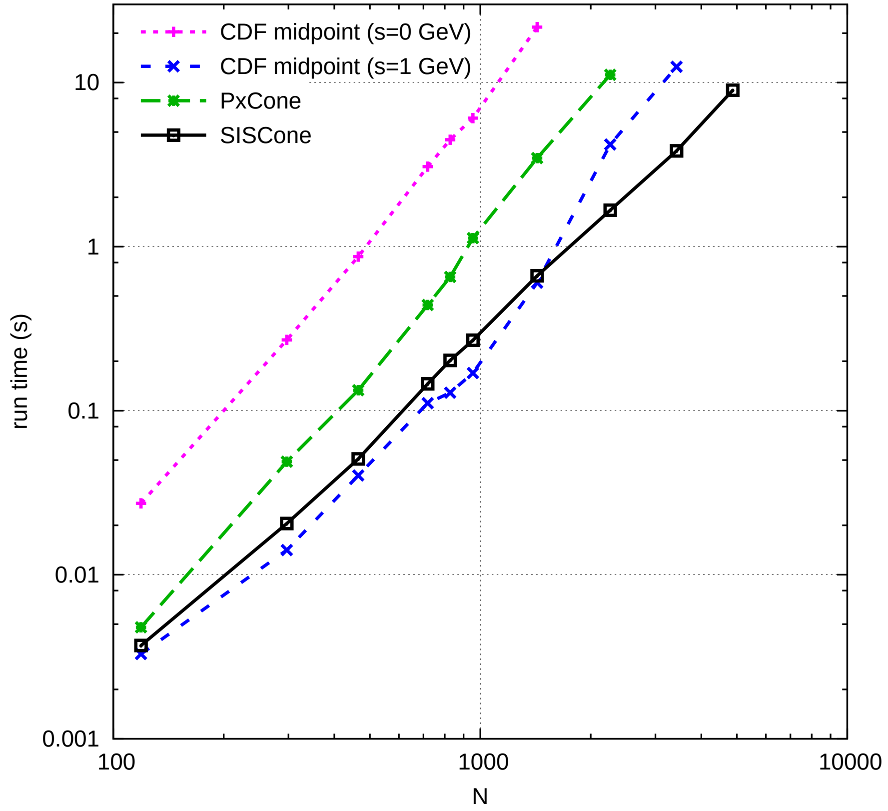
$\longrightarrow$ C++ implementation: Seedless Infrared-Safe Cone algorithm (SISCone)
G.Salam, G.S., JHEP 04 (2007) 086; http://projects.hepforge.org/siscone

NB.: also available from FastJet
[M.Cacciari, G.Salam, G.S.]; http://www.lpthe.jussieu.fr/∼salam/fastjet

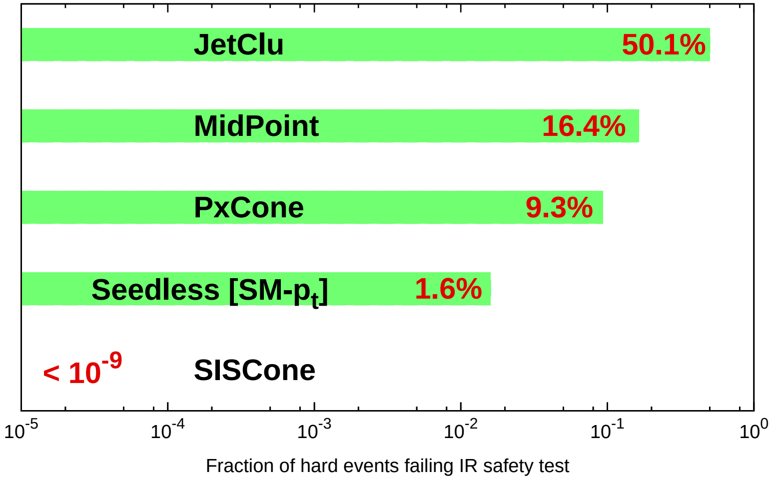
Physical impact

Execution timings:



Random hard & soft partons

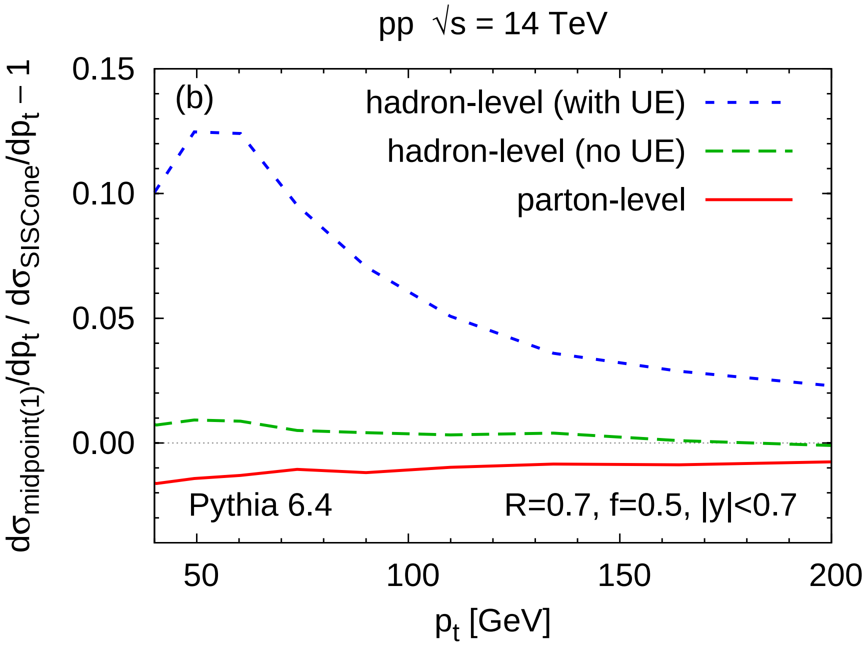
fraction with "hard $\neq$ hard+soft"



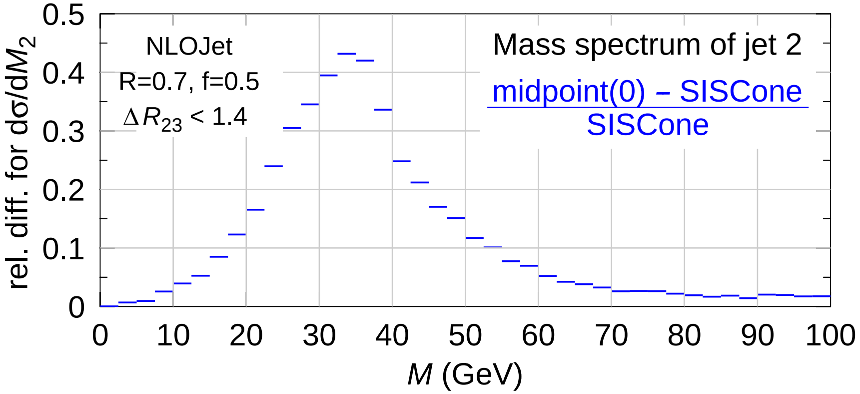
- at least as fast as midpoint cones
- IR safe
  - JetClu,ATLAS cone: 50% failure
  - MidPoint: 15% failure

Physical impact (2)

Inclusive (midpoint/SISCone-1)

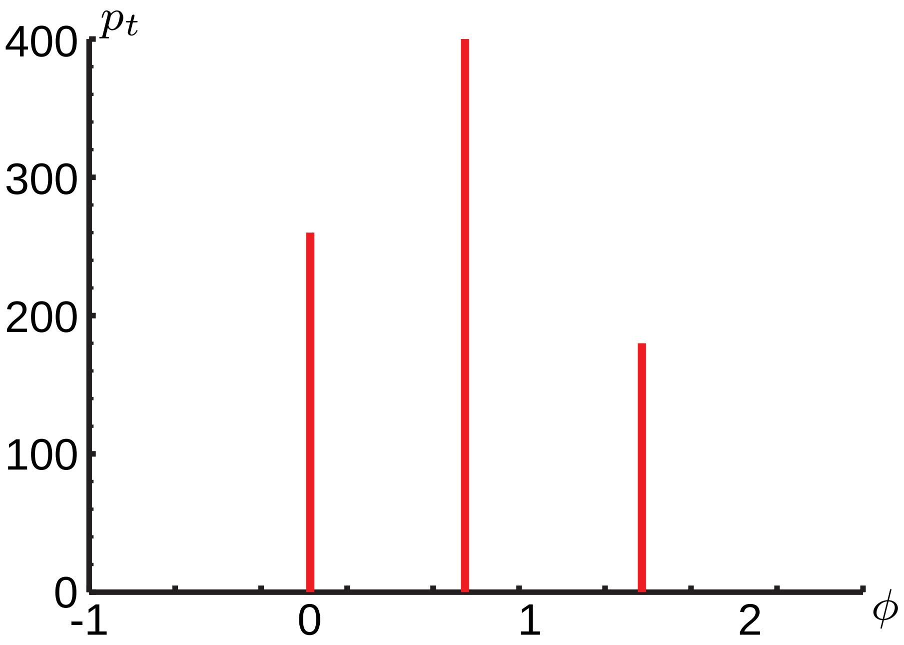


Masses in 3-jet events

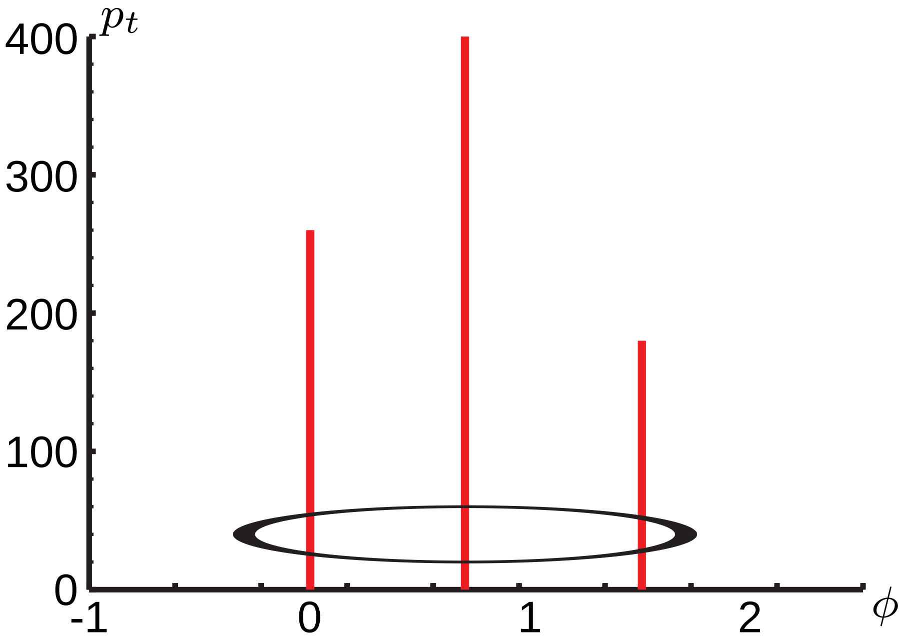


- Inclusive cross-section:
  - Effect of a few percents
  - Less sensitivity to the UE
- More exclusive processes: effects $\sim 45\%$ (Important for LHC!)

Coll. unsafety of the iterative cone



Coll. unsafety of the iterative cone



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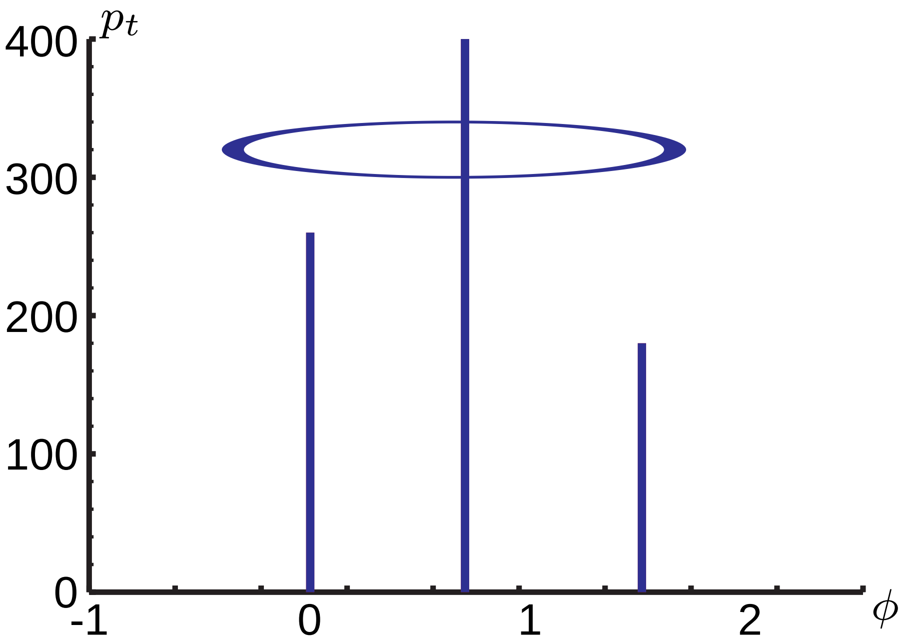

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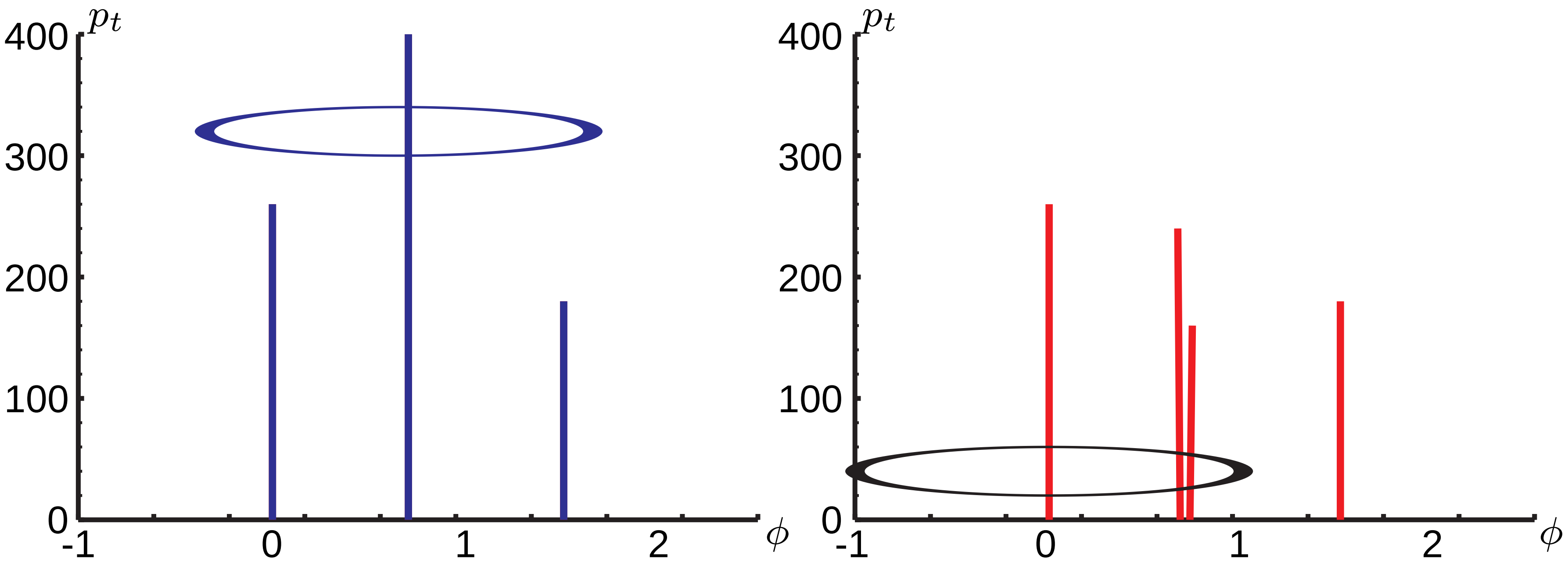

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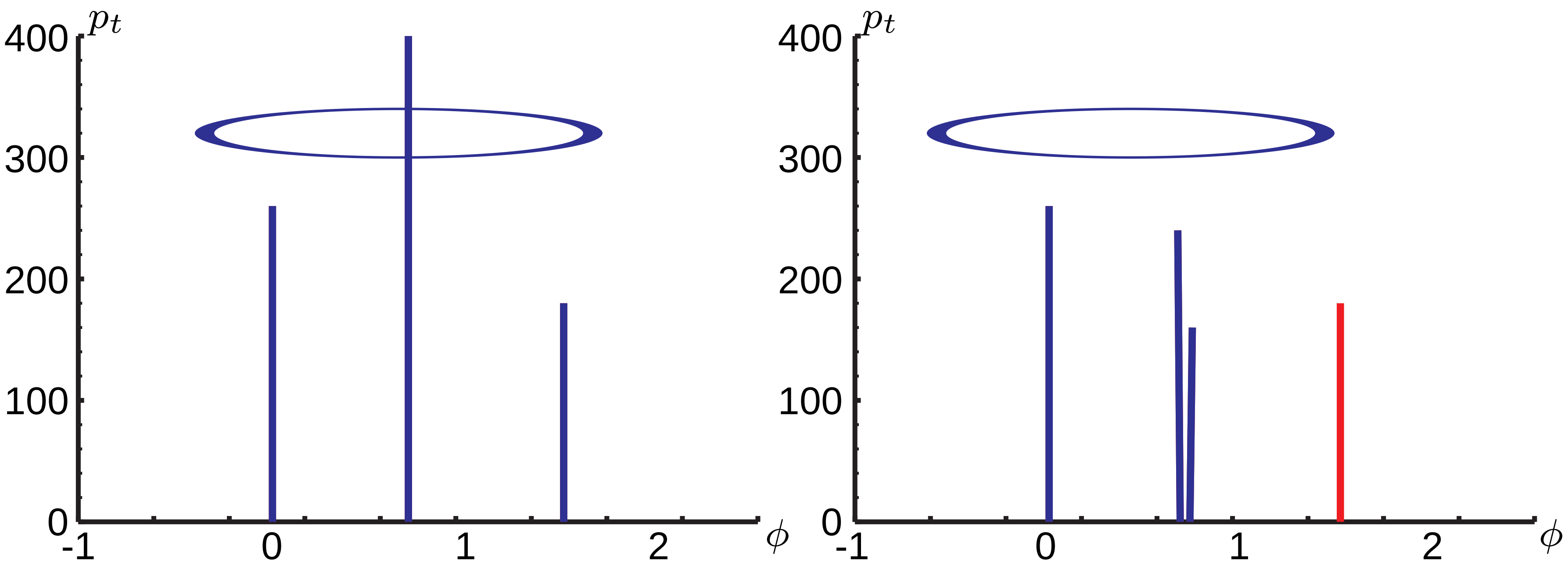

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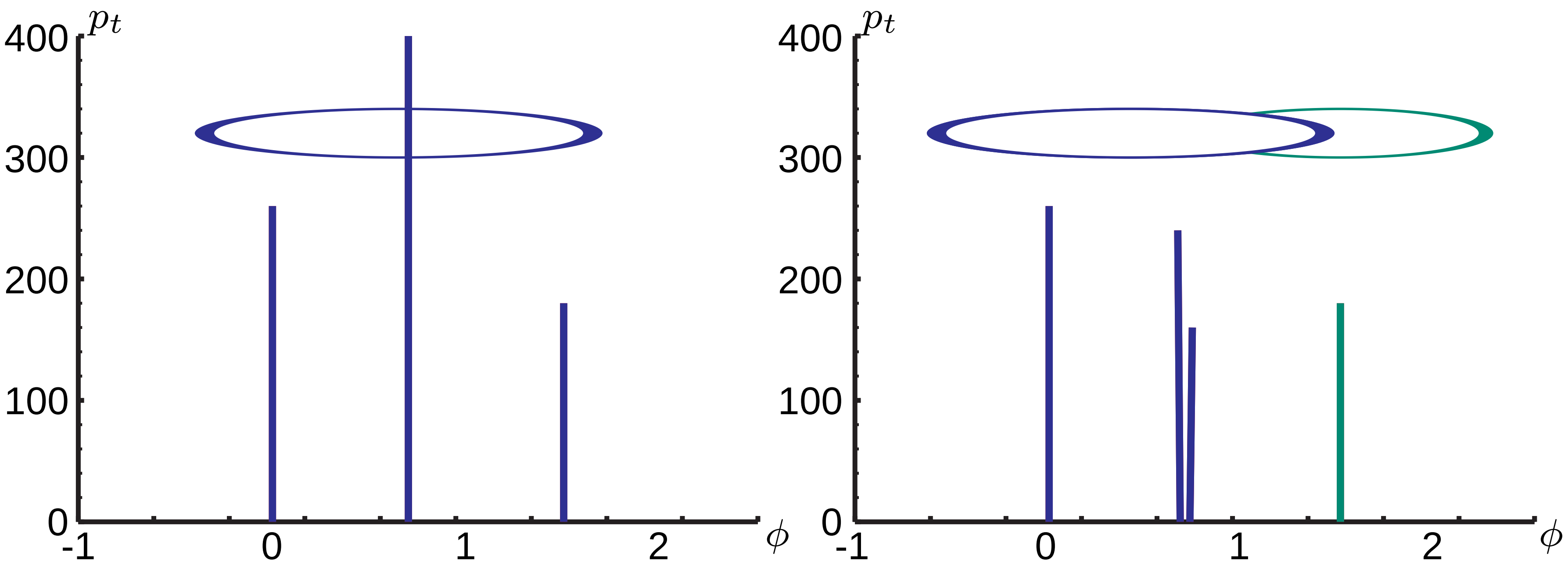

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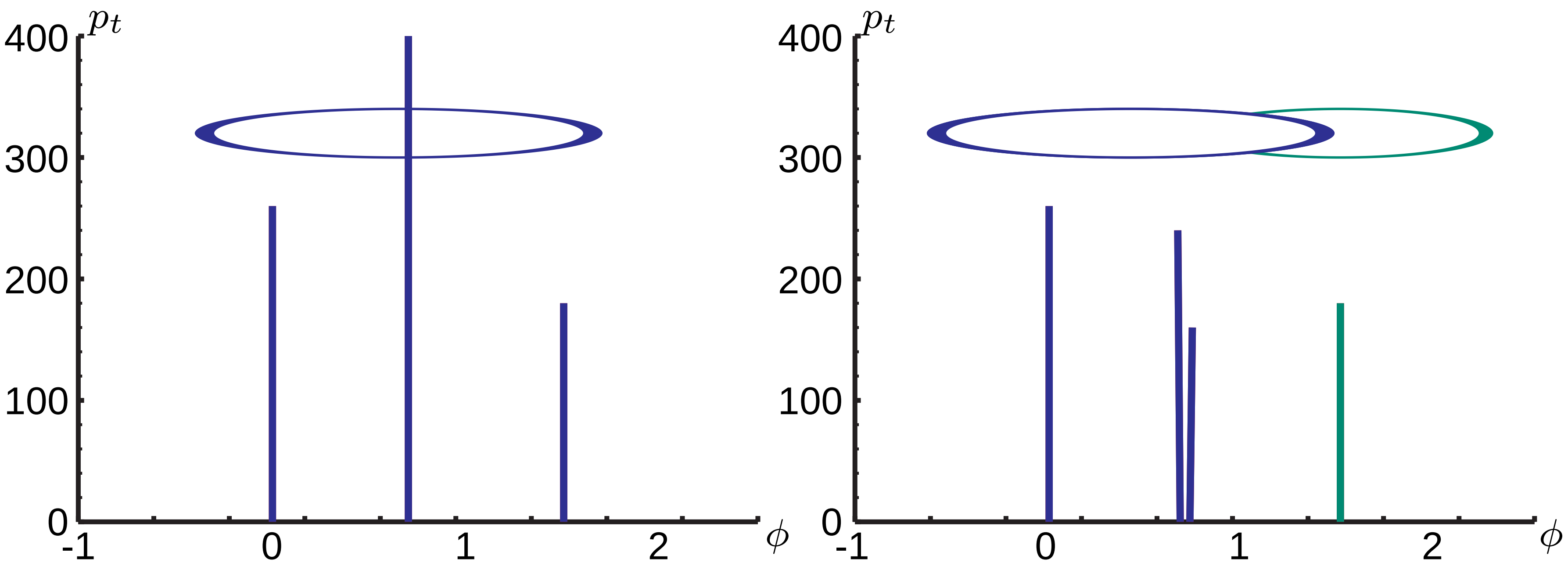

- Before collinear spliting: 1 jet
- After collinear spliting: 2 jets

$\longrightarrow$ **collinear unsafety of the iterative cone algorithm**

Anti-k_t

Come back to recombination-type algorithms:

$$d_{ij} = \min(k_{t,i}^{2p}, k_{t,j}^{2p})\ (\Delta\phi_{ij}^2 + \Delta\eta_{ij}^2)$$

- $p = 1$: k_t algorithm
- $p = 0$: Aachen/Cambridge algorithm

Anti-k_t

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- $p = 1$: k_t algorithm
- $p = 0$: Aachen/Cambridge algorithm
- $p = -1$: anti-k_t algorithm [M.Cacciari, G.Salam, G.S.,JHEP 04 (08) 063]

Anti-k_t

Come back to recombination-type algorithms:

$$d_{ij} = \min(k_{t,i}^{2p}, k_{t,j}^{2p})\ (\Delta\phi_{ij}^2 + \Delta\eta_{ij}^2)$$

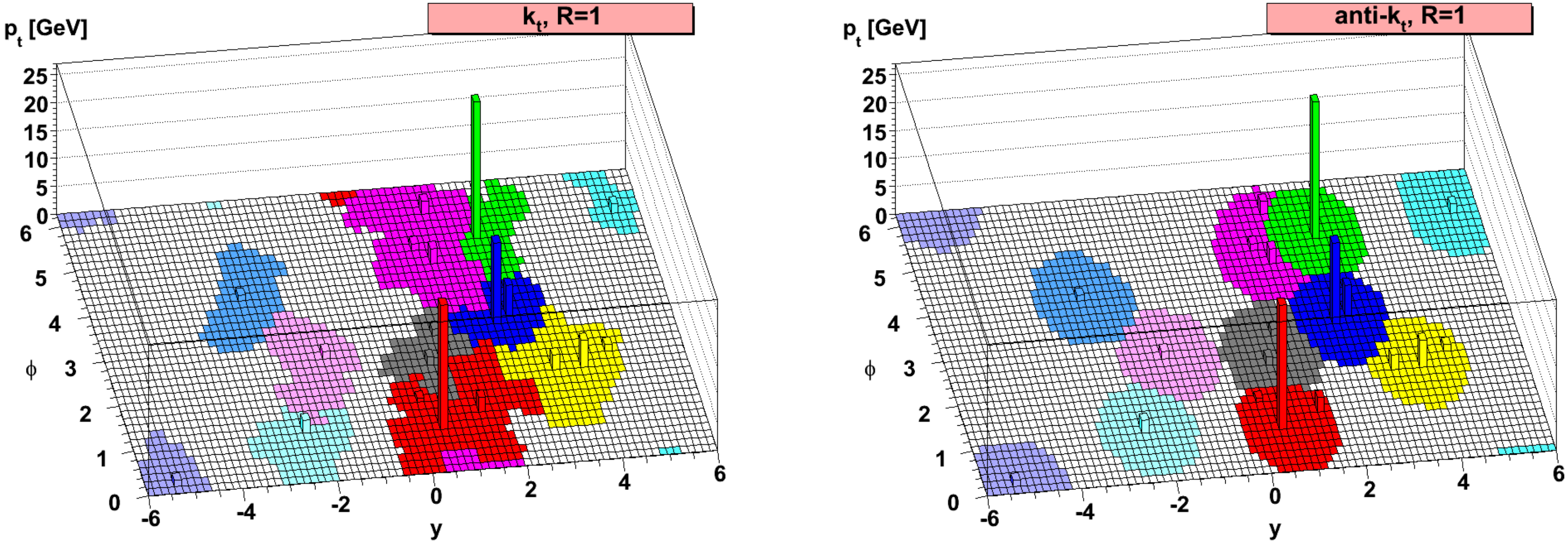
- $p = 1$: k_t algorithm
- $p = 0$: Aachen/Cambridge algorithm
- $p = -1$: anti-k_t algorithm [M.Cacciari, G.Salam, G.S.,JHEP 04 (08) 063]

Why should that be related to the iterative cone ?!?

- "large $k_t \Rightarrow$ small distance"
  i.e. hard partons "eat" everything up to a distance R
  i.e. circular/regular jets, jet borders unmodified by soft radiation
- infrared and collinear safe

anti-k_t

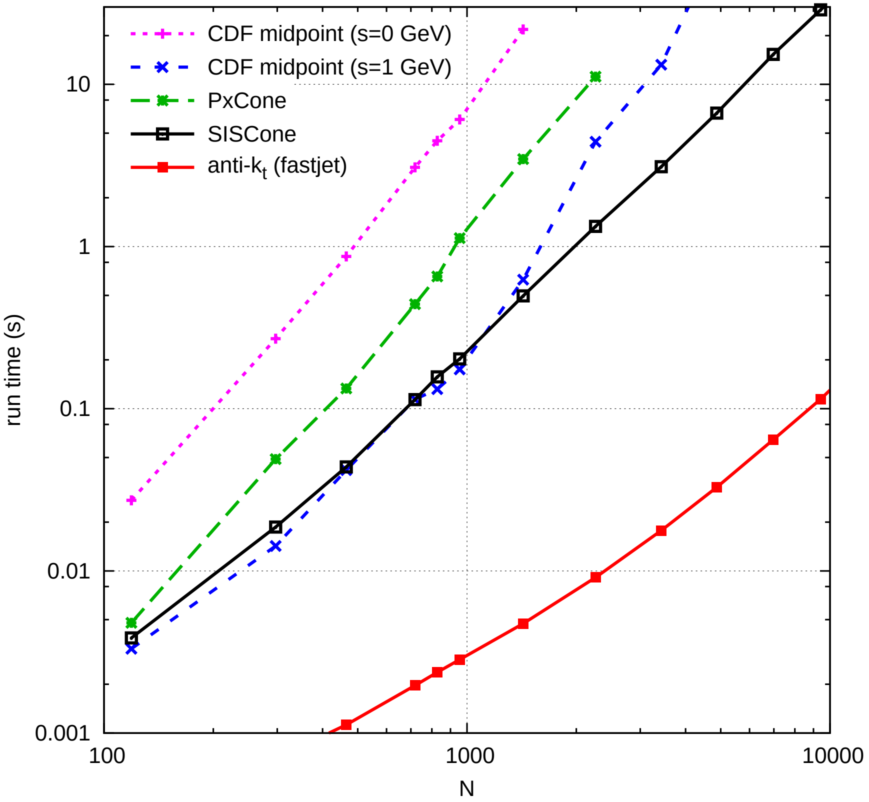
Hard event + homogeneous soft background



anti-k_t is soft-resilient

more later in this talk...

Execution timings:



As fast as the (fast) k_t ([M. Cacciari, G. Salam, 06])

IRC safety is mandatory

Midpoint and the iterative cone IR or Collinear unsafe (at $\mathcal{O}(\alpha_s^4)$)

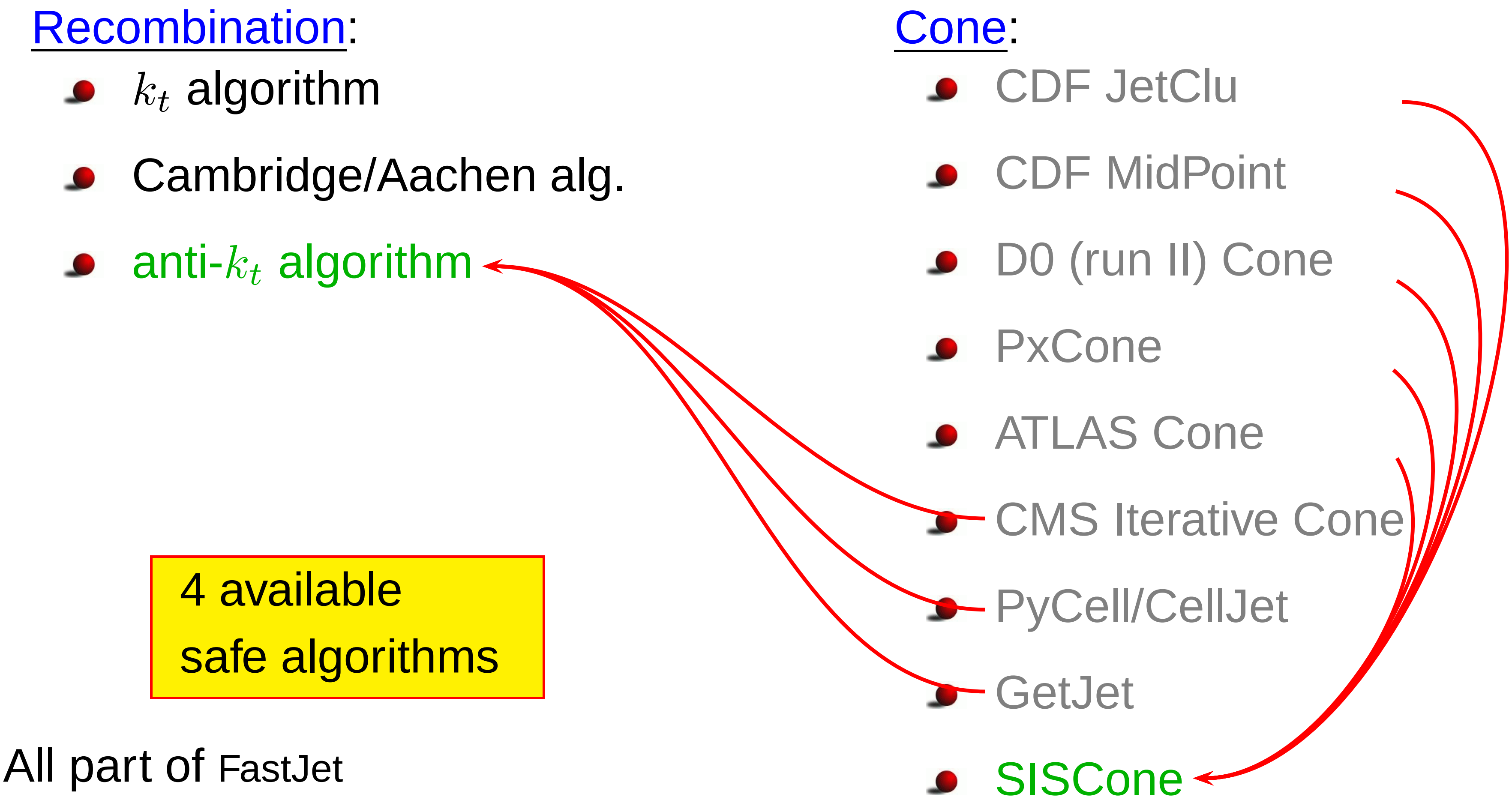
Observable	1st miss cones at	Last meaningful order
Inclusive jet cross section	NNLO	NLO
3 jet cross section	NLO	LO (NLO in NLOJet)
$W/Z/H$ + 2 jet cross sect.	NLO	LO (NLO in MCFM)
jet masses in 3 jets	LO	none (LO in NLOJet)

$\Rightarrow$ The IRC-unsafety issue will matter at LHC

+ We do not want the theoretical efforts to be wasted

Note: 1 order worse for JetClu of the ATLAS Cone!

21st century jet finders



Recombination:

- k_t algorithm
- Cambridge/Aachen alg.
- anti-k_t algorithm

Cone:

- CDF JetClu
- CDF MidPoint
- D0 (run II) Cone
- PxCone
- ATLAS Cone
- CMS Iterative Cone
- PyCell/CellJet
- GetJet
- SISCone

4 available
safe algorithms

All part of FastJet

[M.Cacciari,G.Salam,G.S.]

Filtering using jet substructure

More refined clustering ("2^{nd} generation of algorithms")

Cambridge+Filtering algorithm:

- Cluster with Aachen/Cambridge and radius R
- For each jet, recluster it with Aachen/Cambridge and radius R_{sub} keep only n_{sub} hardest sub-jets of the initial jet

Aim: remove the soft background

Properties:

- Proven to improve jet reconstruction, in $H \to b\bar{b}$
  [J.Butterworth, A.Davison, M.Rubin, G.Salam, 08]
- Additional parameters that deserve appropriate studies
- We will use the simplest choice: $R_{\text{sub}} = R/2$, $n_{\text{sub}} = 2$

Part 1

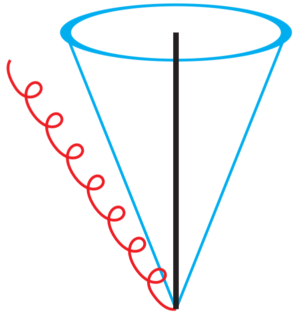
Jets in pp collisions

Choosing the adapted jet definition

Underlying physics

Competition between

- catching perturbative radiation



Out-of-jet radiation:

$$\sim \int_R \frac{d\theta}{\theta} \sim \log(1/R)$$

- not catching soft background radiation

Soft contents $\sim$ jet area $\sim R^2$

more detailed analytic computation in progress...
only numerical results at present time

Sample processes to study

We analyse 3 processes:

- $Z' \to q\bar{q} \to$2 jets: (ficticious narrow Z')

  simple environment: identify 2 jets and reconstruct $M_{Z'}$
  source of monochromatic quark jets
  scale dependence: mass of the Z' between 100 GeV and 4 TeV

- $H \to gg \to$2 jets: (ficticious narrow Higgs)

  simple environment: identify 2 jets and reconstruct M_H
  source of monochromatic gluon jets
  scale dependence: mass of the Higgs between 100 GeV and 4 TeV

- $t\bar{t} \to W^+bW^-\bar{b} \to q\bar{q}bq\bar{q}\bar{b} \to$6 jets:

  complex environment: identify 6 jets and reconstruct 2 top
  balance between reconstruction efficiency and identification

with

- the 5 IRC-safe algorithms: k_t, Cambridge, anti-k_t, SISCone, Cam+filtering
- jet radius varied between 0.1 and 1.5

figure of merit for quality measure

Measure of the jet reconstruction efficiency

- Forget about measures related to parton-jet matching, use the reconstructed mass peak
- Forget about fits depending on the shape of the peak
- we shall maximise the signal over background ratio ($S/\sqrt{B}$):

$$Q^w_{f=z}(JA,R) = \text{minimal width of a window containing a fraction } f = z \text{ of the events}$$

figure of merit for quality measure

$Q^w_{f=z}(JA,R) =$ minimal width of a window containing a fraction $f = z$ of the events

- it intuitively does what it should
- for a constant background,

$$\frac{Q^w_{f=z}(JA_1,R_1)}{Q^w_{f=z}(JA_2,R_2)} = \frac{B_{JA_1,R_1}}{B_{JA_2,R_2}} = \left[\frac{(S/\sqrt{B})_{JA_1,R_1}}{(S/\sqrt{B})_{JA_2,R_2}}\right]^{-2}$$

smaller width $\equiv$ better signal-to-background ratio

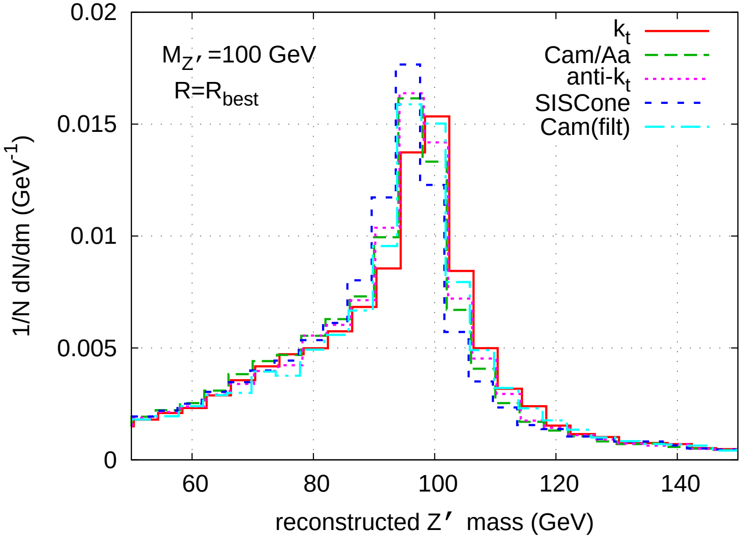
- we can associate an effective luminosity ratio

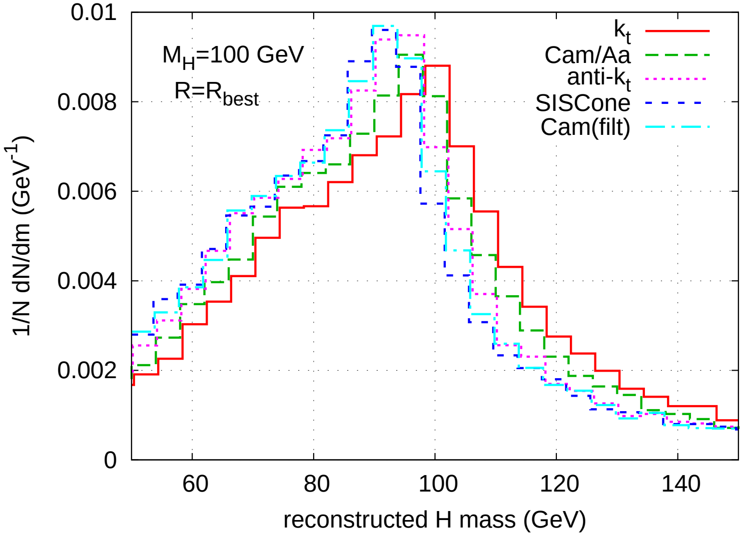
$$\rho_{\mathcal{L}} = \frac{\mathcal{L}_1}{\mathcal{L}_2} = \left[\frac{(S/\sqrt{B})_{JA_1,R_1}}{(S/\sqrt{B})_{JA_2,R_2}}\right]^{2} = \frac{Q^w_{f=z}(JA_2,R_2)}{Q^w_{f=z}(JA_1,R_1)}$$

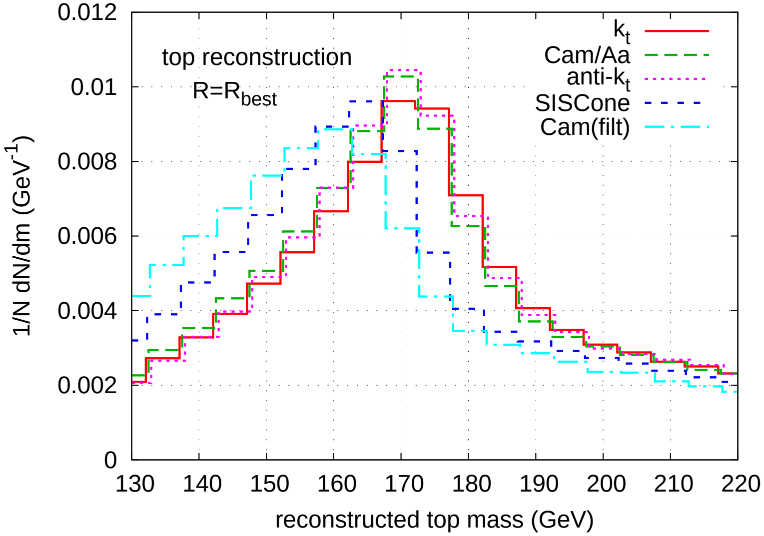
e.g. if $Q^w_{f=z}(JA_2,R_2) = 2Q^w_{f=z}(JA_1,R_1)$, (JA_2,R_2) will need twice the luminosity of (JA_1,R_1) to achieve the same discriminative power.

results (1)

we see peaks...

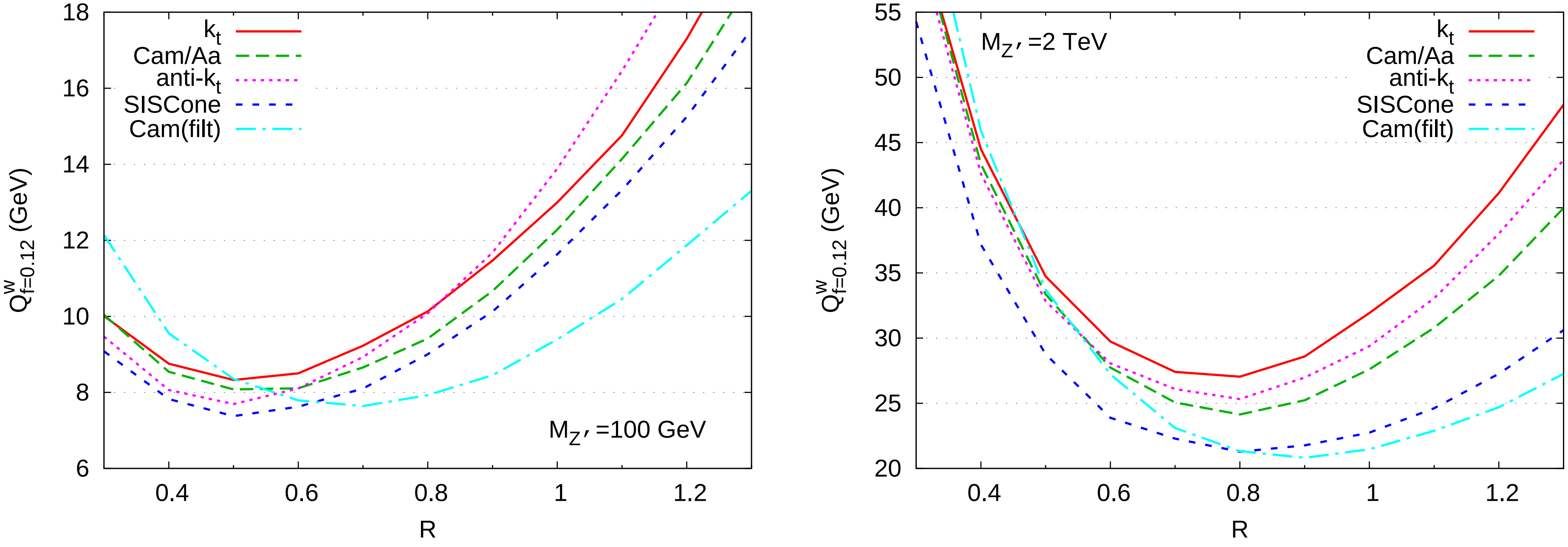






results (1)

Message 1: there is a strong R dependence



At 100 GeV,

using $R = 0.8$ instead of $R = 0.5$ means a discr. power loss of 20% ($\rho_{\mathcal{L}} \approx 0.8$)

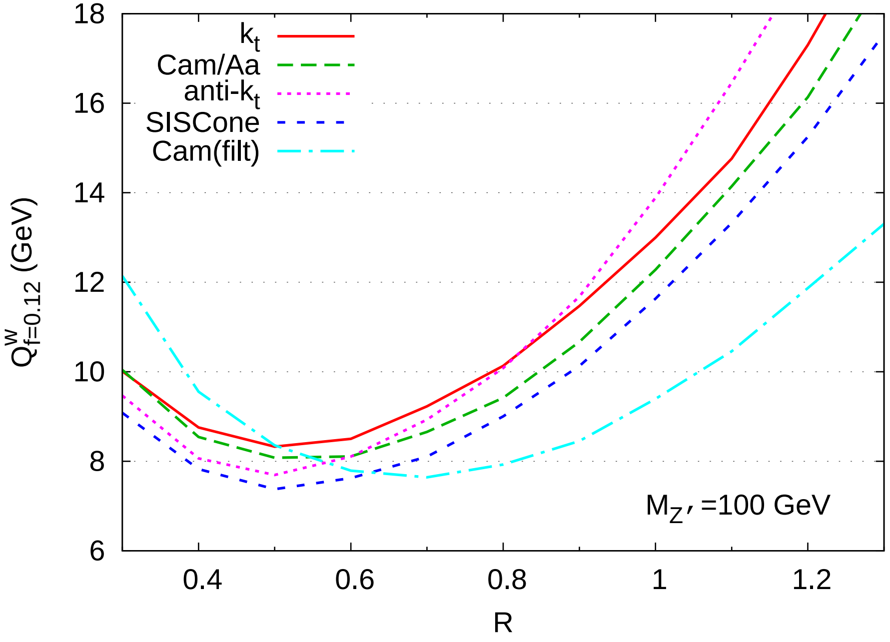
$R = 1$ 40% ($\rho_{\mathcal{L}} \approx 0.6$)

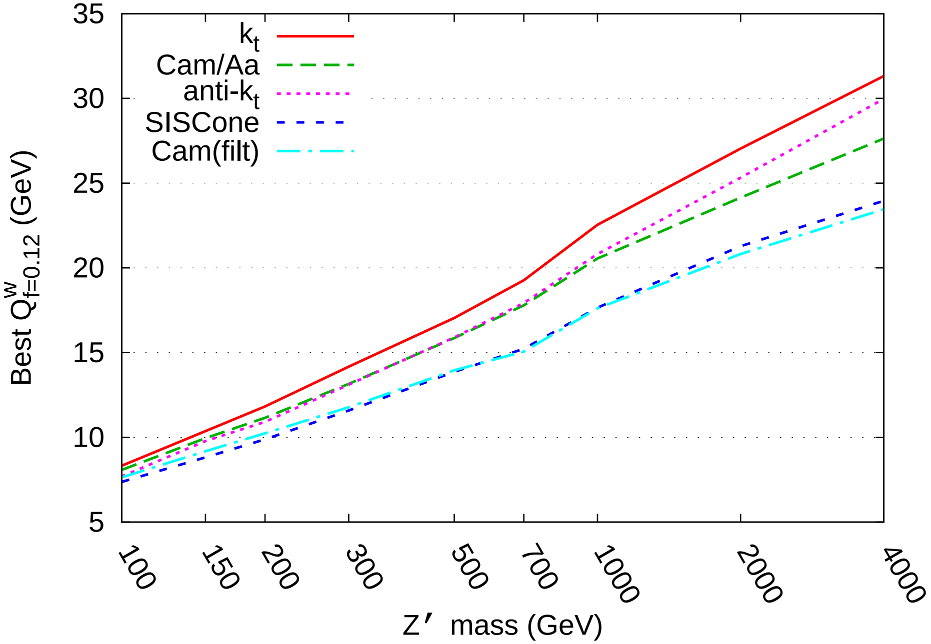
At 2 TeV,

using $R = 0.5$ instead of $R = 0.8$ means a discr. power loss of 20% ($\rho_{\mathcal{L}} \approx 0.8$)

results (1)

Message 2: SISCone and Cam+filt do a slightly better job





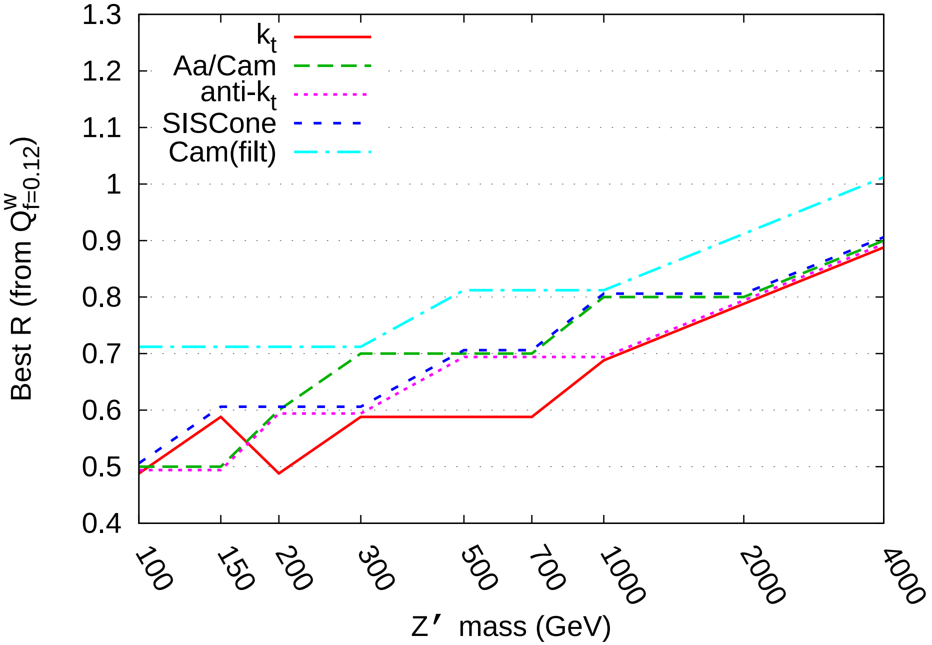
Using k_t instead of SISCone means a discr. power loss of

15% at 100 GeV ($\rho_{\mathcal{L}} \approx 0.85$)

20% at 2 TeV ($\rho_{\mathcal{L}} \approx 0.8$)

results (1)

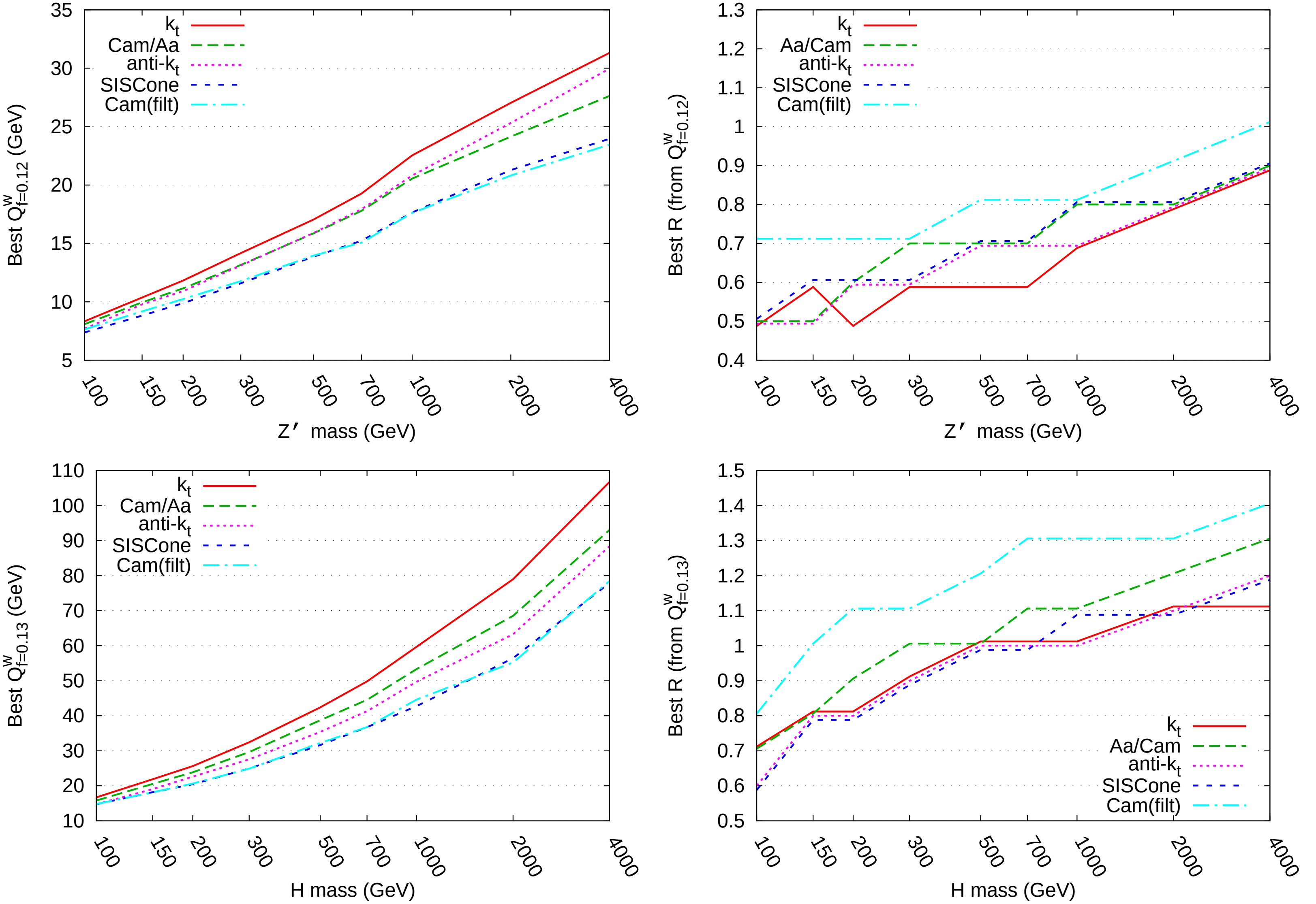
Message 3: The parameters vary with the scale



The preferred value for R increase with the mass scale (typically like $\log(M)$)

results (1)

Message 4: same for the gluon jets, though with a larger R



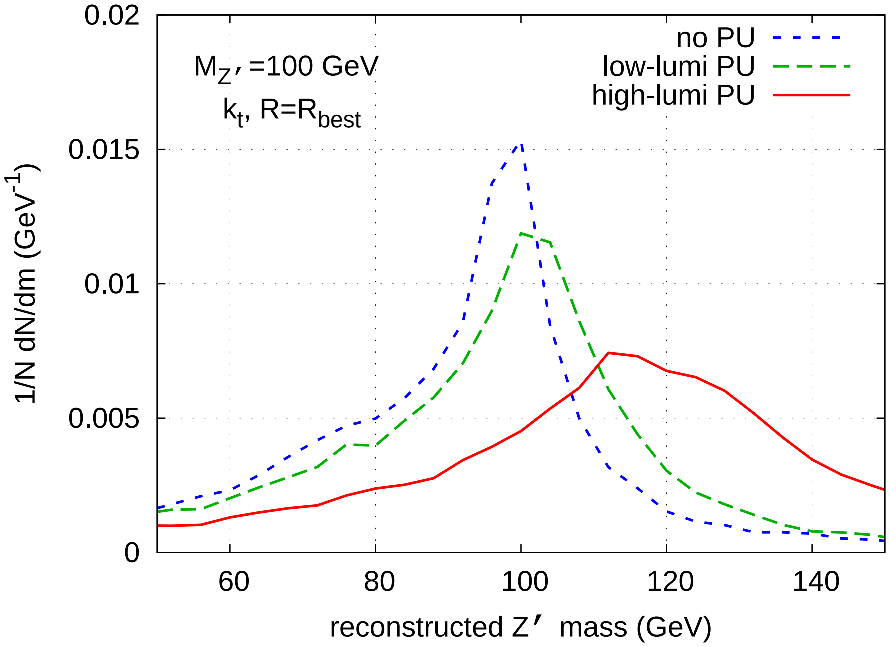
Part 1

Jets in pp collisions

Subtracting pileup background using jet areas

Need for subtraction

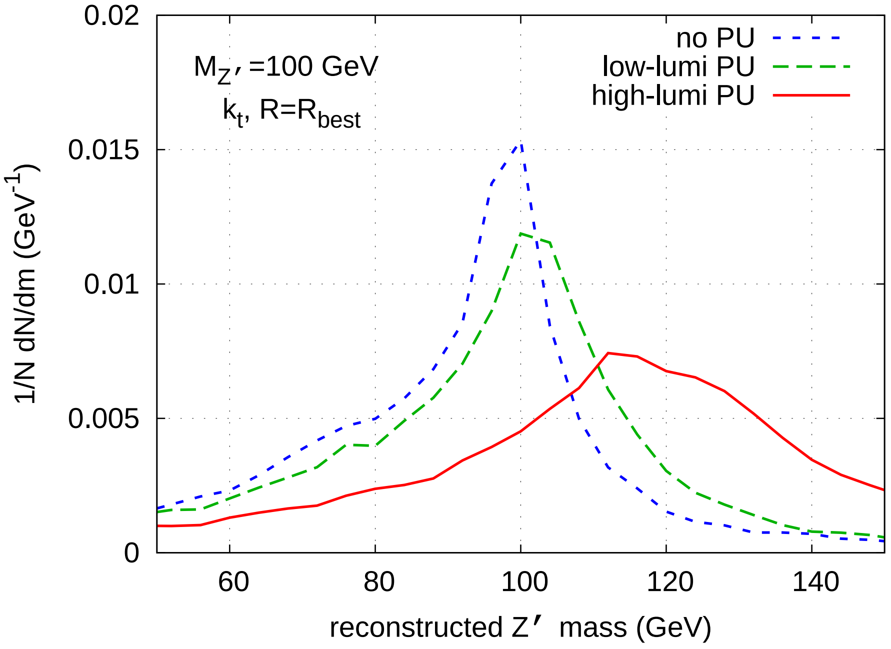
Pileup $\approx$ uniform soft background that shifts jets to higher p_t



... that needs to be subtracted!

Need for subtraction

Pileup $\approx$ uniform soft background that shifts jets to higher p_t



... that needs to be subtracted!

$\Rightarrow$ Using jet areas!

Area definition

- Idea: add soft particles (ghosts) and look in which jets they are caught

jet area = region where it catches ghosts

Area definition

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jet area = region where it catches ghosts

- 2 definitions
  - Passive area

    add one ghost and look where it ends. repeat to cover the (y, ϕ) plane
  - Active area

    add a large amount of ghosts and cluster everything

    also gives purely ghosted jets

Area definition

- Idea: add soft particles (ghosts) and look in which jets they are caught

jet area = region where it catches ghosts

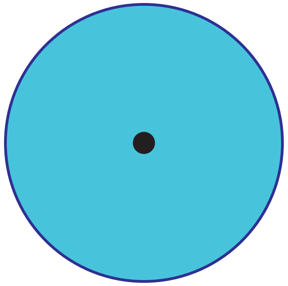
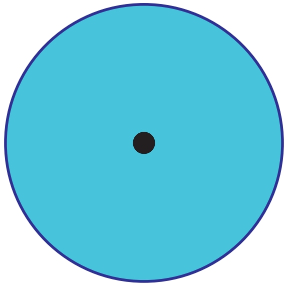
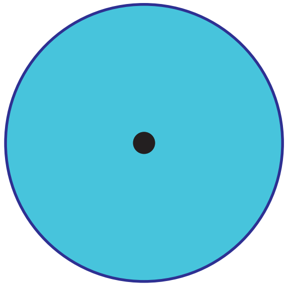
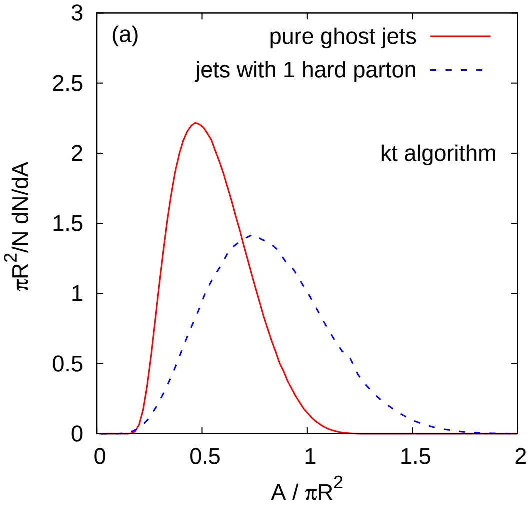
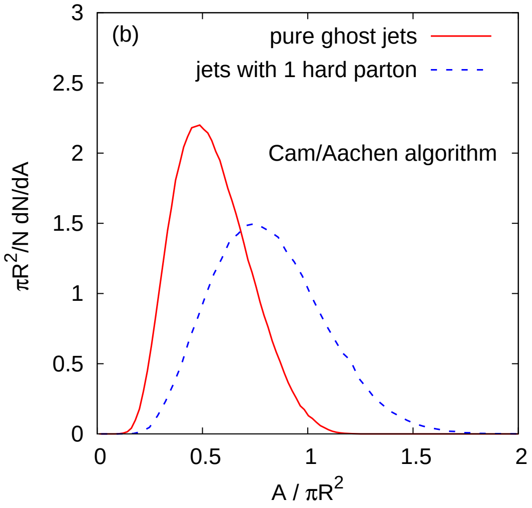
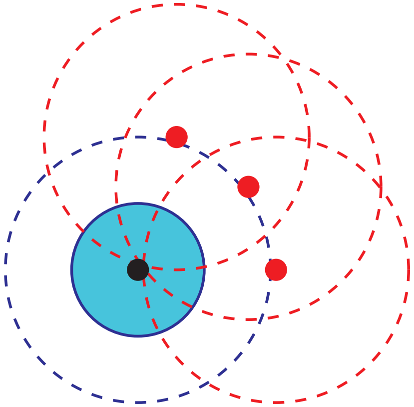
- 2 definitions
  - Passive area

    add one ghost and look where it ends. repeat to cover the (y, ϕ) plane
  - Active area

    add a large amount of ghosts and cluster everything

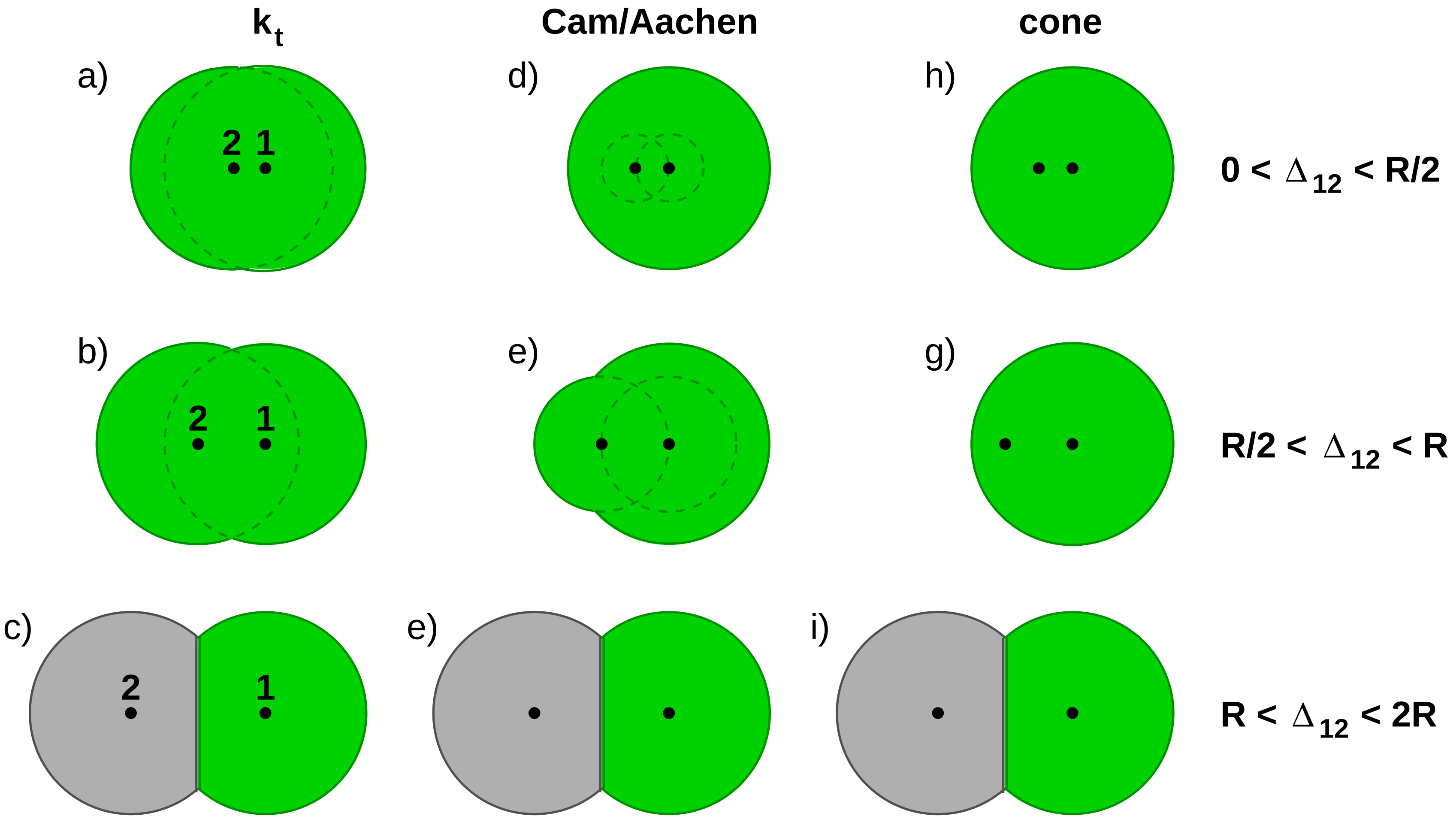
    also gives purely ghosted jets
- Both definitions agree for dense events
- Both practical and tractable analytically

Examples: 1-particle cases

	k_t	Aac/Cam	cone
Passive	 πR^2	 πR^2	 πR^2
Active	 $\frac{A_{\text{hard}}}{\pi R^2} \approx 0.812 \pm 0.277$ $\frac{A_{\text{ghost}}}{\pi R^2} \approx 0.554 \pm 0.174$	 $\frac{A_{\text{hard}}}{\pi R^2} \approx 0.814 \pm 0.261$ $\frac{A_{\text{ghost}}}{\pi R^2} \approx 0.551 \pm 0.176$	 $\frac{A_{\text{hard}}}{\pi R^2} = 0.25$

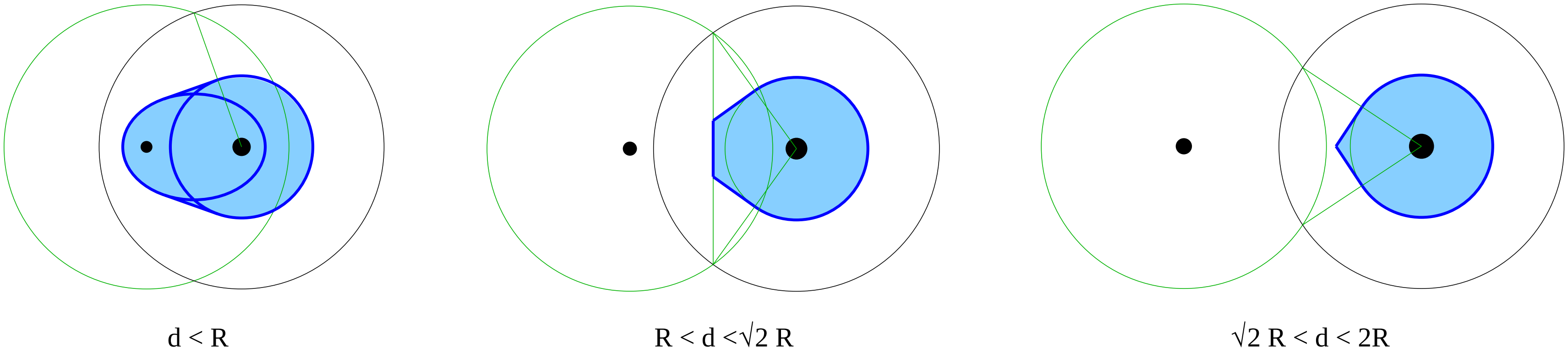
2-particle cases

Passive area: 1 hard particle + 1 soft



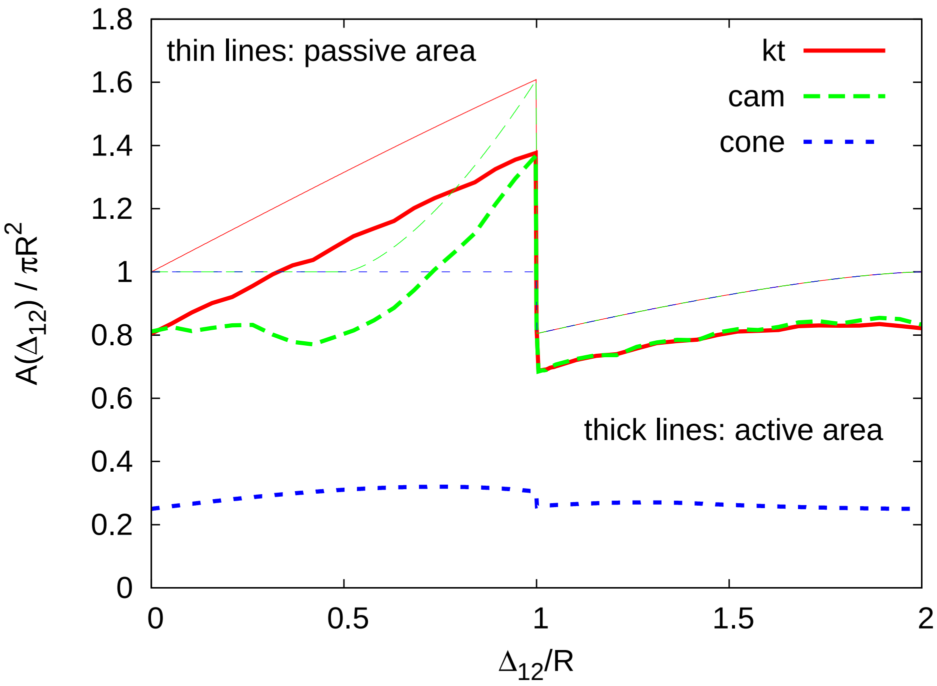
2-particle cases

Active area: 1 hard particle + 1 soft: analytic result for cone only



Alltogether, we have:

- Area $\neq$ cst. πR^2
- Δ_{12} dependence under control



Area scaling violations

QCD probability of emitting a small-angle soft gluon:

$$\frac{dP}{d\Delta_{12} dp_{t,2}} = C_{F,A} \frac{2\alpha_s}{\pi} \frac{1}{\Delta_{12}} \frac{1}{p_{t,2}}$$

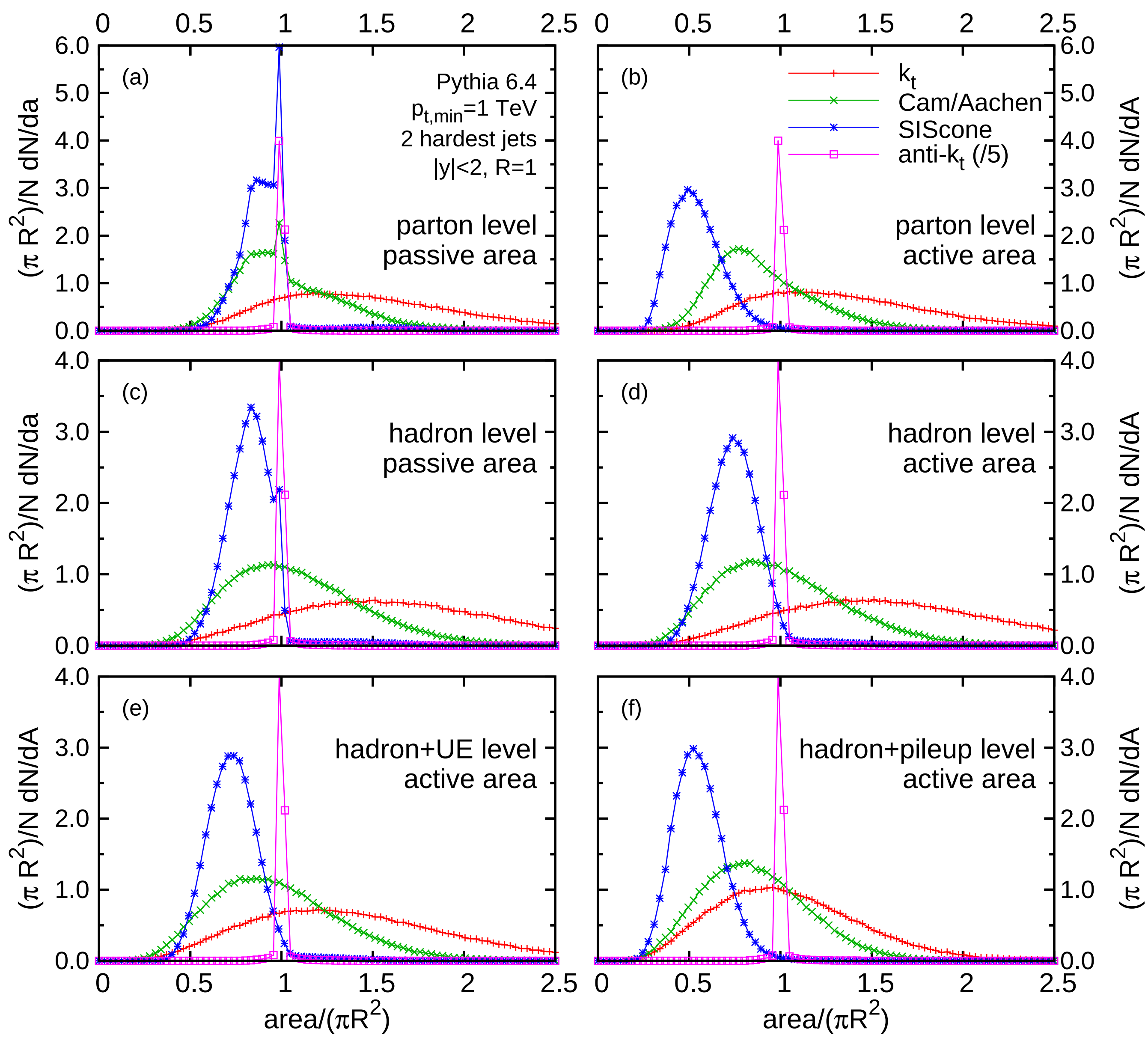
Hence the average area is

$$\begin{aligned} \langle \mathcal{A}(p_{t,1}, R) \rangle &= \mathcal{A}_{\text{1hard}}(R) + \int d\Delta \, dp_{t,2} \frac{dP}{d\Delta_{12} dp_{t,2}} \left[\mathcal{A}_{\text{hard+1 soft}}(\Delta, R) - \pi R^2 \right] \\ &= \frac{C_{F,A}}{\pi b_0} \log\left(\frac{\alpha_s(Q_0)}{\alpha_s(Rp_t)} \right) \pi R^2 \, d \end{aligned}$$

d	passive	active
k_t	0.5638	0.519
Cam	0.07918	0.0865
SISCone	-0.06378	0.1246
anti-k_t	0	0

- Scaling viloation
- gluon > quark
- with know LO anomalous dimension

Area histograms



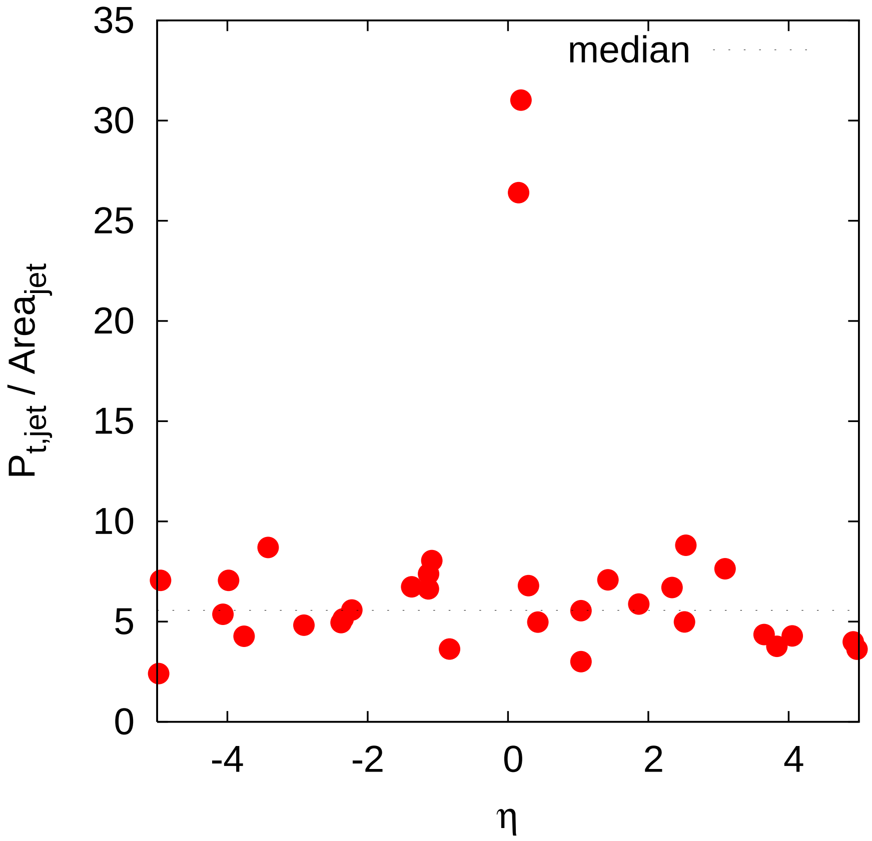
Pileup subtraction

Basic idea:

$$p_{t,\text{subtracted}} = p_{t,\text{jet}} - \rho_{\text{pileup}} \times \text{Area}_{\text{jet}}$$

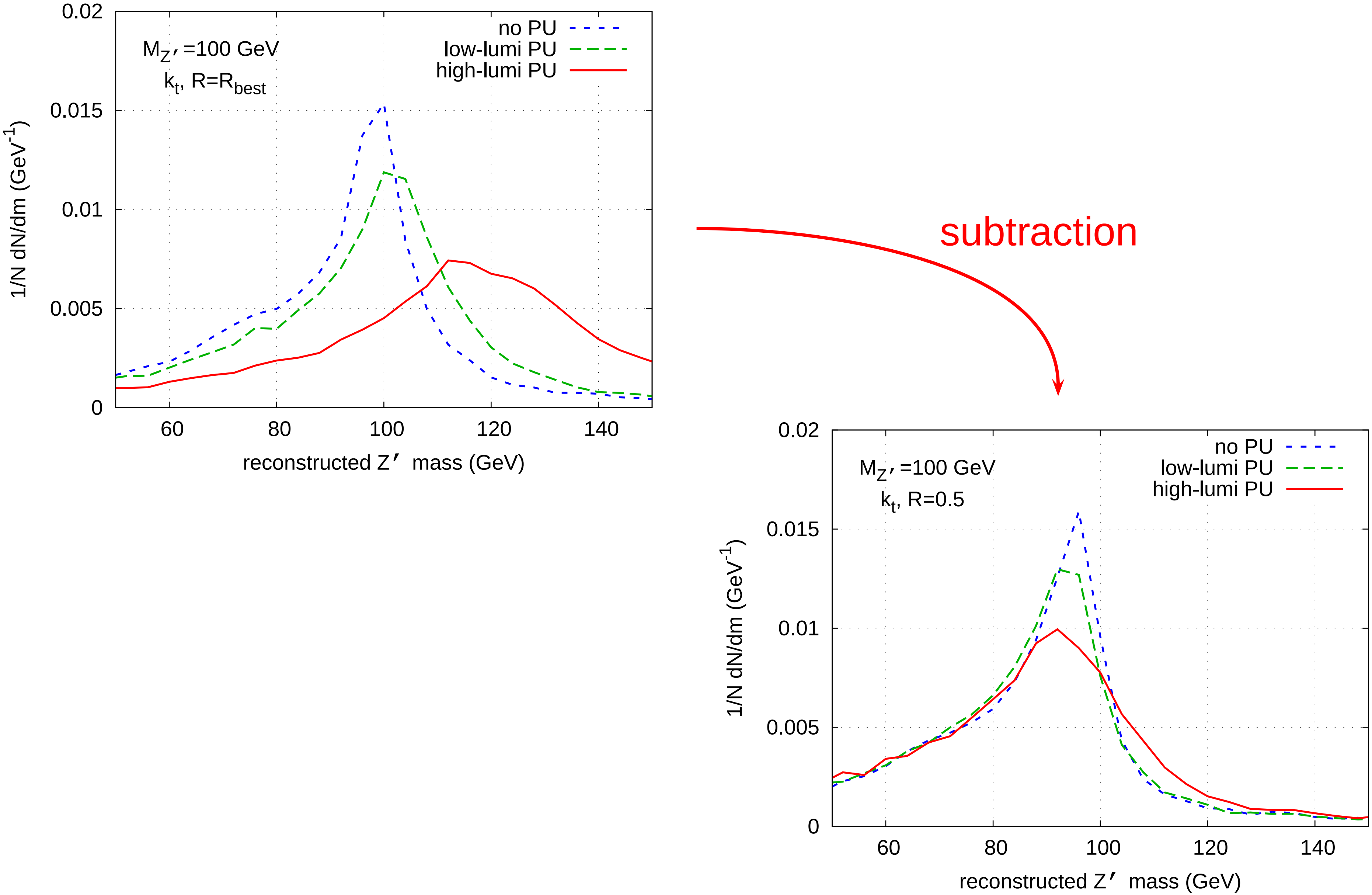
- Jet area:
  - region where the jet catches infinitely soft particles
  - tractable analytically
- Pileup density per unit area: ρ_{pileup}

  e.g. estimated from the median of $p_{t,\text{jet}}/\text{Area}_{\text{jet}}$



implemented in FastJet
on an event-by-event basis

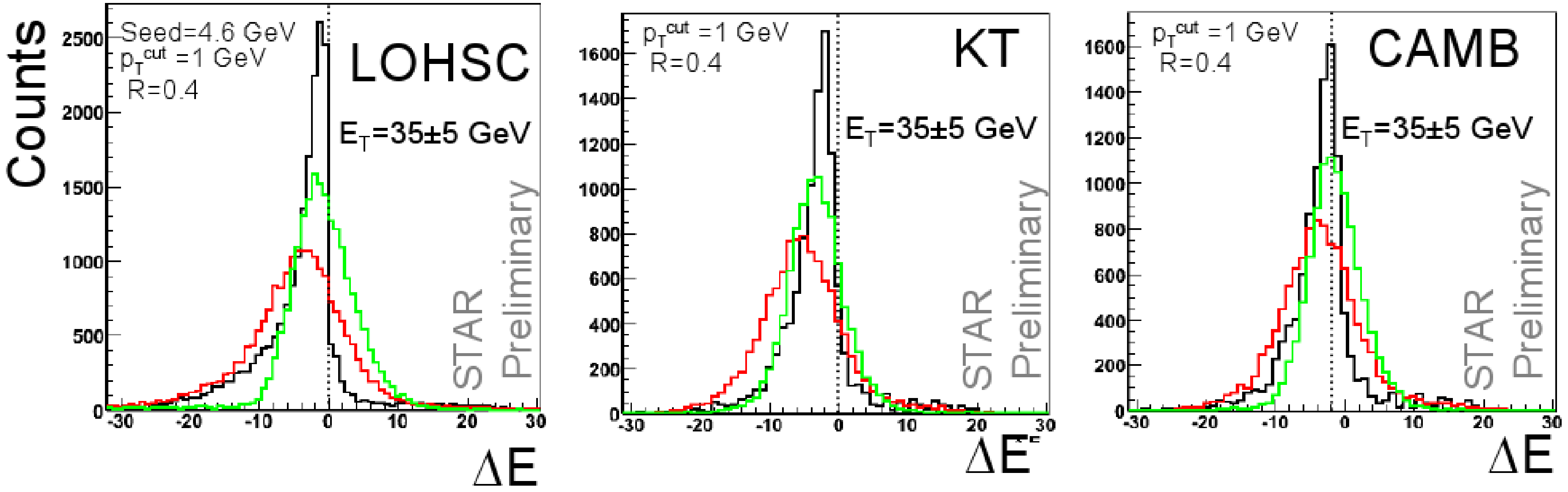
Subtraction recipe



Part 3: heavy-ion background:
subtracting more complex background

Jets at RHIC

[S. Salur, J. Putschke, ... (STAR)]



Red curves: $\Delta E = E^{\text{PyEmbed}} - E^{\text{PyDet}}$

with PyDet $\equiv$ Pythia pp + detector effects

PyEmbed $\equiv$ same with real AuAu event added

$\Rightarrow$ measure of the subtraction efficiency

Jets at RHIC

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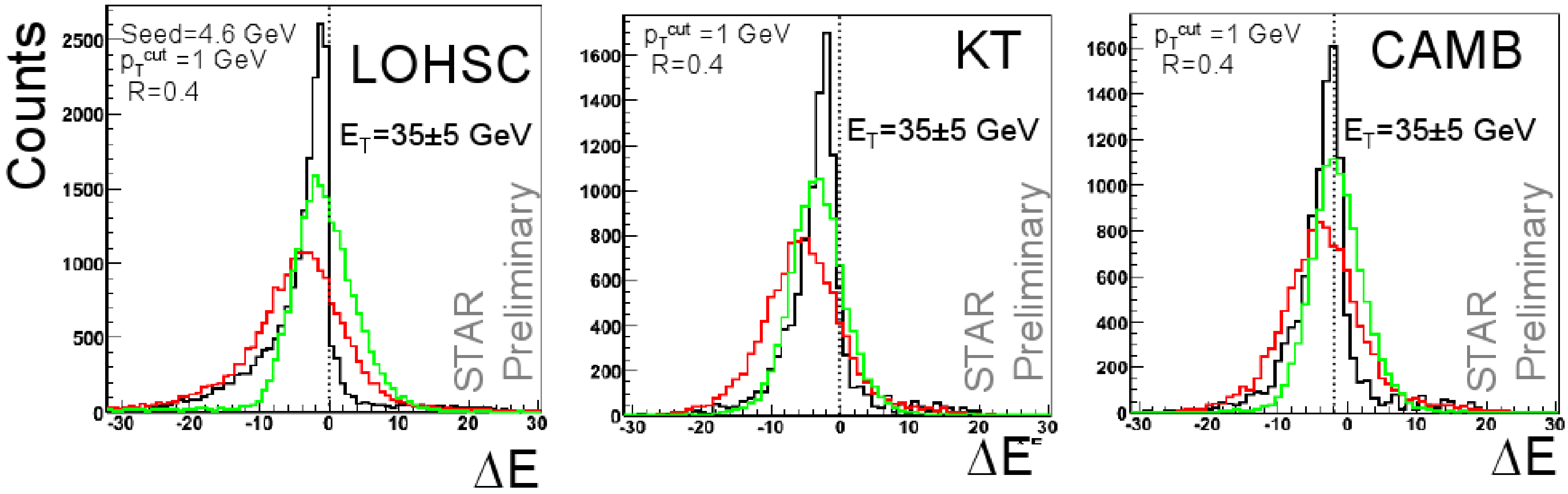


Red curves: $\Delta E = E^{\text{PyEmbed}} - E^{\text{PyDet}}$

with PyDet $\equiv$ Pythia pp + detector effects

PyEmbed $\equiv$ same with real AuAu event added

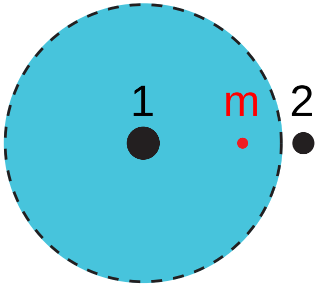
$\Rightarrow$ measure of the subtraction efficiency

Work under progress: removing the last few GeV shift in ΔE

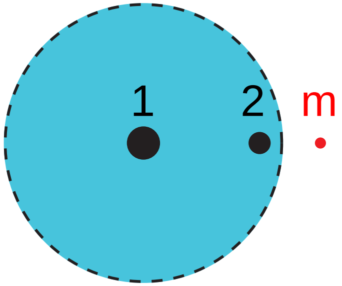
Back-reaction & HI UE

Additional soft background has 2 effects:

- Throw soft particles in the hard jet: dealt with by subtraction
- Modify the hard scattering (back-reaction)
  - can be pointlike or diffuse
  - gain: p_2 gained when adding p_m



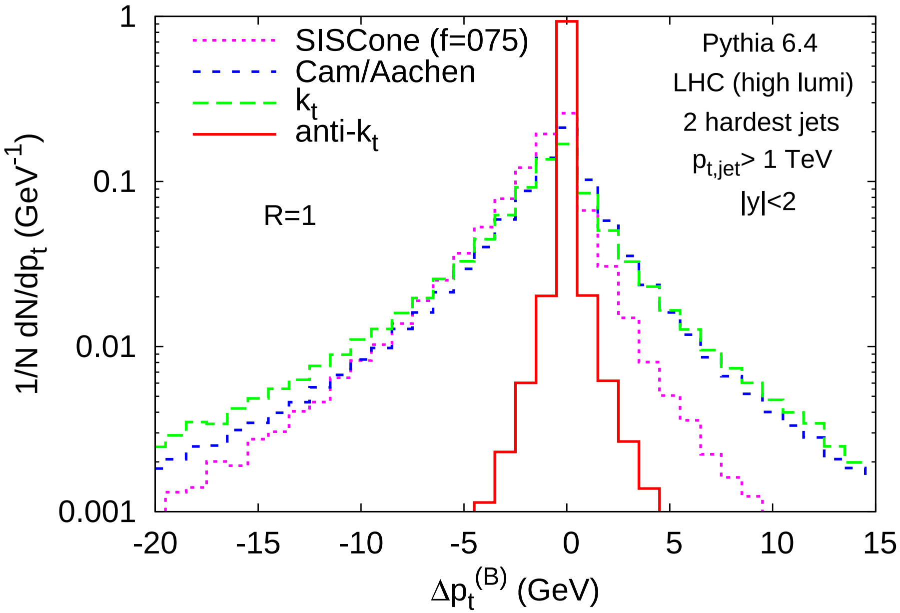
  - loss: p_2 lost when adding p_m



Back-reaction & HI UE

Additional soft background has 2 effects:

- Throw soft particles in the hard jet: dealt with by subtraction
- Modify the hard scattering (back-reaction)
  - can be pointlike or diffuse
  - tractable analytically (similar to areas)
  - $k_t \gtrsim$ Cambridge > SISCone $\gg$ anti-k_t



Back-reaction & HI UE

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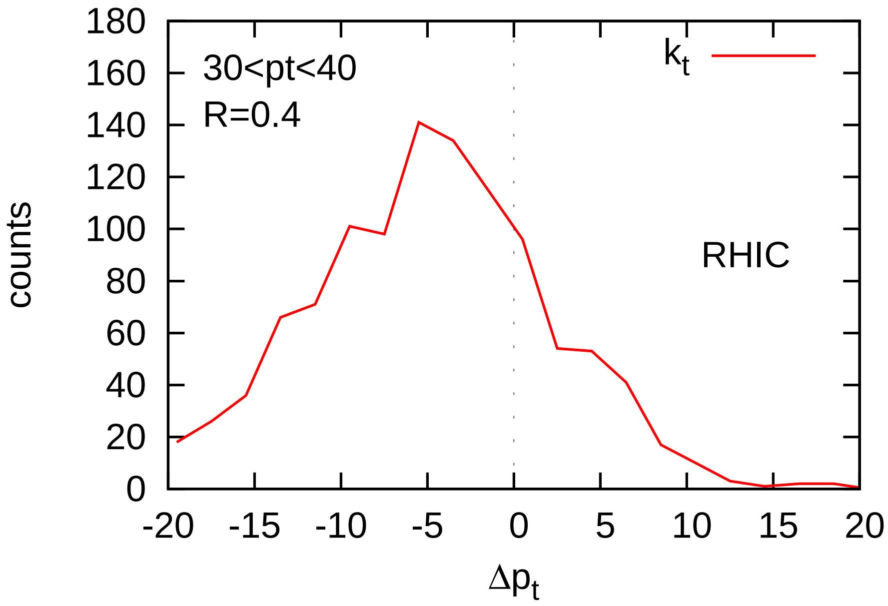
+ For heavy-ion collisions: fluctuating underlying event background

$\longrightarrow$ median estimation of ρ might oversimplified

Results (3)

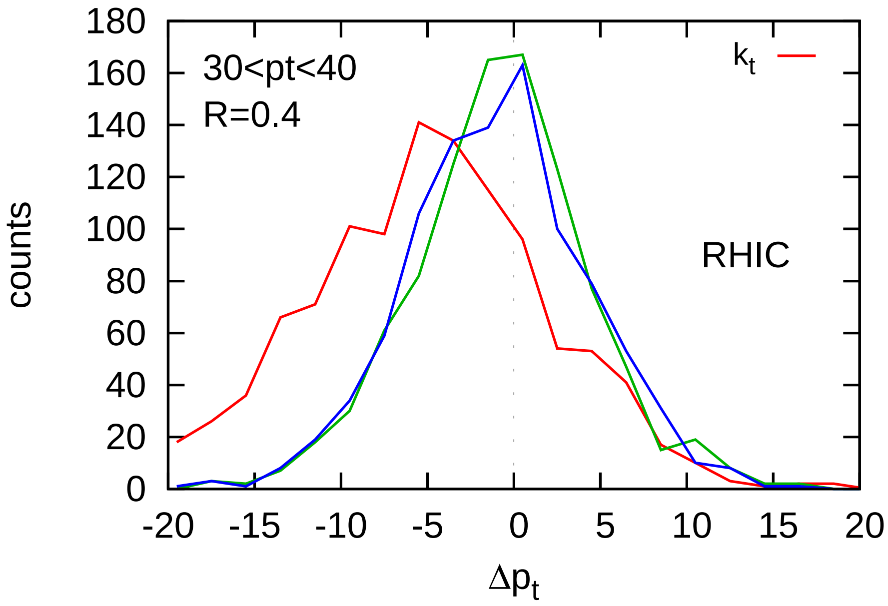
Work under progress: test subtraction using
Pythia pp hard event + HYDJET AA background



Similar shifts than STAR

Results (3)

Work under progress: test subtraction using
Pythia pp hard event + HYDJET AA background



Target: reach a precision of 1-2 GeV by

- carefully tuning the algorithm to reduce UE sensitivity and back-reaction
- carefully tuning the subtraction to deal with the fluctuating background

Conclusions

- Use IRC-safe algorithms!
- Jet-finding in pp at the LHC
  - SISCone and Cam+filt. do a slightly better job
  - strong R dependence: important to choose R_{best}
  - R_{best} increases with the scale
  - same for quark and gluon jets, larger R_{best} for gluons

  $\Rightarrow$ flexibility in jet finding at the LHC
- Subtraction using jet areas
  - Jet areas: clearly defined, analytic control
  - Simple systematic pileup subtraction
- Jet-finding in AA at RHIC and the LHC
  - First measurement at RHIC
  - Work under progress: improve subtraction down to 1-2 GeV