Progress in defining jets for the LHC

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arXiv:0704:0292, arXiv:0802:1188, arXiv:0802:1189, arXiv:0810.1304

Plan

- Foreword: why jets? what are they?

  introducing the basic terminology/concepts

- Part 1: building solid jet definitions

  new algorithms to meet the fundamental requirements

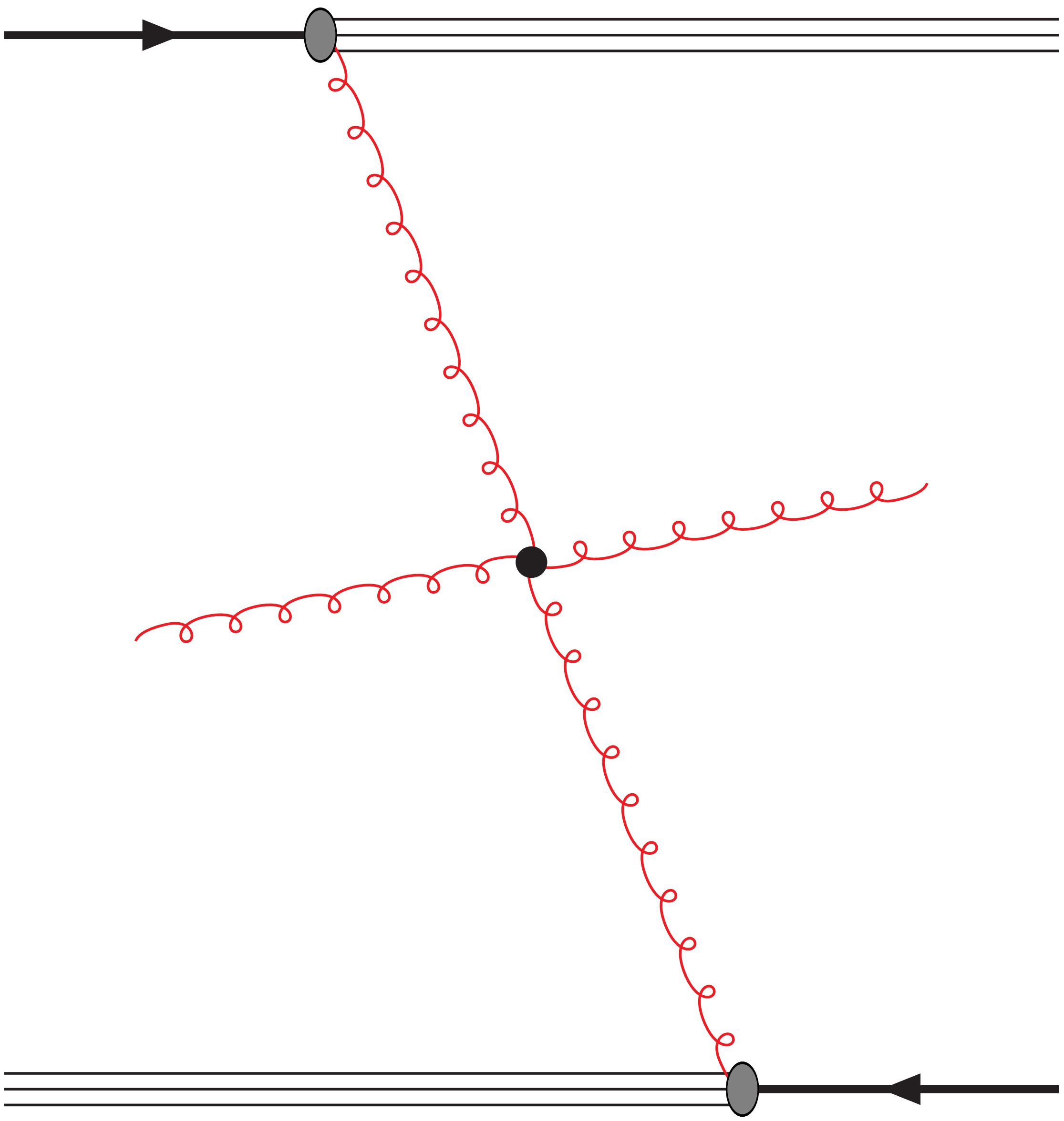
- Part 2: optimizing jets in pp collisions

  which jet algorithm is best suited?

  - how to quantify the reconstruction efficiency
  - Results without pileup
  - Results with pileup (subtraction)

Foreword: why jets? what are they?

General (over)simplified picture



Hard scattering ($2 \to n$)

computed exactly at $\mathcal{O}(\alpha_s^p)$

$gg \to gg$, $gg \to ggg$,
$gg \to gggg$,
$gg \to H \to b\bar{b}$,
$gg \to t\bar{t} \to \mu\nu_\mu b\bar{b}q\bar{q}$,
$gg \to Z' \to q\bar{q}$, ...

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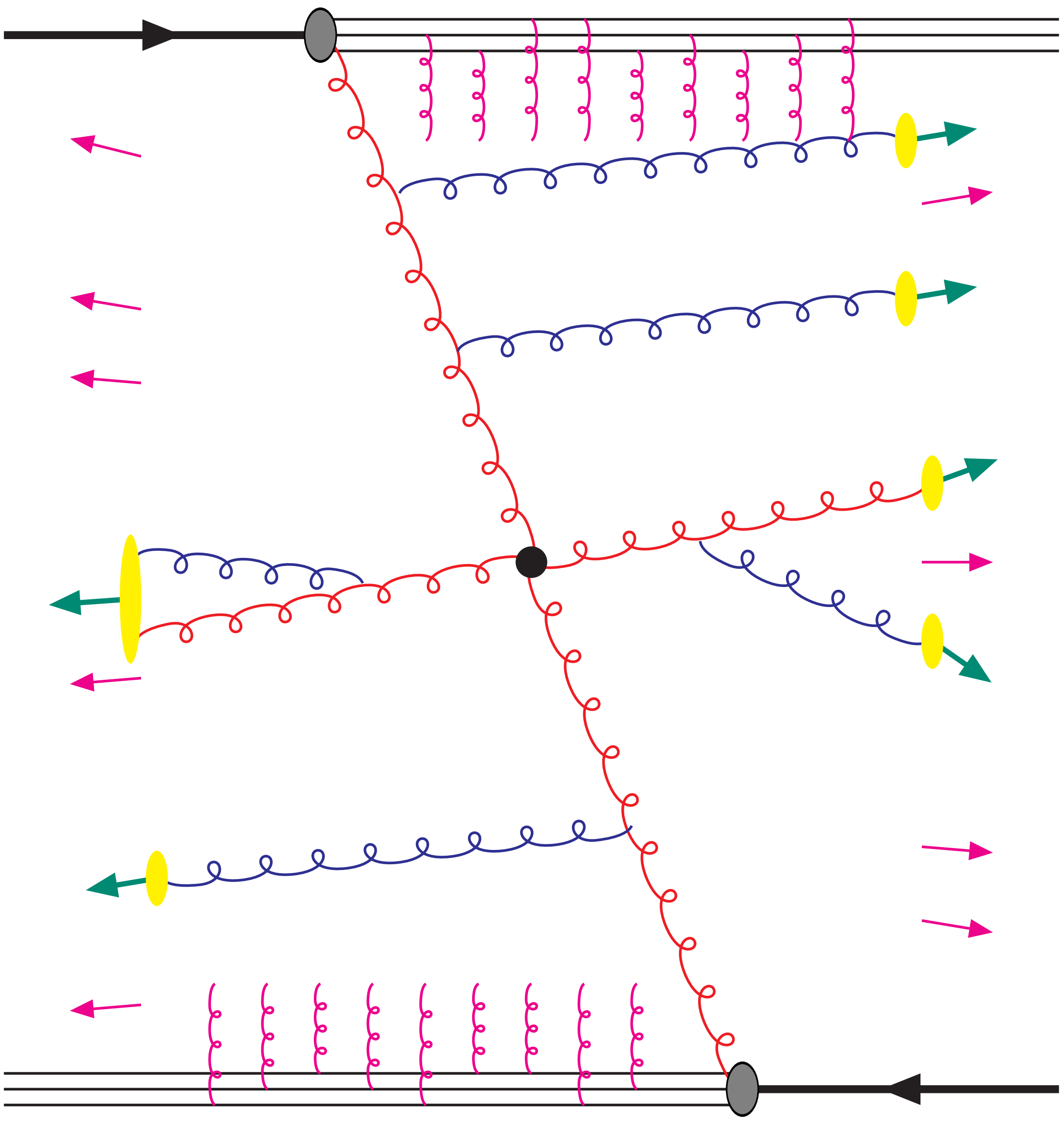

Hard scattering $(2 \to n)$

Parton level

$\approx$ resummed collinear div.

$\sum_i \alpha_s^i \log^i(p_t^2/\mu^2)$

Hadron level: hadronisation

Underlying event

beam remnants interactions

$\Rightarrow$ soft background

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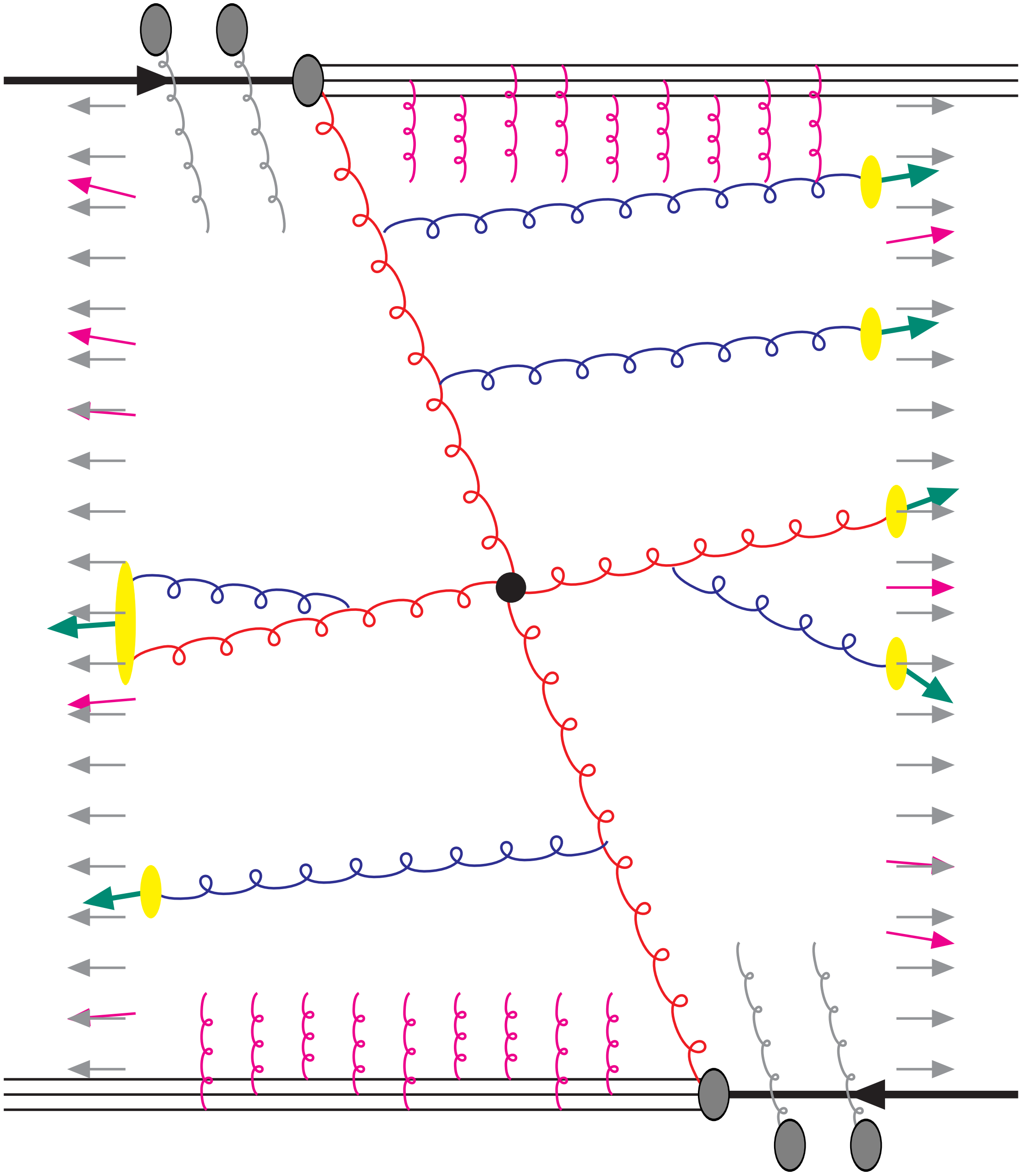

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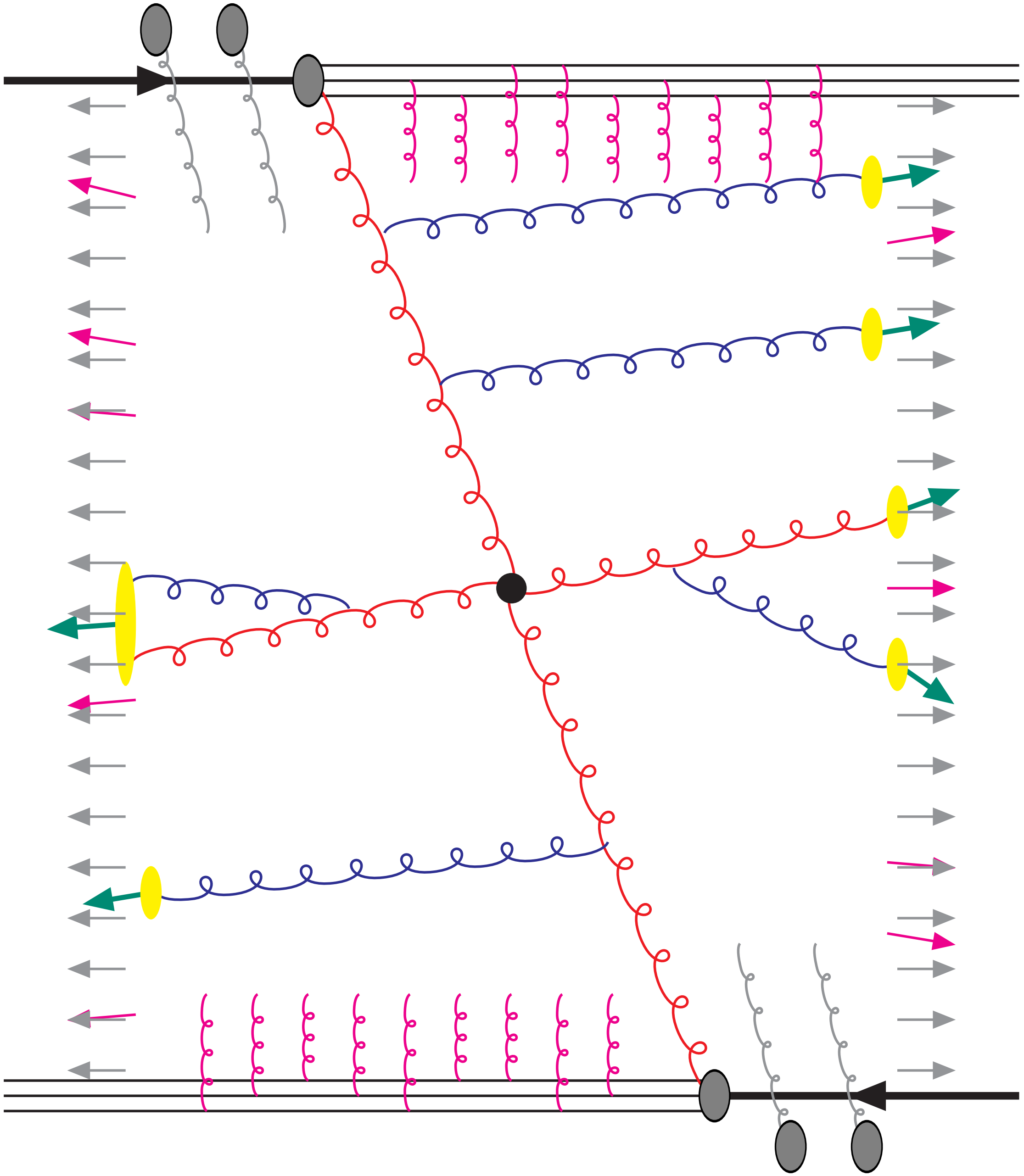

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"Jets" $\equiv$ hard partons

Parton ambiguous

$\Rightarrow$ multiple jet definitions

Two classes of algorithms

Class 1: recombination	Cass 2: cone
Successive recombinations of the "closest"$^{(a)}$ pair of particle	find directions of energy flow $\equiv$ stable cones$^{(b)}$
Nice perturbative behaviour	Small sensitivity to soft radiation (UE,PU)
Often used in $e^\pm e^\pm$, $e^\pm p$	Often used in pp

$^{(a)}$ Distance: (stop when $d_{\min} > R$)

$$k_t: \quad d_{i,j} = \min(k_{t,i}^2, k_{t,j}^2)(\Delta\phi_{i,j}^2 + \Delta y_{i,j}^2)$$

$$\text{Aachen/Cam.:} \quad d_{i,j} = \Delta\phi_{i,j}^2 + \Delta y_{i,j}^2$$

$^{(b)}$ **stable cones** (radius R) such that:
the total momentum of its contents points in the direction of its centre

How the cone works...

- Seeded (iterative) approaches: iterate from an initial position until stable
  - seed = initial particle
  - seed = midpoint between stable cones found at first step
- One has to deal with overlapping stable cones: 2 subclasses

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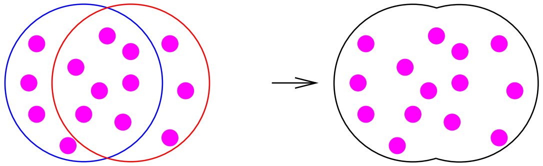
Class 2(a): cone with split-merge (ex.: JetClu, Atlas, MidPoint):

$\tilde{p}_{t,\text{shared}} > f\tilde{p}_{t,\text{min}}$



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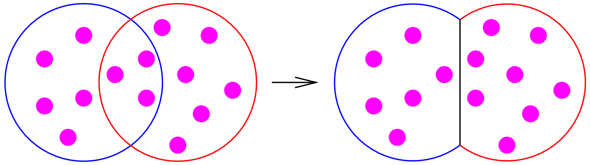

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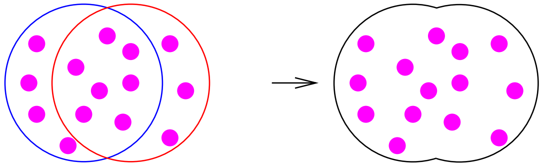

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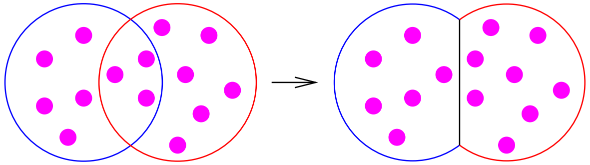


Class 2(b): cone with progressive removal (ex.: Iterative Cone)

- iterate from the hardest seed
- remove the stable cone as a jet and start again

Idea: “regular/circular” jets

20th century jet finders

Recombination:

- k_t algorithm
- Cambridge/Aachen alg.

Cone:

- CDF JetClu
- CDF MidPoint
- D0 (run II) Cone
- PxCone
- ATLAS Cone
- CMS Iterative Cone
- PyCell/CellJet
- GetJet

Part 1
21st century: towards a solid toolkit

SNOWMASS accords, Tevatron 1990 (i.e. old!):

Several important properties that should be met by a jet definition are [3]:

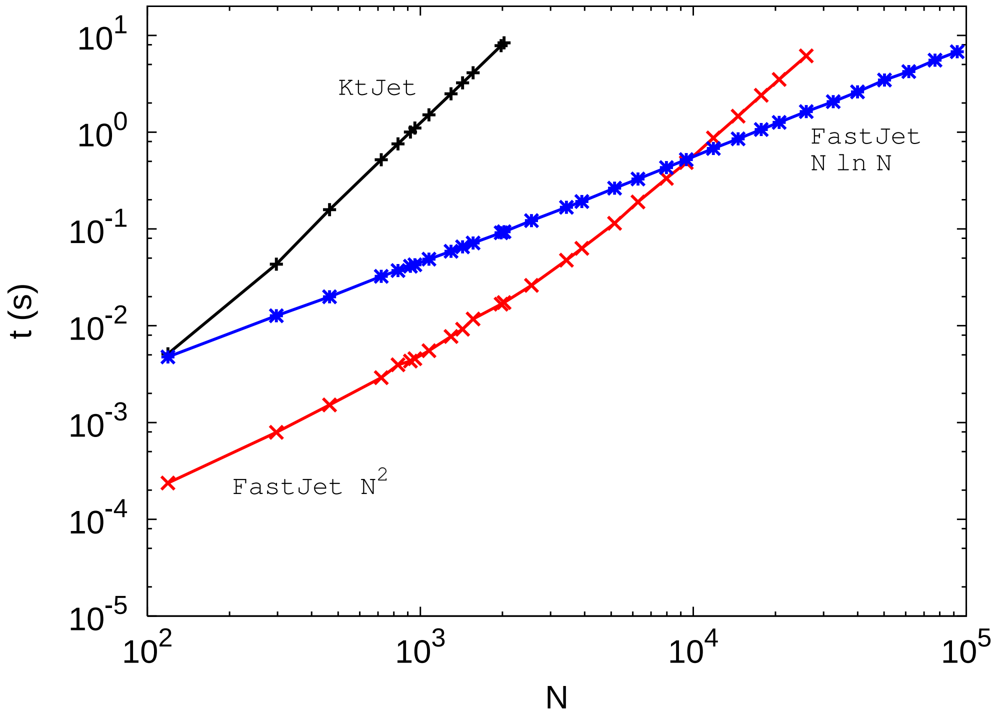
1. Simple to implement in an experimental analysis;
2. Simple to implement in the theoretical calculation;
3. Defined at any order of perturbation theory;
4. Yields finite cross section at any order of perturbation theory;
5. Yields a cross section that is relatively insensitive to hadronization.

i.e. usable by theoreticians (*e.g.* finite perturbative results)
and experimentalists (*e.g.* fast enough, not much UE sensitivity)

Speed improvement

[M. Cacciari, G. Salam, 06]

- Speeding up the k_t and Cam/Aachen algorithms
  using computational-geometry techniques: $\mathcal{O}\left(N^3\right) \to \mathcal{O}\left(N \log N\right)$



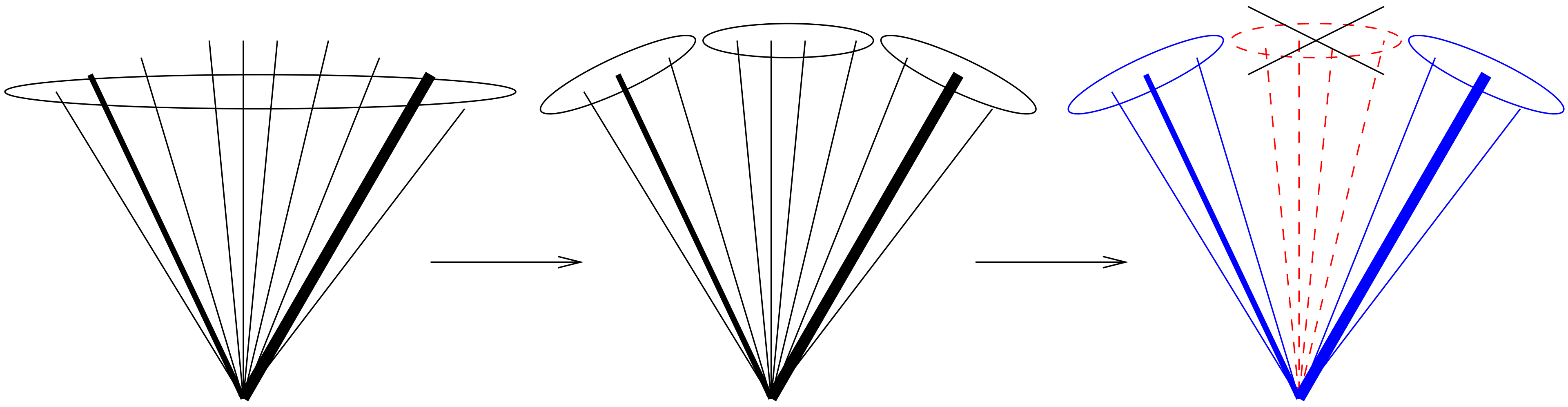
- C++ implementation in `FastJet`
  `http://www.fastjet.fr` (M. Cacciari, G. Salam, G.S.)

Filtering using jet substructure

More refined clustering ("2^{nd} generation of algorithms")

Cambridge+Filtering algorithm:

- Cluster with Aachen/Cambridge and radius R
- For each jet, recluster it with Aachen/Cambridge and radius R_{sub}
  keep only n_{sub} hardest sub-jets of the initial jet



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Aim: remove the soft background

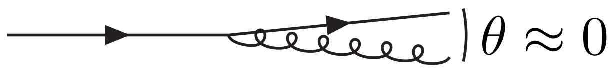
Properties:

- Proven to improve jet reconstruction, in $H \to b\bar{b}$
  [J.Butterworth, A.Davison, M.Rubin, G.Salam, 08]
- Additional parameters that deserve appropriate studies
- We will use the simplest choice: $R_{\text{sub}} = R/2$, $n_{\text{sub}} = 2$

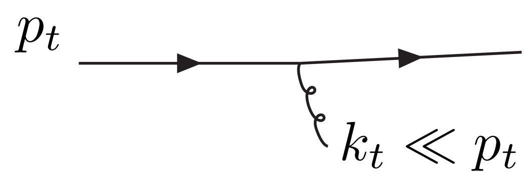
QCD probability for gluon bremsstrahlung at angle θ and $\perp$-mom. k_t:

$$dP \propto \alpha_s \frac{d\theta}{\theta} \frac{dk_t}{k_t}$$

Two divergences:



Collinear



Soft

QCD probability for gluon bremsstrahlung at angle θ and $\perp$-mom. k_t:

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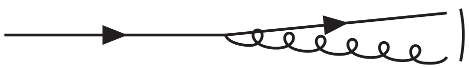

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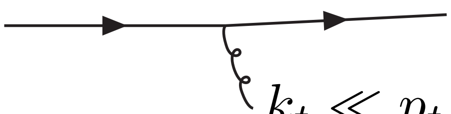
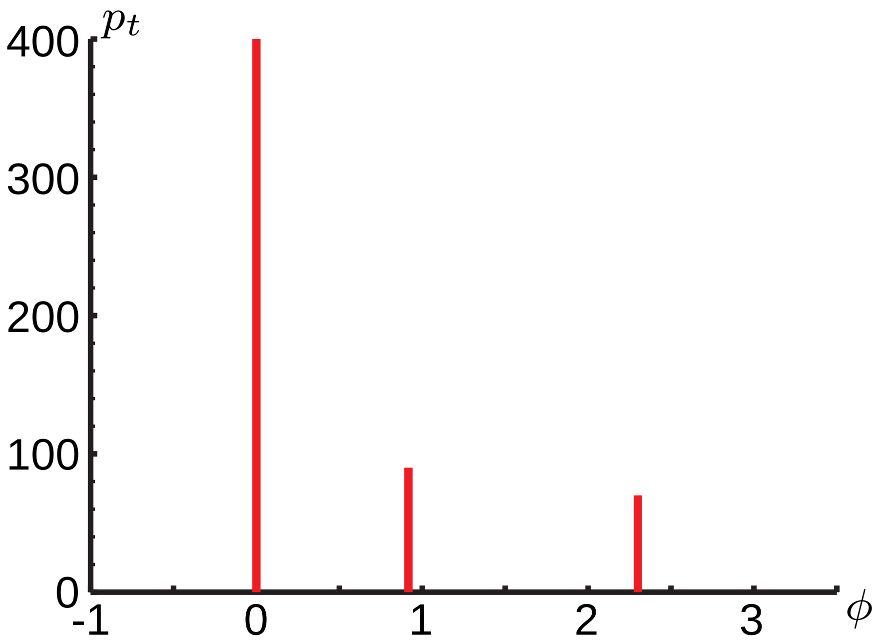


Soft

For pQCD to make sense, the (hard) jets should not change when

- one has a collinear splitting
  i.e. replaces one parton by two at the same place (η, ϕ)
- one has a soft emission *i.e.* adds a very soft gluon

IR unsafety of the Midpoint alg



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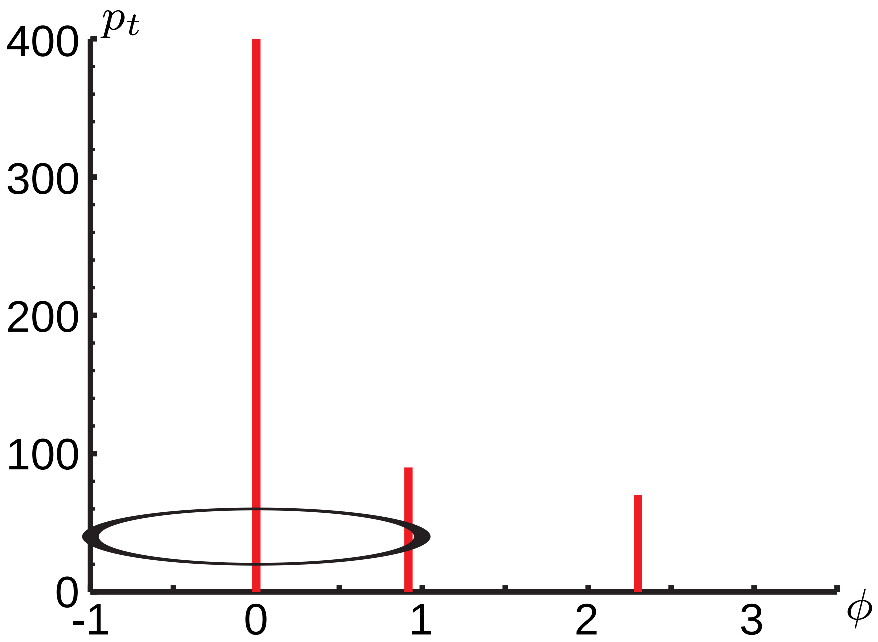

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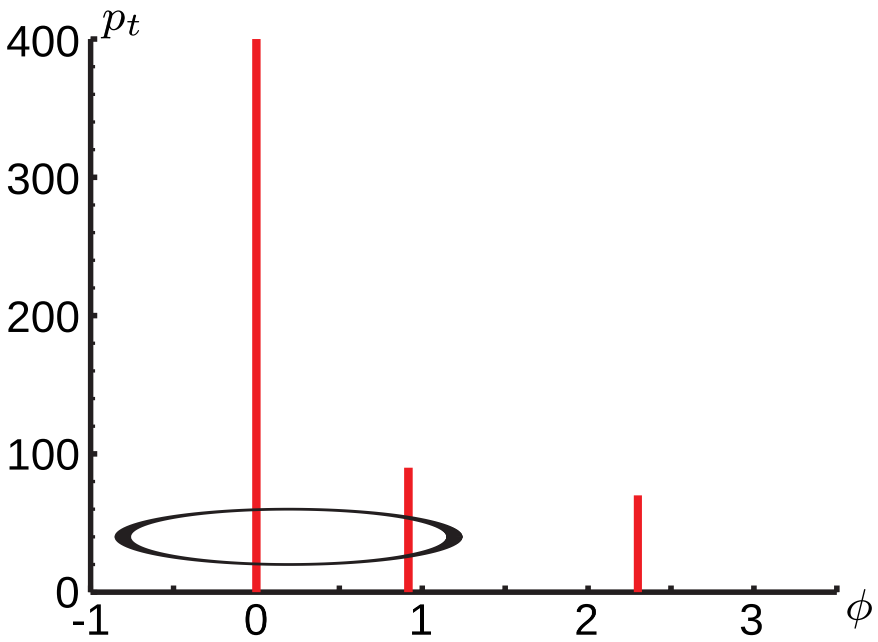

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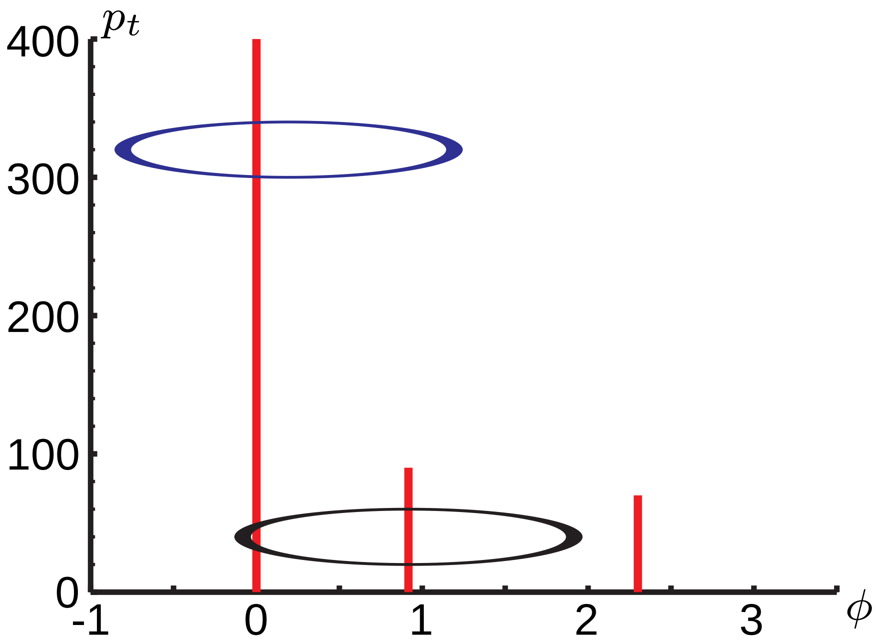

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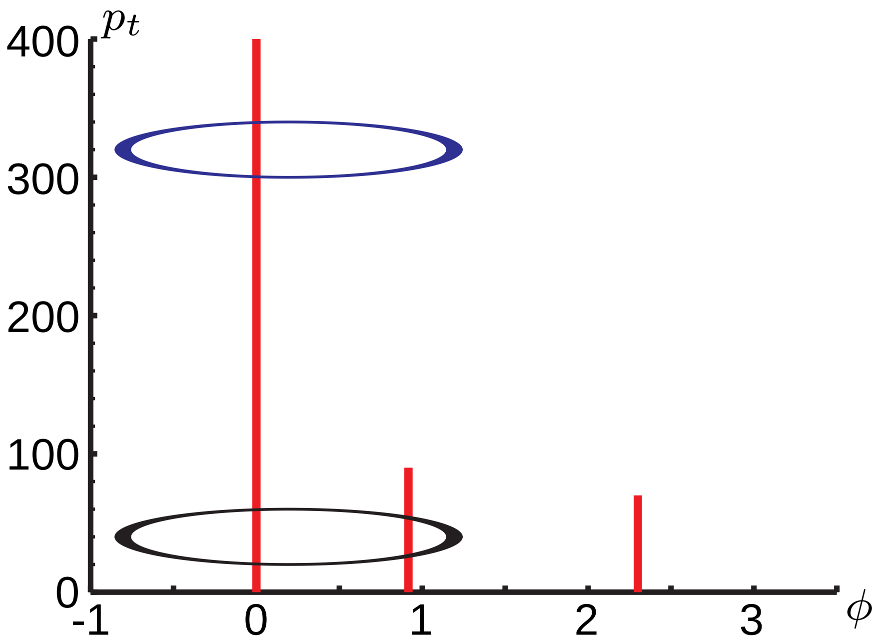

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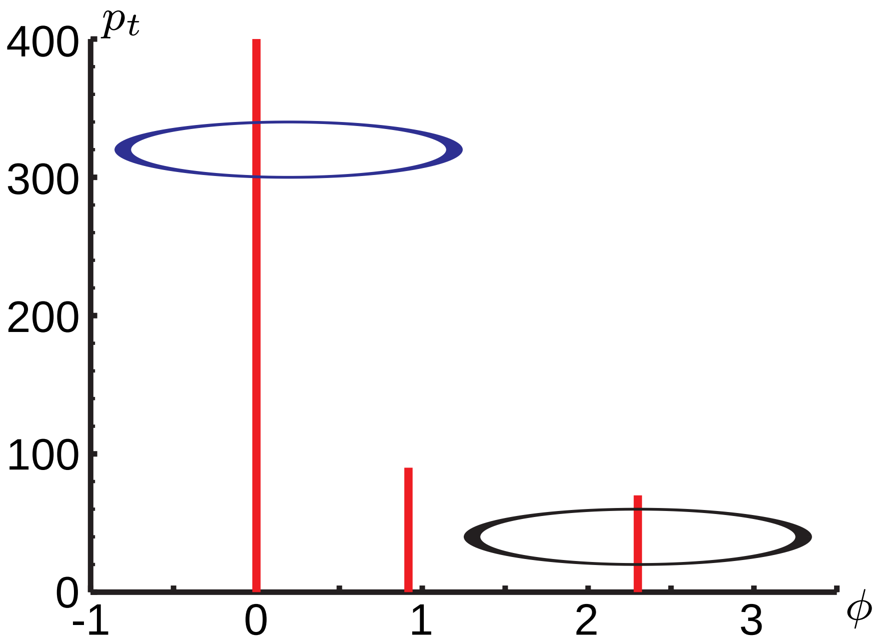

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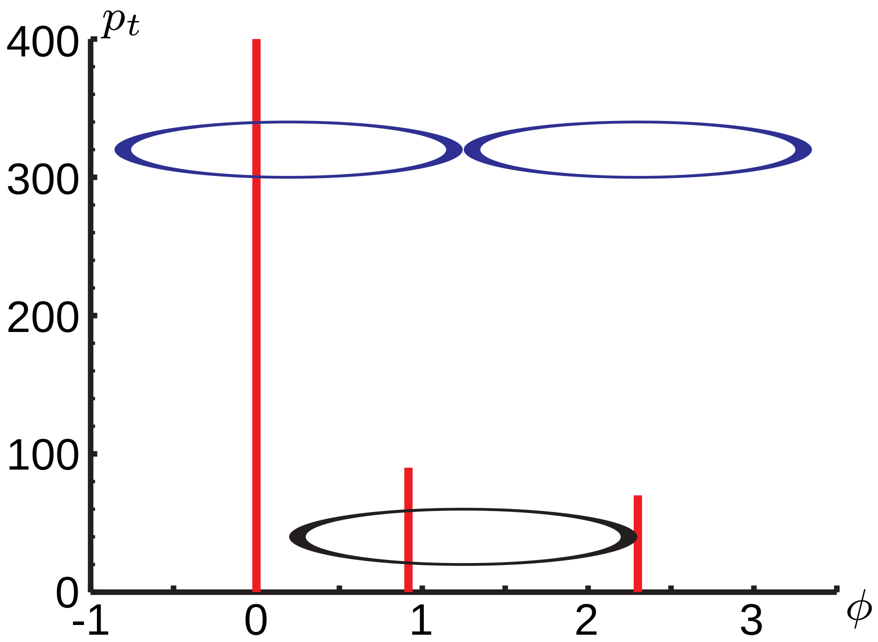

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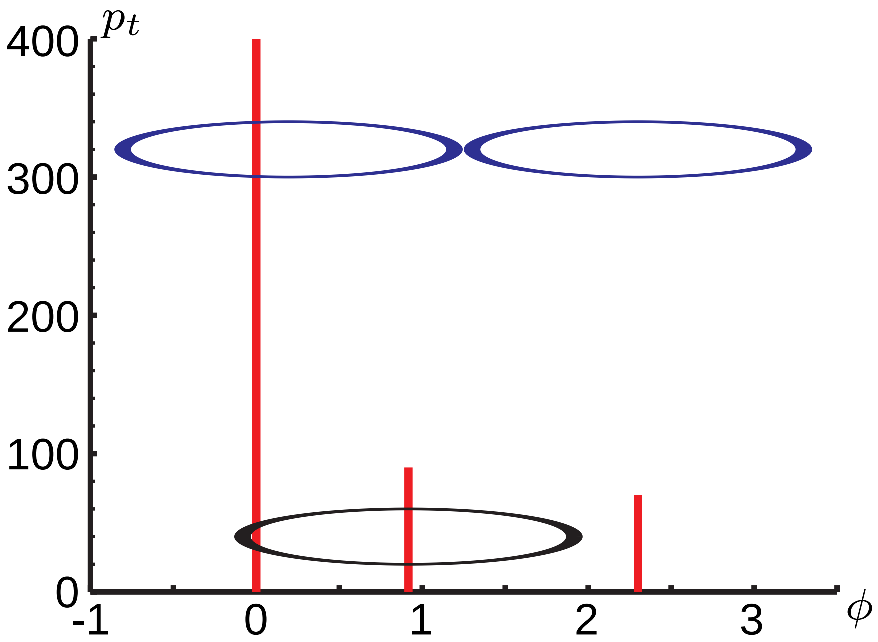

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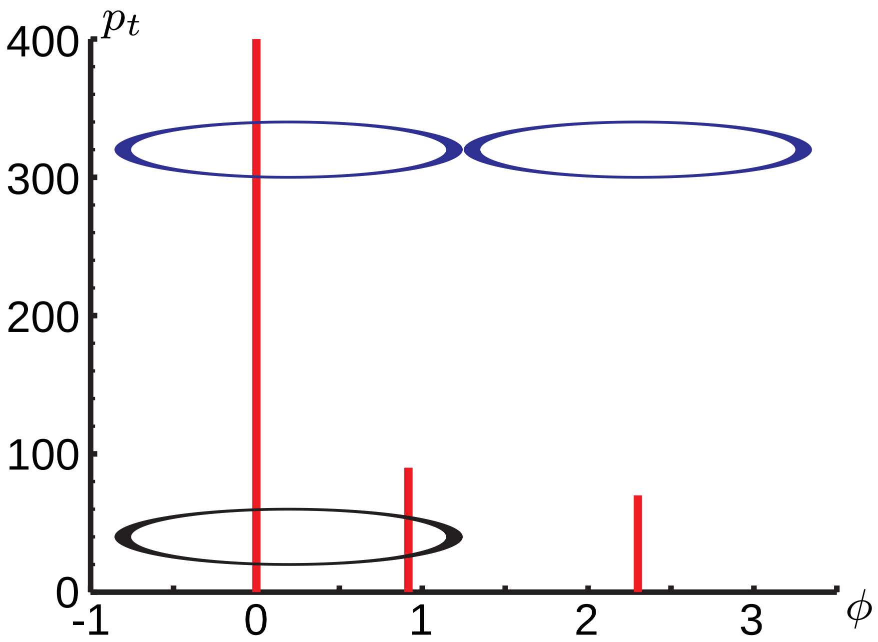

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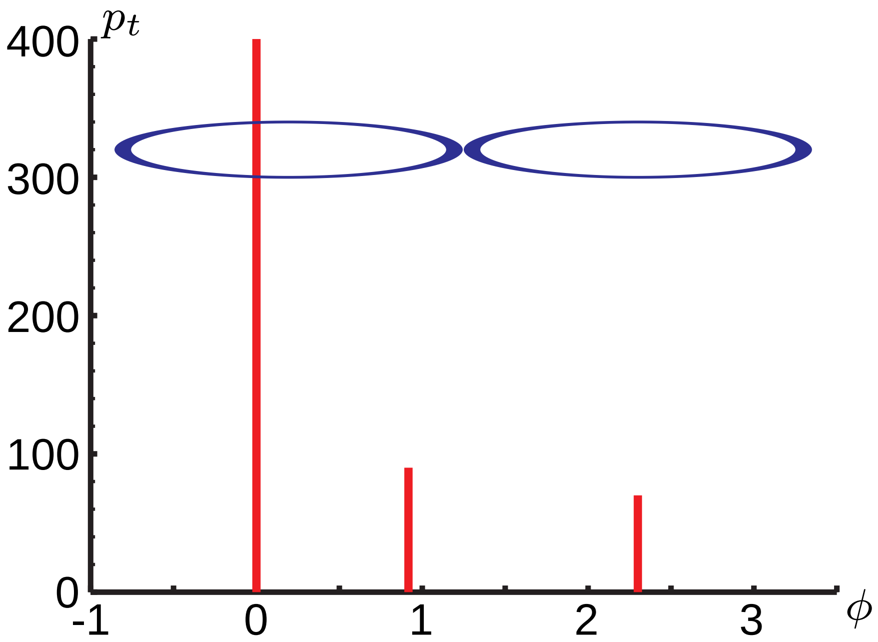

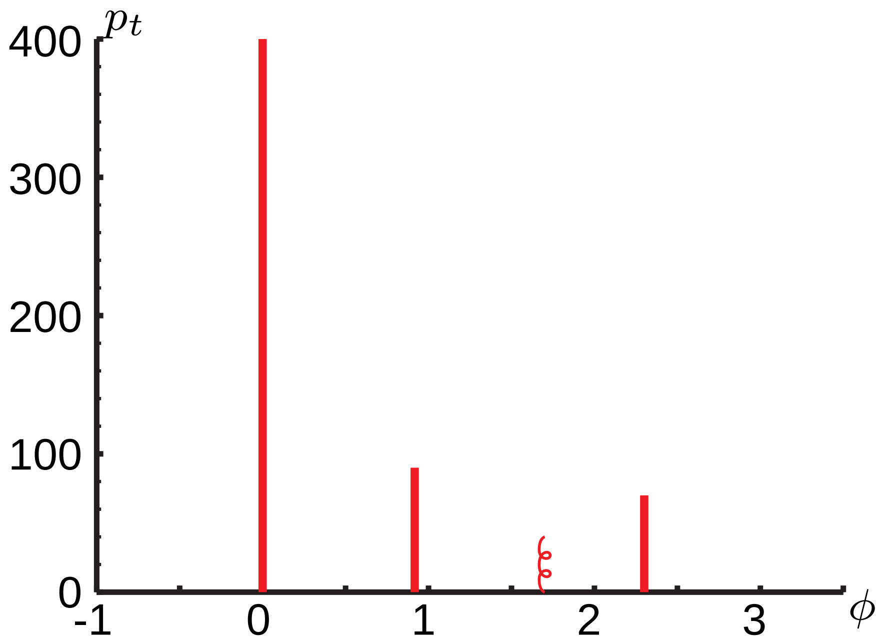

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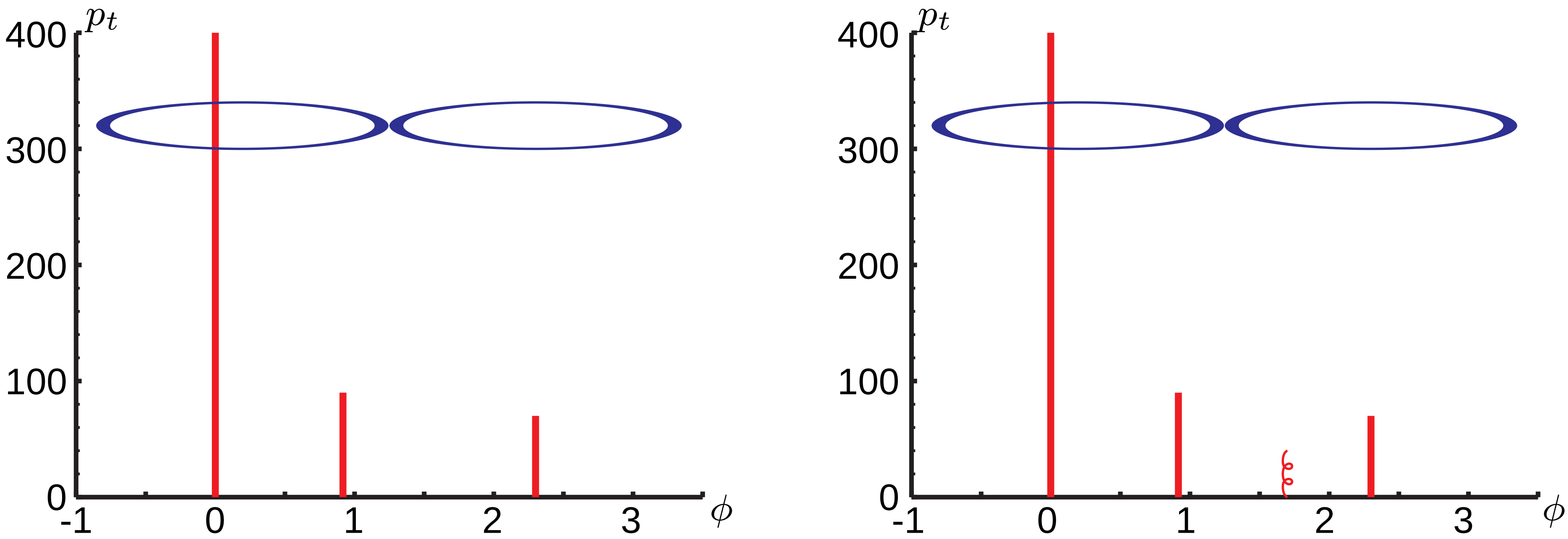

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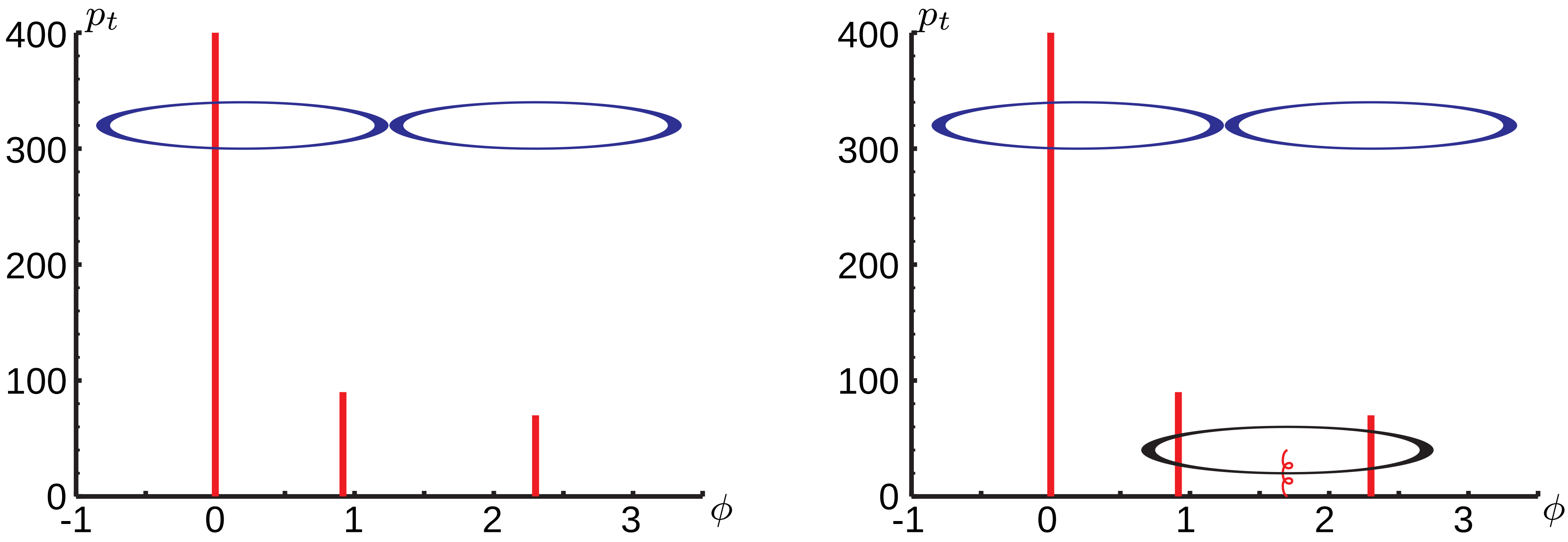

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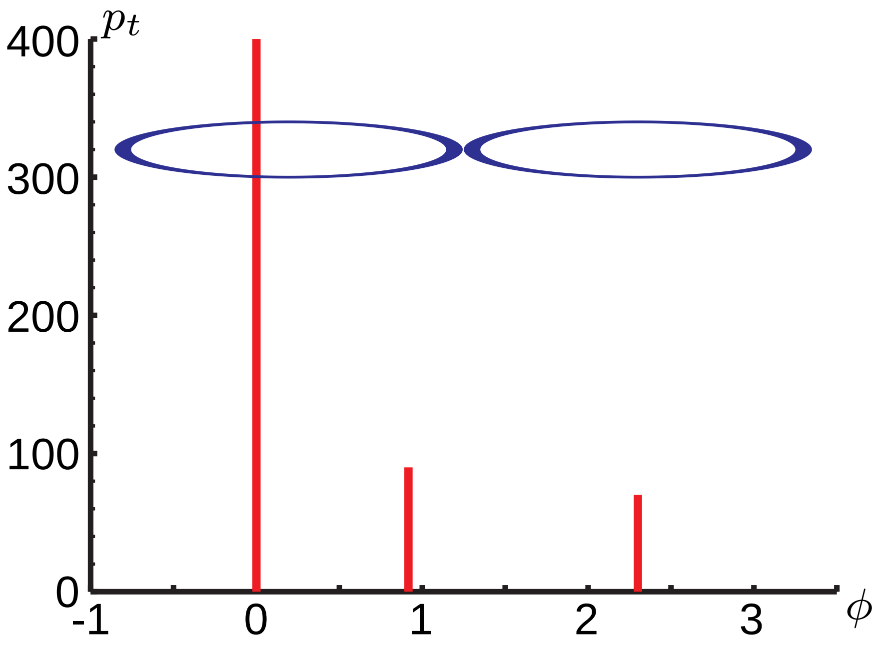

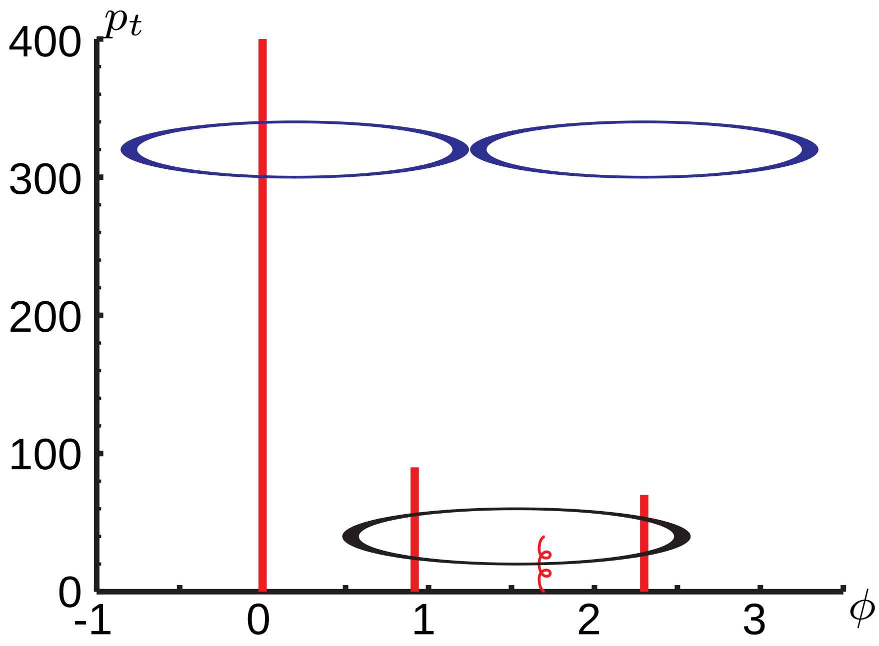

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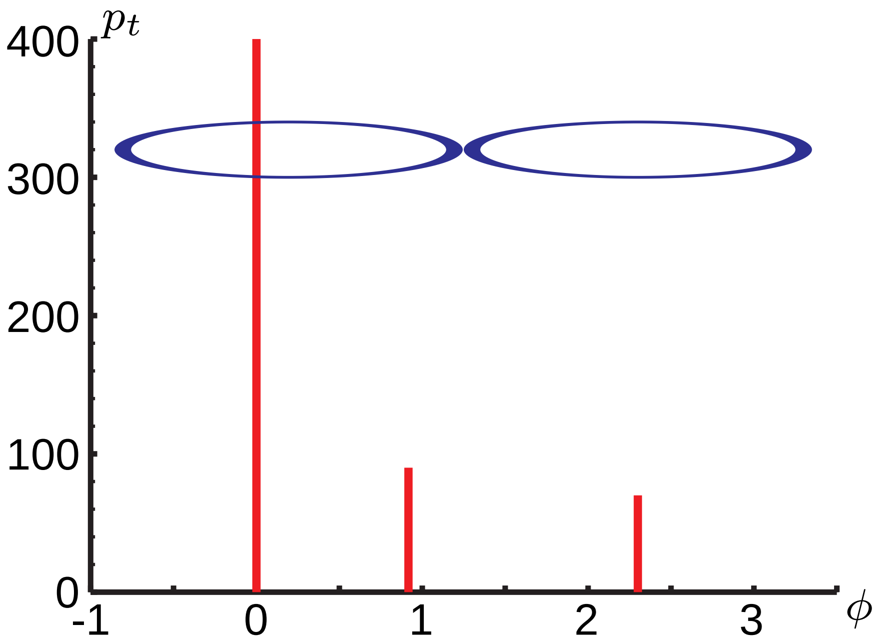
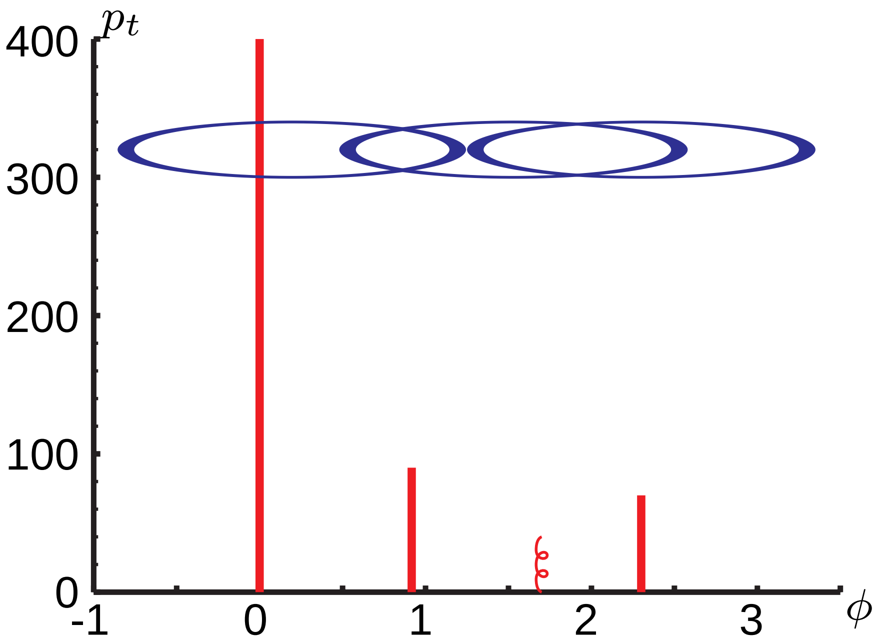
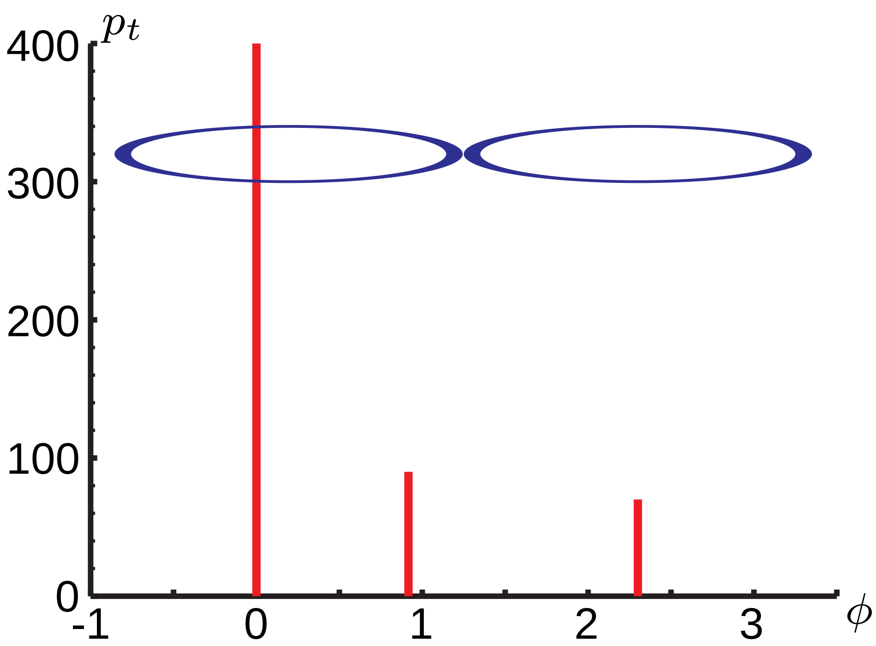
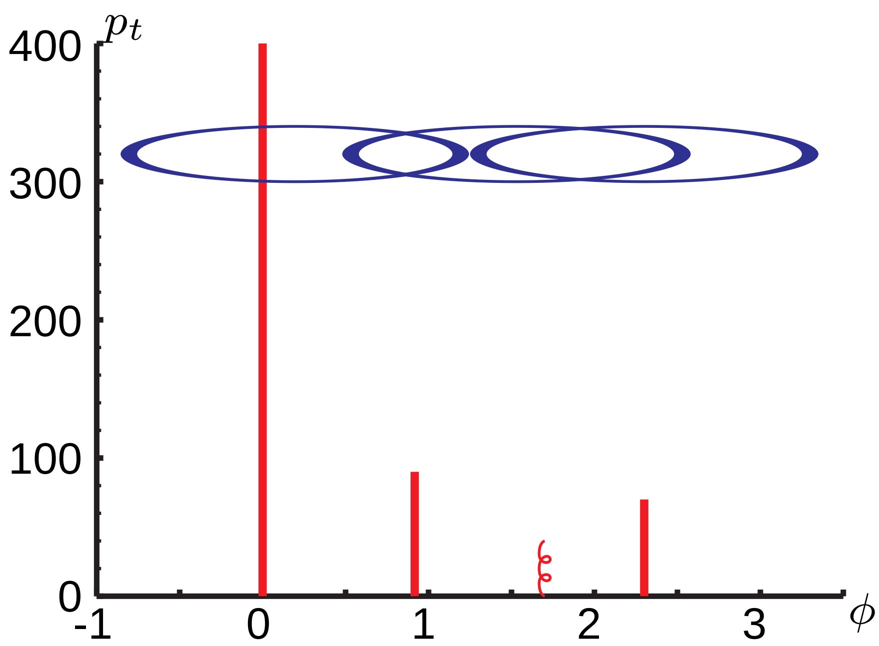




Stable cones:

Midpoint: {1,2} & {3} | {1,2} & {3} & {2,3}





Stable cones:

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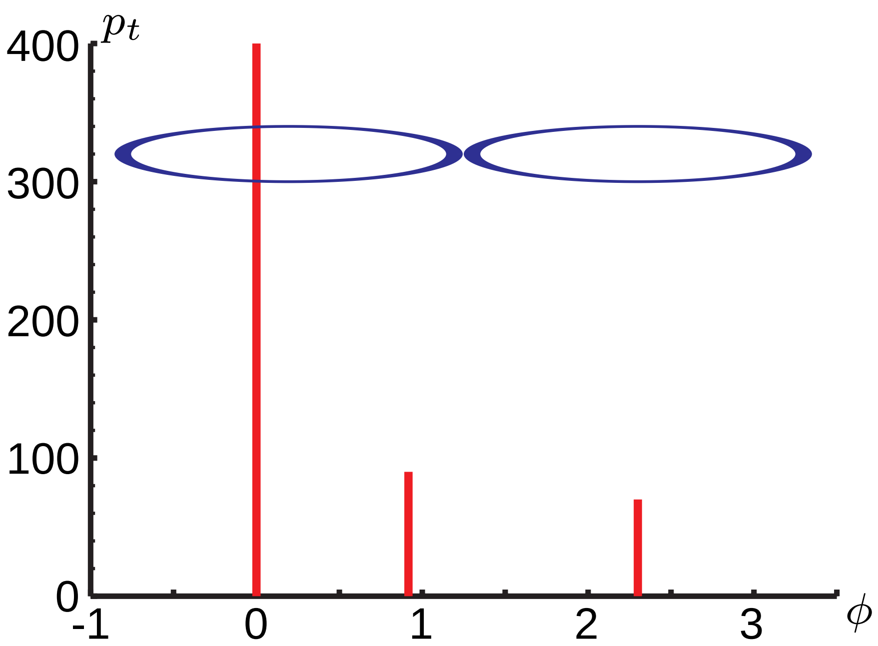
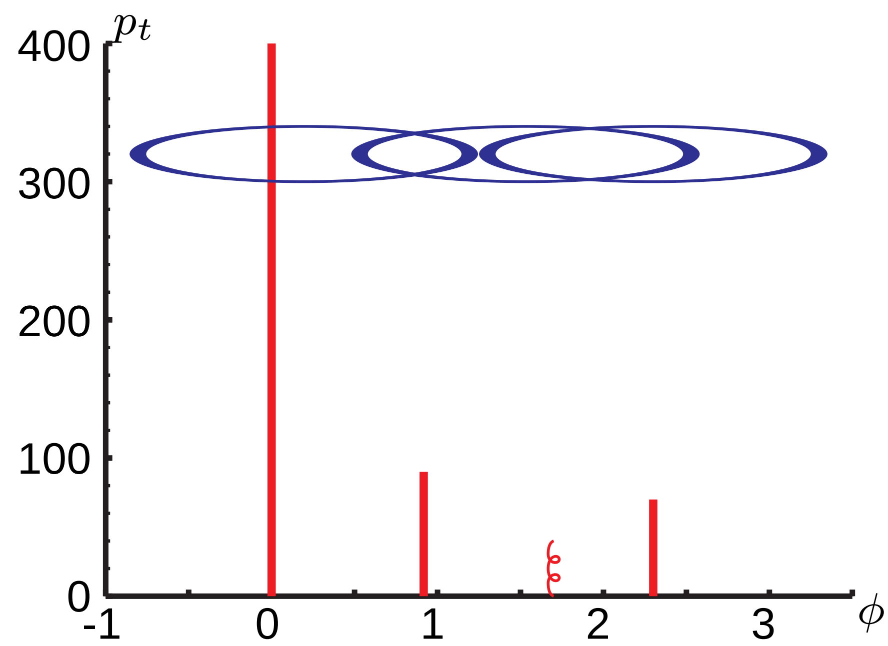
Jets: ($f = 0.5$)

Midpoint:	{1,2} & {3}	{1,2,3}

IR unsafety of the Midpoint alg





Stable cones:

Midpoint:	{1,2} & {3}	{1,2} & {3} & {2,3}
Seedless:	{1,2} & {3} & {2,3}	{1,2} & {3} & {2,3}

Jets: ($f = 0.5$)

Midpoint:	{1,2} & {3}	{1,2,3}
Seedless:	{1,2,3}	{1,2,3}

Stable cone missed $\longrightarrow$ IR unsafety of the midpoint algorithm

Solution: SISCone

- Solution: use a seedless approach, find ALL stable cones
- Naive approach: check stability of each subset of particle

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- Naive approach: check stability of each subset of particle
  Complexity is $\mathcal{O}\left(N2^N\right)$
  $\Rightarrow$ definitely unrealistic: 10^{17} years for $N = 100$
- Midpoint complexity: $\mathcal{O}\left(N^3\right)$

Solution: SISCone

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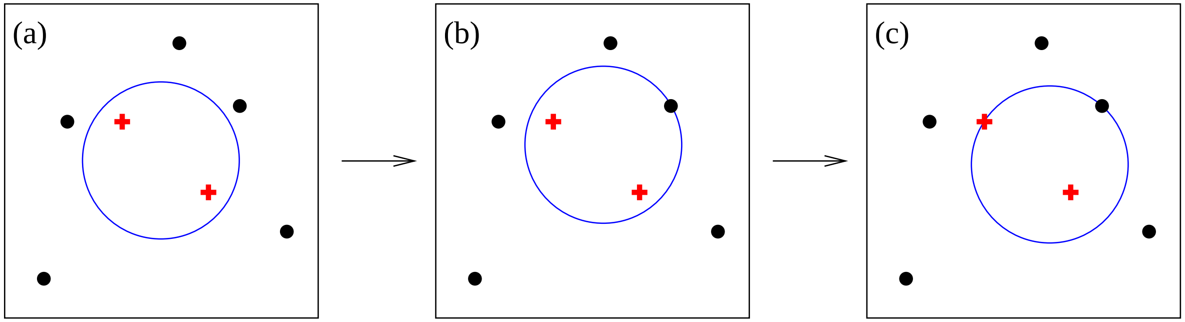
- Midpoint complexity: $\mathcal{O}\left(N^3\right)$

Idea: use geometric arguments



- Each enclosure can be moved (in any direction) until it touches a point
- ... then rotated until it touches a second one

> $\Rightarrow$ Enumerate all pairs of particles
> with 2 circle orientations and 4 possible inclusion/exclusion
> $\longrightarrow$ find all enclosures

Solution: SISCone

- Solution: use a seedless approach, find ALL stable cones
- Midpoint complexity: $\mathcal{O}\left(N^3\right)$

Idea: use geometric arguments

$\Rightarrow$ Enumerate all pairs of particles
with 2 circle orientations and 4 possible inclusion/exclusion
$\longrightarrow$ find all enclosures

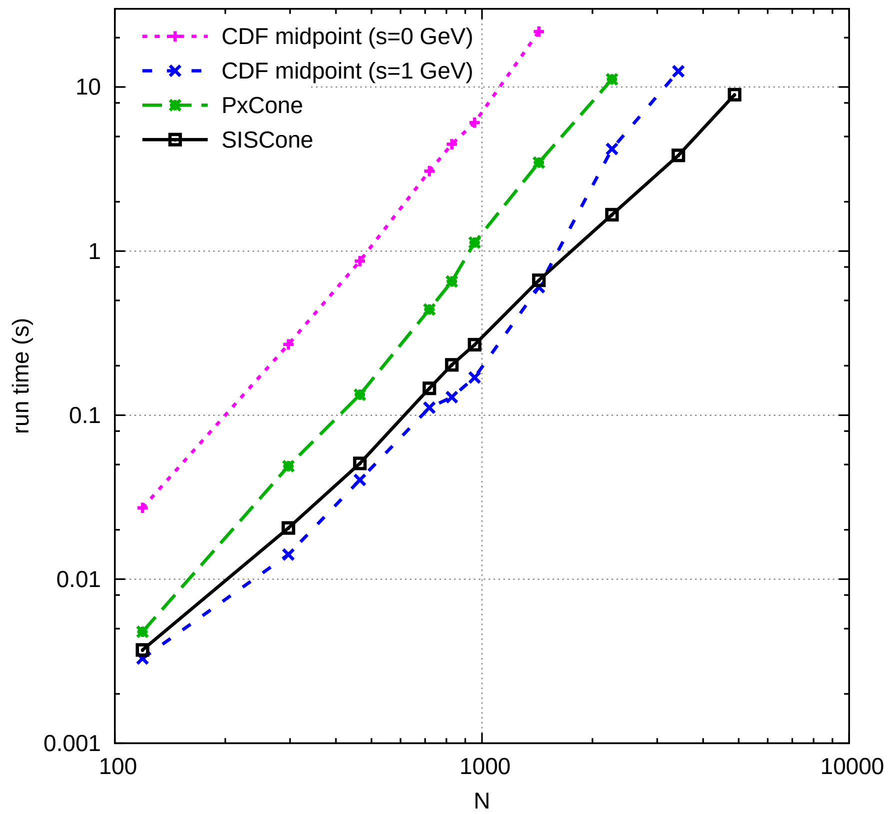
- Complexity: $\mathcal{O}\left(N^3\right)$, with improvements: $\mathcal{O}\left(N^2\log(N)\right)$

$\longrightarrow$ C++ implementation: Seedless Infrared-Safe Cone algorithm (SISCone)
G.Salam, G.S., JHEP 04 (2007) 086; http://projects.hepforge.org/siscone

NB.: also available from FastJet

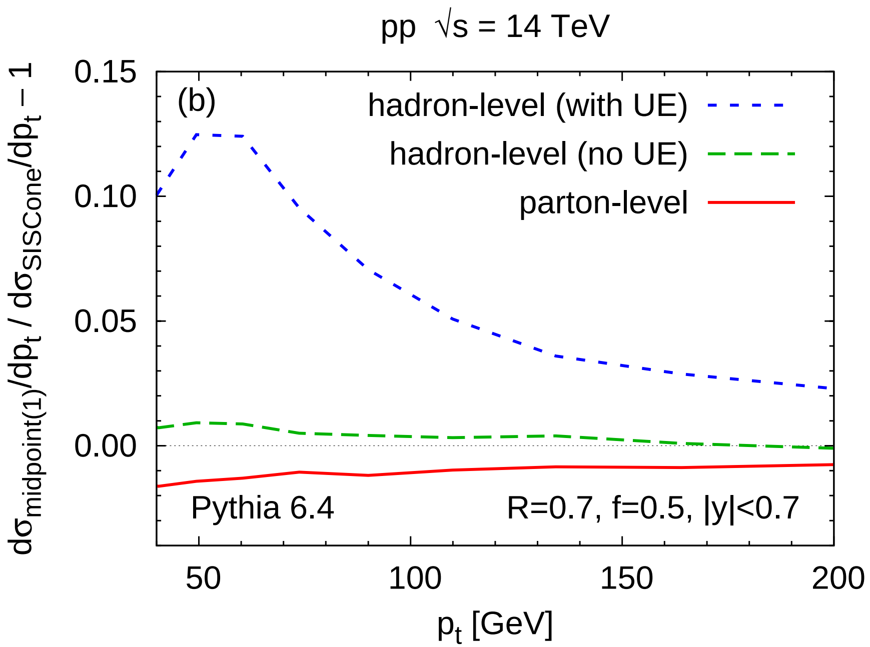
Physical impact

Execution timings:



- faster than midpoint without seed threshold
- at least as fast as as midpoint with seed threshols

(Midpoint-SISCone)/SISCone

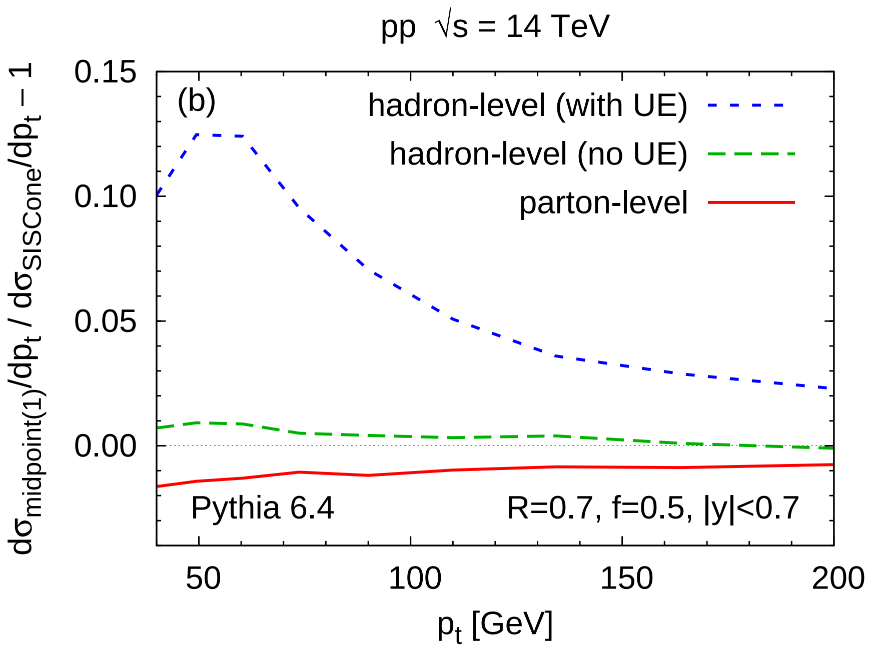


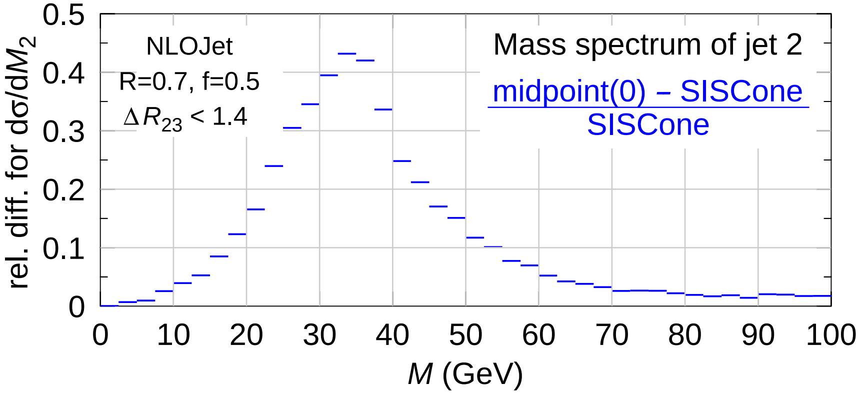
Inclusive cross-section:

- effect of a few %
- less UE sensitivity

Physical impact (2)

(Midpoint-SISCone)/SISCone





Inclusive cross-section:

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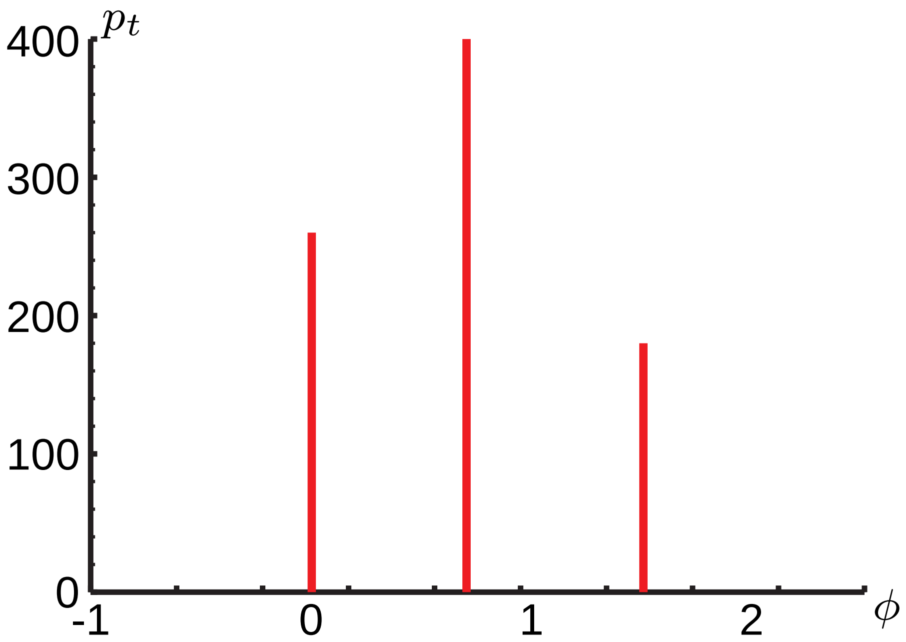
Masses in 3-jet events:

- effects $\sim 45\%$
- Important for LHC!

Coll. unsafety of the iterative cone



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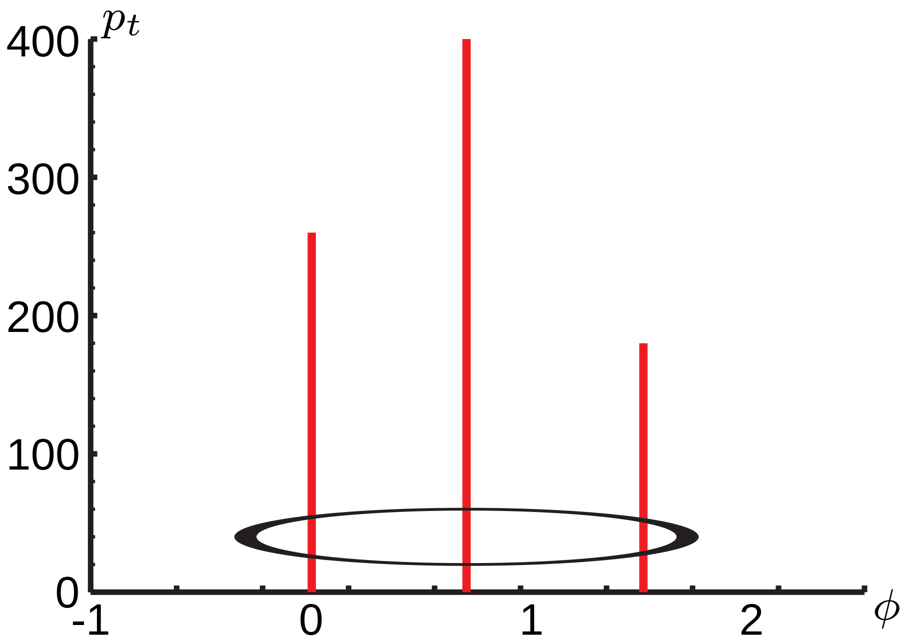

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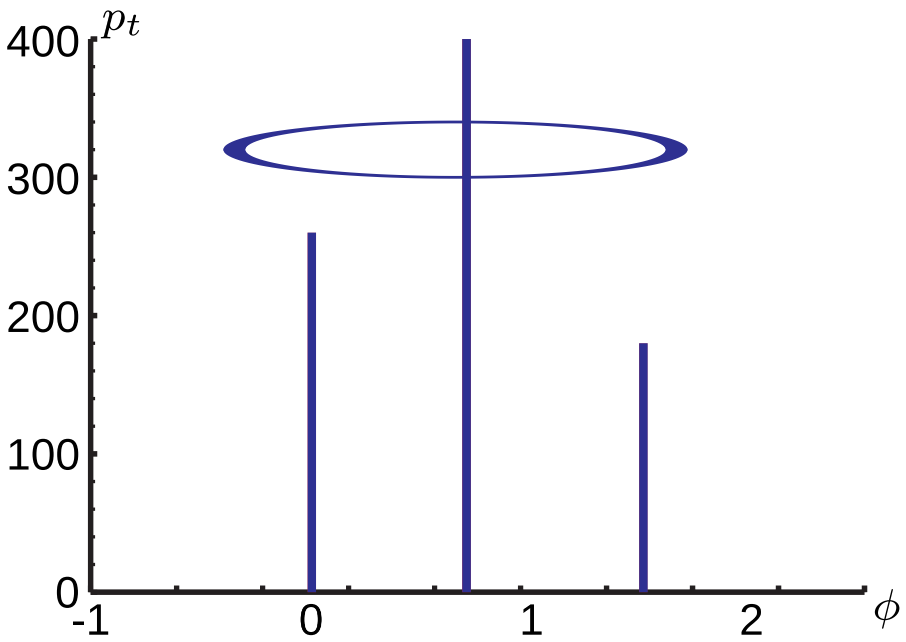

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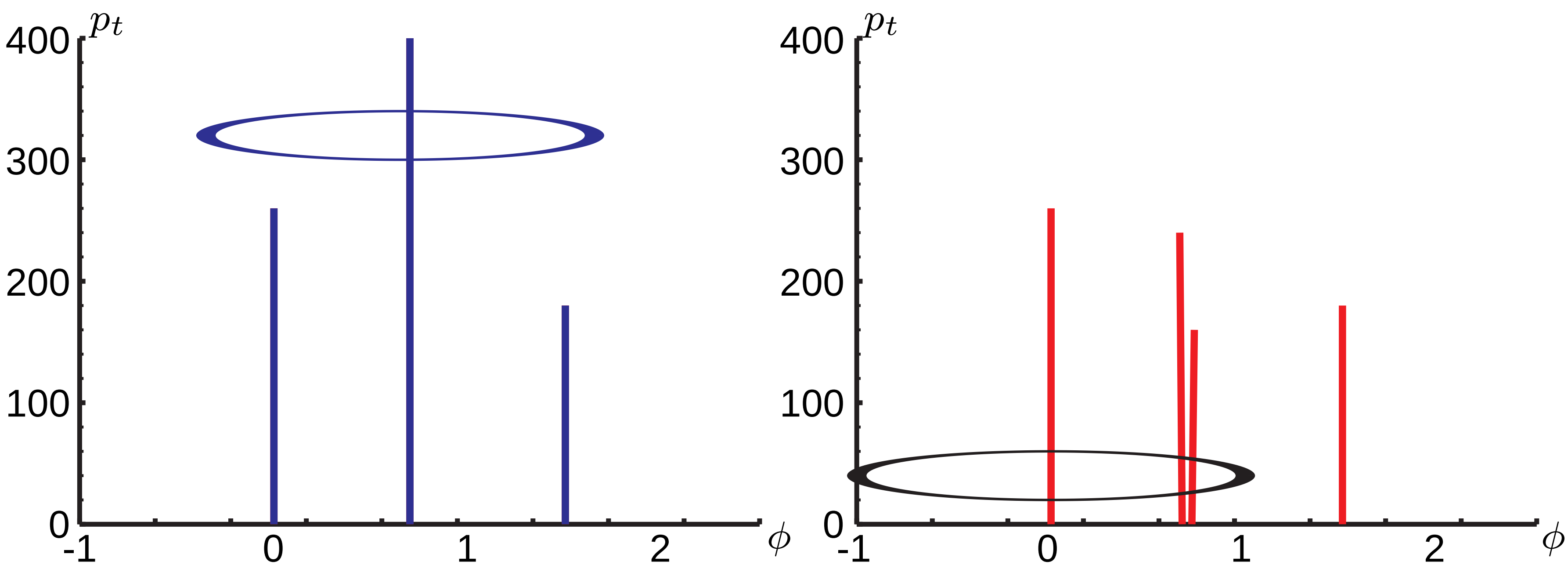

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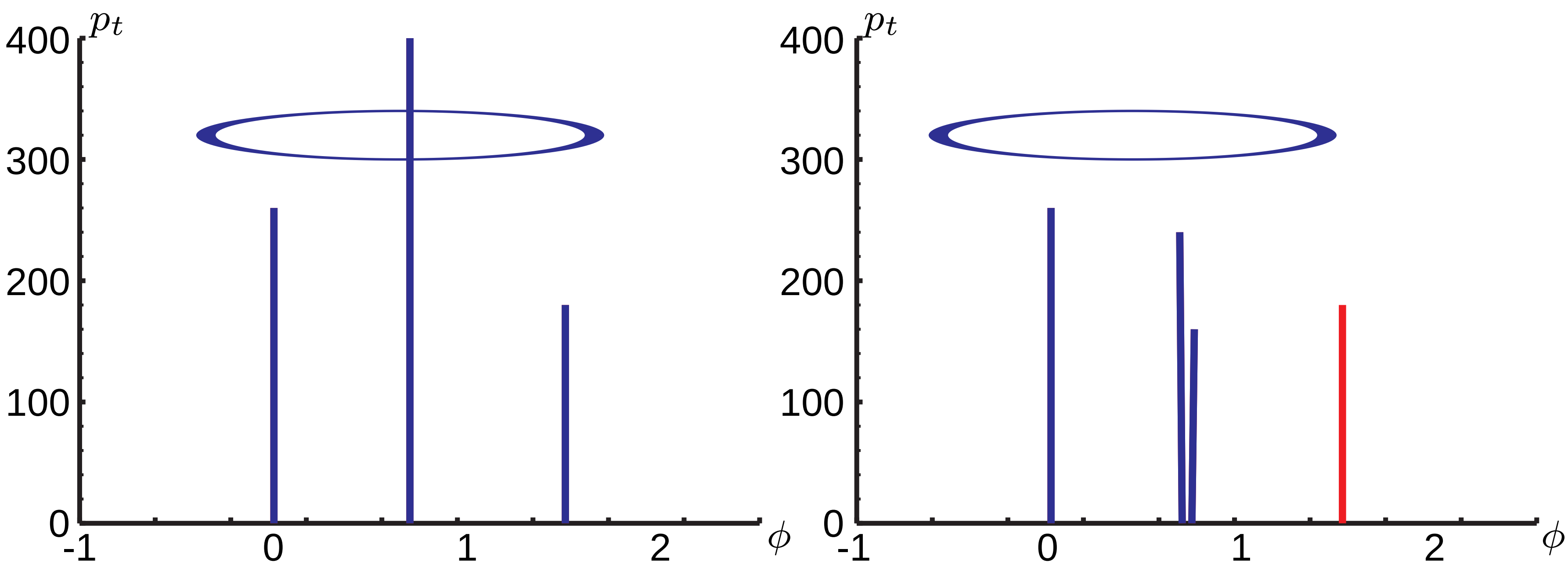

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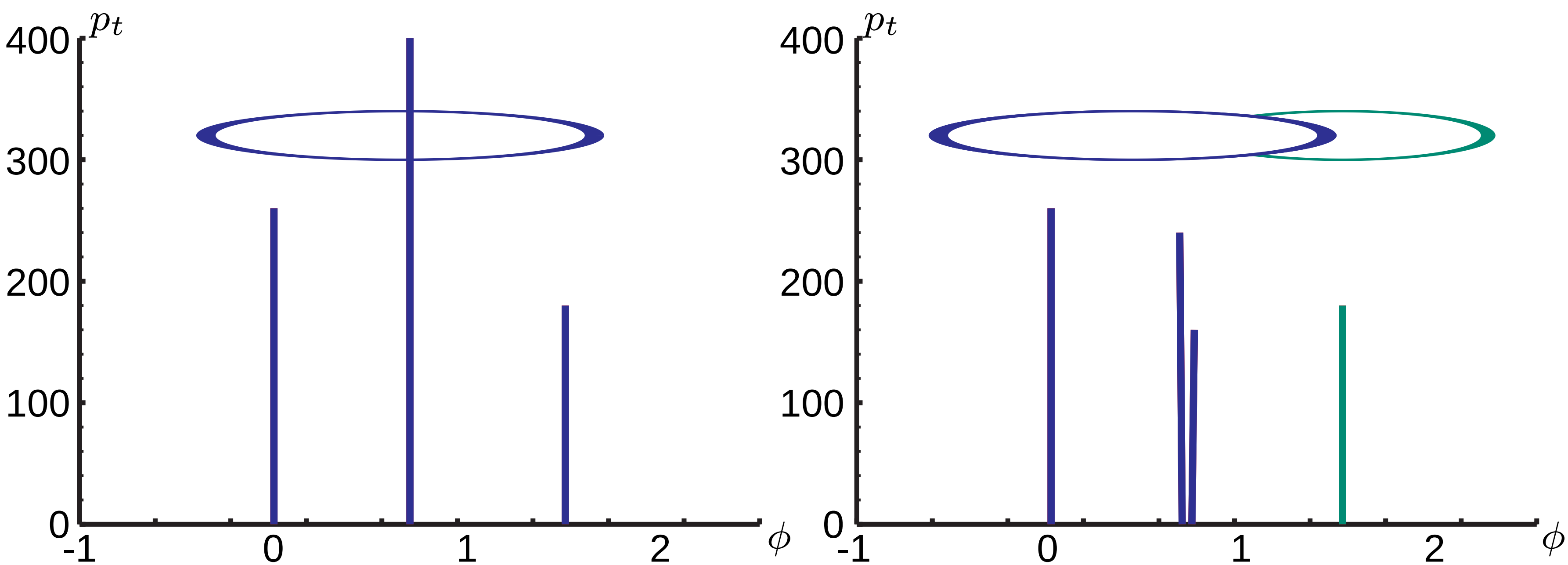

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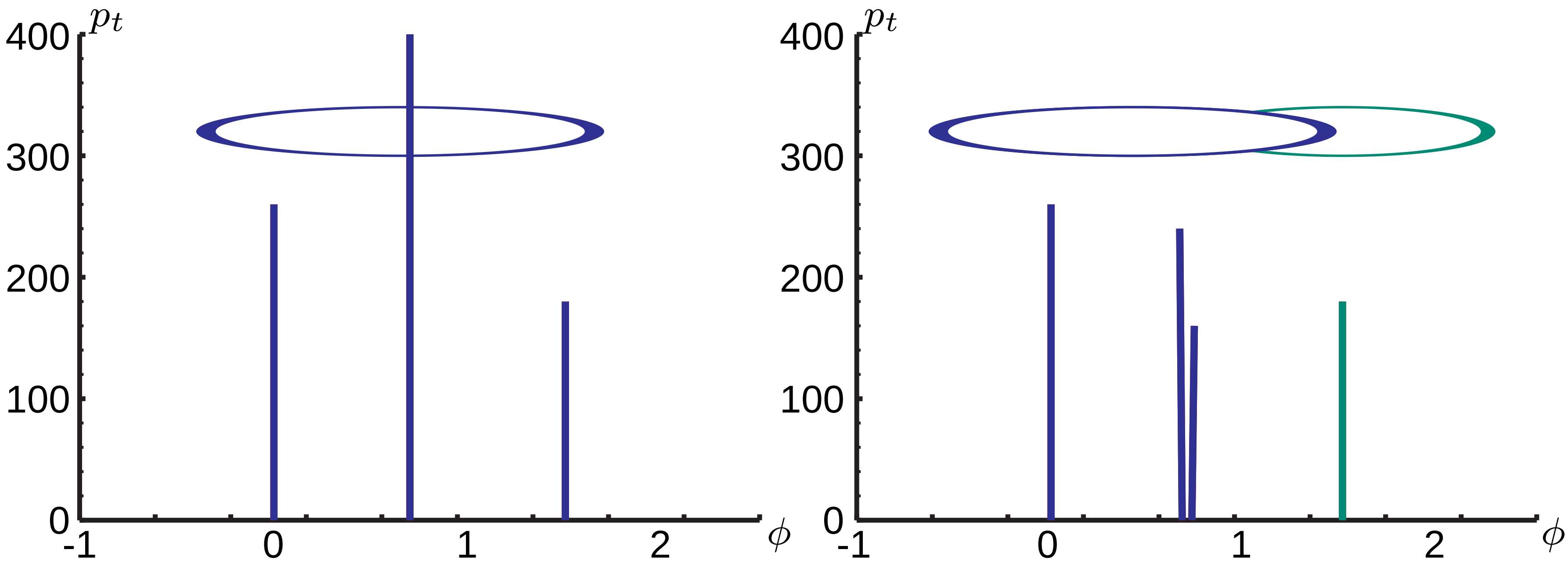

- Before collinear spliting: 1 jet
- After collinear spliting: 2 jets

$\longrightarrow$ collinear unsafety of the iterative cone algorithm

Come back to recombination-type algorithms:

$$d_{ij} = \min(k_{t,i}^{2p}, k_{t,j}^{2p})\ \left(\Delta\phi_{ij}^2 + \Delta\eta_{ij}^2\right)$$

- $p = 1$: k_t algorithm
- $p = 0$: Aachen/Cambridge algorithm

Anti-k_t

Come back to recombination-type algorithms:

$$d_{ij} = \min(k_{t,i}^{2p}, k_{t,j}^{2p}) \; (\Delta\phi_{ij}^2 + \Delta\eta_{ij}^2)$$

- $p = 1$: k_t algorithm
- $p = 0$: Aachen/Cambridge algorithm
- $p = -1$: anti-k_t algorithm [M.Cacciari, G.Salam, G.S.,JHEP 04 (08) 063]

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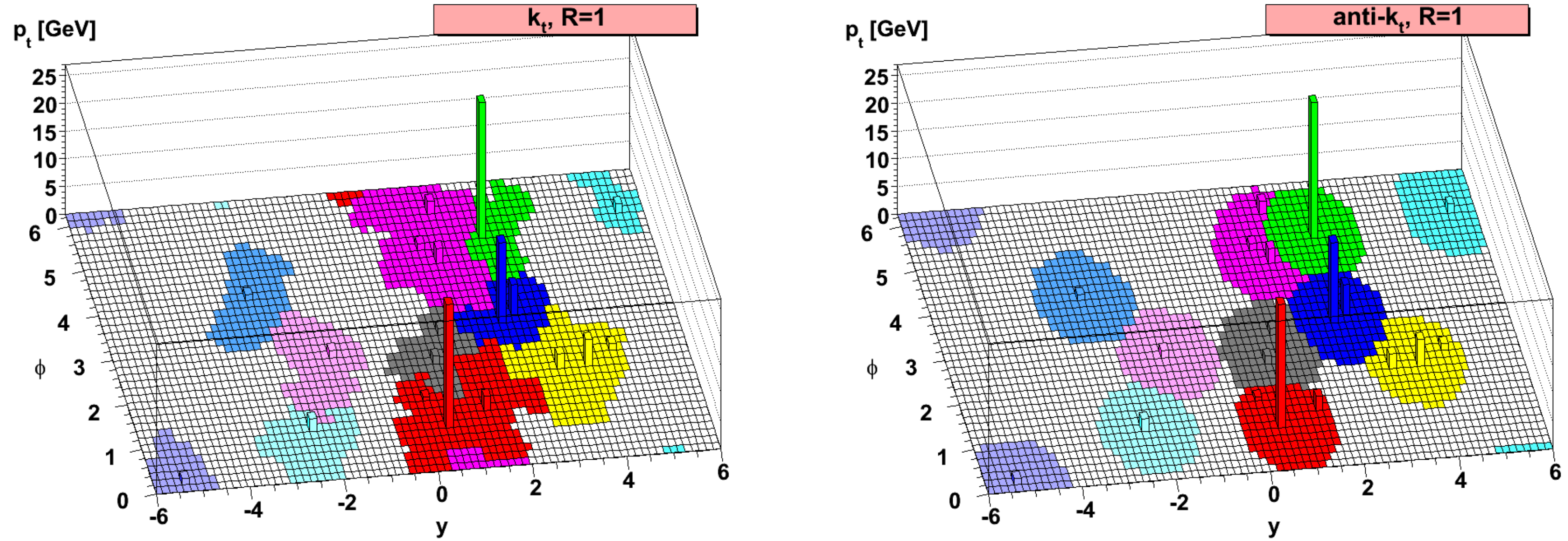
Why should that be related to the iterative cone ?!?

- "large $k_t \Rightarrow$ small distance"
  i.e. hard partons "eat" everything up to a distance R
  i.e. circular/regular jets, jet borders unmodified by soft radiation
- infrared and collinear safe

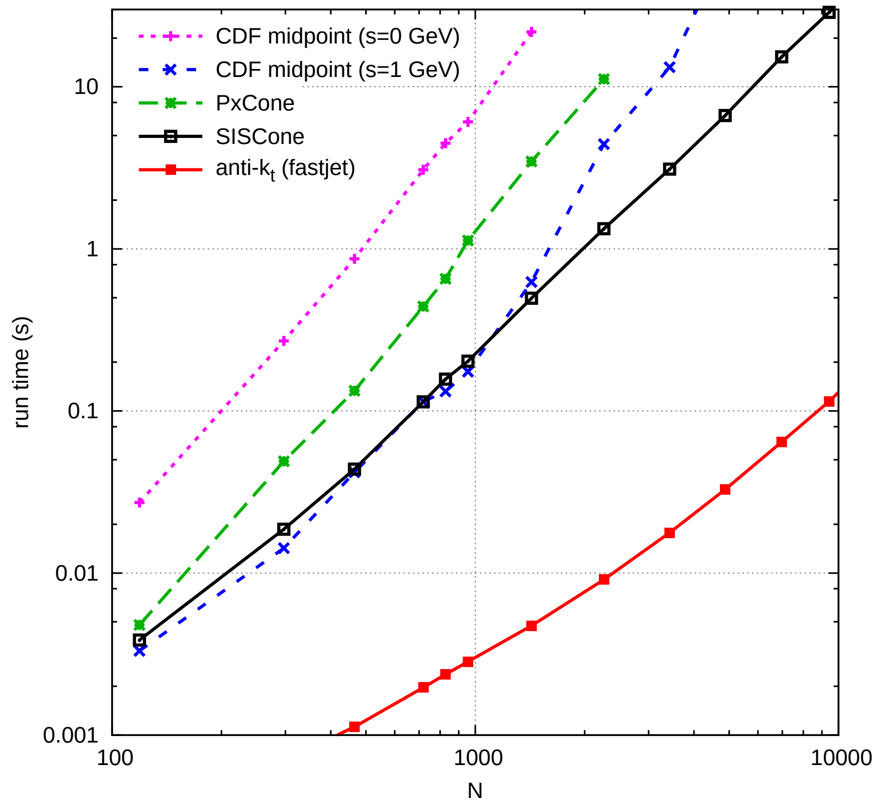
Hard event + homogeneous soft background



anti-k_t is soft-resilient

more later in this talk...

Execution timings:



As fast as the (fast) k_t ([M. Cacciari, G. Salam, 06])

21st century jet finders

Recombination:

- k_t algorithm
- Cambridge/Aachen alg.
- anti-k_t algorithm

Cone:

- CDF JetClu
- CDF MidPoint
- D0 (run II) Cone
- PxCone
- ATLAS Cone
- CMS Iterative Cone
- PyCell/CellJet
- GetJet
- SISCone

4 available
safe algorithms

All accessible from FastJet

Part 2
Jets in pp collisions
(a) Choosing the adapted jet definition

[M.Cacciari,J.Rojo,G.Salam,G.S., arXix:0810.1304]

Sample processes to study

We analyse 3 processes typical of kinematic reconstructions:

- $Z' \to q\bar{q} \to$2 jets and $H \to gg \to$2 jets:

  simple environment: identify 2 jets and reconstruct $M_{Z',H}$
  source of monochromatic quark/gluon jets
  scale dependence: mass of the Z'/H varied between 100 GeV and 4 TeV
  ficticious narrow Z', H

- $t\bar{t} \to W^+bW^-\bar{b} \to q\bar{q}bq\bar{q}\bar{b} \to$6 jets:

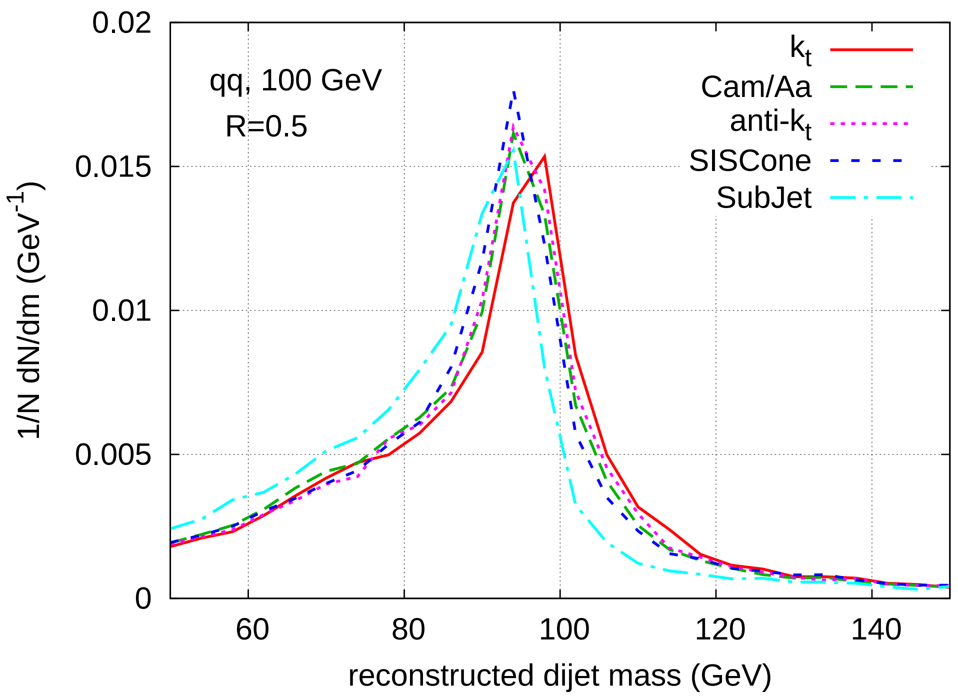
  complex environment: identify 6 jets and reconstruct 2 top
  balance between reconstruction efficiency and identification

with

- the 5 IRC-safe algorithms: k_t, Cambridge, anti-k_t, SISCone, Cam+filtering
- jet radius varied between 0.1 and 1.5

Histograms

- We reconstruct histograms



- How can we quantify the reconstruction efficiency?

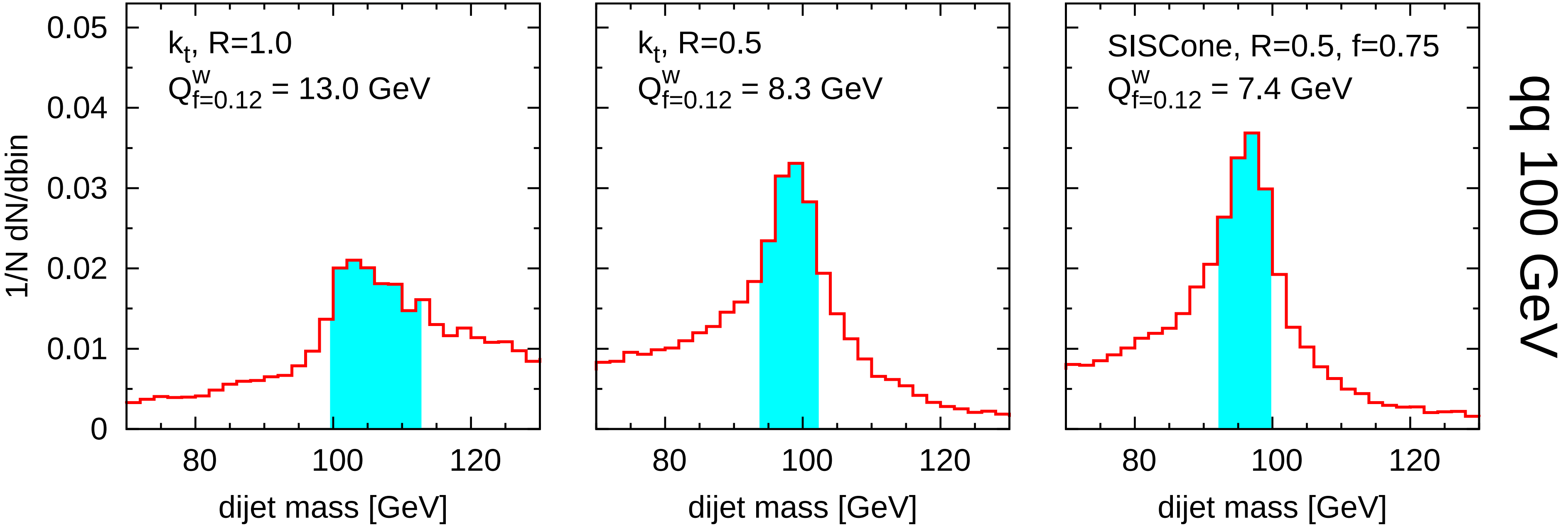
Measure of the jet reconstruction efficiency

- Forget about measures related to parton-jet matching,
  $\longrightarrow$ use the reconstructed mass peak
- Forget about fits depending on the shape of the peak

$\Rightarrow$ maximise the signal over background ratio ($S/\sqrt{B}$):

figure of merit for quality measure

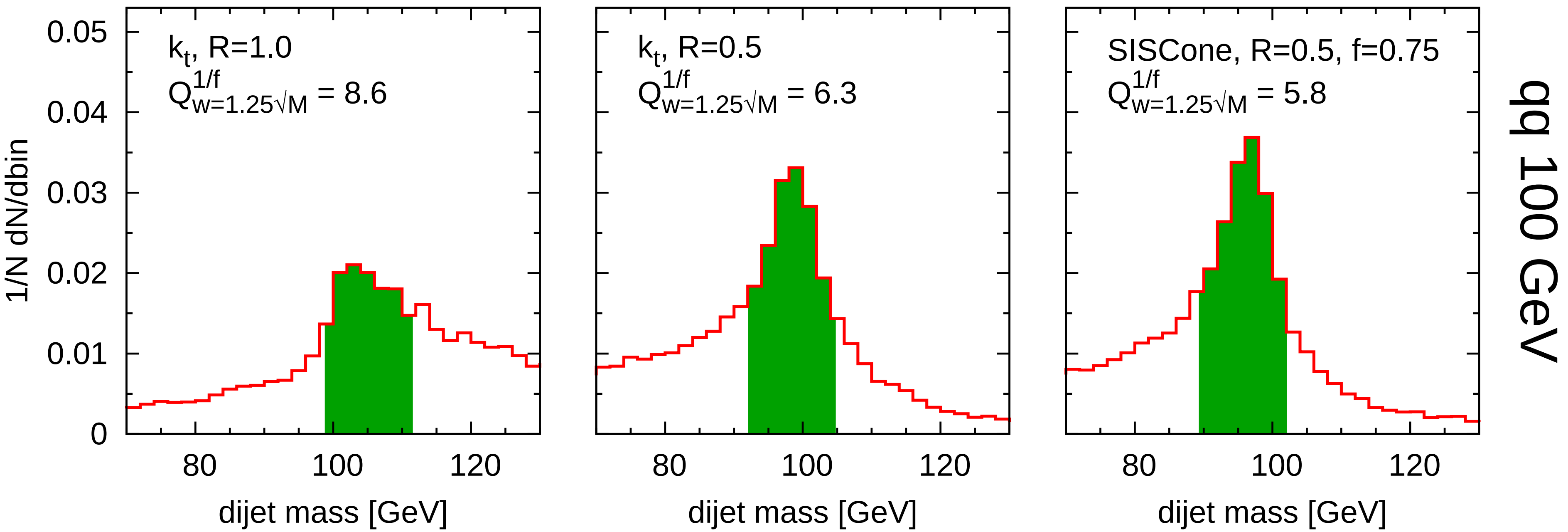
$Q^w_{f=z}(JA,R) =$ minimal width of a window containing a fraction $f = z$ of the events



Fixed signal, minimal width(background)

figure of merit for quality measure

$$Q_f^{w=x\sqrt{M}}(JA, R) = \text{(1/) maximal number of events in a window of width } x\sqrt{M}$$



Maximal signal, fixed width(background)

figure of merit for quality measure

- it intuitively does what it should

figure of merit for quality measure

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- relates to a signal significance(assuming constant background)

$$\frac{\Sigma(\mathrm{JD}_1)}{\Sigma(\mathrm{JD}_2)} = \left[\frac{N_{\text{signal}}}{\sqrt{N_{\text{bkg}}}}\right]_{\mathrm{JD}} = \sqrt{\frac{Q^w_{f=z}(\mathrm{JD}_2)}{Q^w_{f=z}(\mathrm{JD}_1)}} = \frac{Q_f^{w=x\sqrt{M}}(\mathrm{JD}_2)}{Q_f^{w=x\sqrt{M}}(\mathrm{JD}_1)}$$

minmimal $Q \equiv$ better signal-to-background ratio

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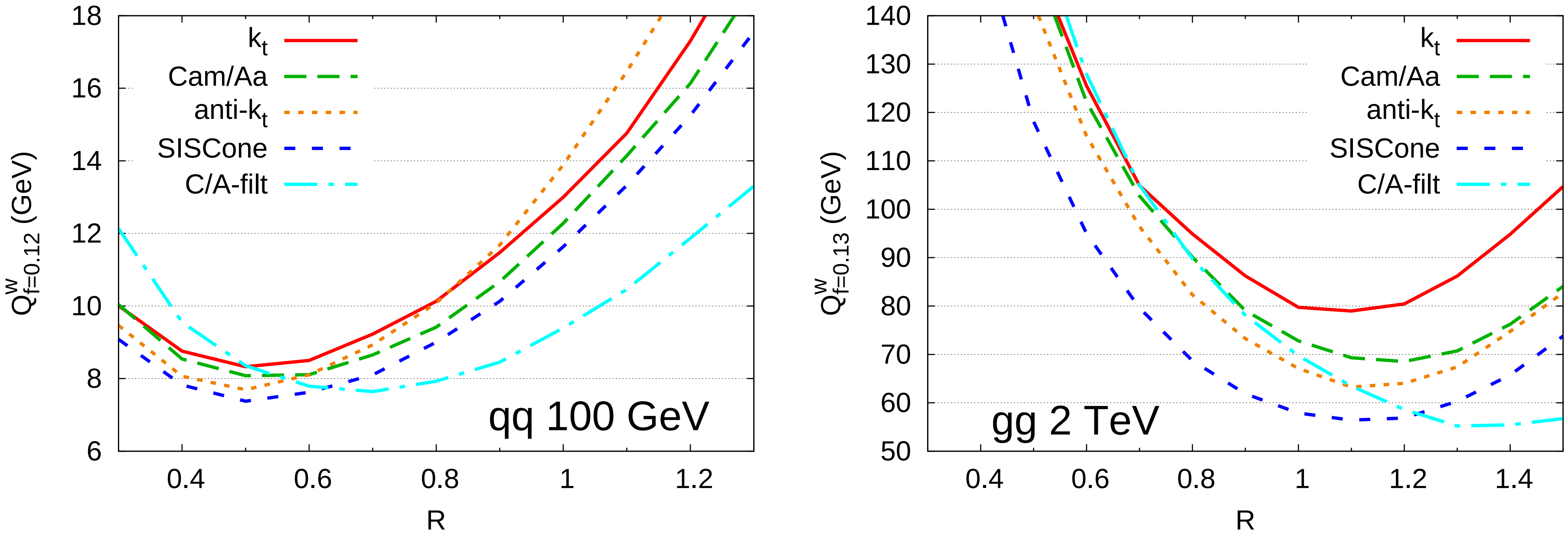
  minmimal $Q \equiv$ better signal-to-background ratio

- we can associate an effective luminosity ratio

$$\rho_{\mathcal{L}}(\mathrm{JD}_2/\mathrm{JD}_1) = \frac{\mathcal{L}\text{ needed with }\mathrm{JD}_1}{\mathcal{L}\text{ needed with }\mathrm{JD}_2} = \left[\frac{\Sigma(\mathrm{JD}_1)}{\Sigma(\mathrm{JD}_2)}\right]^2$$

  e.g. $\rho_{\mathcal{L}} = 2 \equiv \mathrm{JD}_1$ has $\sqrt{2}$ the significance of JD_2
  $\equiv \mathrm{JD}_2$ requires 2 times the integrated luminosity
  to achieve the same discriminative power.

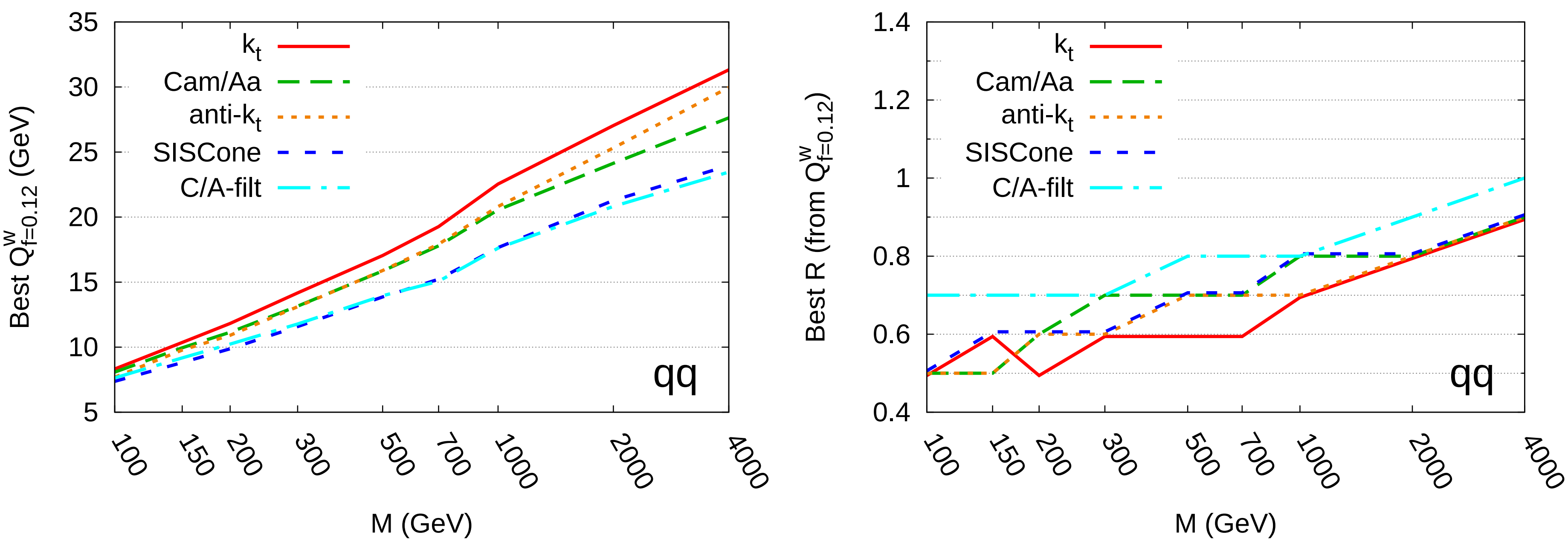
Examples: best quality measures



Allows to

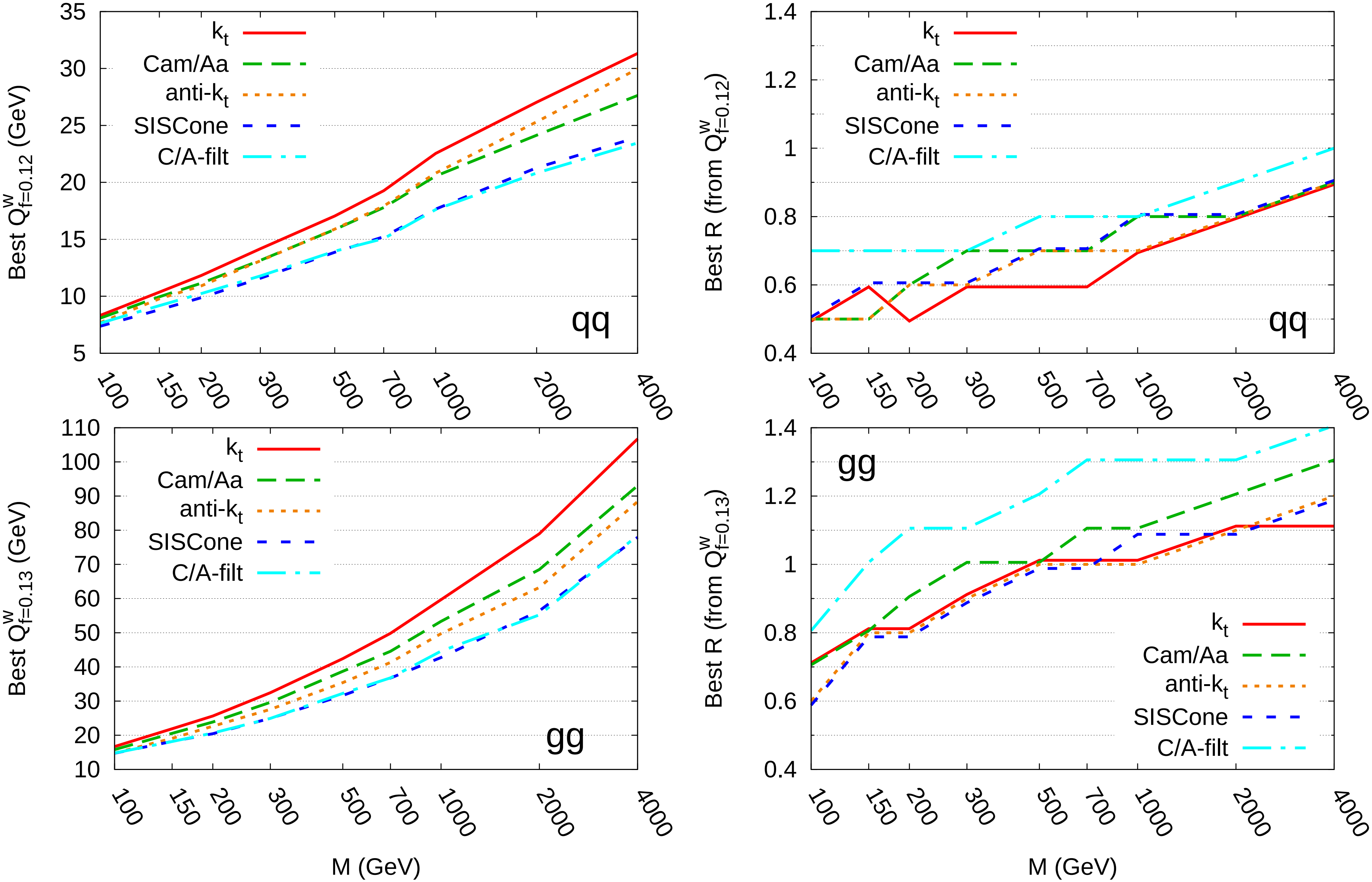
- extract the best radius R_{best}
- compare the different algorithm

Best choices



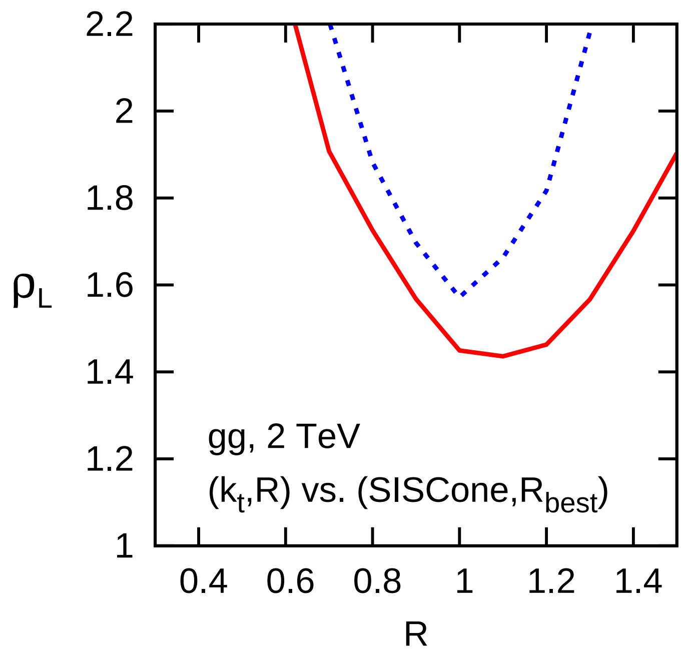
- SISCone and Cam+filtering perform better
- R_{best} strongly depends on the mass

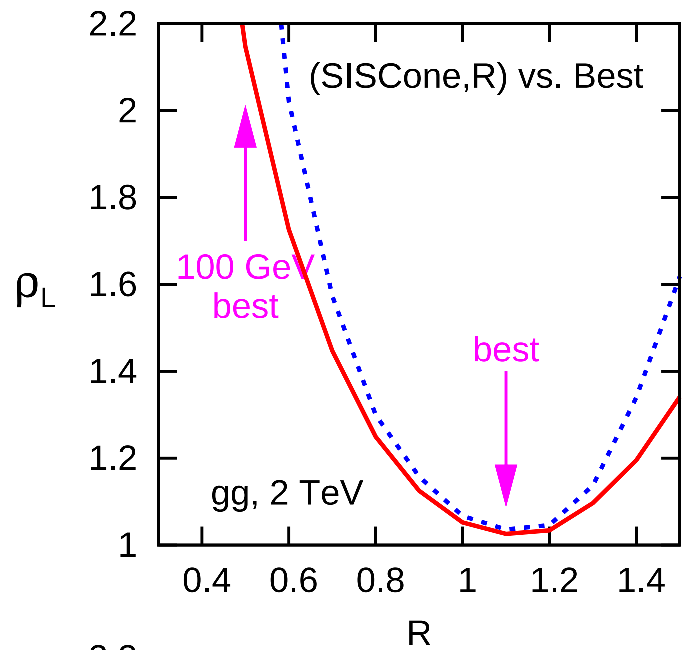
Quarks vs. gluons

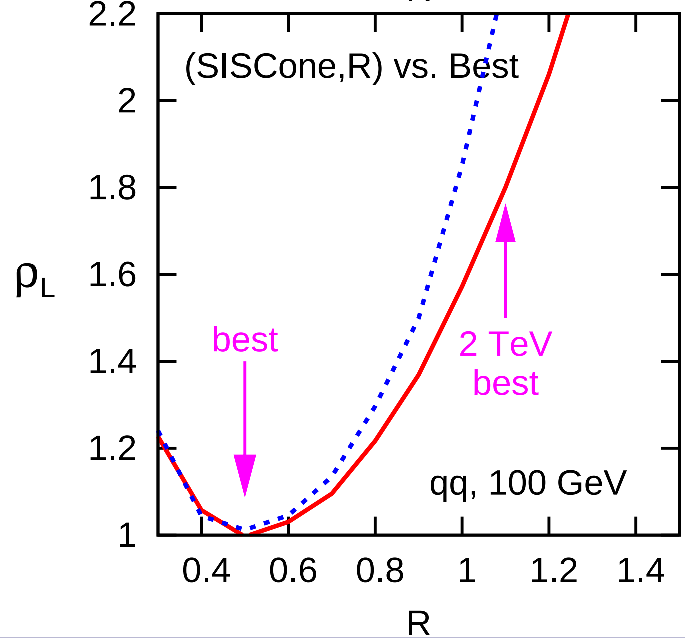


Same conclusions for gluon jets with slightly larger R

Luminosity ratios







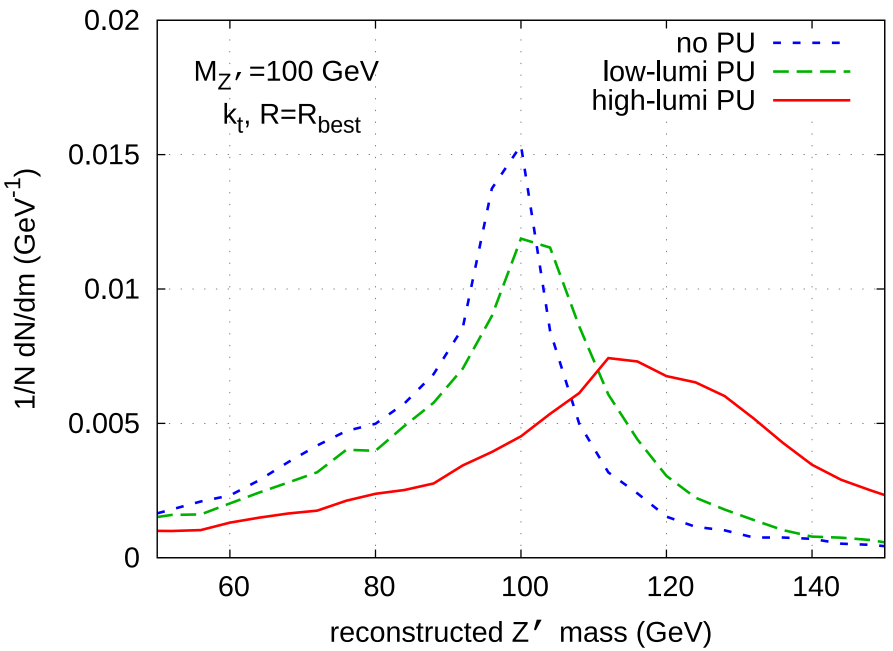
Mandatory at the LHC:
Not choosing the best alg.
AND R can be very costly
for new discoveries

Note: typical choice, $R \sim 0.5$

Part 2
Jets in pp collisions
(b) pileup effect (jet areas & subtraction)

Need for subtraction

Pileup $\approx$ uniform soft background that shifts jets to higher p_t



... that needs to be subtracted!

$\Rightarrow$ Using jet areas!

Pileup subtraction

Basic idea: [M.Cacciari, G.Salam, 08]

$$p_{t,\text{subtracted}} = p_{t,\text{jet}} - \rho_{\text{pileup}} \times \text{Area}_{\text{jet}}$$

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- Jet area: [M.Cacciari, G.Salam, G.S., 08]
  - region where the jet catches infinitely soft particles (active/passive)
  - tractable analytically in pQCD

    Example: area corrections from QCD radiation

$$\langle \mathcal{A}(p_{t,1}, R)\rangle = \mathcal{A}_{1\text{ parton}}(R) + \frac{C_{F,A}}{\pi b_0} \log\left(\frac{\alpha_s(Q_0)}{\alpha_s(Rp_t)}\right) \pi R^2\, d$$

- area $\neq \pi R^2$
- area scaling violations

d	passive	active
k_t	0.5638	0.519
Cam	0.07918	0.0865
SISCone	-0.06378	0.1246
anti-k_t	0	0

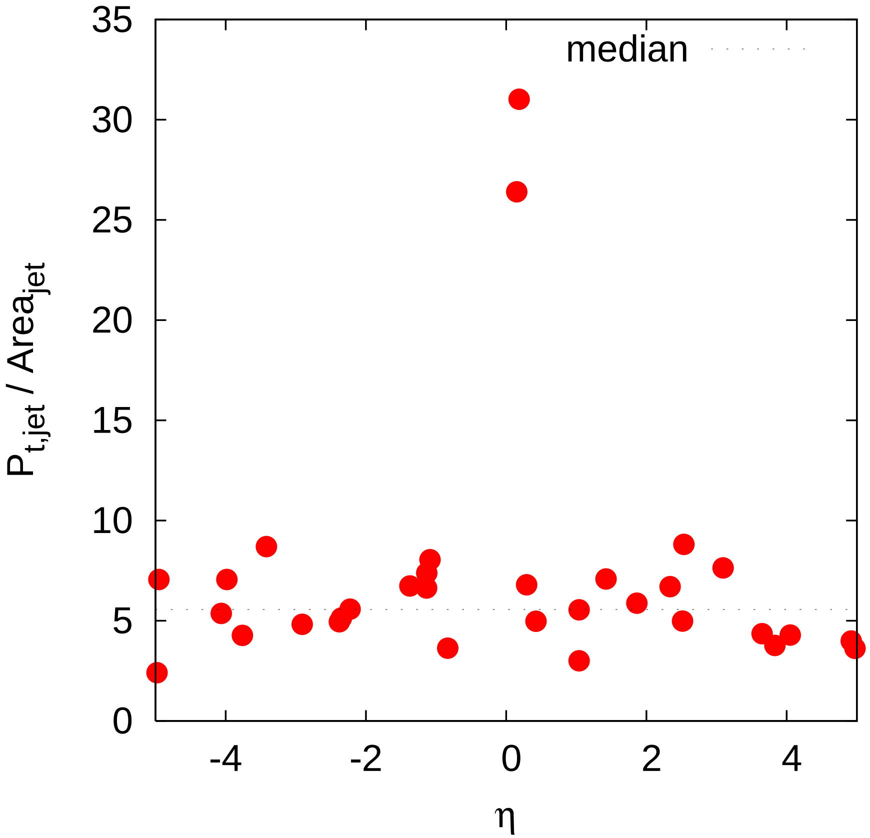
Pileup subtraction

Basic idea: [M.Cacciari, G.Salam, 08]

$$p_{t,\text{subtracted}} = p_{t,\text{jet}} - \rho_{\text{pileup}} \times \text{Area}_{\text{jet}}$$

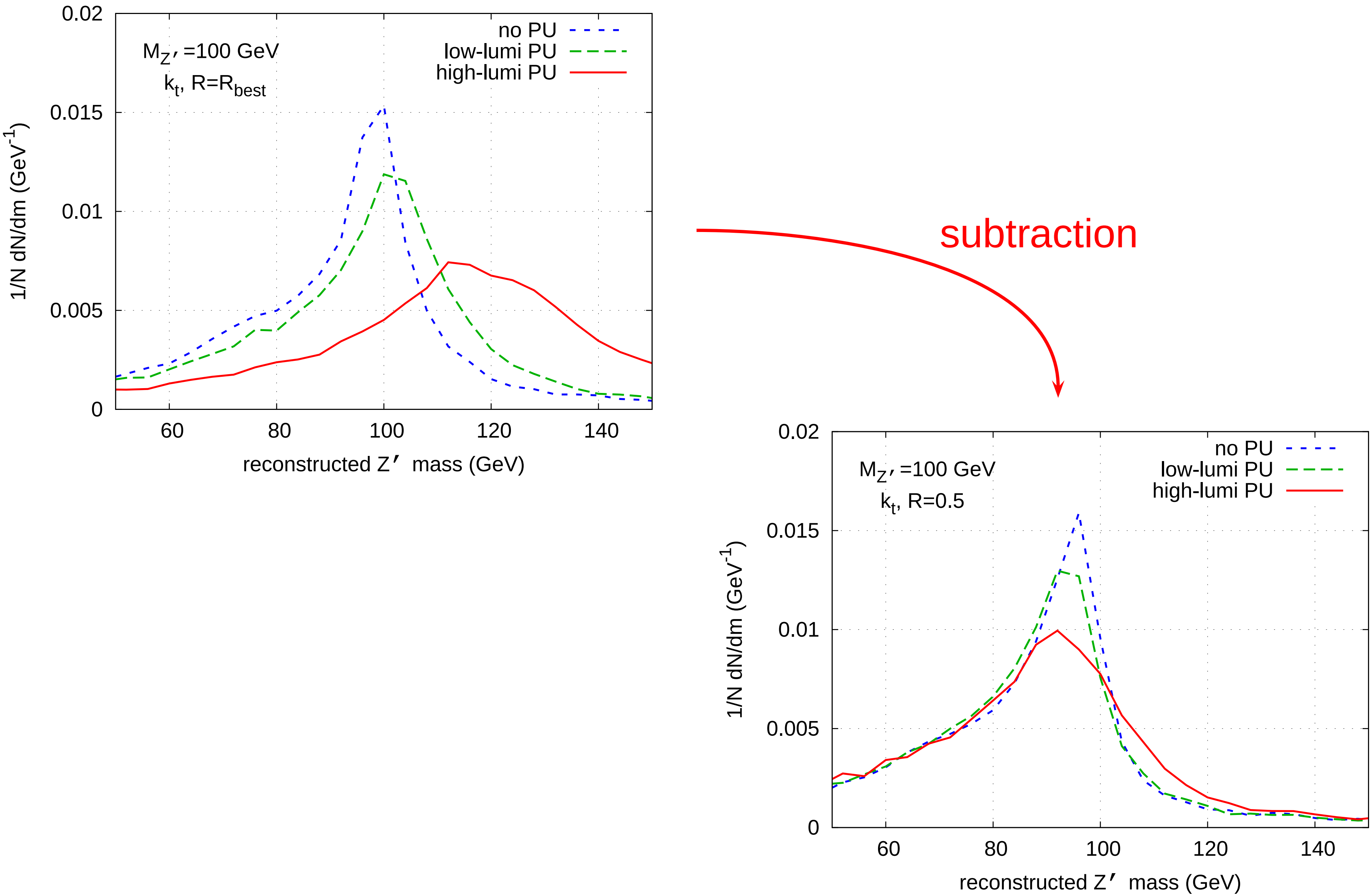
- Jet area: [M.Cacciari, G.Salam, G.S., 08]
  - region where the jet catches infinitely soft particles (active/passive)
  - tractable analytically in pQCD
- Pileup density per unit area: ρ_{pileup}

  e.g. estimated from the median of $p_{t,\text{jet}}/\text{Area}_{\text{jet}}$

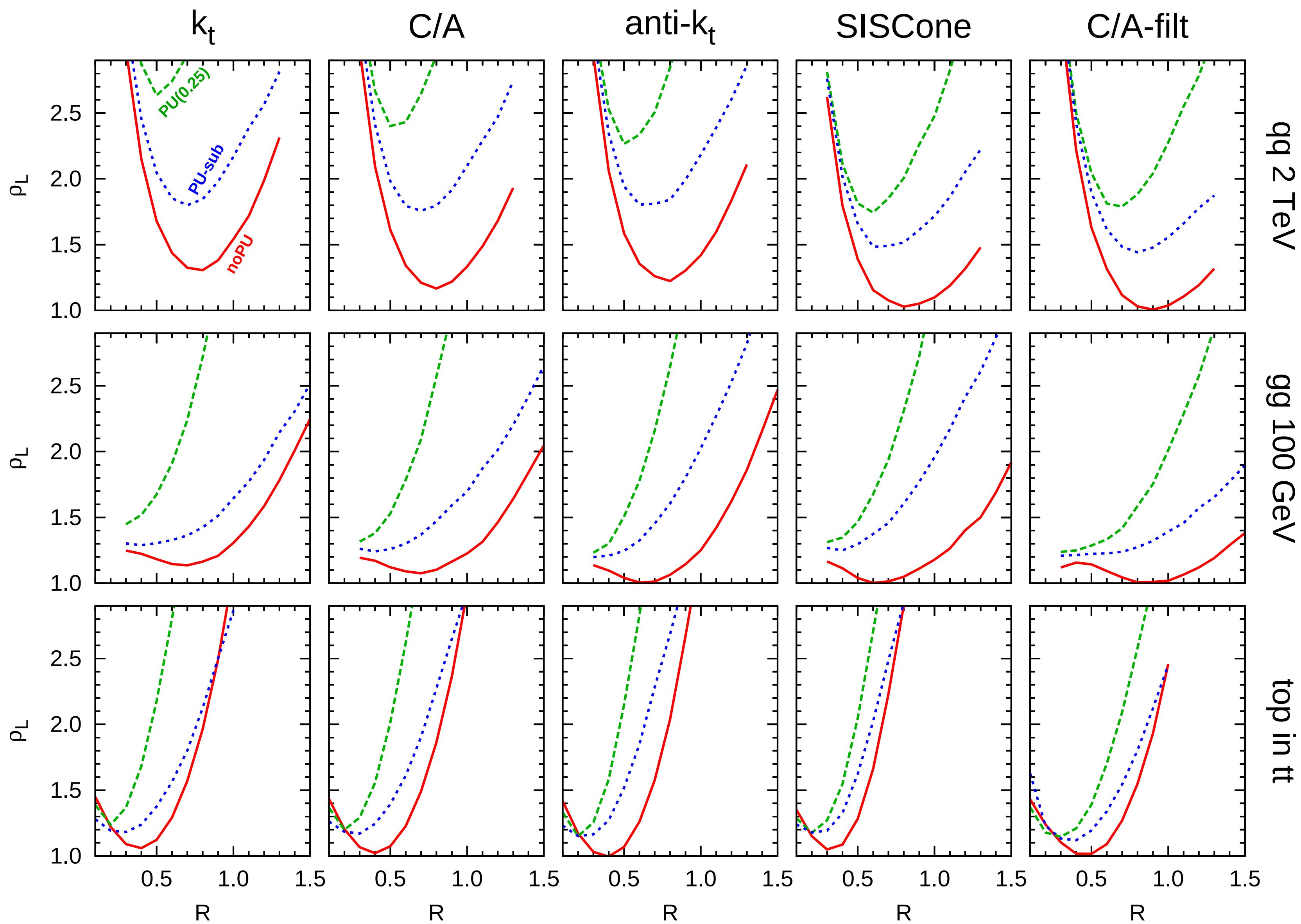


implemented in FastJet
on an event-by-event basis

Subtraction at work



PU effects summary

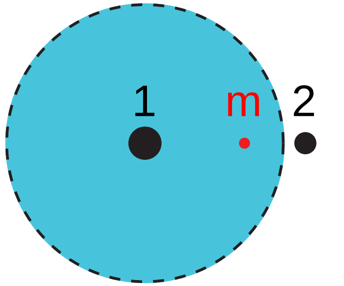


Subtraction ⇒ (i) large improvement, (ii) $R_{\text{best}} \sim$ unchanged

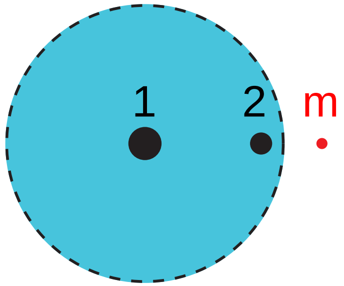
Back-reaction

Additional soft background has 2 effects:

- Throw soft particles in the hard jet: dealt with by subtraction
- Modify the hard scattering (back-reaction)
  - can be pointlike or diffuse
  - gain: p_2 gained when adding p_m

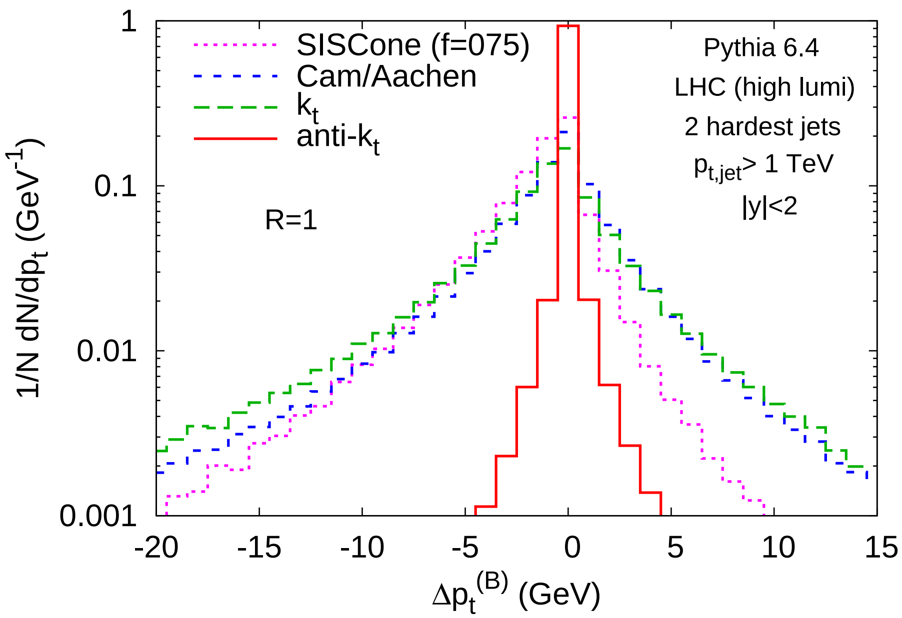


  - loss: p_2 lost when adding p_m



Additional soft background has 2 effects:

- Throw soft particles in the hard jet: dealt with by subtraction
- Modify the hard scattering (back-reaction)
  - can be pointlike or diffuse
  - tractable analytically (similar to areas)
  - $k_t \gtrsim$ Cambridge > SISCone $\gg$ anti-k_t



Conclusions

Message 1: IRC safety is mandatory

Midpoint and the iterative cone IR or Collinear unsafe (at $\mathcal{O}(\alpha_s^4)$)

Observable	1st miss cones at	Last meaningful order
Inclusive jet cross section	NNLO	NLO
3 jet cross section	NLO	LO (NLO in NLOJet)
$W/Z/H$ + 2 jet cross sect.	NLO	LO (NLO in MCFM)
jet masses in 3 jets	LO	none (LO in NLOJet)

+ We do not want the theoretical efforts to be wasted

- Note: 1 order worse for JetClu of the ATLAS Cone!
- All IRC-safe algorithms available from FastJet (http://www.fastjet.fr)

Conclusions

Message 2: flexibility in jet finding at the LHC

- Optimal jet definition (see also http://quality.fastjet.fr)
  - $R_{\text{best}} \sim 0.5$ at 100 GeV, $R_{\text{best}} \sim 1$ at 1 TeV
  - important to choose R_{best}, SISCone and Cam+filt. slightly better
  - same for quark and gluon jets, larger R_{best} for gluons
  - TODO: understand this analytically/ improve clustering (e.g. filtering)

- Pileup subtraction using jet areas
  - Jet areas: clearly defined, analytic control
  - Simple systematic pileup subtraction
  - Same conclusions as without pileup
  - TODO: deal with fluctuating backgroubd (e.g. heavy ions)