Analytic Amplitude Models for Forward Scattering

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Abstract

We report on fits of a large class of analytic amplitude models for forward scattering against the comprehensive data for all available reactions. To differentiate the goodness of the fits of many possible parameterizations to a large sample of data, we developed and used a set of quantitative indicators measuring statistical quality of the fits over and beyond the typical criterion of the χ^2/dof. These indicators favor models with a universal $\log^2 s$ Pomeron term, which enables one to extend the fit down to $\sqrt{s} = 4$ GeV.

1 Introduction

This presentation is based on the results of the recent comprehensive studies on the fits of the comprehensive analytic amplitude models for the high energy forward scattering amplitude against all available data of the total cross sections and real part of the hadronic amplitudes by the COMPETE [1–3] collaboration. The data set is accumulated and maintained by some of us (COMPAS) for the PPDS data bases [4].

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The problem of universal description of forward scattering and rising total cross section has been with us for over two decades ever since the ISR experiments. Interest in this topic has been revived by the recent activities at HERA on deep-inelastic and diffractive scattering and also at LEP on $\gamma\gamma$ scattering. The rising behavior of total cross sections in energy was suggested theoretically before the ISR results in connection with the rigorous proof of the Pomeranchuk theorem [5].

Theoretically the total cross section is related to the imaginary part of the forward scattering amplitude. Unitarity and analyticity relates the real part to the imaginary part of the scattering amplitude. The real part of the forward amplitude F^{ab} can be obtained from the imaginary part via the well-known substitution rule $s \to se^{-i\pi/2}$ or, in an equivalent way, from the derivative dispersion relations [6]. A slow rise of total cross sections with energy is possible if the leading Regge trajectory having vacuum quantum numbers has an intercept slightly larger than 1, *i.e.*, the intercept of the Pomeron as a simple Reggeon is $\alpha_P(0) = 1 + \varepsilon$, ε being a small positive number, which was proved to be useful for studying the data at non-asymptotic energies [7]. But as the Pomeron intercept has a bearing on the extrapolation of total cross section to higher energies, such Pomeron term will violate eventually the Froissart bound [8] $\sigma_{tot} \le c \log^2 s$, s being the square of the C.M. energy, a consequence of the unitarity and positivity of the imaginary part of the scattering amplitudes in the Lehmann ellipse. In addition, the simple Pomeron model does not offer a simple and automatic extension to the off-shell particle scattering and in particular to deep inelastic scattering (DIS). It is generally believed that the BFKL re-summation of energy logarithms is relevant to high energy hadron processes but the BFKL scheme offers no simple clue for the off-shell extension either.

The Pomeron intercept is also a crucial element in HERA DIS analyses and provides the starting point at low x and low Q^2, from which perturbative QCD evolution can be performed and can be tested. Since the Regge-Pomeron behavior in s originates from that of the large t-channel scattering angle, $\cos\theta_t$, in hadron-hadron scattering, the same $J-$ plane singularity is affecting the corresponding (x, Q^2) region in DIS at HERA. Though there is some overlap region of the large s behavior of the soft Pomeron in hadronic scattering and the small x behavior from perturbative QCD evolution, the study of the singularity structure of forward hadronic scattering lies mostly outside the realm of perturbative QCD. In pQCD, one may expect that a higher order effects would unitarize the amplitude and tame the fierce rise observed at large Q^2 to something compatible with the Froissart bound. But no one has derived reliably unitarized QCD amplitudes. Moreover such unitarization will involve necessarily multi-gluon exchanges between the quarks and therefore will require detailed quark structure, *i.e.,* the hadronic wave functions. In doing so, one is likely to lose the nice properties that the simple Regge-Pomeron exhibits. It is simply the reality that no one has the QCD-based understanding and derivation of the forward hadronic scattering amplitudes.

The basic idea of the analytic amplitude method for the forward ($t = 0$) hadron scattering is to treat this non-perturbative domain by implementing the general principles as much as possible, such as analyticity, unitarity, crossing-symmetry and positivity of total cross-sections, and by supplementing the scheme by some well-established strong interaction properties. Good analytic models must describe not only the desired rising behavior at high energies but also the correct energy behavior of the cross sections when extended to medium energies where a smooth analytic behavior of the amplitude has set in and the secondary Reggeon contributions are needed. Such general approach, unfortunately, does not provide a unique answer. There can be many models that satisfy these theoretical criteria. In addition, good models are required to explain all available forward scattering data of all reactions and their applicability must be judged on a common ground. It is therefore clear that one needs to develop and use a common procedure and decision-making criteria to differentiate the goodness of the fits of the models. In fact a decision-making procedure was initiated in Ref. [9] for pp and $\bar{p}p$ scattering, which led us to conclude that the exchange-degenerate Reggeons were not preferred by the forward scattering data and pursued further in Refs. [4] for the collisions $p^{\mp}\,p$, $\pi^{\mp}\,p$, $K^{\mp}\,p$, $\gamma\,p$, and $\gamma\,\gamma$, which however led to the conclusion that the forward data could not discriminate between a simple Regge-Pomeron fit and asymptotic $\log^2 s$ and $\log s$ fits for $\sqrt{s} \geq 9$ GeV.

- We have therefore developed a set of statistical indicators which measure quantitatively the goodness of the fits and which complement the usual χ^2 criterion. The values of these indicators enable us to differentiate the diverse analytic amplitude models on the basis of characteristics of their fits.
- The data set has been improved somewhat by eliminating some preliminary data on the ρ parameter and by adding newly published data from SELEX ($\pi^- N$ and $\Sigma^- N$ at 600 GeV/c) [10] and OPAL ($\gamma\,\gamma$) [11]. Thus new SELEX data of the ($\Sigma^- p$) collision and the published OPAL data are added to the current simultaneous scan-fits. We have however excluded all cosmic data points [12,13] as in the previous studies [4] because the original numerical Akeno(Agasa) data are not available (only data read from graph) and there are contradictory statements concerning the cosmic data points of both Fly's Eye and Akeno(Agasa) in the literature.
- Recently a two-component soft Pomeron picture was rediscovered [14], in which the first component has intercept 1 and comes from $C = +1$ three-gluon exchanges and the second component may be thought of as a unitarized Pomeron with intercept larger than 1, the well-known BFKL Pomeron associated with 2-gluon exchanges[15,16]. The first component can take the quark counting rule into account while the second one is responsible for the universal rising of total cross sections with energy.

We see that this procedure changes the picture considerably and can differentiate the models that can be applied to the energy region as low as $\sqrt{s} = 4$ GeV.

2 Analytic Parameterizations for the Forward Scattering Amplitudes

The different variants of parameterization can be classified basically into three exemplary classes depending on the asymptotic behaviors of the cross section, in the limit $s \to \infty$, as a constant, as $\log s$ or as $\log^2 s$. We introduce the following notations accordingly for the imaginary part of the forward scattering amplitude:

$$ImF^{ab} = s\sigma^{a\mp b} = \mathrm{R}^{+ab}(s) \pm \mathrm{R}^{-ab}(s) + \mathrm{P}^{ab} + \mathrm{H}^{ab}(s), \tag{1}$$

where the $\pm$ sign in formula corresponds to anti-particle(particle) - particle collisions,

$$\mathrm{R}^{\pm ab}(s) = Y_{\pm}^{ab} \cdot (s/s_1)^{\alpha_{\pm}},$$

$R_{\pm}$ representing the effective secondary-Reggeon $((f, a_2), (\rho, \omega))$ contributions to the even(odd)-under-crossing amplitude, $\mathrm{P}^{ab} = sZ^{ab}$, and $\mathrm{H}^{ab}(s)$ stands for either $\mathrm{E}^{ab} = X^{ab}(s/s_1)^{\alpha_{\wp}}$, or $\mathrm{L}^{ab} = s\,(B_{ab}\ln(s/s_1) + A_{ab})$, with $s_1 = 1$ GeV ^{2}, or $\mathrm{L2}^{ab} = s(B^{ab}\ln^2(s/s_0) + A_{ab})$ with an arbitrary scale factor s_0.

In our work we combine the constant A_{ab} with Z^{ab} in the P^{ab} term and rewrite $\mathrm{P}^{ab} + \mathrm{L}^{ab} = s\,(B_{ab}\ln(s/s_1) + Z_{ab})$ and also $\mathrm{P}^{ab} + \mathrm{L2}^{ab} = s(Z_{ab} + B_{ab}\ln^2(s/s_0))$. Analyticity determines the real part of F^{ab} from the formula (1) via the rule as mentioned already. Most of the analytic amplitude models proposed in the literature [6,17,18] are some variants of the RRPH parameterization or those constrained further by imposing such conditions as the degeneracy of the leading $C = \pm$ Regge trajectories, the universal rising $B_{ab} = B$ of total cross sections, the factorization of the Regge residues of H^{ab}, $H_{\gamma\gamma} = \delta H_{\gamma p} = \delta^2 H_{pp}$, and the quark counting rule for the residues of the Regge-Pomeron terms in H^{ab}, in the interest of reducing the number of parameters. Models constrained by such additional conditions are indicated by appropriate symbols d, u, qc and nf (in the case of not imposing the factorization of H^{ab}) as supplementary subscript or superscript.

In general there are seven (six) adjustable parameters for each pair of collisions $\overline{a}\ b$ and $a\ b$ in the analytic parameterizations RRPH where $\mathrm{H} = \mathrm{E}, \mathrm{L2}$ (L) so that there are 24 or 25 parameters in total to adjust the simultaneous fits for the collisions $p^{\mp}\ p$, $\Sigma^-\ p$, $\pi^{\mp}\ p$, $K^{\mp}\ p$, $\gamma\ p$, and $\gamma\ \gamma$. We considered more than 256 different variants of the analytic amplitudes [19]. We summarize in Table 1 the results of six representative cases that give a χ^2/dof smaller than 1.5 for all all cross section and ρ data for $\sqrt{s} \geq 4$ GeV. Because of the large number of points, slight upward deviations of the χ^2/dof from 1 would imply a very low confidence level. The area of applicability of the models, *i.e.*, the low-energy cut-offs for which $\chi^2/dof \leq 1.0$ are shown with numbers in bold.

	$\sqrt{s_{min}}$ in GeV and number of data points						
Model code (N_{par})	4 (742)	5 (648)	6 (569)	7 (498)	8 (453)	9 (397)	10 (329)
RRE_{nf}	1.4	1.1	1.1	1.1	1.1	1.0	1.0
$RRL_{nf}(19)$	1.1	**0.97**	**0.97**	**1.0**	**0.96**	**0.94**	**0.93**
RRPL(21)	1.1	**0.98**	**0.98**	**0.99**	**0.94**	**0.93**	**0.91**
$(RR)^d\ P_{nf}\ L2(20)$	1.2	**1.0**	**1.0**	**0.99**	**0.94**	**0.93**	**0.92**
$RRPL2_u(21)$	1.1	**0.97**	**0.97**	**0.97**	**0.92**	**0.93**	**0.92**
$(RR)^d\ PL2_u(17)$	1.3	1.0	**1.0**	**0.98**	**0.94**	**0.93**	**0.93**

Table 1: Six representative models in three classes fitting all cross section and ρ data down to 5 GeV. Numbers in bold represent the area of applicability of each model.

As can be seen from Table 1, the data are compatible with many possibilities for $\sqrt{s} \geq 9$ GeV and cannot differentiate the models at this level, let alone the nature of the Pomeron. Also it seems that sub-leading trajectories and other non-asymptotic characteristics do not manifest themselves. The two classes of logarithmic increases seem to fit better than simple powers. Also reasonable degeneracy of the leading Reggeon trajectories can be implemented only for the class of $PL2$ models having a $\log^2 s$-type effective Pomeron. Such degeneracy is in fact expected to hold in global fits to the forward scattering data of all hadronic processes that include pp and K^+p scattering, which have exotic s-channel, in view of the Reggeon-particle duality.

3 Statistical Indicators Measuring the Quality of the Fits

To distinguish further the nature of the fits in these models, we need a statistical procedure. This procedure will enable us to compare and rank the quality of the analytic amplitude models.

The best known quantity is certainly the χ^2/dof, or more precisely the confidence level (CL). However, because Regge theory does not apply in the resonance region, no model is expected to reproduce the data down to the lowest measured energy. The energy cutoff $\sqrt{s_{min}}$ in Table 1 is *ad hoc*. Clearly the *range* of energy over which the model can reproduce the data with a $\chi^2/dof \leq 1.0$ must be a part of the indicators. Also the quality of the data varies depending on which quantity or which process one considers. An unbiased way of taking into account the quality of the data is to assign a weight to each process or quantity. Given that this will be done to compare models together, the weights as determined by the best fit is a reasonable choice for the weight, *i.e.*, we introduce

$$w_j = \min\left(1, \frac{1}{\chi_j^2/nop}\right)$$

where $j = 1, \ldots, 9$ refers to the process, and we define the renormalized $\chi_R^2 \equiv \sum_j w_j \chi_j^2$. The number of parameters in the model should be a consideration too, given the data sample in the range of applicability. Finally, if a fit is physical in a given range, then its parameters must be stable with respect to the sub-part of the range: different determinations based on a sub-sample must be compatible. Hence certain criteria for the *stability* of the fits should be taken into consideration as indicators.

We have developed a set of statistical quantities that enable us to measure the above features of the fits. All these indicators are constructed so that the higher their values are the better is the quality of the data description.

(1) The Applicability Indicator: It characterizes the range of energy which can be fitted by the model with a confidence level bigger CL > 50%. This range can in principle be process-dependent, but we consider the simplest case here:

$$A_j^M = w_j \log(E_j^{M,high}/E_j^{M,low}), \qquad A^M = \frac{1}{N_{sets}} \sum_j A_j^M \tag{2}$$

where j is the multi-index denoting the pair (data subset, observable); $E_j^{M,high}$ is the highest value of the energy in the area of applicability of the model M in the data subset j; $E_j^{M,low}$ is the lowest value of the energy in the area of applicability of the model M in the data subset j, and w_j is the weight determined from the best fit in the same interval (hence w_j will depend itself on $E_j^{M,high}$ and $E_j^{M,low}$). In our case the applicability indicator takes the form:

$$A^M = \frac{1}{15}\left(A^M_{pp,\sigma} + A^M_{\bar{p}p,\sigma} + A^M_{\pi^+p,\sigma} + A^M_{\pi^-p,\sigma} + A^M_{K^+p,\sigma} + A^M_{K^-p,\sigma} + A^M_{\Sigma^-p,\sigma}\right.$$
$$\left. + A^M_{\gamma p,\sigma} + A^M_{\gamma\gamma,\sigma} + A^M_{pp,\rho} + A^M_{\bar{p}p,\rho} + A^M_{\pi^+p,\rho} + A^M_{\pi^-p,\rho} + A^M_{K^+p,\rho} + A^M_{K^-p,\rho}\right).$$

The fit results in detail show that for L, PL and $PL2$ class of models we obtain rather good fits to all cross sections starting from $E_{min} = 4$ and to cross section and ρ data from 5 GeV but in some cases with negative contributions to the total cross sections from terms corresponding to the exchange of the Pomeron-like objects in low energy part of the area of applicability as defined above. This is unphysical: we are forced to add an additional constraint to the area of applicability and exclude the low energy part where at least one collision process has a negative contribution from the Pomeron-like (asymptotically rising) term. It turned out that some models have an empty area of applicability once this criterion was imposed.

(2) Confidence-1 Indicator: $C_1^M = CL\%$
where the CL refers to the whole area of applicability of the model M.

(3) Confidence-2 Indicator: $C_2^M = CL\%$
where the CL refers to the intersection of the areas of applicability of all models qualified for the comparison (we choose here $\sqrt{s} \geq 5$ GeV for the fits to the cross sections but without ρ-data, and $\sqrt{s} \geq 9$ GeV for the fits to both cross sections and ρ data.

(4) Rigidity Indicator 1: We propose to measure the rigidity of the model by the indicator

$$R_1^M = \frac{N_{dp}^M(A)}{1 + N_{par}^M} \qquad (3)$$

The most rigid model has the highest value of the number of data points per adjustable parameter.

(5) Reliability Indicator 2: This indicator characterizes the goodness of the parameter error matrix.

$$R_2^M = \frac{2}{N_{par}(N_{par}-1)} \cdot \sum_{i>j=1}^{N} \Theta(90.0 - C_{ij}^R) \qquad (4)$$

where C_{ij}^R – is the correlation matrix element in % calculated in the fit at the low edge of the applicability area. For the diagonal correlator this indicator is maximal and equals 1.

(6) Uniformity Indicator: This indicator measures the variation of the χ^2/nop from bin to bin for some data binning motivated by physics:

$$U^M = \left\{ \frac{1}{N_{sets}} \sum_j \frac{1}{4} \left[\frac{\chi_R^2(t)}{N_{nop}^t} - \frac{\chi_R^2(j)}{N_{nop}^j} \right]^2 \right\}^{-1}, \qquad (5)$$

where t denotes the total area of applicability, j is a multi-index denoting the pair (data set, observable). In our case we use the calculation of the χ_R^2/nop for each collision separately, i.e. the sum runs as in the case of the applicability indicator.

(7) Stability-1 Indicator:

$$S_1^M = \left\{ \frac{1}{N_{steps} N_{par}^M} \sum_{steps} \sum_{ij} (P^t - P^{step})_i (W^t + W^{step})^{-1}_{ij} (P^t - P^{step})_j \right\}^{-1} \quad (6)$$

where: P^t - vector of parameters values obtained from the model fit to the whole area of applicability; P^{step} - vector of parameters values obtained from the model fit to the reduced data set on the *step*, by which we mean shift in the low edge of the fit interval to the right by 1 GeV. If there are no steps then $S_1^M = 0$ by definition; and W^t and W^{step} are the error matrix estimates obtained from the fits to the total and to the reduced on the step s data samples from the domain of applicability.

(8) Stability-2 Indicator:

$$S_2^M = \left\{ \frac{1}{2N_{par}^M} \sum_{ij} (P^t - P^{t(no\,\rho)})_i (W^t + W^{t(no\,\rho)})^{-1}_{ij} (P^t - P^{t(no\,\rho)})_j \right\}^{-1} . (7)$$

The indicator S_2^M characterizes the reproducibility of the parameters values when fitting to the reduced data sample and reduced number of observable but with the same number of adjustable parameters and might be strongly correlated with the uniformity indicator U^M. In this case, we fit the whole set of the model parameters to the full area of applicability (superscript t) and the same set of parameters but to the data sample without ρ-data (superscript $t(no\ \rho)$).

To complement the usual χ^2 criterion, we have developed and used these quality measure indicators to rank the models via the "league comparison" between the models in eight disciplines of the quality indicators with equal weight

$$I_k^m = (A^m, C_1^m, C_2^m, R_1^m, R_2^m, U^m, S_1^m, S_2^m)$$

where the index m describes the model and index k describes the indicator type.

With all the calculated components of the indicators, it is easy to assign score points in each discipline to a given model M:

$$P_k^M = \sum_{m \neq M} (2\Theta(I_k^M - I_k^m) + \delta_{I_k^M, I_k^m}). \quad (8)$$

The rank of models is decided by the total score points:

$$P^M = \sum_k P_k^M = \sum_k \sum_{m \neq M} (2\Theta(I_k^M - I_k^m) + \delta_{I_k^M, I_k^m}) \quad (9)$$

The scores of the **ACCRRUSS** league comparison are given in Table 2 for the five high ranking representative models, out of 21 models that passed the high CL tests of the fit comparison of the $\sigma_{tot}(s)$- and $\rho(s)$-data.

Model Code	A^M	C_1^M	C_2^M	U^M	R_1^M	R_2^M	S_1^M	S_2^M	rank P^M
RRPL2$_u$(21)	**2.2**	68.	**85.**	**23.**	29.	**0.90**	0.22	0.10	**222**
(RR)^d P$_{nf}$L2(20)	**2.2**	50.	82.	18.	31.	**0.90**	0.27	0.41	178
(RR)^d PL2$_u$(17)	2.0	50.	**83.**	16.	**32.**	0.88	**0.30**	0.67	174
RRL$_{nf}$(19)	1.8	**73.**	81.	17.	**32.**	0.78	**0.29**	**1.3**	**222**
RRPL(21)	1.6	**67.**	82.	**26.**	29.	0.75	0.21	**1.1**	173

Table 2: Quality indicators in five representative models fitting all forward data.

4 Results of the Quality Tests of the Fits

The clearest outcome of the applicability indicator A^M test is that all models belonging to the class of a simple Regge-Pomeron are eliminated from the exclusive group that meets $\chi^2/dof \leq 1$ for $\sqrt{s} > 5$ GeV. The best χ^2/dof for these is 1.12 for RRE$_{nf}$, which is rejected at the 98% C.L. as we have a large number of data points, 648, for $\sqrt{s} > 5$ GeV. However, upon checking where these values of χ^2/dof come from, we see that the main difference comes from fitting the ρ parameter data, which is much worse for the class of RRE than the others, though no model in all variants can fit the ρ data perfectly and in particular those of πp and pp.

As we can see from Table 2, the two classes of models having double poles or triple poles achieve comparable levels of quality, and one cannot decide which is better with these indicators. Clearly we shall need further physics arguments to differentiate these two effective Pomeron.

We found that imposing the Johnson-Treiman-Freund relation for the cross section differences $\Delta\sigma(N) = 5\Delta\sigma(\pi), \Delta\sigma(K) = 2\Delta\sigma(\pi)$ never has led to an improvement of the fit and in some case degraded the fit considerably, though they produced two parametrisations with fewest parameters.

It turns out that the original cosmic experimental data are best fitted by our high-rank models quoted in Table 2.

The class of models with a $\log s$ type Pomeron gives excellent fits to the soft data without violating the unitarity and can be extended to deep-inelastic scattering [20] without any further singularity. But it suffers from several drawbacks: First of all, the Pomeron term becomes negative below 9.5 GeV, and the split of the leading meson trajectories is somewhat bigger than what a normal duality-breaking estimate or a linear extrapolation of the known resonances would allow [21]. As a result, the Pomeron in this class of variants is inevitably compromising with the crossing even Reggeon in the Regge region, effectively counter-balancing the excessive contribution of the $C = even$ Reggeon and thus taming the medium energy behavior while describing the asymptotic behavior of the amplitude. Though the quark counting rule seems to be respected to a very good approximation by this effective Pomeron , *i.e.*, by the coefficients of the $\log s$ and of the constant term, this only reinforces the problem of negativity as it is very difficult to conceive a non factorizing pole which would nevertheless respect quark counting.

Finally, we conclude that the best fits are given by the class of $PL2_u$ models that contain a triple pole at $J = 1$ besides a simple pole with the intercept exactly 1, which thus produce $\log^2 s$, $\log s$ and constant terms in the total cross section. Barring the details of the best model $RRPL2_u(21)$ and its parameters to [1,2], the most interesting properties of this model may be that the constant term respects the quark counting rule to a good approximation, whereas the $\log^2 s$ term can be taken as universal, i.e. independent of the process, as rediscovered in [14] (see also [22]). The universality of the rising term is expected in the case of the eikonal unitarisation of a bare Pomeron with the intercept larger than 1, because the coefficient of the rising term turns out to depend only on the intercept and slope of the bare Pomeron [23]. But for the J-plane singularities of double and triple pole types considered in this paper, the structure of such a singularity and the origin of its universality is less obvious. Nevertheless, such a singularity at $J = 1$ may in fact have a theoretical explanation: recently, Bartels, Lipatov and Vacca [15,16] discovered that there are, in fact, two types of Pomeron in LLA: besides the well-known BFKL Pomeron associated with 2-gluon exchanges and with an intercept bigger than 1, there is a second one associated with $C = +1$ three-gluon exchanges and having an intercept precisely located at 1. Also the factorization of the rising components of the cross section, $(H_{\gamma p})^2 \to H_{\gamma\gamma} \times H_{pp}$, is well satisfied by the $PL2$ Pomeron. Furthermore, the degeneracy of the lower trajectories is respected to a very good approximation, and the model seems extendible to deep inelastic scattering [24]. This model also respects unitarity by construction. Hence this solution is the one that currently meets all phenomenological and theoretical requirements.

A few remarks as for the future directions are in order: a remaining problem in the analysis of the forward data is the difficulty in adequately fitting the data for the ρ parameter in pp and in π^+p reactions. While extraction of the ρ data from the measurements of the differential cross sections data at small t is a delicate problem, re-analysis of these data will call for simultaneous fits to the total cross section data and to the elastic differential cross sections in the Coulomb-nuclear interference region and in the diffractive cones and thus an extension of the parametrisations considered here to the non-forward region. One could also consider a class of analytic models not incorporated in our fits and ranking procedures, class in which the rising terms would turn on at some dynamical threshold s_t (demanding the use of exact dispersion relations), or add lower trajectories to the existing models. Both approaches would lead to many extra parameters, and will be the subject of a future study. Secondly the inclusion of other data may very well allow one to decide finally amongst the various possibilities. One can go to deep-inelastic data, but the problem here is that the photon occupies a special position in Regge theory, and hence the singularities of DIS amplitudes do not need to be the same as those of hadronic amplitudes. One can also extend the models to non-forward data and off-diagonal amplitude such as those of diffractive scattering. Such steps will involve new parameters associated mainly with form factors, but there are many data, hence there is the hope that this kind of systematic study may be generalized. Thirdly it is our intention to develop the ranking scheme further, probably along the lines of [25], and to fine-tune the definition of indicators, in order that a periodic cross assessments of data and models be available to the community [19].

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