Dark Matter and Particle Physics

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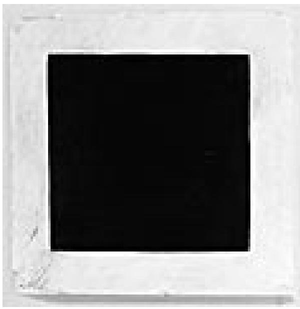
2nd June 2005



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Aurore Courtoy en vue de
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Hemos soñado el mundo.
Lo hemos soñado fuerte, misterioso, visible, ubicuo en el espacio y seguro en el tiempo; pero hemos permitido tenues y eternos intersticios de sinrazón en su estructura para poder saber que es falso.

Jorge Luis Borges

Acknowledgments

Whatever are the words elected, hardly chosen, spontaneous, the deeper acknowledgment I could offer my friends is an ellipsis. Like an author feeling that the story goes beyond his role, only a silence or a blank page is needed leaving the powerless words sleeping.

Nevertheless, imagination is sometimes missing. So I continue for my friends to know how much they are, were and will be important in my whole life, included my carreer. According to they know physics, english or simply my fantasy, I am more than grateful for having shared my doubts or impressions with them. In particular, I thank Christophe and Carles -Kill the Neutrino- for their reading.

Moreover, I would like to thank my supervisor, Maxim, as well as the Fundamental Theoretical Physics group for providing me with their surroundings. In particular, I thank Jean-Philippe, my "officemate", whose help and advices allowed me to go ahead with this dissertation.

Contents

Introduction

“Je n’ai pu percer sans frémir ces portes d’ivoire et de corne qui nous séparent du monde invisible” said de Nerval. So do I.

On the edge, hardly holding between two worlds, the metaphoric and the scientific, lies the concept of Dark Matter. Those two worlds, visible or invisible, mythic or meaningful, metaphoric or roughly scientific, ... as many plausible qualificatives dubbing indeed trying to label the Unknown and the Known parts of the Universe; one could say trying to fulfill the lack of our knowledge.

In the same way, the explanation of the unexpected shape of the rotation curves of spiral galaxies found a good issue in introducing the revolutionary idea of a missing mass. Thus this has fast been taken for granted by the scientific community. Nonetheless authors of science fiction and other kinds of scientific vulgarisation could not resist incorporating this mysterious component, awaring the common run of people that the “stuff” we are supposed to be made of is not the leader.

And that is the point. Mankind needs a reason to cling to. Historically, when first leaving the geocentrism for heliocentrism, one conservative idea quickly skipped to another. But our solar system is nor the only one nor the center of the galaxy; our galaxy -the Milky Way- is not the only one. And the matter we are composed of seems not to be the principal matter component neither.

Even though it is tantilizing choosing suggestive words to describe, one could say that by labelling you do not fix. But through the suggestive expression “Dark Matter” one however only catches a glimpse of its nature, as long as the word “nature” is relevant.

And, sure, I am part of mankind, moreover, a part intending to elaborate a description of the backdrop of the scene. From this crossing point, I faced several possible directions.

Since this dissertation aims to expose the present situation binding the Dark Matter concept to particle physics, shall I blindly write about the latter? Shall I pay attention to cosmology as well?

Despite my “undergraduate-student” experience in particle physics, I thought I would rather avoid writing a “laius” on the Standard Model of particles and its extensions, in particular on supersymmetry. Although naivete is neither the good way, this dissertation simply goes, step by step, the way my mind took.

The reasons for Dark Matter obviously open the dinner. Astrophysical observations of unexpected shapes in the rotation curves, the confirmation of a missing mass, at a larger scale, by lensing are first exposed. The homogeneous and isotropic Friedmann-Lemaître

Universe is therefore introduced in order to explain the cosmological needs for Dark Matter. Actually, low energy density Friedmann-Lemaître Universes are incompatible with the data.

The constraints given by the Cosmic Microwave Background fluctuations, the Big Bang Nucleosynthesis, the Supernovae behaviour provide us with some cosmological parameter values, hence favouring a cosmological model. And therefore the constraints on Dark Matter total energy density contributions are suggested. Following those constraints, several candidates are introduced according to their properties. Namely, Dark Matter particle candidates are divided into baryonic and non-baryonic particles. The latter category abounds in more or less "exotic" particles which properties could fit with Dark Matter requirements. Neutrinos, axions and neutralinos are of those. Scenarios of evolution of Large Scale Structures imply different condition according to some Dark Matter properties, e.g. "hot" and "cold". The apparent evidence for Cold Dark Matter is noticed. However, concerning cosmological probes, namely the topic of the Chapter 2, one should advice to stay eyes wide open. It is in fact worth underlining the uncertainties still existing about cosmological parameters.

Since, as above mentioned, this dissertation deals with particle physics, we will concentrate on particle candidates for Dark Matter. This implies several assumptions on cosmological scenarios without which plenty of other models could avoid the present issue. For instance, homogeneity of the Universe and inflation are not discussed in here; more precisely, they are assumed.

In this optic, the "main dish" intends to expose what is our current knowledge in particle physics and in what it could or could not include Dark Matter candidates. A description of the theoretical framework, from the particle content to the spontaneous breaking of symmetry, is followed by the Glashow-Salam-Weinberg Standard Model of particle physics. This GSW Standard Model is in fact the first version of an unification of the electroweak and the strong interactions.

Leaving little by little the GSW model, we go towards relic neutrinos as Hot Dark Matter and towards the importance of their mass. From neutrino theoretical introduction and the oscillations, we take a winding path to its role in the evolution of the Universe. This neutrino puzzle weighted for a long time on the painting I was drawing in my mind. Actually the neutrino puzzle does not seem to be solved yet.

Afterwards, we go beyond the GSW Standard Model with the explanation of the strong CP problem. This latter is in fact briefly discussed in order to introduce the axion, hypothetical pseudo-Nambu-Goldstone boson arising from the breaking of a global symmetry. Its role as Dark Matter is then exposed.

Finally, last but not least, supersymmetry is shortly described in order to introduce the Lightest SuperParticle as WIMP candidate. Actually, Weakly Interacting Massive Particle is a generic name dubbing the would-be favoured Dark Matter candidate. More precisely, the neutralino would potentially suit. Its contribution as Dark Matter is therefore shortly described. Nevertheless, since this particle is still hypothetical, several experiments and their results are also exposed.

Is it possible to choose between Saint Thomas' behaviour and blind faith in cosmological data?

This is finally the question underlying the following pages. The conclusion intends to gather the principal ideas of this dissertation and to open the way to further points of view.

Units and Conventions

In this dissertation,we work with "natural" units as it is usually done in Quantum Field Theory. Namely, the speed of light and the Planck constant are, from now,

$$\begin{aligned} c &= 1, \\ \hbar &= 1. \end{aligned} \tag{1}$$

In such a scheme, the unit of energy, the electronvolt (eV), is principally used

$$[\text{energy}] = [\text{mass}] = [\text{length}]^{-1} = [\text{time}]^{-1}. \tag{2}$$

Nevertheless it is sometimes difficult to imagine or simply to change units. Thus, for information, the following values are given

$$\begin{aligned} 1\text{eV} &\sim 1.6 \times 10^{-19}\,\text{J} \\ &\sim 1.78 \times 10^{-36}\,\text{kg}. \end{aligned} \tag{3}$$

Several astrophysical notions will be discussed in this dissertation. We therefore the following change of distance units. The astronomic distances are reported in *parsec* which corresponds to the distance of an object seen under 1 arcsecond.

$$1\text{pc} = 3.26\,\text{lyr} \sim 30.8 \times 10^{12}\ \text{km}, \tag{4}$$

with 1 kpc $= 10^3$ pc for galactical distances and 1Mpc $= 10^6$ pc for cosmological distances.

Other units or constant values will be given in the text.

Chapter 1

Why do we need Dark Matter?

The problem of Dark Matter is one of the most evasive and fascinating enigmas in modern physics. Evidences for an unseen substance have been gathered over the past few years. Nowadays, both astronomers and cosmologists are convinced that more than 90% of the total mass of the Universe is due to non-luminous matter. The first queries, the first doubts, as well as the first explanations are exposed here.

1.1 Astrophysical Reasons

Who has never been impressed and amazed by the starlight? Looking at the ceiling of the sky, the celestial sphere, men have been intrigued for centuries by the beauty of what surrounds us. Scientists, dreaming scientists, are of these. As a consequence, exploration of our Universe has begun via the visible light that reaches us, giving birth to astronomy and observation of celestial motions, to astrophysics and a deeper understanding of bodies and larger scales. The Dark Matter problem belongs to the actual puzzling questions that should be an important step to improve our knowledge of astrophysics.

1.1.1 Rotation Curves

In the early time of the exploration of the night sky, astronomers observed objects via the visible light that reaches us. Specific notions and vocabulary were so in close relation to light. For instance, the mass-to-luminosity ratio M/L enables us to extract information on the visible Universe. However, a larger set of tools has been developed.

Nowadays, rotation curves are obtained by redshift analysis. They are the principal tool to deduce distribution of mass in disk galaxies. In the 1970's, astronomers have noticed anomalous rotation curves for some spiral galaxies.

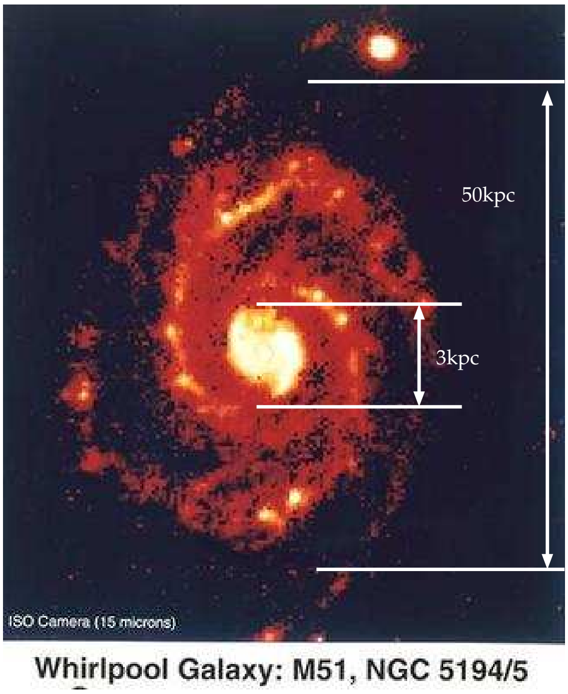


Figure 1.1: Typical size of a spiral galaxy. The diameter of the bulge is about 3 kpc and the diameter of disk about 50 kpc.The dark halo might be as large as 200 kpc. Image ESA, modified.

As Newton's law predicts, one should expect the velocity to decrease with the distance from the bulge.

Indeed the newtonian problem, for a simple body, leads to the following equation, equating circular motion of average distance r from the center with average velocity $v(r)$ and the attraction by $M(r)$, the total mass inside the orbit of radius r:

$$\frac{v^2(r)}{r} = \frac{GM(r)}{r^2} \tag{1.1}$$

which obviously gives

$$v(r) = \sqrt{\frac{GM(r)}{r}} \tag{1.2}$$

where $G = 6.67 \times 10^{-11}$ m^3/kg.s^2 is the gravitational constant.

Generalized to galaxies, this relation can be used as an estimate if they present a spherical or ellipsoïdal mass distribution. The last equation means that, if the mass of the galaxy is concentrated in its visible part, the velocity should vary as $1/\sqrt{r}$ for distances far beyond the visible radius. The typical size of a spiral galaxy are shown on Fig. (1.1).

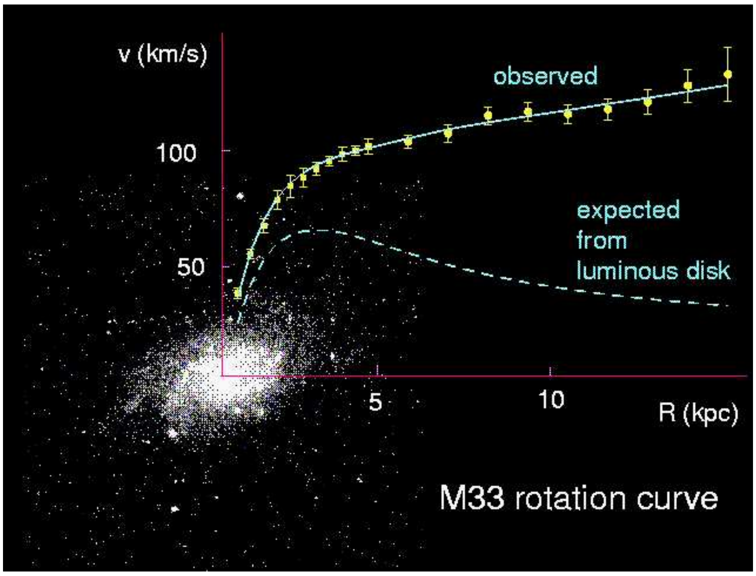


Figure 1.2: Observed rotation curve of the nearby dwarf spiral galaxy M33, superimposed on its optical image [1].

However, $v(r)$, and so by extension the ratio $M(r)/r$, is observed to be roughly constant for large distances: the rotation curves are "flat", as shown by Fig. (1.2). Hence, we deduce that the mass increases linearly with r outside the disk of the galaxy, opening the idea of a hidden mass. To explain this, physicists have postulated the presence of Dark Matter: galaxies would be immersed in an extended Dark Matter halo.

The main contribution to the study of these rotation curves has been made by Vera Rubin. She determined the velocities of spiral galaxies via the measurement of Doppler shifts. In fact, velocities of clouds of ionized hydrogen inside the galaxies are plotted as a function of the distance from the galactic center. Their H-α emission lines enable the measurement of redshift. Together with W.K. Ford, they published their detailed observations of the Andromeda galaxy.

In a more precise way, one could use the rigid-body behaviour of inner rotation curves. Works like those of Vera Rubin or Sofue [2] argue that high-mass galaxies show almost constant rotation velocities from the center to the outer edge. Besides the expected contributions of the nucleus, the bulge and the disk, it seems obvious to introduce a halo component. Comparing the surface mass density, obtained from the rotation curves, with the observed surface luminosity, we derive the mass-to-luminosity ratio M/L. The latter is found not to be constant nor within the bulge or the disk but, and this is what we are mostly interested in here, neither within the halo. Furthermore, the M/L ratio steeply increases towards the halo. This is interpreted like the presence of a Massive Dark Halo, leading to the notion of Dark Mass Fraction (DMF), i.e. the ratio between distributions of dark and visible matter.

Sofue observed that the M/L ratio and the DMF gradually increase from the inner disk to the outer disk. The gradient also increases with the radius. Moreover the M/L ratio and the DMF increase drastically from the outer disk towards the edge, showing the credibility of the idea of a Massive Dark Halo.

First observed by studying spiral galaxies, Dark Matter can also be detected, even though problematically, in galaxies such as ellipticals or lenticulars. And, as a matter of fact, rotation curves are much better known for external galaxies than for ours. However analysis have concluded that indeed the Milky Way contains large amounts of Dark Matter.

From the historical point of view, astronomers like Jan Oort, at the beginning of the XXth century, noticed the orbital velocities in the Milky Way do not decrease with increasing distance from the center of the galaxy. However, he has not anticipated the existence of an unvisible mass. This idea of a missing mass has already been mentioned by Fritz Zwicky in 1933. Nowadays, this lack is widely assumed to be part of the galaxies masses. Dark Matter is believed to account for more than 90 % of the total mass, i.e. energy density, of the Universe.

For instance, considering the visible matter of our galaxy, the density of Dark Matter at the Sun position, i.e. 8.5 kpc from the center of the Milky Way, has been approximatively computed. The mean density of elementary particles gravitationally trapped in the galaxy is expected to be [1, 35]

$$\begin{aligned} \rho_{dark} &\sim 0.5 \times 10^{-24} \mathrm{g\ cm}^{-3} \\ &\sim 0.3\,\mathrm{GeV\ cm}^{-3}. \end{aligned} \tag{1.3}$$

More than an astrophysical artifact, the Dark Matter notion, as disturbing as it is, is acknowledged. As Vera Rubin said during a meeting in 1997: "The identification of dark matter problem via rotation curves of stars in galaxies will one day occupy the same place in the history of human thought that the absence of interference fringes in the Michelson-Morley experiment, and the subsequent falsification of the luminous ether, did. I suspect that we are at the stage where the Michelson-Morley like experiment has been done and Lorentz and Poincaré invariance has been written out by their respective authors, but an Einstein has not yet come along and invented or reinterpreted Lorentz and Poincaré invariance into the theory of special relativity, yet-or if they have, the general scientific community has not embraced their radical theory yet"[1].

Finally, on a critical point of view, we would like to emphasize the fact that some authors deny the necessity of Dark Matter. The study of rotation curves is one indication on a typical scale. And this argument could possibly be misled by other theories, rejecting the notion of Dark Matter. However, observations on larger scales argue in favour of models including Dark Matter. In fact, for now, no theory avoiding the concept of Dark Matter has been revealed relevant on all scales.

Evidence for Dark Matter on larger scales, evoked just above, is discussed in the next subsection.

1.1.2 Gravitational Lenses

From the theory of General Relativity, we know that every massive body deflects the trajectory of propagating light rays: the larger the mass, the more curved the space-time. Actually, light rays follow geodesics of this curved space-time, what can also be viewed as a bending of electromagnetic waves by the gravitational field. This bending by massive objects should give birth to images of distant objects, comparable to the atmospherical mirages. The observation of images of distant galaxies have been predicted, already by Newton -considering light as granules- and also by Einstein [2].

[1]see: `http://capitalastronomers.org/vera_talk_intro.htm`

[2]Einstein's prediction, in 1915, that a light ray passing near the solar limb should be deflected by an angle of 1.75 sec was a big triumph for the theory of General Relativity when it was confirmed by Eddigton

In 1979, a long time after theoretical predictions, Walsh, Carswell and Weynman discovered the first “mirage”, in scientific words: gravitational lens systems.

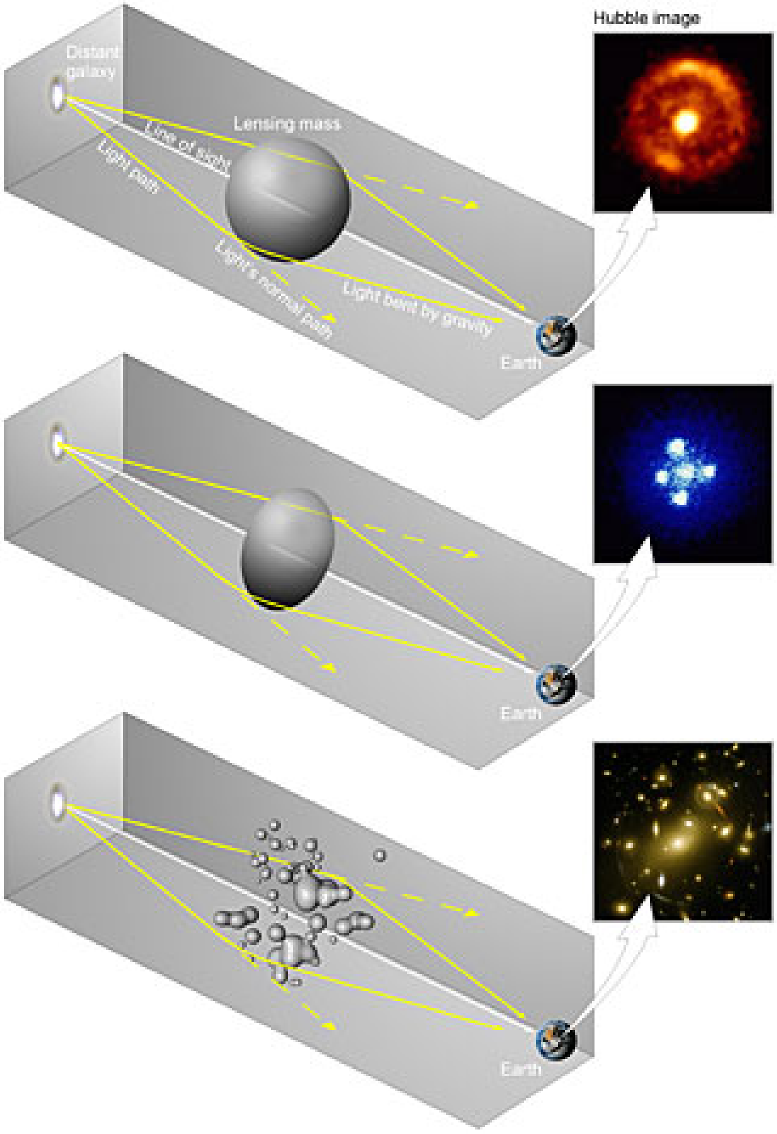


Figure 1.3: Illustration of gravitational lensing by a galaxy or a cluster. The first drawing illustrates the particular case of a perfect alignment of the source, the deflector, with a spherical mass distribution, and the observer, which gives an Einstein ring. On the second drawing, we can see an Einstein cross which appears with a mass distribution of lower symmetry. And finally, gravitational arcs and arclets, portion of Einstein rings are shown. Image ESA.

Galaxies, clusters or every massive object of our Universe can act as a lens and therefore be a “natural telescope” giving astronomers a glimpse of what happens on large scales. Multiple images of distant galaxies or quasars become now accessible. And the possibility of weighing the mass of distant galaxies emerged.

Indeed, for small deflection angle, the Einstein deflection, $\alpha(\xi)$, of light ray passing near

during the solar eclipse in May 1919.

a compact mass at a distance ξ, the impact parameter, is given by

$$\alpha(\xi) = \frac{4GM(\xi)}{\xi}. \tag{1.4}$$

The angle is a function of the mass of the deflecting galaxy. We can also define the Schwarzschild radius R_{sc} telling that only the mass contained in a cylinder defined by the impact parameter ξ contributes to the light deflection (by Gauss theorem).

$$\alpha(\xi) = \frac{2}{\xi} R_{sc} \tag{1.5}$$

In case of perfect alignment between source, deflector and observer, a so-called Einstein ring is observed. The angular radius θ_E of this ring can be measured: $\xi = D_{od}\theta_E$, with D_{od} is the distance between the observer and the deflector and in the small angle deflection condition. The deflection angle α is obviously related to the radius of the Einstein ring θ_E by $\alpha = \theta D_{os}/D_{ds}$, see Fig. (1.4). The separation of observed images, $2\theta_E$ (approximation used even if the alignment condition is not fulfilled), can therefore lead to the value of M/D_{od} or to the value of MH_0, with H_0 the Hubble constant at the present time[3].

Actually, knowing that

$$D = \frac{D_{od}D_{ds}}{D_{os}} = \frac{c}{H_0} z_d \frac{z_s - z_d}{z_s}, \tag{1.6}$$

with D_{od}, D_{os}, D_{ds} , respectively the distances between the observer and the deflector, the observer and the source, the deflector and the source; z_d, z_s the redshift, see Eq. (1.9) below, of, respectively, the deflector and the source, we can write Eq. (1.4) in terms of θ_E:

$$\theta_E \frac{D_{os}}{D_{ds}} = \frac{4GM(r)}{c^2 \theta_E D_{od}}, \tag{1.7}$$

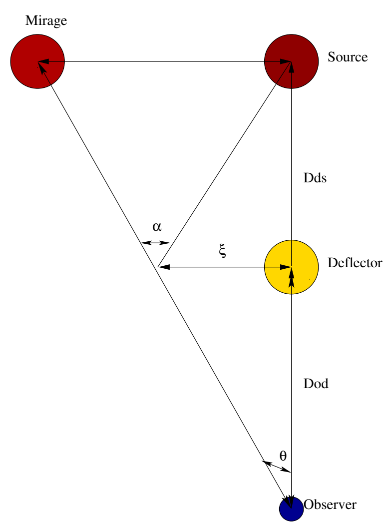


Figure 1.4: Illustration of gravitational lensing in case of a perfect alignment and under small deflection angle assumption. The angles and the distances.

[3] See next Section.

If the distances and the redshifts are known, we can effectively deduce the value of MH_0 according to

$$\theta_E = \sqrt{\frac{4GM(r)D_{ds}}{c^2 D_{od} D_{os}}}, \tag{1.8}$$

where r is smaller than $D_{od}\theta_E$, i.e. we consider the mass inside the impact parameter.

If we know H_0, we may determine the mass of the deflecting galaxy or cluster. Comparing this mass with the visible matter quantity, we deduce that more than 90% of the matter is invisible. This matter seems to be incredibly concentrated in the center of the clusters while there is no significant galaxies assembly near the center. The dark matter-to- visible matter ratio is also observed to increase with the distance.

In summary, the deformation of galaxies, among clusters, enables us to rebuild a “density field” by deducing the mass contained in those clusters. As a consequence, the Dark Matter distribution on large scales can be found.

1.2 Cosmological Reasons

The notion of Dark Matter first appeared with the problem of rotation curves, as introduced above. However, the development of general relativity brought more tools to understand the behaviour of our Universe. The gravitational lensing effects illustrate the geometrical implication of the theory of general relativity. However, the application of this theory gave also birth to cosmology, relating the history of our Universe. Nowadays, the development of cosmology has revealed lots of astonishing needs, for instance non-trivial mass densities. In this section, we shall develop some basic cosmology and introduce the lacks in our classical point of view, in order to introduce the notion of Dark Matter. The energy density, leading to cosmological parameters, will be discussed.

1.2.1 A Short Story of Cosmology

There is no need to recall who was Johann Christian Doppler and the famous effect bearing his name. If a body in motion emits some wave, a fixed observer will measure a shifted wave length. This shift is proportional to the velocity of the emitting body with respect to the observer. The Doppler effect has been usefully applied to many domains of physics and, in particular, in star motion. In this way, we can derive the projection along the line of sight of star velocities, v_r,

$$z \equiv \frac{\triangle\lambda}{\lambda} = \frac{v_r}{c} \tag{1.9}$$

where λ is the emitted wave length, $\triangle\lambda$ represents the difference between measured and emitted wave lengths and z is called the redshift. If stars go away from us, the wave length they emit will be shifted towards the higher λ.

In the XX-th century, more possibilities of observations appeared with the development of technology. Thus, gathering results, it has been observed a tendency to redshifts in nebulae spectra. Astronomers were in front of one of the biggest step in the knowledge of our Universe: the majority of nebulae seem to go away from us. This, together with the assumption that galaxies are separated by large voids, suggested the Universe to be in expansion, on large observable scales. Anyway, for mankind, the idea of expansion led to a big problem: does it exist a center or, in other words, a privileged point? The negative answer to this philosophical question has been formulated by Milne. This "Cosmological Principle" postulates the Universe as a whole should be homogeneous; this homogeneity is, of course, assumed over large scales.

Hubble added a fantastic contribution to this cosmological principle in supposing, and checking, that galaxy redshifts should be proportional to their distances. Which leads to the following relation:

$$\vec{v} = H\vec{r} \tag{1.10}$$

where the coefficient of proportionality H is called the Hubble parameter and is generally parametrized as $H = 100h$ km s^{-1}Mpc^{-1}. Hubble concluded the Universe should be in homogeneous expansion.

To, this principle has been added the condition of isotropy, verified by the Cosmological Microwave Background (CMB). In fact, this background is isotropic in the whole Universe.

1.2.2 The Friedmann-Lemaître Model

It has been mentioned above that the Universe is supposed to be homogeneous and isotropic, this assumption is known as the Cosmological Principle. However, another essential question appeared: is the Universe static or not? The Hubble law is a first clue. Moreover, this question should lead to different cosmological models, depending on the behaviour of parameters, called "cosmological parameters", that could all be included by the Friedmann-Lemaître model. Actually, our Universe, whatever it corresponds to a model or another, could be described by the equations, elaborated in the 1920's, of the russian Friedmann and the most famous belgian physicist Lemaître. In this subsection, the Robertson-Walker metric, the extended notion of redshift and also the Hubble parameter are going to be broached, with the goal of describing the so-called Friedmann-Lemaître Universe.

The Robertson-Walker Metric

In Special Relativity, we shall use the metric, with a negative signature [4]:

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{1.11}$$

However, in General Relativity, due to the curvature of the Universe, this metric has to be modified. One of the main mind revolution introduced by Einstein's theory of General Relativity is its view of gravity. Matter curves the space-time, which guides matter in its motion in return.

The system of coordinates employed is a particular system of spherical coordinates (r, θ, ϕ) called "comoving" coordinates. To answer to the question whether is the Universe static or not, an expansion parameter is introduced. The role of this parameter will enable us

[4] Notice that the sign of the metric is different if it is used by a cosmologist or a particle physicist.

to answer to this question. Because they are motionless in a 3-dimensional space, massive objects need to be at rest in this specific system, this is the definition of the comoving coordinates. So, we define a system of coordinates such that matter keeps the same coordinates with running time. In this optic, the comoving coordinates take the expansion into account via the parameter $R(t)$. Then, the physical distance dl between two infinitesimally close objects with coordinates $(r,\ \theta,\ \phi)$ and $(r + dr,\ \theta + d\theta,\ \phi + d\phi)$ is given by:

$$dl^2 = R^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta \ d\phi^2) \right], \tag{1.12}$$

and so, as the metric is given by[5]

$$ds^2 = dt^2 - dl^2, \tag{1.14}$$

the Robertson-Walker metric is

$$ds^2 = dt^2 - R^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta \ d\phi^2) \right], \tag{1.15}$$

where $R(t)$ is the scale factor and k is a constant number. The space-time is “sliced” into hypersurfaces homogeneous and isotropic[6].

Assuming homogeneity, the total curvature can, indeed, be decomposed as follows: a spatial and a space-time curvature. The spatial curvature is easier to behold as it is an usual 3-dimensional space curvature at fixed time. Three maximally solutions can be found, corresponding to euclidean, hyperboloïc or elliptic space. This space curvature enters in the Robertson-Walker metric as the constant k. If k equals 0, the Universe is flat and, so, euclidean; if $k < 0$, the Universe is open and, conversely, if $k > 0$, it is closed, and correspondingly hyperboloïc or elliptic.

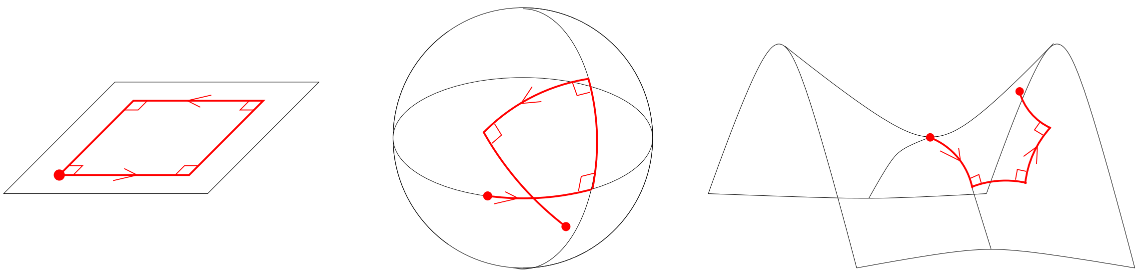


Figure 1.5: Spatial curvature for some 2-dimensional spaces: euclidean, elliptic and hyperboloïc spaces, or, correspondingly, flat, closed and open spaces [3].

[5]As already mentioned, the sign of the metric can be different. Thus, we will find, in general relativity

$$ds^2 = -dt^2 + dl^2. \tag{1.13}$$

[6]This metric, whatever is the k value, does not depend on the origin: the expression for physical length are left unchanged by change of coordinates. A good analogy can be done by projecting a globe on a map under different point of view.

The second curvature is the one responsible for the expansion and is a 2-dimensional curvature (t, x), so-called "space-time curvature". Assuming the isotropy of the Universe, we can say that (t, y) and (t, z) are similar to (t, x). It is, awkwardly, less obvious to apprehend. The scale factor, $R(t)$, is tightly related to this curvature.

In case of open or closed Universe, we can define the radius of curvature as

$$R_c(t) = \frac{R(t)}{\sqrt{|k|}}. \tag{1.16}$$

The scale factor obviously leads to an infinite radius for our Universe when $k = 0$.

The Redshift

In the comoving coordinates, matter, in particular non-relativistic matter, is locally at rest. Geodesics[7] followed by non-relativistic bodies in space-time are given by $dl = 0$, while photons obey to $ds^2 = 0$ or, in other terms:

$$\begin{aligned} dt^2 &= dl^2 \\ \Leftrightarrow dt^2 &= R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\ d\phi^2)\right], \end{aligned} \tag{1.17}$$

for the photons.

We are now able to re-express the redshift in terms of the scale factor. Taking a photon with constant (θ, ϕ) (radial geodesics) and extrapolating on large scales in integrating Eq. (1.17) between t_1 and t_2, we obtain the equation of propagation of light

$$\begin{aligned} dt^2 &= R^2(t)\frac{dr^2}{1-kr^2} \\ \Rightarrow \int_{r_1}^{r_2} \frac{dr}{\sqrt{1-kr^2}} &= \int_{t_1}^{t_2} \frac{dt}{R(t)}, \end{aligned} \tag{1.18}$$

which gives us the change in comoving coordinates for a photon.

So, for two observers with fixed comoving coordinates and with λ_1 the measured wave length at t_1 the emission time, and λ_2 the measured wave length at t_2 reception time, we can apply Eq. (1.9) of Doppler effect.

As emission at $t_1 + dt_1$ and reception at $t_2 + dt_2$ does not change the left hand side of the previous relation, for any value and sign of k, we find

$$\frac{dt_1}{R(t_1)} = \frac{dt_2}{R(t_2)}, \tag{1.19}$$

or,

$$\frac{\lambda_2}{\lambda_1} = \frac{R(t_2)}{R(t_1)}, \tag{1.20}$$

[7] Geodesics are the shortest paths between two points; it corresponds to the straight line in euclidean geometry. Geodesics of a surface are mathematically defined as curves for which the variation of their tangent vector is orthogonal to the considered surface.

which leads to

$$z = \frac{R(t_2)}{R(t_1)} - 1. \tag{1.21}$$

Choosing t_2 the actual time to be t_0, which is the reception time, and t_1 to be t, the emission time, as well as defining R_0 the present scale factor, we are left with the following relation:

$$z = \frac{R_0}{R(t)} - 1. \tag{1.22}$$

Thus, the scale factor can be reinterpreted as a stretching of light in curved space-time. The observed redshifts of galaxies, and other distant objects, teach us about the expansion. At small redshifts, where the distance L that separates us from the galaxy we study [8] can be approximated by dl, with $dl = dt$, we recover the Hubble law:

$$z = \frac{R(t_0)}{R(t_0 - dt)} - 1 = \frac{\dot{R}(t_0)}{R(t_0)}dt, \tag{1.23}$$

because $R(t_0 - dt) \sim R(t_0) - \dot{R}(t_0)dt$ by Taylor expansion. And, as a consequence, we define the Hubble parameter as the expansion rate of the scale factor:

$$H(t) = \frac{\dot{R}(t)}{R(t)}. \tag{1.24}$$

Anyway, the present value of $H(t)$ is noted H_0. This latter is also parametrized by h, which is a dimensionless quantity,

$$H_0 = 100\, h_0 \text{ km s}^{-1}\text{Mpc}^{-1}, \tag{1.25}$$

where $0.5 < h_0 < 0.8$[9].

Let us define $\tilde{H} \equiv H/H_0$ the Hubble expansion rate relative to its present value. This also lets us with a dimensionless quantity.

Time is often expressed as a function of the scale factor $R(t)$ or simply the redshift z, parametrized by h, obviously. The convention for the origin of the time axis is chosen by extrapolating $R(t = 0) = 0$. In fact, by differentiating Eq. (1.22), we can express the Hubble parameter as a function of the redshift:

$$dt = -\frac{dz}{(1+z)H(z)}. \tag{1.26}$$

The Friedmann-Lemaître Law

The homogeneous Universe is curved by its own matter content. The space-time curvature, explaining this behaviour, is also related to the matter density. The homogeneous energy

[8] Studies of galaxies are possible via the light we receive, as already mentioned, that is why Eq. (1.22) is so important. Overduin and Wesson [6] mention it as the EBL, Extragalactic Background Light, which overflows of hidden information.

[9] The values of the cosmological parameters will be discussed in the next chapter.

density of the Universe $\rho(t)$ related together with the spatial curvature k and the scale factor $R(t)$, both describing the space-time curvature, gives the Friedmann-Lemaître equation[6]:

$$\tilde{H}^2 = \frac{8\pi G}{3H_0^2}\rho_{tot} - \frac{k}{R(t)^2 H_0^2}, \tag{1.27}$$

where ρ_{tot} is the total density of all forms of matter-energy.

The newtonian problem amazingly leads to a solution which is quite close to the Friedmann-Lemaître equation. We want to express the motion, in the Universe, of a "particle" located at a distance $r(t)$ from us. As the Universe is assumed to be isotropic, Gauss theorem can be employed. So, we simply consider the matter contained inside of a sphere of radius $r(t)$: this radius is chosen to vary with time to mimic the expansion. The motion of a particle located at such a distance is given according to the following newtonian equation of motion:

$$\ddot{r}(t) = -\frac{GM(r(t))}{r^2(t)}, \tag{1.28}$$

where, even thought r(t) varies, the mass is constant because of the spherical symmetry imposed by homogeneity and isotropy: no particle can leave or enter the sphere.

Integrating, we find:

$$\frac{\dot{r}^2(t)}{2} = \frac{GM(r(t))}{r(t)} - \frac{k}{2}, \tag{1.29}$$

where k, which we have defined as the spatial curvature, appears naturally here as an integration constant.

Introducing the homogeneous mass density $\rho_{mass}(t)$ as the mass $M(r(t))$ multiplied by the volume of the sphere of radius $r(t)$, the solution of the newtonian problem reads:

$$\left(\frac{\dot{r}(t)}{r(t)}\right)^2 = \frac{8\pi G}{3}\rho_{mass}(t) - \frac{k}{r^2(t)}. \tag{1.30}$$

The expansion rate $\dot{r}(t)/r(t)$ appears naturally here. The definition of the evolution of the scale factor with time makes a link between the newtonian solution and the Friedmann-Lemaître equation: the $r(t)$ corresponds to the $R(t)$ in general relativity. However, neither the dimension of the space nor the mathematical objects are the same; the density is no longer a scalar in General Relativity.

Moreover, in the newtonian problem, the mass density evolves as the third inverse power of the radius $r(t)$. It means we have accounted non-relativistic matter only.

The Universe contains also relativistic particles, e.g. photons and light neutrinos, and a hypothetically vacuum energy density which probably has to be associated with the cosmological constant introduced by Einstein. We have to include these contributions in ρ_{tot}. Moreover, coming back to Eq. (1.27), we want to write explicitly the energy density. The relation between $R(t)$ and $\rho(t)$ depends on the type of matter; the energy of each type gets diluted by the expansion on its own way.

Under assumption of homogeneity and isotropy, the matter-content of the Universe can be modelled by an energy-momentum tensor of the perfect fluid. The equation of state of this fluid is

$$p = (\gamma - 1)\rho \tag{1.31}$$

with γ the adiabatic index. This index values vary with the type of energy. So, for non-relativistic matter, also called "dust-like matter", γ equals 1; while for radiation energy, approximated by an ultra-relativistic perfect gas, it is equal to 4/3.

In the comoving frame, ρ and p are left unchanged. We express the energy-momentum tensor in this frame, in the following way:

$$T_{\mu\nu} = (\rho + p)\, U_\mu U_\nu - p g_{\mu\nu}, \tag{1.32}$$

where $U_\mu = (1, 0, 0, 0)$ is the component of the 4-dimensional velocity of the comoving frame, p is the pressure and ρ the energy density.

This tensor, combined with the equation of state, enables us to find a conservation equation, via the resolution of the Einstein equations. This resolution is beyond the scope of this dissertation, only the final solution interests us here,

$$\frac{1}{R^3}\frac{d}{dt}(R^3(\rho + p)) = \frac{dp}{dt}. \tag{1.33}$$

Inserting Eq. (1.31), we find[10]:

$$\begin{aligned} \frac{1}{R^3}\frac{d}{dt}(R^3\gamma\rho) &= (\gamma - 1)\frac{d\rho}{dt} \\ \Leftrightarrow 3\gamma\frac{\dot{R}}{R} &= -\frac{\dot{\rho}}{\rho} \end{aligned} \tag{1.35}$$

as γ is a constant. Integrating, it reads:

$$\rho(t) = \text{cste} R^{-3\gamma}(t) \tag{1.36}$$

The researched relation between the scale factor and the energy density is expressed, for dust-like matter, for which pressure is negligible,

$$\rho(t) \propto R^{-3}(t), \tag{1.37}$$

and, for radiation matter,

$$\rho(t) \propto R^{-4}(t), \tag{1.38}$$

due to the γ values.

For the vacuum, the adiabatic index is zero, which implies $p_v = -\rho_v$. The vacuum energy density is, so, never diluted and its density remains constant. The cosmological constant, associated to the vacuum energy, will be denoted Λ below.

In summary, the energy density contribution of non-relativistic matter, or dust-like matter[11], is obvious from a newtonian point of view. Anyway, to this contribution, we have to add those of the energy density of relativistic matter, called radiation, and the ambiguous cosmological constant, which relative energy density is given by $\Lambda/8\pi G$.

[10] This relation is often found in the following form, without the simplification dued to Eq. (1.31):

$$\dot{\rho} = -3H(\rho + p), \tag{1.34}$$

with the relation between H and the scale factor given by Eq. (1.24).

[11] The energy is related to the mass via the so well-known Einstein mass-energy relation: inertia of a body depends on its energy.

1.2.3 Matter Budget

The Friedmann-Lemaître equation, which was the subject of the previous section, includes all the types of energy mentioned above, namely, the radiation matter, the dust-like matter, the cosmological constant, as well as a spatial curvature component. Nevertheless, the dominating component of the density is not yet known. We can usefully rewrite Friedmann-Lemaître equation in detailed terms:

$$H^2 = \frac{8\pi G}{3}\rho_r + \frac{8\pi G}{3}\rho_m - \frac{k}{R^2} + \frac{\Lambda}{3}, \tag{1.39}$$

or, equivalently, conserving the dimensionless notations of Eq. (1.27)

$$\tilde{H}^2 = \frac{8\pi G}{3H_0^2}\rho_r + \frac{8\pi G}{3H_0^2}\rho_m - \frac{k}{R(t)^2 H_0^2} + \frac{\Lambda}{3H_0^2}. \tag{1.40}$$

The right-hand side terms vary, correspondingly, like R^{-4}, R^{-3}, R^{-2}, R^0. If the scale factor keeps growing, these four components, if all present, dominate the expansion one after each other. Actually, a radiation stage that decreases as R^{-4} should arrive before a matter-dominating era, decreasing as R^{-3}, a curvature dictature, followed by R^{-2}, and a final cosmological constant domination. During each stage, one component strongly dominates the others. We would like to notice that the role played by the spatial curvature is assumed to be important, and even predominant for our Universe. Some authors adopt the point of view where k is considered as a density like those of dust-like matter, for example. However, this usage should encourage one to think of curvature as a contribution to the energy density of the Universe, which is not correct. And, inserting Eq. (2.6) in Friedmann-Lemaître equation, we are able to predict the behaviour of the Hubble factor. First, neglecting the curvature and cosmological constant terms, we integrate easily Eq. (1.39) and obtain,
for the radiation domination:

$$\begin{aligned} H^2(t) &\propto R^{-4}, \\ \frac{\dot{R}(t)}{R(t)} &\propto R^{-2}(t), \\ R &\propto t^{1/2} \\ \Rightarrow H(t) &= 1/(2t); \end{aligned} \tag{1.41}$$

and for the matter domination:

$$\begin{aligned} H^2(t) &\propto R^{-3}, \\ \frac{\dot{R}(t)}{R(t)} &\propto R^{-3/2}(t), \\ R &\propto t^{2/3} \\ \Rightarrow H(t) &= 2/(3t), \end{aligned} \tag{1.42}$$

what means the Universe, at early times, was decelerating as a power-law.

Furthermore, by interpreting the spatial constant term $-k/(R^2)$ as a "total energy" (and still neglecting Λ), we see that the evolution of the Universe is governed by a competition between the potential energy $8\pi G/3\ (\rho_r + \rho_m)$ and the kinetic term $H^2(t) = (\dot{R}/R)^2$.

So, the "fate" of our Universe seems to be determined by the curvature constant k. Nevertheless, the Universe must be expanding or contracting. Correspondingly: for $k > 0$, it will collapse whereas for $k = 0$ and $k < 0$, it will infinitely expand. However, the cosmological constant Λ can dominate the "fate" of the Universe, whatever is the k value, if Λ is non-zero.

We should expose what happened in the early times. In the 1940's, Gamow and his collaborators, Alpher and Herman, proposed a first formulation of the Big Bang model. The Big Bang occured, by definition, when the scale factor goes to zero. Our actual knowledge in physics allows us to describe the Universe from the Planck time $t \sim 10^{-43}$ s [3], i.e. when gravity became a classical theory. Anyway, they proposed that the early Universe was once very hot and dense such that matter and radiation existed in a nearly perfect thermal equilibrium. At the very beginning, an inflationary stage took place at $t \sim 10^{-35}$ s. After, during a reheating stage, the scalar field responsible for inflation decayed into a thermal bath of Standard Model particles like quarks, leptons, gauge bosons and Higgs bosons. At $t \sim 10^{-4}$s, the baryogenesis began combining quarks into baryons and mesons, generically hadrons. The thermal equilibrium between radiation and matter could allow the synthesis of hydrogen. This stage is called the radiation era. The appropriate equation of state to be used for this era is the one corresponding to a gas of radiation, namely "relativistic or ultra-relativistic particles". This early Universe has expanded and cooled to a more understood state, see Appendix A. Around $t \sim 10^4$ years, dust-like matter took the principal part, instead of radiation. The decoupling of photons caused by the recombination of atoms takes place at $t \sim 10^5$ years.

An important contribution of vacuum-like dark energy is not excluded in the future.

During the cosmological constant domination, neglecting the radiation and dust-like matter energy density components, we would have an exponentially accelerated expansion, in opposition to the decelerating power-law expansion in the first two cases,

$$\begin{aligned} H^2(t) &= \frac{\Lambda}{3}, \\ \frac{\dot{R}(t)}{R(t)} &= \sqrt{\frac{\Lambda}{3}}, \\ R &= \exp(\sqrt{\Lambda/3}\,t) \\ \Rightarrow H &= \sqrt{\frac{\Lambda}{3}}. \end{aligned} \tag{1.43}$$

The Hubble parameter is, in this case, time independent, but governed by the cosmological constant. This last, once introduced by Einstein, has often been assumed to be zero, to make calculations easier. However, it has been reintroduced as a fundamental constituent of the energy density. More than a simple constant, the presence of this vacuum energy can radically change the future of our Universe. Indeed, independently of the sign of the spatial curvature k, Λ could be the dominating term in the Friedmann-Lemaître equation. It means, if its sign is negative, the Universe could eventually re-collapse. The role played by this term is not quite clear and is regarded with particular suspicion.

In this paragraph, the aim is to evaluate the present values of the four right-hand side terms of Eq. (1.39), in order to define relative contributions.

At the present time, Eq. (1.27) and, by extension, Eq. (1.39) are

$$1 = \frac{8\pi G}{3H_0^2}(\rho_{r,0} + \rho_{m,0}) - \frac{k}{R_0^2 H_0^2} + \frac{\Lambda}{3H_0^2}, \tag{1.44}$$

dued to $H(t=0)/H_0 = 1$.

In order to define dimensionless quantities, or rather quantities parametrized by h^2, also dimensionless, we introduce the notion of critical energy density. This critical density is the energy density of a spatially flat Universe, i.e. $k=0$, when Λ is nul.

$$\begin{aligned}\rho_c \equiv \frac{3H^2(t)}{8\pi G} &= 1.88\,10^{-26} h^2 \;\; \text{kg m}^{-3} \text{ or equivalently } 10^{-29} h^2 \;\; \text{g cm}^{-3} \\ &= 1.05\,10^{-5} h^2 \;\; \text{GeV cm}^{-3}.\end{aligned} \tag{1.45}$$

Actually, if the energy density of the Universe is larger than this critical value, the situation would be comparable to a Big Crunch, corresponding to a negative spatial curvature, or, in other words, the Universe would re-collapse. At contrary, with a lower density, large-scale structures observed could not develop. However, this critical density should not be able to give a satisfactory explanation.

In fact, it is widely believed that an inflationary period, appeared around $t \sim 10^{-35} s$, i.e. in an earliest stage of our Universe, would have driven ρ_{tot} to the critical density.

Small inhomogeneities fall down by gravitational effect, giving birth to stars, galaxies and so on. However, those inhomogeneities have to be unstable, i.e. time dependent. The Cosmic Microwave Background, namely CMB, the fading glow of the Big Bang itself, “ashes and smoke of creation” as Lemaître used to say, tells us about such inhomogeneities. Unfortunately, inhomogeneities compatible to what we observe today should be larger than the ones observed in the CMB fluctuations. Many scenarios have been proposed to face this problem, among others some including Cold Dark Matter, which is part of the topic of this dissertation. Nevertheless, physicists suggested that inhomogeneities such as those observed in the CMB would have been enlarged by an inflationary stage. A scalar field should be involved even if its nature is still unknown. Alan Guth and Andreï Linde were the pioneers of this theory.

Therefore, with increasing exponentially the scale factor, the inflation could lead to a spatially flat, or at least close to it, Universe. Our daily vision of a flat Earth whereas we know this Earth to be an ellipsoïd is due to the scale difference. That is what happened during the inflation: the curvature radius of the sphere goes to high values and mimic a flat Universe.

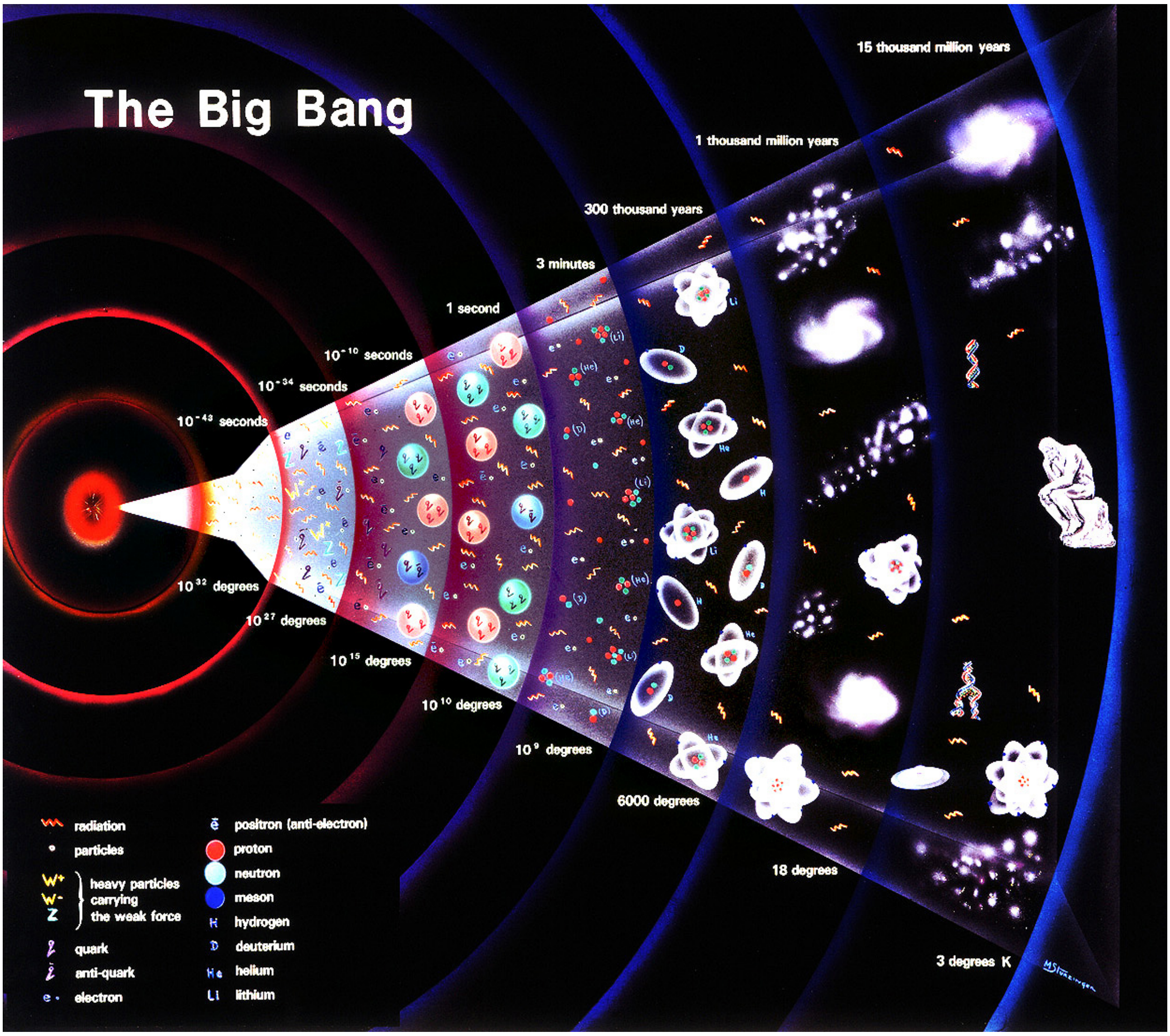


Figure 1.6: The Big Bang. Image CERN.

For the present time, we can re-write the Friedmann-Lemaître equation in a simpler form.

The "cosmological parameters" $\Omega_r, \Omega_m, \Omega_\Lambda, H_0$ are the present-day density parameters: the energy densities, of each type of energy matter, relative to their critical values[12] and are, in this way, dimensionless.

Ω_{tot}, the total density, is $\Omega_r + \Omega_m + \Omega_\Lambda$, so that Eq. (1.44) becomes the matter budget equation:

$$\frac{k}{H_0^2 R_0^2} = \Omega_{tot,0} - 1\,, \tag{1.46}$$

or,

$$\sum_i \Omega_{i,0} + \Omega_{\Lambda,0} - 1 = \frac{k}{R_0^2 H_0^2}, \tag{1.47}$$

[12]For instance, $\Omega_r = \rho_r/\rho_c$ and $\Omega_{tot,0} = \rho_{tot}/\rho_{c,0}$ where $\rho_{c,0}$ is the present critical density.

which explicitly writes the k term and immediately recovers the condition $\Omega_{tot,0} = 1$. The sum is over all the matter species i.

Putting the previous relation expliciting k into Eq. (1.27),

$$\tilde{H}^2 = \Omega_{tot} - (\Omega_{tot,0} - 1)\,\tilde{R}^{-2}, \tag{1.48}$$

where $\tilde{R} = R/R_0$.

By Eq. (2.6), $\rho_r = \rho_{r,0}\tilde{R}^{-4}$ and $\rho_m = \rho_{m,0}\tilde{R}^{-3}$. Using this and Eq. (1.22) $(1+z) = \tilde{R}^{-1}$, we can rewrite the Friedmann-Lemaître equation

$$\tilde{H}(z) = [\Omega_{r,0}\,(1+z)^4 + \Omega_{m,0}\,(1+z)^3 + \Omega_{\Lambda,0}\; - (\Omega_{tot,0} - 1)\,(1+z)^2]^{1/2}. \tag{1.49}$$

This version of the Friedmann-Lemaître equation explicitly shows the dependance with the scale factor, or at least the redshift. As the redshift is related to the age of the Universe, it can also be understood as an indication on time.

Now that the main notions have been introduced, we shall approach the concept of Dark Matter and, principally, we shall answer to the question "why would we need Dark Matter?".

So, it has been said above that the total energy density, $\Omega_{tot,0}$, should be equal to 1 for many reasons. However, the determination of the density of our Universe, via the measures of the four cosmological parameters, i.e. $\Omega_\Lambda, \Omega_m, \Omega_r$ and H_0, is one of the main task of modern observational cosmology.

The density of luminous matter has been deduced in particular from the EBL, extragalactic background light [6]. Values given by Overduin and Wesson are

$$\begin{aligned} \rho_{lum} &= 4 \times 10^{-32} \;\; \mathrm{g} \quad \mathrm{cm}^{-3} \\ &= 4 \times 10^{-29} \;\; \mathrm{kg} \quad \mathrm{m}^{-3}. \end{aligned} \tag{1.50}$$

Comparing these values with the critical energy density Eq. (1.45), we immediately notice that ρ_{lum} represents less then 0.5% of the critical energy density,

$$\Omega_{lum,0}\, h_0^2 = 4 \times 10^{-3}. \tag{1.51}$$

In fact, even though there are uncertainties lying on h_0 value, its value could never match with the two quantities, i.e. $\Omega_{tot,0}$ and Eq. (1.51). The lack of luminous matter is that huge that we cannot deny it. This is why the notion of "non-luminous" density, namely Dark Matter, has been evoked.

1.3 In Summary...

On galaxy or cluster scales, analysis of velocity curves or bending of light rays through their gravitational effects do not fit with the actual knowledge. Among the models trying to explain those observed behaviour, the assumption of a missing mass has been showed to be relevant.

Moreover, cosmological observations tend to adopt a spatially flat Universe model, i.e. $k = 0$. The Friedmann-Lemaître law illustrates the evolution of the scale factor $R(t)$, and in particular the Hubble parameter $H(t)$, according to the energy density. The zero value of k implies a simpler formulation of this Friedmann-Lemaître law, i.e. Eq. (1.49). In fact, the energy density of each type of matter with corresponding dependance on the redshift are explicitly shown.

The energy density of luminous matter, Eq. (1.51), cannot fit with the spatial flatness condition. Moreover, even if the spatial flatness is not assumed in all the theories, the luminous energy density contribution is so negligible that it could not fit nor with the rotation curves nor with low density cosmological models, i.e. $\Omega_{tot,0} < 1$. In that scheme, non-luminous matter energy density has to be taken into account.

The reasons for Dark Matter are uncontestable in our present point of view of Physics and Cosmology. However, "Dark Matter's nature" is still a mystery.

Chapter 2

Dark Matter Candidates, a Short Introduction to the Main Dish

In the previous chapter, the reasons for Dark Matter have been exposed. An uncontestable lack in the energy density as well as an overwhelming observational evidence for invisible mass led physicists to view “dark” components.

While cosmological parameters are not well known, while the evolution of the scale factor depends on those parameters, lots of cosmological models have been proposed. Indeed the possible values of those cosmological parameters lead each to different models. Moreover, once the repartition between the type of matter, i.e. dust-like, vacuum-like or radiation matter, fixed, their own content is still a mystery. The search for the solution to this overwhelming problem provides an important interaction between particle physics and cosmology, now called “astroparticle” physics.

Actually it seems that only elementary particles should be reliable for Dark Matter in our current understanding of the Universe and in the scenario we have adopted here, i.e. assumption of homogeneity and inflation.

In fact, we are seeking for matter that has no significant interactions with other matter, namely weakly interacting or “noticed” by ordinary matter primarily via their gravitational influence. Otherwise, the effect of those interactions would already have revealed its nature. The Standard Model of particles should thus include some candidates.

In this section, the Dark Matter candidates are shortly described, in order to bind cosmological needs and particle physics help.

From now the Hubble parameter at the present time h_0 will be denoted h. The values given above are obviously the present day values.

2.1 The Four Elements of Modern Cosmology

We begin here with a brief overview of the current situation and will follow by a closer look at the arguments for all the parts of ”Nature’s dark side”.

Actually, indirect evidences over the past few years have increasingly suggested that there are in fact four distinct categories of Dark Matter and Energy. The Dark Matter should fulfill the lack in $\Omega_{m,0}$ while the Dark Energy plays the part of the cosmological constant, i.e. $\Omega_{\Lambda,0}$.

From the Supernovae Hubble program, current data indicate that the vacuum energy is indeed the largest contributor to the cosmological density budget, with

$$\Omega_{\Lambda,0} = 0.72 \pm 0.05, \tag{2.1}$$

and,

$$\Omega_{m,0} = 0.28 \pm 0.05, \tag{2.2}$$

with $k = 0$ [8, 24].

Let us shortly develop the $\Omega_{m,0}$ term. The Standard Model of particles distinguishes two families of elementary particles[1], i.e. leptons and hadrons. These names come from the greek expressions "leptos", thin or light, and "hadros", strong. Indeed, oppositely to hadrons, leptons are non-strongly-interacting particles.

The hadrons are divided into mesons ("mesos": middle) and baryons ("baryos": heavy). In a pedagogic goal, baryons are also often denoted by "the stuff we are made of". Indeed, mesons are made of a quark and an anti-quark while baryons are composed of three quarks. As the nucleons are baryons, common matter is called baryonic matter. Hence, Dark Matter can be divided into baryonic and non-baryonic particles.

So the baryonic energy density -including both ordinary matter and a supposed fraction of Dark Matter-, the non-baryonic energy density and the Vacuum or Dark energy should enable the total energy density $\rho_{tot,0}$ to reach its critical value. In other words, $\Omega_{tot,0}$ should equal 1.

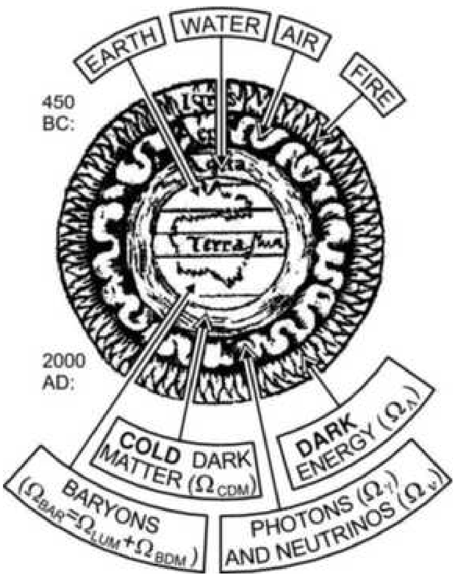


Figure 2.1: "The four elements of ancient and modern cosmology". Earth, water, air and fire for ancient cosmology attributed to Empedocles. The modern counterparts to these 4 elements, viewed by Overduin [6].

[1] An extended description of the Standard Model of particle physics is the topic of the following section.

2.1.1 Baryonic Energy Density

Since ordinary matter is baryonic, the most straightforward possibility is to assume also this composition for Dark Matter, especially at the galaxy scale.

Contribution of the unseen gas clouds to the energy density are not enough. X-ray observations imply that, even in rich clusters, hot gas makes up less than 20% of the total mass of the system[13]. Moreover, matter contained in planets, "failed" stars, brown dwarfs[2] or primordial black holes which are too dim to be seen accounts for baryonic matter. So, these astrophysical bodies, this hidden galactic baryonic matter called MAssive Compact Halo Objects (MACHOs[3]) could be part of the Dark Matter, at least in the galactic dark matter halos. Thus we define the baryonic energy density as $\Omega_{bar} = \Omega_{lum} + \Omega_{bdm}$, where the index bdm stands for baryonic Dark Matter and we refer to Eq. (1.51) for the values of the luminous energy density. Last values for the energy density of baryonic matter are[8]

$$\Omega_{bar} h^2 = 0.023 \pm 0.001. \tag{2.3}$$

We immediatly notice that, considering Eq. (2.2), the baryonic matter, even if it is dark, cannot solve the problem exposed above.

Moreover, the Big Bang nucleosynthesis confirms the necessity of non-baryonic Dark Matter with giving an upper limit to the baryon density.

The origin of the elements after the Big Bang, i.e. the primordial abundances on light elements (deuterium, lithium and helium), is explained by the theory of the synthesis of nuclei, namely nucleosynthesis. Between $t \sim 1$ s and $t \sim 200$ s, the first nuclei gathered, as seen in Fig. (1.6). Due to a dissymmetry between neutron and proton masses, the ratio "number of neutrons" by "number of protons" does not remain constant. This sets a limit to the number of baryons that can exist in the Universe [8],

$$0.012 \leq \Omega_{bar}\, h^2 \leq 0.025. \tag{2.4}$$

The only free parameter in this theory is the baryon-to-photon ratio. Thus, primordial abundance measurements determine this ratio. In comparing those results with theoretical predictions and assuming that the number of photons remains constant, i.e. the number of "free" photons is negligible in front of the CMB content, the previous value is found. It is a key for our understanding of the matter budget of the Universe.

If the Universe is flat, the conclusion of this section can be summarised in this way: there is an uncontestable evidence for non-baryonic matter. Furthermore, the data show that the fraction of baryonic Dark Matter, the ratio Ω_{bar}/Ω_m evaluated at 0.17 ± 0.06 [11] , is not consistent with a dust-like matter dominated Universe. And so the theory of nucleosynthesis fits with the values given by Eq. (2.1) and Eq. (2.2). Dark baryons would play the role of "reservoir" for galaxies.

From a critical point of view, we notice that, for instance, the mass estimates of Galactic Dark Matter Objects (GDMO), obtained via microlensing observations, should lead to different conclusions. Oldershaw [14] suggests a Self-Similar Cosmological Model (SSCM).

[2]Low luminosity stars are shown to contribute less than 6% of the unseen matter in the galactic halo [13].

[3]While it is difficult to detect those objects through their luminosity properties, we should detect them through their gravitational effects. So, gravitational lensing or microlensing searches are driven.

The SSCM proposes that Nature is organised into discrete hierarchical scales which exhibit self-similarity. From the coincidence of similarity between the Dark Matter mass function inside the bulge and those inside the halo of galaxies, he assumes that there should be a universal mass function. So atomic, stellar and galactic scale systems would constitute the three equally fundamental cosmological scales, indeed the total number of cosmological scales could possibly be infinite. This fractal model involving discrete self-similarity would have made unique predictions that are consistent with the "unusual observational findings". Moreover, the SSCM would predict that the GDMO populations should constitute virtually all of the galactic Dark Matter. Nevertheless, we emphasize that this paper deals with galactic Dark Matter since microlensing techniques are used. However, according to what has been said above, those GDMO would be sufficient to solve the Dark Matter enigma.

2.1.2 Non-Baryonic Energy Density

Two categories of non-baryonic matter can be distinguished. In fact, another type of division can be made into relativistic and non-relativistic particles. Particles which velocity is close to the speed of light when they decouple from the primordial fireball, i.e. relativistic or "hot", have different properties than those leaving with lower velocities, in that case called non-relativistic or "cold".

From the analysis of the measurements of the anisotropy of the CMB and the spatial distribution of galaxies, we deduce, knowing Eq. (2.2) and Eq. (2.3), that

$$\Omega_{nbm}\, h^2 = 0.111 \pm 0.006, \tag{2.5}$$

where *nbm* stands for non-baryonic matter.

Hot Dark Matter

The Standard Model of particles and their interactions should be able to explain the Hot Dark Matter (HDM) and to predict its contribution to the total energy density. The HDM energy density is often referred as the neutrino plus the photon energy density Fig. (2.1).

If neutrinos are massless then they remain relativistic throughout the history of the Universe and behave for dynamical purposes like photons. Otherwise, massive neutrinos could be part of the pressureless Dark Matter. In such a case, they would have played an important role in the formation of large-scale structure in the early Universe. These neutrinos belong to the HDM because they were moving with relativistic velocities at the time the galaxies started to form.

Recent experiments suggest that their contribution to the energy density should be lower than the contribution of baryons.

A glimpse of the neutrinos enigma will be exposed below.

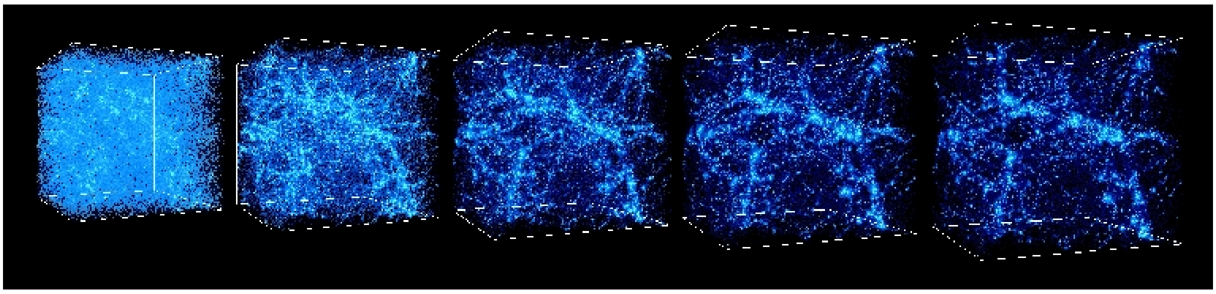
Cold Dark Matter

From the behaviour of visible matter on large scales, like galaxies and clusters[4], we guess that the missing mass should consist of particles whose interactions with ordinary matter

[4]See section 1.1.1 about Rotation Curves.

are so weak that they are primarily seen via their gravitational influence. Moreover, it would mean that those particles have a negligible velocity, this is why they are called “cold”. These unknown particles, interacting really weakly with electromagnetic radiation, are classified as Cold Dark Matter (CDM). But Dark Matter candidates have to satisfy more conditions. In fact, candidates for CDM must obviously be stable on cosmological time scales otherwise they would have decayed by now. And, finally, they must have the right relic density. The relic density corresponds to the density after the decoupling from the thermal bath[5].

Furthermore, simulations of the Large Scale Structure formation show that Cold Dark Matter is required. Actually models suggesting small structures grew up before large, i.e. small-scale damping models, are favoured. Therefore, Hot Dark Matter could not be relevant considering the structures as we know them today.



The figure above, Fig. (2.1.2) from [7], shows a simulation of Large Scale Structure formation, i.e. clusters and large-scale filaments, within the assumption of Cold Dark Matter and Dark Energy, i.e. the ΛCDM model. On the left, we can see an homogeneous distribution of matter for $z = 30$. We go to the present time, i.e. $z = 0$, on the right where such structures are observed.

Beyond the Standard Model, there should exist a theory, probably not yet highlighted, implying such a type of particle. Candidates include Axions, Weakly Interacting Massive Particles (WIMPs) and other exotic particles. The existence of axions was first postulated to solve the strong CP problem of QCD as it will be discussed in a next section. WIMP is a generic name for particle which could have a sufficient mass and cross section corresponding to the one of the weak interaction. The WIMP candidate currently considered is the lightest superparticle (LSP). Widely beyond the Standard Model, the theory of SuperSymmetry, often denoted by SUSY, could propose some Dark Matter candidates, what will be exposed below.

2.1.3 Dark Energy

Observation of distant Supernovae suggests that the Dark Energy is not only a real component of the total energy density but that its present energy density $\Omega_{\Lambda,0}$ exceeds that of all other forms of matter and radiation put together. This component has many alternative

[5]See Appendix A.

names: the zero-point field, vacuum energy, quintessence or the cosmological parameter Λ. Actually, Einstein introduced first this constant quantity in a completely different way and with other motivations. However, its composition stays mysterious even if lots of cosmological theories tend to understand its behaviour.

One could say the concept of Dark Energy is a resurrection of a forgotten integration constant.

Nowadays, the cosmological constant appears to be an important contribution in the most promising models. Nevertheless, its raison d'être is not well established and still controversial. Supernovae seems to be the only real evidence for Dark Energy and other phenomena are capable to explain what is observed, see Blanchard [41].

In spite of the interest Dark Energy could gather, only Dark Matter cases will be discussed in this dissertation.

2.2 The Cosmological Models

In order to understand the evolution of the Universe, as far as we can go, we would like to study the solutions of the Friedmann-Lemaître equation. However, these theoretical solutions depend on the values chosen for the cosmological parameters, i.e; the priors.

At the beginning of development of cosmology, some simple models have first been proposed. Actually, flat single-component and empty models contain, namely, one critical component or nothing at all.

Among those models are the radiation model, which obviously means that the Universe was dominated by a radiation energy density, i.e. $\Omega_{r,0} = 1$ and all the rest is zero. The Einstein-de Sitter model has been popular for a long time. It implies $\Omega_{m,0} = 1$, namely a dust-like matter dominated Universe. The de Sitter model chooses a vacuum energy critical density. This model is quite intriguing in implying no Big Bang for $\Omega_{\Lambda,0} > 1$ and one has instead a non-zero scale factor at the beginning of the expansionary phase, i.e. a big bounce. Finally, we can mention the Milne model, which describes an empty Universe.

However, even if these models are convenient, they fail at describing observations. In fact, to consider only luminous matter cannot fit with a dust-like matter critical energy density model, at least, without inserting a missing mass.

The Universe has now finished its radiative era, it means we can neglect the radiation energy density in models describing the actual and future Universe. However, the presence of relativistic neutrinos, as well as photons, need to take into account a term of radiation energy density $\Omega_r = \Omega_\nu + \Omega_\gamma$. Indeed the main contribution to the radiation energy density is due to the CMB. Moreover, Ω_r is the only parameter measured directly. The temperature of the CMB has been determined to be $T = 2.725 \pm 0.001$ K, which corresponds to [11], also see Appendix A,

$$\Omega_r h^2 = 2.47 \times 10^{-5}. \tag{2.6}$$

Nevertheless, this term is assumed to be very small. And, on the other hand, if neutrinos are sufficiently massive, they are no longer relativistic and will so belong to the category of dust-like matter[6].

Observations are consistent with spatial flatness and indeed the inflation models automatically generate spatial flatness. From now, we can set $k = 0$. So, most of the relevant models imply $\Omega_{tot} \sim 1$, i.e.[24].

$$\Omega_{tot} = 1.02 \pm 0.02. \tag{2.7}$$

The four "test" models considered in the report of Overduin and Wesson [6] describe a Universe containing dust-like matter and vacuum energy. Those The Einstein-de Sitter (EdS) model, above mentioned, is referred as the Standard Cold Dark Matter model (SCDM).

However, from observation of supernovae, which indicates that $\Omega_{\Lambda,0} > \Omega_{m,0}$, its position of favoured model has been released. The Open Cold Dark Matter (OCDM) is more consistent with the above mentioned value of $\Omega_{m,0}$ even though it implies an open Universe. From the assumed hypothesis of flatness, this model holds appeal for those who accept the possibility of a non-zero vacuum-energy.

Therefore, a model combining CDM and cosmological constant, called ΛCDM, has been verified to agree best with both observations of Supernovae and CMB. However, even if this model has rapidly been assumed as the new cosmological standard model[24], it hints we find ourselves at a moment in cosmic history when both $\Omega_{m,0}$ [6] and $\Omega_{\Lambda,0}$ have the same order of magnitude, while they are supposed to evolve differently with time (Eq. (2.6)). Moreover this model includes no Hot Dark Matter.

Putting $\Omega_{m,0}$ to the upper limit of the baryon energy density and the vacuum energy density to the critical value, i.e. 1, we obtain the ΛBDM model. The name of this model obviously refer to a Baryonic Dark Matter component only plus a domination by the Dark Energy.

In spite of the remark about our coincident happening in the ΛCDM model, the present data set of cosmological parameters seems to elect this model and so gives it the name of Standard Cosmological Model or Concordance Model.

Those models span the widest range possible in the parameter space defined by $\Omega_{m,0}$ and $\Omega_{\Lambda,0}$.

Cosmological Model	$\Omega_{m,0}$	$\Omega_{\Lambda,0}$
EdS	1	0
OCDM	0.3	0
ΛCDM	0.3	0.7
ΛBDM	0.03	1

Table 2.1: The 4 Cosmological Test Models.

The currently most accurate, if somewhat indirect, determination of the Dark Matter energy density comes from global fits of cosmological parameters to a variety of observations, for instance the measurement of the anisotropy of the Cosmic Microwave Background (CMB).

[6]The neutrinos contribution to Dark Matter constitutes a section of this dissertation. This introduction aims to overview the present knowledge.

Particulary prominent are the measurement of cosmic microwave anisotropies, leading to the first results from the Wilkinson Microwave Anisotropy Probe (WMAP)[24] announced in February 2003[7].

We will refer to this paper and use their "priors", i.e. value of cosmological parameters, even if we know this is still a puzzling point.

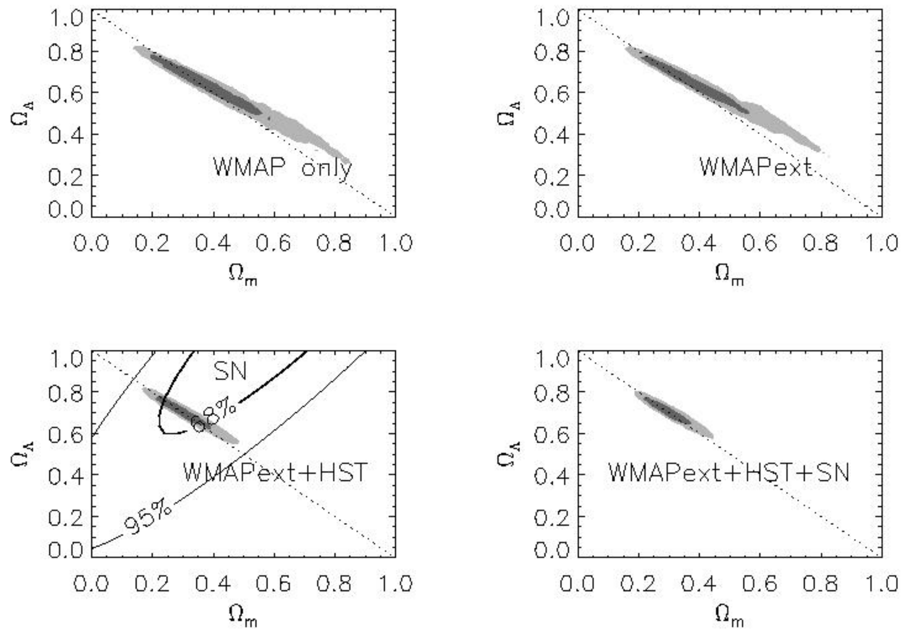


Figure 2.2: Constraints on the geometry of the Universe. The $\Omega_\Lambda - \Omega_m$ plane with cumulating data sets[24].

Even if we keep the parametrization by h^2 in the energy density values, it is sometimes necessary to specify the value of h. In their review [6], Overduin and Wesson adopt $h = 0.75 \pm 0.15$. The Particle Data Group [8], referring to last data, adopt the following convention

$$h = 0.73 \pm 0.03, \tag{2.8}$$

which shows less uncertainties. In the same way, adopting the previous convention, we find

$$h^2 = 0.53 \pm 0.04. \tag{2.9}$$

[7]Most of the values given in this section are taken from the Particle Data Group report 2004[8]. The authors of the 21th review "The Cosmological Parameters" themselves refer to a dataset known as WMAPext+2dF.

Chapter 3

To the Frontier of the Standard Model

The nowadays physics offer large possibilities of understanding of the Universe we live in. But, what seems more important is the projection into future, the dreams and reveries it leads to.

An overview of the present gained knowledge in particle physics will be introduced in order to include the notion of Dark Matter. The most straightforward way to first include this mysterious component is to begin with the Standard Model. In this section, a short history and a glimpse of the concepts of this model are proposed.

In other words, this section could be named, if not "a part of the main dish", "What is the Standard Model of Elementary Particles and why do we have to modify it", as said Vissani's paper[15], in order to include the notion of Dark Matter.

3.1 The Particles & the Interactions

The question of elementary has always been present in mind of human being. Behind the idea of "being" laid the need of unicity.

The presocratics are known to be the first philosophers as well as the first physicists. Around 500 BC, Democritus brought the unique and unswerving being and the perpetual idea of motion into conformity. Conciliating the concepts of Parmenide and Heraclitus, he thought the world being constituted of compact bodies: the smallest individable bodies surrounded by a vacuum, place where those bodies move in a continuity. Those "atoms", namely individable, fill the whole visible and invisible world. This atomist sight of the world has never left science in progress.

Nowadays, a long philosophic way covered, the questions of elementarity and unicity are still unsolved. Particle physics has been that developed that it is comparable to a zoo: lots of unknown and unwished particles appeared in accelerators collisions[1]. Finally, taking courage, particle physicists classified them according to their properties and the forces they interact through. Moreover, ever-lastingly amazed by the beauty of simplicity and attracted to idea of unifying interactions through symmetries, particle physicists dream of a "perfect" theory able to describe the Universe as a whole in simple terms.

[1] In the 1960's, J. Robert Oppenheimer said in exasperation that the Nobel Prize should be given to the physicist who did not discover a new particle.

In this framework, a first step to an unified model is proposed, namely the Standard Model. Based on the gauge group $SU(3) \times SU(2) \times U(1)$, this model can apparently describe all known fundamental forces, excluding gravity. Unfortunately, the Standard Model, although it is not in a conflict with experiments, faces several unanswered questions. Dark Matter is probably one of those. And this is what we are going to discuss.

3.1.1 The Particle Content

We introduced above different categories of particles. More fundamentally, we know there are two groups of spin 1/2 particles, i.e. fermions, which are called quarks and leptons. For now, they are, together with gauge bosons, assumed to be the "Democrite's atoms", it means, the fundamental, individable particles.

All leptons and quarks are brought together in three generations Tab. (3.1), which differ only by their masses.

Category	Charge	1st generation	2nd generation	3rd generation
Leptons	0	ν_e	ν_μ	ν_τ
	-1	e	μ	τ
Quarks	2/3	u	c	t
	-1/3	d	s	b

Table 3.1: The three generations of fermions.

In this way, the first generation is the lightest. Each negatively charged lepton has a corresponding neutrino.

So, in the first generation, the electron e is accompanied by a electronic neutrino ν_e. The two quarks corresponding are the up quark u and the down quark d.

The second generation, with an intermediate mass, is constituted by the muon μ and its neutrino ν_μ as well as the charm quark c and the strange quark s.

The heaviest generation is the third. The tauon τ and its neutrino ν_τ are in correspondence with the top quark t and the bottom or beauty quark b.

The mass range is the 0.511 MeV for the electron, 105 MeV for the muon and 1777 MeV for the tauon. Our knowledge of neutrino masses is rather limited, but it is expected that these masses are small. Therefore in a first approximation, we can consider them as massless.

The quark masses are also an awkward point due to the fact that free quarks have never been directly observed. It is thought that they can manifest themselves only in very energetic processes and they necessarily bind to form hadrons due to phenomenon of confinement. The binding of hadrons is provided by the strong interaction. For instance, the nucleons, i.e. proton and neutron, are a system of three quarks bound via the strong interaction[2].

The huge variety of hadrons that surround us is due to a combination of elementary quarks. In this sense the hadrons are not elementary particles. It is important that fundamental degrees of freedom – quarks– provide us with useful classification scheme of hadron.

[2]We recall there are 4 fundamental interactions are, with increasing intensity, the gravitation, the weak interaction, the electromagnetic interaction and the strong interaction.

The $q\bar{q}$ states are called mesons and are bosons, while the baryons, qqq states, are fermions[3].

Mesons	Baryons	Exotics
$q\bar{q}$	qqq	$qqqq\bar{q}$

Table 3.2: Hadrons.

The fundamental fermions interact among themselves with the help of integer-spin fundamental particles, i.e. the gauge bosons, which are "mediators" of the interactions. The Standard Model includes several of those gauge bosons. The photon γ has a spin 1 and is responsible for the electromagnetic interaction. This interaction naturally deals with electrically charged particles. The gluon g, spin-1 particle too, carries the strong interaction. As only quarks are strongly interacting particles, the gluons can be exchanged exclusively between quarks. To mediate the weak interaction, one introduces three particles of spin 1: W^+, W^- and Z^0 which couple to both leptons and quarks. We mention the existence of the graviton, hypothetical spin-2 particle carrying the gravitational forces, even if it is not strictly speaking part of the Standard Model.

Fundamental Interactions	Gauge Bosons
Weak Interaction	W^+ W^- Z^0
Gravitation	Graviton
Electromagnetic Interaction	Photon γ
Strong Interaction	Gluon g

Table 3.3: Gauge Bosons.

Finally, to close this view of particles, we mention the hypothetical Higgs boson. This scalar particle, i.e. spin-0, should be present in the Standard Model in order to generate masses of gauge bosons and fermions[4].

At the present time, all the Standard Model particles have been discovered excepted the Higgs boson.

3.1.2 The Interactions

The fermionic particle content together with the concept of gauge invariance enable us to fix the form of interactions. In other words, the symmetry principles lead to definite predictions for forces between particles. In fact, particles interact by exchange of gauge bosons, what is usefully described by Quantum Field Theory (QFT).

Let us briefly discuss the main ideas of Quantum Field Theory (QFT) from the modification of the Schrödinger equation to the symmetries of the Standard Model.

[3]In Tab. (3.2) here after, we have added the exotic hadrons, bound states of a baryon and a meson such as the Θ^+.

[4]The Higgs mechanism is related to the concept of spontaneous breaking of gauge symmetry. This mechanism is shortly described below.

Quantum Field Theory aims to unify special relativity (the velocities of the particles are not negligible anymore in front of the speed of light) with quantum mechanics in order to explain the interactions. Moreover, the word "field" hints that we go from a discrete number of degrees of freedom (dof) to an infinite. From a classical point of view, it is comparable to going from the Newton's equations of motion to the action principle lying under the Lagrange formalism. The Lagrangian is a function of both the position and the velocity of the particle, $L(q^i, \dot{q}^i)$ and is expressed by

$$L = \frac{1}{2}m(\dot{q}^i)^2 - V(q). \tag{3.1}$$

Indeed the motion of a particle is determined by minimizing the action, which is the integral of the Langrangian.

The wave equation in Quantum Mechanics formalism is the well-known Schrödinger equation,

$$i\frac{\partial\Psi(\vec{r},t)}{\partial t} = \hat{H}\Psi(\vec{r},t), \tag{3.2}$$

where $\Psi(\vec{r},t)$ is the wave function and the operator $\hat{H} = -\frac{1}{2m}\Delta + V(\vec{r})$ is the Hamiltonian. This equation is in agreement with the non-relativistic "mass-shell" condition $E = \vec{p}^2/2m$: without any potential, the energy corresponds to the classical kinetic energy.

In the same way, for a relativistic wave equation, we introduce the relativistic energy-momentum relation, i.e. the momentum 4-vector[5],

$$p^\mu = (E, \vec{p}). \tag{3.3}$$

This momentum 4-vector satisfies the mass-shell condition[6]

$$p^2 \equiv p_\mu p^\mu = E^2 - \vec{p}^2 = m^2. \tag{3.6}$$

The simplest field is a spin-0 bosons fields, i.e. scalar field ϕ. We expect an appropriate wave equation to describe these fields. From the relativistic mass-shell condition, we find the Klein-Gordon equation[7]

$$(\Box + m^2)\phi(\vec{x}, t) = 0, \tag{3.7}$$

[5] We are in the framework of special relativity and so with the metric of the Minkowski space-time, i.e. Eq. (1.11), with the particle physics convention of sign.

[6] The indices conventions are

$$\begin{aligned} x^\mu &= (t, x, y, z), \\ x_\mu &= (t, -x, -y, -z), \end{aligned} \tag{3.4}$$

and, for the derivatives,

$$\begin{aligned} \partial_\mu &\equiv \frac{\partial}{\partial x^\mu}, \\ \partial^\mu &\equiv \frac{\partial}{\partial x_\mu}. \end{aligned} \tag{3.5}$$

[7] The d'Alembertian operator is defined by $\Box \equiv \partial_\mu\partial^\mu = \frac{\partial^2}{\partial t^2} - \vec{\nabla}^2$.

and its Lagrangian is

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2. \tag{3.8}$$

However, this equation leads to nonphysical solutions: negative energies and negative probabilities.

Dirac has modified this equation in order to obtain positive-definite probability density. It requires an equation linear in $\partial/\partial t$. Moreover, the researched equation should be Lorentz covariant. The Dirac equation describes the free propagation of a relativistic spin-1/2 massive particle:

$$(i\gamma^\mu \partial_\mu - m)\psi(\vec{x}, t) = 0, \tag{3.9}$$

where $\psi(\vec{x}, t)$, or $\psi(x)$ using the conventions mentioned above, is a wave function of 4 components and describes a spin-1/2 particle of mass m. The γ^μ are 4×4 matrices including the Pauli matrices related to the spin[8] And thus, the Lagrangian is

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi. \tag{3.11}$$

The γ-matrices obey the following anticommutation relations,

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}. \tag{3.12}$$

Gauge Invariance

The properties of fields are in close relation with those of particles. This is the reason why fields correspond to particles. The question crossing our minds now is: "What is it going to happen if we transform the field?". The notion of gauge transformation will answer this question. Gauge transformations are the foundation of the Standard Model. It is assumed that every physical theory should be gauge invariant. The concept of gauge invariance is not new, actually Maxwell theory of electromagnetism is the first gauge theory.

Symmetries are known to play an important role in nowadays physics. More than simplifying equations, symmetries imply conservation principles: time independence of the Hamiltonian leads to energy conservation, translation to momentum conservation and so on [9].

---

[8]The γ matrices are given by

$$\begin{aligned} \gamma^0 &= \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \\ \text{and } \gamma^i &= \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \end{aligned} \tag{3.10}$$

where the index μ includes 0 for the time component and i for the spatial components. The σ^i matrices are the usual Pauli matrices.

[9]We refer to Noether's theorem.

The symmetry consists in the fact that the transformed wave function still obeys its corresponding field equation, for instance the spinors must still obey the Dirac equation. In other words, equations are to be covariant under the transformation

$$\psi_a \rightarrow \Lambda_a^b \, \psi_b, \tag{3.13}$$

the fields, each of them describing a particle, are included in ψ. In fact, they are written under a vector form, ψ, which one transforms invariantly under the considered transformation Λ_a^b.

Two types of transformations are distinguished: transformations which vary from point to point in space-time are called "local" transformations, for which $\Lambda = exp(i\alpha(x)\tau/2)$. And those which are constant over all space-time are called "global" transformations, for which $\Lambda = exp(i\alpha\tau/2)$. The τ are the "generators" of the group of symmetry.

We define the "covariant derivative" which implies a compensating field when going from a global to a local gauge, in order to impose covariance to local transformations.

Let us illustrate this on the example of electromagnetism, where the compensating field is, in fact, the electromagnetic potential. We know from Maxwell theory that the action should be invariant under

$$A_\mu \rightarrow A_\mu - \partial_\mu \Lambda(x), \tag{3.14}$$

where $A_\mu = (V, \vec{A})$ is the electromagnetic potential. The action should also be invariant under the transformation given by Eq. (3.13). However, since $\Lambda(x)$ varies from point to point, the simple derivative ∂_μ will no longer ensure the covariance, we define the above-mentioned covariant derivative,

$$\partial_\mu \rightarrow D_\mu \equiv \partial_\mu + ieA_\mu, \tag{3.15}$$

so that the transformation lets us with

$$D_\mu \psi_a \rightarrow \Lambda_a^b(x) D_\mu \psi_b. \tag{3.16}$$

Althought the required covariance has been re-established, the particle described by the field is no longer free but interacting. Actually, Dirac's equation has become

$$(i\gamma^\mu D_\mu - m)\psi(x) = 0, \tag{3.17}$$

or, equivalently,

$$(i\gamma^\mu(\partial_\mu + ieA_\mu) - m)\psi(x) = 0. \tag{3.18}$$

The dynamics are intimately related to local invariance. In fact, the history of the considered particle is memorised in the wave function phase, according to the imposed initial conditions. Since the phase convention varies from point to point, it describes dynamics. In the example of electromagnetic field, the charged particles interact with the electromagnetic field described by A_μ. In other words, the gauge invariance dictates the presence of the field

A_μ. In such a case, this field is called a "gauge field". Each type of gauge invariance is associated to an interaction. The gauge principle indeed modifies the equations and introduces gauge fields and their coupling constants to fermions.

Above we considered the simplest example of $U(1)$ gauge group. It can be easily generalized to more complicated groups, e.g. $SU(n)$. Generically the number of gauge bosons we need is equal to the number of generators of the group. For instance the number of generators of a $SU(n)$ group is $n^2 - 1$, with n the dimension of the group. For the $U(1)$ group, associated to the electromagnetic interaction and giving birth to Quantum Electrodynamics (QED), we have one particle, the photon γ, and one coupling constant, the electron charge e.

Gauge fields are in the following form, obeying Maxwell's equations,

$$\Box A^\nu - \partial^\nu(\partial_\mu A^\mu) = j^\nu, \tag{3.19}$$

with j^ν the current. However, we would like to introduce a mass term to this massless gauge field. This could be done by the simple replacement $\Box \to \Box + m^2$. Since we notice this mass term does not respect gauge invariance, we therefore introduce the mass through the concept of spontaneous symmetry breaking.

Spontaneous Breaking of Symmetry

Some of the symmetries are spontaneously broken. Namely, it means that the hamiltonian of an interaction is invariant under a given symmetry while the fundamental eigenstate, i.e. its vacuum state, is not invariant under this symmetry.

The most common example that illustrates this phenomenon is given by a Lagrangian[10] with a ϕ^4-potential, i.e.

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\ \partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4, \tag{3.20}$$

which preserves the symmetry $\phi \to -\phi$.

This potential, i.e. $\frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4$, shows one minimum at $\phi_0 = 0$ with an obviously positive mass squared. However, if the mass squared is negative, in other words, if the mass is imaginary, the potential presents two minima at

$$\phi_0 = \pm\sqrt{-6m^2/\lambda}. \tag{3.21}$$

The vacuum "eigenvalue"of the scalar field ϕ is expected to be zero,

$$\langle 0|\phi|0\rangle. \tag{3.22}$$

If we keep the vacuum state at $\phi_0 = 0$ for a negative square mass, the vacuum expectation value of the studied field will no longer vanish. We then simply "shift" the vacuum to a new minimum position, i.e.

$$\tilde{\phi} = \phi - v, \tag{3.23}$$

[10] This Lagrangian corresponds to a scalar particle.

so that its vacuum expectation value vanishes. In other words, the new vacuum is located at

$$v = \pm\sqrt{-6m^2/\lambda}. \tag{3.24}$$

The symmetry of the initial ϕ^4-potential has been "broken" by shifting the ϕ-field according to Eq. (3.23).

This spontaneous breaking of continuous symmetry "gives birth" to massless bosons. Since the example above deals with spontaneous breaking of discrete symmetry, let us illustrate this using a n-dimensional scalar theory with the same potential Eq. (3.20). The solutions for the minimum lead to the so-called "mexican hat" representation, namely a ring of degenerated solutions.

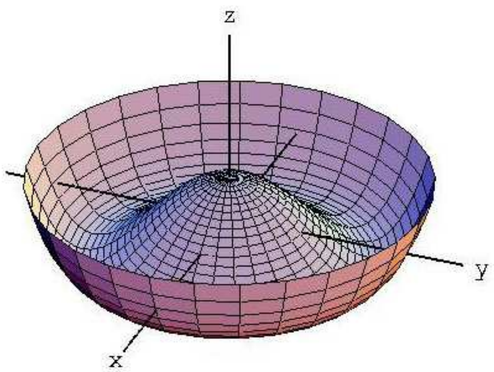


Figure 3.1: Ring of degenerated solutions for the new vacuum states for $m^2 < 0$. Also called "mexican hat".

Facing this infinite set of solutions, it is useful to select one direction for the vacuum. More precisely, the column vector representing the vacuum-field is chosen to be

$$v_i = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ v \end{pmatrix}. \tag{3.25}$$

From a system with n degrees of freedom, we go to another containing $n-1$ degrees of freedom. This "selection"of one direction for the new vacuum, as we did in first example, breaks the symmetry again when we shift the field to its right vacuum position. The trivial parametrisation of the new field by Eq. (3.23) directly shows this breaking.

However, we should usefully parametrise it in a different way. The original set of fields ϕ^i can also be shared out into two different sets, i.e. $n-1$ fields ξ_i and one field σ. The σ-field is chosen in the selected direction for the vacuum. This parametrisation can be written in the following way,

$$\phi = e^{i\xi_j\tilde{\tau}_j/v} \begin{pmatrix} 0 \\ 0 \\ \vdots \\ v + \sigma(x) \end{pmatrix}. \tag{3.26}$$

At the first order of expansion, the fields are, for $1 < i < n-1$,

$$\phi_i = \xi_i, \tag{3.27}$$

and, for $i = n$,

$$\phi_n = v + \sigma. \tag{3.28}$$

Breaking the symmetry, the number of generators of the symmetry group considered decreases. In our example, we go from $n(n-1)/2$ generators, corresponding to the $O(n)$ symmetry group, to $(n-1)(n-2)/2$ generators for the $O(n-1)$ symmetry group. This leaves us with $n-1$ broken generators, namely the number of massless bosons.

Writing the Lagrangian for such a new "shifted" field, we notice that the $n-1$ ξ_i fields have become massless while σ remains massive. In fact, the particles are massive if the coefficient multiplying the square field is non-zero (see, for instance, Eq. (3.8)), what is not the case for the ξ_i fields. Massless bosons have appeared with the spontaneous breakdown of the symmetry. Those bosons are called "Nambu-Goldstone bosons".

Moreover, from this point, we could straightforwardly go to the famous Higgs mechanism. In fact, the Nambu-Goldstone theorem was exposed in the case of scalar fields. What happens if gauge symmetry is broken down?

As we discussed previously, in gauge theories, vector gauge bosons are massless. However, the situation changes if one introduces scalar fields which break the gauge symmetry spontaneously. Actually breaking a symmetry to a lower one, Nambu-Goldstone bosons are generated. In this case, the Nambu-Goldstone degrees of freedom mix with gauge bosons. Since massless scalar particles only have two transverse polarization states, the above mentioned mechanism gives rise to the longitudinal polarization component of the vector field and hence to the mass of the gauge bosons. One could also say the massless bosons are "eaten up" by the broken gauge fields. The price we have to pay is the introduction of new scalar degrees of freedom and associated with them particles. This is the essence of the Higgs mechanism[11]. The reamining σ-field is, in gauge theories, the Higgs boson.

3.2 The GSW Standard Model

After the success Maxwell encountered in his unification of electricity and magnetism, it has been believed possible to unify the whole set of fundamental interactions. The Standard Model went born in this optic. The quantization of Maxwell's theory led to good experimental results: gauge principle can effectively be used as a tool.

The gauge principle of the electroweak interaction of quarks and leptons has been a first step to the Standard Model. This theory is often called GSW theory, from the names of Glashow, Salam and Weinberg. This theory tends to unify the Quantum Electrodynamics, which symmetry is $U(1)$, with the weak interaction. Its gauge group is

$$SU(2)_L \times U(1), \tag{3.29}$$

[11] We refer to the undergraduate dissertation of Cédric Lorcé for more details [31].

where L stands for "left". Actually the "helicity" implies that left-handed and right-handed parts of a particle behave independently, if they are massless. The neutrinos exist only under left-handed representation, i.e. we have ν_L[12].

The $SU(2)$ group imposes the fields to transform as doublets and as singlets under a gauge transformation such as the one described by Eq. (3.13). For the first generation of leptons[13], we have

$$\begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \times \begin{pmatrix} e_R \end{pmatrix}, \tag{3.32}$$

the left-handed neutrino and the left-handed electron are combined in the doublet while the right-handed electron forms the singlet.

The fields transform as singlets under the $U(1)$ group. Keeping our example for the first generation of lepton[14], we have

$$(e_L) \times (\nu_L) \times (e_R). \tag{3.34}$$

It is time now to give an explicit example of the above described Higgs mechanism.

Gauge bosons are theoretically supposed to be massless as long as the symmetry they obey to is not broken. The electroweak symmetry is in fact spontaneously broken. More precisely, one needs to introduce four scalar fields to break this $SU(2) \times U(1)$ symmetry down to $U(1)$.

The Nambu-Goldstone mechanism tells us three of those four fields are massless, i.e. the number of broken generators is here $((2^2 - 1) + 1) - 1 = 3$.

Moreover the gauge bosons associated to $SU(2)_L \times U(1)$ are the photon γ, for QED, and $W^{\pm}$ and Z^0 vector bosons mediating the weak interaction. The association of the degrees of freedom of the three massless scalar fields with the gauge bosons generates the mass of

[12] See next Section "Neutrinos in the Standard Model". Anyway, we can say now that, for instance, the anti-neutrinos are right-handed, according to CP-parity, $\bar{\nu}_R$. Indeed the weak interaction should be CP-invariant but, this symmetry is violated by the neutral kaons oscillation. So, the weak interaction is in fact CPT-invariant.

[13] The other doublets are generally denoted by

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \text{ or } q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \tag{3.30}$$

for the three generations of quarks even though we have chosen the first generation in our example. And the leptons weak doublets are also sometimes denoted by

$$l_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \tag{3.31}$$

valid for the three generations too, see Tab. (3.1).

[14] For the quarks, both the u and d quarks exist as right-handed states, i.e.

$$(u_R) \text{ and } (d_R). \tag{3.33}$$

three of the four weak gauge bosons. And actually the $W^{\pm}$ and Z^0 intuitively have to be massive because the weak interaction has a very short range so that only the photon remains massless since the electromagnetic interaction has an infinite range.

Finally, what happens to the fourth remaining scalar field introduced in order to break the symmetry down? This field is obviously massive and is in fact the famous Higgs field.

Moreover we notice that two coupling constants appear.

The Quantum Chromodynamics (QCD) care about the strong interaction. As the leptons do not interact strongly, QCD deal with quarks. In Tab. (3.1), the quark flavours are shown. Quarks come in six flavours and three colours, where the colour is a quantum number introduced to satisfy Pauli principle, although only the colour participates in the local gauge symmetry. In fact, QCD are invariant under the following local gauge

$$SU(3)_C, \tag{3.35}$$

not to be confused with $SU(3)_F$. Actually, it was thought the quarks belonged to the fundamental representation of this flavour symmetry, but $SU(3)_F$ is a global and so not gauged symmetry. In other words, the quark flavour is not a good quantum number.

The eight generators of the colour group symmetry indicate the existence of eight gluons. These QCD gauge bosons are a priori massless and spin-1 particles. Unlike the electroweak symmetry, the colour group is unbroken and the gluons remain massless. The gluons permute the three colour states, i.e. red, blue and green. In other words, the properties of the quark system is invariant whatever is the local transformation, i.e. $\alpha(x)$, chosen. And it is possible if the gauge bosons are able to transport the colour from a quark to another in such a way the $SU(3)_C$ symmetry is conserved at each point of the space-time. The fundamental representation is, for the u quark,

$$q_C = \begin{pmatrix} u_r \\ u_b \\ u_g \end{pmatrix}, \tag{3.36}$$

where r stands for red, b for blue and g for green.

In summary, the GSW Standard Model is created by splicing QCD with the electroweak model. It allows us to unify all known experimental data concerning particle interaction via the gauge group

$$SU(3)_C \times SU(2)_L \times U(1). \tag{3.37}$$

The gauge fields of colour $SU(3)_C$ are responsible for binding quarks together while gauge fields of $SU(2)_L \times U(1)$ mediate the electroweak and weak interactions.

Altogether, there are 45 matter particles. A few free parameters as well as coupling constants are present due to the fact this model is a juxtaposition of gauge theories. So, the renormalisability of the Standard Model has been difficult to prove.

Nevertheless, the success of this model resides in the fact that no experimental conflicts ever appeared. However, the Standard Model has also some weak points: a large number of parameters, the undetected Higgs boson (which allows masses in the model). Other questions remain unanswered, for instance: why does it contain three generations. In the “minimal” Standard Model, the neutrinos are predicted to be massless, while the experiments operated in underground sites, i.e. Kamiokande, suggest they are massive.

Moreover, we cannot consider this model as the final unifying model as long as gravity is not included. The whole troubles of the Standard Model have not been exposed in here. Formal troubles such as the CP strong problem leading to the existence of axions, Dark Matter candidates, will be discussed later.

Chapter 4

The Neutrinos

The theoretical introduction of neutrino predicted it to be massless. Afterwards the possibility of massive neutrino, mainly motivated by the neutrino oscillations, opened different and wide points of view on its cosmological impact.

Relativistic particles were first thought to fill a non-negligible part of the missing mass of our Universe. Those particles are labelled "Hot Dark Matter". The neutrino, according to its energy at the decoupling time, would be a plausible candidate for Dark Matter.

The neutrino puzzle is going to be carefully introduced in this section. The pro and contra of their contribution to the total energy density of the Universe will be exposed.

4.1 Neutrinos in the Standard Model

In 1930, a new particle, a neutral weakly interacting spin-1/2 particle, was theoretically introduced by Pauli. Even though this "neutrino" was the Pauli solution to the nitrogen catastrophe [1], the neutrino, which mass is smaller than the electron mass, allowed to understand the β-decay spectrum,

$$\begin{aligned} &{}^6He \rightarrow^6 Li + e^- \\ \Rightarrow &{}^6He \rightarrow^6 Li + e^- + \bar{\nu}_e. \end{aligned} \tag{4.1}$$

Actually the expected spectrum of this β-decay was quite different than the one observed. Fermi himself suggested that the principle of energy conservation was not relevant!! In fact, this "light" electronic antineutrino $\bar{\nu}_e$ brought the "missing" energy in order to explain the spectrum.

Experiments aiming to measure the small mass of these neutrinos were only able to put upper bounds. In the 50's, the upper bound of the neutrino mass found the value

$$m_\nu \lesssim (100 - 200)\,\mathrm{eV}. \tag{4.2}$$

[1] The neutron was not known at that time. Therefore its discovery few years later solved lots of theoretical troubles. The one we consider now is that the ${}^{14}N$ was thought to be composed of 14 protons and 7 electrons. It means the nitrogen should have an half-integer spin, while experiments shown it was integer.

However, their mass is not really known yet. It seems that, up to now, this neutrino has always been a troubling particle. A closer look at the Standard Model point of view will be exposed here.

Neutrinos are spin-1/2 particle belonging to the lepton category, see Tab. (3.1). Nowadays, three generations of neutrinos are known. The electronic neutrino was the first to be discovered in the 50's. In the 60's, experiments showed the existence of a muonic neutrino. And the tauonic neutrino was the very last particle to be discovered. As neutral leptons, they interact through the gravitational and the weak interactions. The weak interaction should so conserve the lepton number. In Tab. (4.1), the first-generation-lepton number is illustrated. The second and third generations follow the same scheme.

Lepton number	e^-	e^+	ν_e	$\bar{\nu}_e$
L_e	1	-1	1	-1
L_μ	0	0	0	0
L_τ	0	0	0	0

Table 4.1: The three lepton numbers, illustrated for the first generation.

The GSW model assumes that only negative helicity component of the neutrino participates in weak interaction.

Let us introduce the concept of helicity. The helicity λ is the projection of the spin $\vec{S}$ along the direction of motion, i.e. the momentum $\vec{p}$:

$$\lambda = \frac{\vec{S}.\vec{p}}{|\vec{p}|}. \tag{4.3}$$

So, a spin-1/2 particle can occur with two helicity states, i.e. left-handed $\lambda = -1/2$ (see Fig. (4.1)) and right-handed $\lambda = 1/2$.

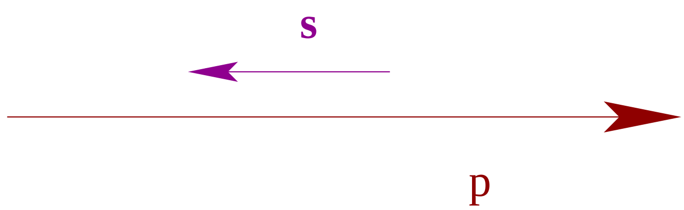


Figure 4.1: Representation of left-handed helicity state.

The helicity is a pseudo-scalar quantity so its transformation under P-parity[2] gives a negative helicity state for an initial positive positive state. For instance, a left-handed neutrino could become a right-handed one. Actually the direction of motion changes under this transformation while the spin $\vec{S}$ remains the same.

The weak interaction violates the P-parity[3], i.e. considering our previous example: $\nu_L \rightarrow \nu_R$. For theory to fit with the observations, i.e. the fact that only left-handed

[2]The P-parity is comparable to parity while watching through a mirror.

[3]The P-parity is not a "good" quantum number as its operator does not commute with the Lagrangian of the weak interaction.

neutrinos have been observed, Landau, Lee, Yang and Salam proposed, in 1957, the "two-component neutrino theory". Let us rewrite the Dirac's equation, i.e. Eq. (3.9), with a different representation of the γ matrices, namely the Weyl representation. The representation of the γ-matrices by Weyl is

$$\begin{aligned} \gamma^0 &= \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix}, \\ \text{and } \gamma^i &= \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \\ \text{and } \gamma^5 &= \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \end{aligned} \tag{4.4}$$

where γ^5 is intimately related to the helicity,with which we deal now, and is defined as

$$\gamma^5 = \gamma_5 = -\frac{i}{4!}\epsilon_{\mu\nu\sigma\rho}\gamma^\mu\gamma^\nu\gamma^\sigma\gamma^\rho. \tag{4.5}$$

We can write down two projection or "chiral" operators, P_R and P_L,

$$\begin{aligned} P_R &= \frac{1+\gamma_5}{2} = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}, \\ P_L &= \frac{1-\gamma_5}{2} = \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix}. \end{aligned} \tag{4.6}$$

So a 4-component spinor as

$$\psi = \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix}, \tag{4.7}$$

itself shared out into right-handed and left-handed 2-vectors, go decoupled under the transformation given by Eq. (4.6).

In fact[4],

$$\frac{1+\gamma_5}{2}\psi = \begin{pmatrix} \psi_R \\ 0 \end{pmatrix}, \tag{4.9}$$

$$\frac{1-\gamma_5}{2}\psi = \begin{pmatrix} 0 \\ \psi_L \end{pmatrix}. \tag{4.10}$$

[4]The hermitian conjugates lead to

$$\begin{aligned} \bar{\psi}_L &= \bar{\psi}\,\frac{1+\gamma_5}{2} \\ \bar{\psi}_R &= \bar{\psi}\,\frac{1-\gamma_5}{2}. \end{aligned} \tag{4.8}$$

Thus, this Weyl representation leads to the operators P_R and P_L. The application of these operators allows the right-handed and the left-handed components to stay decoupled. So, looking at the Dirac Lagrangian Eq. (3.11), we must impose $m = 0$, in order for the states to remain decoupled. The reason that these "chiral" fermions must be massless is because the mass term $m\bar{\psi}\psi$ is not invariant, i.e. mixes the right- to the left-handed neutrinos,

$$\bar{\psi}\psi = \bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L, \tag{4.11}$$

what is not observed. Finally, we are left with the two decoupled Weyl's equations instead of the Dirac's equation,

$$i\gamma^\mu\partial_\mu\nu_{R,L}(x) = 0. \tag{4.12}$$

In summary, the Glashow-Weinberg-Salam Standard Model was build under the assumption of massless two-component neutrinos[5] and, in the same way, the lepton number was assumed to be a "good" quantum number.

Moreover, Eq. (4.12) shows $\nu_R(x)$ and $\nu_L(x)$ (and their respective antiparticle fields) both could theoretically be neutrino fields. However, the neutrinos just interact through the weak interaction which only deals with left-handed fields. Indeed, they have no electric charge and so do not take part in the electromagnetic interaction. Thus, the right-handed field does not appear naturally neither in this theoretical scheme nor experimentally.

Should we expect more neutrino generations? In fact, neutrinos interact via the neutral current Z^0. The width spectrum of the Z^0 decay allows only three neutrino generations with a mass smaller than $m_{Z^0}/2$, i.e. $m_{Z^0} = 91$ GeV.

Are the neutrinos really massless? If neutrinos are massive, they could be the only candidate for the Dark Matter. Actually none of the other particles in the Standard Model suit for this. That is why neutrinos and their mass deserve a closer look. The "entrée" allows us to drive to massive neutrinos and the reasons for they are or are not good candidates for Dark Matter.

4.2 Massive and "Dark" Neutrinos

4.2.1 Neutrino Mass

There are various possibilities to figure out whether neutrinos are massive. The most direct way is the study of energy spectra in β-decays. However this method is sensitive only to the mass of electronic neutrino and would require a not very small neutrino mass.

Luckily, the possible neutrino mass is closely related to the oscillation phenomenon. Actually observation of neutrino oscillations gave a first proof that neutrinos are massive.

[5] The left-handed fields are present for all the weak interaction, more precisely for the electroweak interaction. The $SU(2)$ doublets and singlets are valid not only for neutrinos but for other leptons and quarks too. See previous chapter.

The idea of neutrino mass, intimately related to the notions of mixing and oscillations, was suggested in 1957 by Pontecorvo[22].

The Schrödinger equation implies the wave function to evolve with time. If the state is not found to remain the same after some time but a combination of states, we face an "oscillation" and that is what is known to happen with the kaons. Pontecorvo thought the same phenomenon could occur with neutrinos, what is only possible if they are massive[6]. The neutrino oscillations can occur, for instance, between two species,

$$\nu_{\mu,L} \rightleftarrows \nu_{e,L} \ \text{ or } \ \nu_{e,L} \rightleftarrows \bar{\nu}_{e,L}. \tag{4.13}$$

If neutrinos really have a non-zero mass, they will obviously mix between in the above frame and we could experimentally observe their evolution with time through their oscillations. Actually, the observation of a mass difference would confirm the neutrinos not to be massless.

To describe the first phenomenon of Eq. (4.13), we define, in the Dirac representation, two eigenstates of the CP-parity, which conserve the lepton number. The mixing between the two eigenstates $\nu_{1,L}$ and $\nu_{2,L}$[7] is

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}, \tag{4.14}$$

where it is understood that the fields are left-handed. The eigenmass values are m_1 and m_2, respectively.

Knowing that the neutrinos are ultra-relativistic particles[8], their energy is approximatively given by, see Eq. (3.6),

$$E \cong p + \frac{m^2}{2p}. \tag{4.15}$$

The wave function of ν_1 and ν_2 shows a phase factor $e^{-i(E_{1,2}t-pl)}$. The argument of this phase function becomes $-im^2l/2E$ considering the previous relation and the fact that $t \sim l$, l the covered length, according to the relativistic nature of neutrinos. Thus, after some time, the two components have propagated with their respective phases, i.e.

$$\nu_e \to \nu_1 \cos\theta\, e^{-im_1^2 l/2E} + \nu_2 \sin\theta\, e^{-im_2^2 l/2E}, \tag{4.16}$$

which directly shows that, going from ν_e, we can observe muonic neutrinos ν_μ:

$$\nu_e \to (\cos\theta\, \nu_e - \sin\theta\, \nu_\mu)\cos\theta\, e^{-im_1^2 l/2E} + (\sin\theta\, \nu_e + \cos\theta\, \nu_\mu)\sin\theta\, e^{-im_2^2 l/2E}. \tag{4.17}$$

The oscillation probability is given by the square of the coefficients of the ν_μ term, i.e.

$$P_{\nu_e \to \nu_\mu}(l) = \left| \cos\theta\, \sin\theta \left(-e^{-im_1^2 l/2E} + e^{-im_2^2 l/2E} \right) \right|^2. \tag{4.18}$$

[6] See section "Neutrinos in the Standard Model"

[7] The mixing between the three species is given by the Maki-Nakagawa-Sakata matrix (MNS-matrix).

[8] The observed neutrinos are, up to now, the left-handed ones. So, as their mass is tiny, they can still be considered as ultra-relativistic particles while coming from the sun and the atmosphere.

Defining the mass difference δm^2 as $m_1^2 - m_2^2$ and assuming a small square-mass difference, we obtain

$$P_{\nu_e \to \nu_\mu}(l) = \sin^2 2\theta \, \sin^2 \frac{\delta m^2 l}{4E}. \tag{4.19}$$

Such oscillation effects are shown by some experiments. The atmospheric neutrino observations favour the $\nu_\mu - \nu_\tau$ oscillation. The Super-Kamiokande as well as the Kamiokande experiments are the market leader. The mass difference found is [20]

$$|\delta m_{23}^2| \sim 2.5 \times 10^{-3} \mathrm{eV}^2. \tag{4.20}$$

The solar neutrino oscillations, also shown by the Kamiokande experiments, are consistent with three active[9] light neutrino states and leads to [20]

$$\delta m_{12}^2 \sim 5 \times 10^{-5} \mathrm{eV}^2. \tag{4.21}$$

Those oscillation phenomena should obviously lead to lower bounds[10].

The second phenomenon of Eq. (4.13) is a mixing between both active and sterile neutrinos. The concept of Majorana mass, and therefore Majorana particle, in opposition with Dirac mass could introduce such sterile massive neutrinos.

In summary...

Two main types of experiments allow us to evaluate the neutrino mass range. The neutrino oscillations lead to a lower bound while upper bounds are given by weak-decay analysis.

The oscillations violate the two-component neutrino theory as well as the conservation of the lepton number. Indeed, this quantum number has no natural meaning and its conservation has been assumed ad hoc. To concord with the lepton number conservation, the flavour neutrinos are described by mixed eigenstates, i.e. $\nu_{1,2,3}$ (see Eq. (4.14)).

We present here after the particle physics limits on neutrino masses. The Particle Data Group last report [8] gives the following upper bounds:

$$m_{\nu_e} < 3\,\mathrm{eV}, \tag{4.22}$$

$$m_{\nu_\mu} < 0.19\,\mathrm{MeV}, \tag{4.23}$$

$$m_{\nu_\tau} < 18.2\,\mathrm{MeV}, \tag{4.24}$$

those values are found from the decay of the respective electron-like leptons. Actually, charged particles, e.g. leptons or pions, emitted together with flavour-neutrino decays are analysed giving information on the neutrino square mass. For instance, the tritium β-decay

[9] "Active" is here in opposition to "sterile".
[10] See section "Neutrinos as Cosmological Actor".

seems to be the most stringent experiment up to now.

We notice that it is unfortunate our actual experimental abilities are still limited and prevent the determination of more accurate masses values.

How could this oscillation, this mass influence the evolution of the cosmological parameters? In particular, what is the neutrino participation to the total energy density?

4.2.2 Neutrino as Cosmological Actor

Copious number of neutrinos were produced in the early Universe. Every type of particles is more or less considered like a photon gas during the radiation domination. The radiation domination is characterised by a temperature between $T \sim 10^9 K$ and the temperature corresponding to the energy required for the radiation to decouple from matter (namely, around $T = 6000K$), see Fig. (1.6). According to the statistic they obey, i.e. Bose-Einstein or Fermi-Dirac, as well as their number of degrees of freedom, their state variables can be different, see Appendix A.

Neutrinos were in thermal equilibrium until the temperature dropped out to the critical value. This transition out from equilibrium is called the neutrino decoupling or weak interaction decoupling. From this moment in the history, the Universe has been "transparent" to neutrinos. The energy of those "relic" neutrinos has decreased and their momenta has been "redshifted" by the expansion.

If neutrinos have non-negligible mass, they can make a non-trivial contribution to the total energy density of both radiation and matter domination. However, even if their contribution during the radiation domination is of high interest in structures development, we will concentrate on the matter budget during the matter domination and briefly expose their role in Large Scale Structure construction.

Thus, the neutrino contribution to the energy density of the Universe strongly depends upon the sum of the mass of the light neutrino species[11](or flavours) [24]

$$\Omega_\nu h^2 = \frac{\Sigma_i m_{\nu_i}}{94\text{eV}}, \tag{4.26}$$

where the sum runs on the neutrino flavours. This expression is valid for all families with mass in the range 5×10^{-4} eV to 1 MeV. Indeed, higher masses would need a more sophisticated calculation as they can no longer be consider as ultra-relativistic particles.

Neutrino-oscillation experiments tend to restrict large mass intervals like those given by the Particle Data Group [8]. Moreover, cosmological observations impose more drastic bounds.

[11]The value of the denominator is obtained knowing that

$$\Omega_\nu \equiv \rho_\nu / \rho_c, \tag{4.25}$$

where ρ_c is given by Eq. (1.45).The neutrino density ρ_ν equals $\Sigma_i m_{\nu_i} n_\nu$. And the number of relativistic neutrinos n_ν filling the Universe at the present time is found by statistical mechanics, see Appendix A.

In fact, neutrinos do play a important role in the development of Large Scale Structures if structures growing by gravitational instabilities are assumed[6].

They are part of the so-called Hot Dark Matter, namely those particles were relativistic at the decoupling time and during galaxy formation as well. They began losing their energy just after their decoupling from the thermal bath. The time scale at which they become non-relativistic strongly depends on their mass [37]. At present, relic neutrinos are no longer relativistic. The fact that we have neglected the radiation energy density Ω_r as a component of the present day total energy density is therefore relevant.

Nevertheless, Hot Dark Matter has long been excluded as it does not permit galaxies to form. Neutrinos, moving freely from high to low density regions, would erase fluctuations; and this below their damping-length scale. It would mean large structures formed first, i.e. top-down or large-scale damping models. Nevertheless, those models are not favoured since our galaxy appears to be older than the Local Group [35]. Moreover, the mass of the thermal relics should be above the range of 1 keV [11] to coincide with what is observed today. However, an excessive amount of this free-streaming HDM particles would lead to galaxies being less clumped, especially on small-distance scales.

An energy density that could be reliable with the LSS theories would be as high as $\Omega_\nu \sim 0.2$. But it would imply cosmological parameters really different from those assumed by the ΛCDM model [6][12]. So they cannot reproduce correctly the observed structures in the Universe.

The 2dF Galaxy Redshift Surveys[13] are sensitive to the ratio Ω_ν/Ω_m and, so, provide this ratio with an upper bound. For a four-component model, i.e. an Universe filled with baryons, CDM, massive neutrinos as well as a participation of a cosmological constant, Elgaroy *et al.* [25] show

$$\frac{\Omega_\nu}{\Omega_\nu + \Omega_{CDM} + \Omega_{bar}} < 0.13. \tag{4.27}$$

Based on the fact that the ΛCDM model assumes $\Omega_m = 0.3$, the previous result gives

$$\Omega_\nu < 0.04. \tag{4.28}$$

Then, Elgaroy *et al.*[25] set an upper bound of $\Sigma_i m_{\nu_i} \sim 1.8$ eV.

Assuming a hierarchical mass organisation comparable to the quark-doublet mass hierarchy, the electronic neutrino should be lighter than the tauonic one. The results of oscillation experiments suggest a lower bound of [11]

$$\Omega_\nu \approx 0.001, \tag{4.29}$$

for the neutrino total energy density.

[12] See paragraph about Mixed Dark Matter below.

[13] The analysis are made by comparing the power spectrum of CMB fluctuations derived from the 2dFGRS with power spectra for 4-components cosmological models. However, the critic of those analysis are beyond the scope of this dissertation.

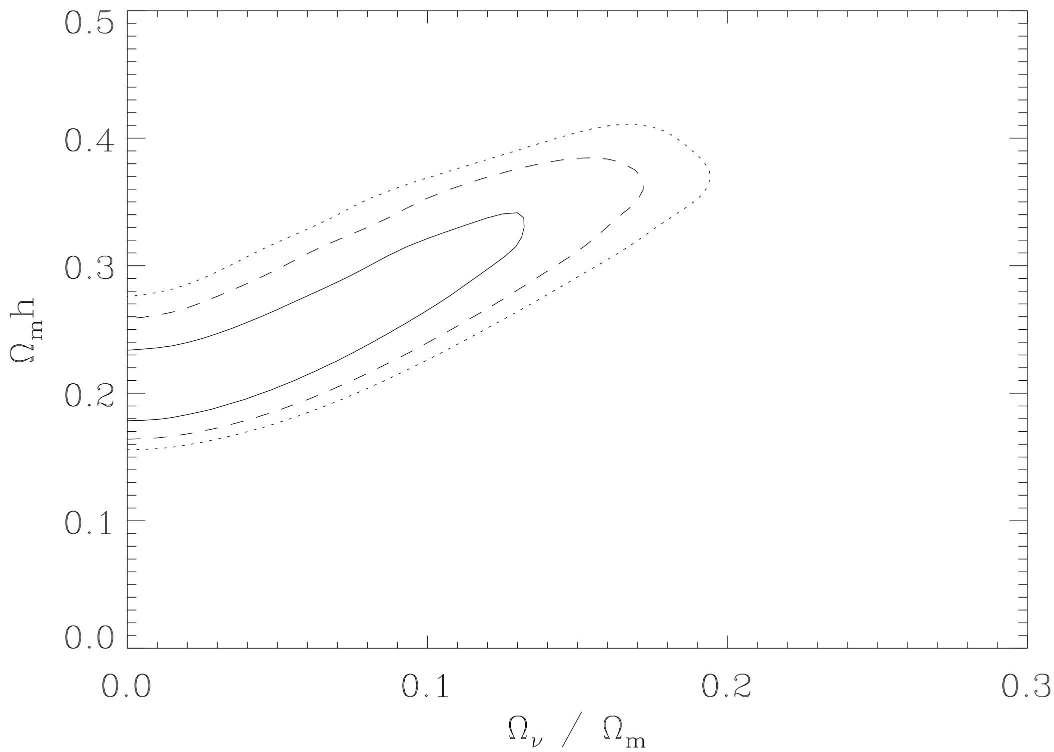


Figure 4.2: 2dF limit on neutrino mass with a "prior" of $0.1 < \Omega_m < 0.5$ [25].

Knowing the square-mass differences Eq. (4.20) and Eq. (4.21), it is straightforward to find a lower bound by assuming a mass equaling zero for the electronic neutrino,

$$\Sigma_i m_{\nu_i} > 0.057\text{eV}. \tag{4.30}$$

This leads to, taking Eq. (2.9) and Eq. (4.26) into account,

$$\Omega_\nu h^2 = 6.07 \times 10^{-4}, \tag{4.31}$$

in other conventions, we have

$$\Omega_\nu \sim 0.001, \tag{4.32}$$

which fits with the value given by [11] and with the cosmological observation constraints Eq. (4.28) too.

Nevertheless, this hierarchy can be accepted ad hoc as it can be rejected. And this assumption is rejected by Spergell *et al.* in their review [24]. Using the WMAPext+2dFGRS+Lyman-α data set, they conclude that the total neutrino mass is driven to $\Sigma_i m_{\nu_i} < 0.7$ eV, which strongly restricts the particle Data Group bounds[14]. This data set leads to

$$\Omega_\nu h^2 < 0.0076, \tag{4.33}$$

[14]The two total mass values given in here, i.e. those of Spergel *et al.* $\Sigma_i m_{\nu_i} < 0.7$ eV and Elgaroy *et al.* $\Sigma_i m_{\nu_i} < 1.8$ eV, are the two extreme values. Indeed, other intermediate values has been proposed.

which is ten times higher than the previous given value, using Eq. (2.9).

Some authors [6] evoke a decaying-neutrino scenario. In fact, the possibility for neutrinos to decay is closely related to their mass. Models incorporating supersymmetry imply a decay time-scale lower than the age of the Universe. The "decaying-neutrino hypothesis", introduced by Sciama, suggests that some of the heavy relic neutrinos could decay into lighter ones plus photon,

$$\nu_\tau \rightarrow \nu_\mu + \gamma. \tag{4.34}$$

According to the fact $m_{\nu_\tau} \sim 29$ eV in his model, resulting photons would bring an energy of 14.4 eV. Thus, an ultra-violet background, more precisely a 86 nm background, is expected.

In their review, Overduin *et al.* [6] study, among others, the relevance of this hypothesis through the bolometric intensity of the Extragalactic Background Light (EBL) [15]. The data may safely be said to exclude the decaying-neutrino hypothesis. The LSS formation matches with this rejection. Thus, those hypothetical decaying neutrino with such a rest mass and decay lifetime cannot provide some of the Dark Matter.

Lahav and Elgaroy [11, 25, 37, 26] resurrect an out-of-fashion model of Mixed Dark Matter (MDM), namely Dark Matter comprising two separate components. If massive neutrinos have a non-negligible effect, Hot Dark Matter could be one of this component. An Einstein-de Sitter cosmological model, i.e. $\Omega_m = 1$, is assumed. The participation of massive neutrinos to the total energy density should be around $\Omega_\nu = 0.2$ and the Hubble parameter $h = 0.45$. The 2dF results as well as a slightly modified WMAP data set seem to match. Those values are different than the cosmological data set favoured today, which Lahav himself exposes in [11]. This model is also "resurrected" by Blanchard [41].

4.2.3 Some Kind of Conclusion - Harsh Way to Digestion

From its theoretical conception to its role in the evolution of the Universe, the neutrino has always been a puzzling particle. Massive or massless, left-handed or both right- and left-handed, cosmologically negligible or not: all those questions have not found answers yet. What can be noticed is: the less detectable the particle is, the most imaginative the hypothesis are.

Nowadays, due to the results of oscillation experiments, it is widely believed they are massive, even if their mass is assumed to be small by "standard" theories and experiments. Astonishingly, whether we consider a hierarchical or non-hierarchical mass ordering, the neutrino masses are restricted more by Eq. (4.26) than by the Particle Data Group values.

Due to they are "puzzling", still mysterious, light, weakly interacting particles with no charge, neutrinos could be thought as a main component of Dark Matter. And we can say they are part of those 25% of missing matter.

[15] The bolometric intensity of the EBL is the energy received by us over all wavelengths per unit of time, per unit area, from all the galaxies which have been shining since a given time, i.e. $t = 0$

Unfortunately, as long as they are considered relativistic, namely mostly as long as their mass is assumed to be small, they enter in conflict with the theory of Large Scale Structure formation. Cosmological constraints on neutrino mass are more restrictive than particle physics experiments. Nonetheless, the interference of the values of neutrino mass depends on the "priors", i.e. the assumption made on the other cosmological parameters like Ω_m. Lots of parameters can effectively be changed, from the number of neutrino species to the cosmological model. More constraints shall be added in order to find more accurate mass values.

Some more models should deserve a closer look. From the Mixed Dark Matter model to supersymmetric heavy neutrino hypothesis, a wide overview on neutrino puzzle is provided.

The difficulty of the conclusion lies in the fact no consensus on neutrino participation are made. We should conventionally adopt the WMAPext+2dFGRS+Lyman-α conclusion, i.e.

$$\Omega_\nu h^2 < 0.0076. \tag{4.35}$$

Nor the Dark Matter problem, neither the neutrino question has been closed. This harsh digestion should be softened in the next sections.

Chapter 5

The Axions

In the short introduction to the Dark Matter problem in Chapter 2, we have introduced a naive point of view through well-defined concepts.

Actually, the standard cosmological model is almost assumed to be the ΛCDM model [24]. This ΛCDM model suggests that most of the missing mass should be cold, i.e. non-relativistic at its decoupling time and a fortiori at the time structures began to form. The structure formation could not have taken place as it did unless the energy density of matter Ω_m is much larger than Ω_{bar}. The Cold Dark Matter would prepare the gravitational potential wells in order to baryons to fall into. Within this sketch of structure formation, i.e. LSS via gravitational instabilities (GI), the Cold Dark Matter hypothesis is required.

As a matter of fact, the particle physics investigations aim for Cold Dark Matter (CDM). The two main candidates for CDM are the axion and the weirdly called WIMP, i.e. weakly interacting massive particle, which is the subject of the next Chapter.

Nonetheless, the frontier between Hot and Cold Dark Matter is not obvious. Depending on the particle mass and the time at which the particles appear and decouple from the thermal bath, this characteristic, first thought sufficient to rule some candidates out, are sometimes not taken into account. For instance the Hot Dark Matter hypothesis, exposed in the Chapter 4, is still problematic considering a cosmological model favoured by both the data and the theorists.

Moreover, the axion, first classified as Cold Dark Matter, could possibly be part of the Hot Dark Matter or both Hot and Cold Dark Matter.

Slightly beyond the GSW Standard Model, the axion comes from the theory of strong interactions, i.e. the Quantum Chromodynamics. Not yet discovered or, one could say still a hypothetical particle, the axion is predicted to interact with a small strength and to have a small mass. Moreover its relic density should allow us to consider it as a good Dark Matter candidate. It is proposed to expose here after the theoretical origin of this intriguing particle as well as its contribution to the total energy density.

Dinner is getting softer little by little.

5.1 The Strong CP Problem

The theory of strong interaction may include in its Lagrangian a term added "by hand", namely the so-called θ-term. This term is required when working with the non-perturbative QCD theory.

In fact, the theory of instantons, largely beyond the scope of this dissertation, proposes a view of the vacuum as a linear combination of different vacua states rotated by an angle θ. The different vacua coefficients are fixed within the true vacuum $|vac\rangle$ by a phase

$$|vac\rangle = \sum_{n=-\infty}^{+\infty} e^{i\,n\,\theta}|n\rangle. \tag{5.1}$$

A gauge transformation allows to go from one of those vacua $|n\rangle$ to the following

$$|n\rangle \to |n+1\rangle. \tag{5.2}$$

The effect of this gauge transformation on the true vacuum is comparable to a multiplication by a phase factor $e^{-i\,\theta}$, namely the vacuum is parametrised by an arbitrary constant θ.

This parametrisation by θ has an impact on the Lagrangian

$$\begin{aligned} \mathcal{L}_{QCD} &\to \mathcal{L}_{QCD} + \mathcal{L}_\theta \\ &\to \mathcal{L}_{QCD} + \frac{\theta}{16\pi^2}\, Tr F_{\mu\nu}\tilde{F}^{\mu\nu}, \end{aligned} \tag{5.3}$$

where $\mathcal{L}_\theta$ is the θ-term. The field strength $F_{\mu\nu}$ is also sometimes written taking the gluon color a into account, i.e. $F^a_{\mu\nu}$. And the dual field strength $\tilde{F}^{\mu\nu}$ is defined by $\tilde{F}^{\mu\nu} = \frac{1}{2}\,\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$.

Nevertheless, the presence of the $\epsilon^{\mu\nu\rho\sigma}$ in the Lagrangian explicitly violates P-parity. Moreover, T-parity is also violated. According to the CPT-conservation theorem, a non-zero θ-term would imply CP violation.

However, and astonishingly, experiments on neutron electric dipole moment lead to the following bound on θ

$$\theta < 10^{-9}. \tag{5.4}$$

This is called the "strong CP problem". Indeed this θ_{QCD} would have been expected to be not far from $\mathcal{O}(1)$, in other words: if another parameter contributes to QCD, why is it so small?

In 1977, Peccei & Quinn proposed to show that θ is effectively zero, which could solve the strong CP problem. They invoke a global $U(1)$ symmetry, also called $U(1)_{PQ}$ or Peccei-Quinn symmetry. This symmetry is preserved by a combined QCD and electroweak theory. Beyond the GSW Standard Model, this symmetry requires the presence of additional Higgs fields. As this symmetry is not observed at the present time, the Peccei-Quinn symmetry is broken.

In Chapter 3, we have seen that a spontaneous breaking of symmetry implies the apparition of Nambu-Goldstone (NG) bosons. If the symmetry is exact, those resulting NG bosons are massless. But if there is a small explicit breaking of the symmetry, either already in the Lagrangian or due to quantum mechanical effects, the "would-be" NG bosons acquire a finite mass and are called pseudo-NG bosons.

So, the breakdown of the Peccei-Quinn symmetry, at the energy scale of the decay constant f_a, generates a pseudo-NG boson, the axion. Axions are spin-0 particles with zero charge.

Let us first expose in what the axion solves the strong CP problem neglecting the quantum correction required by non-perturbative QCD . In fact, the axion field allows the "absorption" of the CP-violated terms and so opens the possibility of solving the strong CP problem[1].

To the overall CP-violated phase factor is added an axion field. Namely, Eq. (5.3) becomes, after some more transformation making the $F\tilde{F}$ disappeared,

$$\theta \rightarrow \theta_{eff} - \frac{\phi_{axion}}{f_a}, \tag{5.5}$$

where ϕ_{axion} is the axion field and θ_{eff} is the CP-violated term coming from the overall phase factor. A simple shift from ϕ_{axion}/f_a to $\phi_{axion}/f_a + \theta_{eff}$ is sufficient to absorb θ, as required.

At first approximation, i.e. if quantum mechanical effects are not taken into account, this simple "shift" does not alter the kinetic term since the axion is massless.

The coupling action with both matter and gluons is inversely proportional to the axion decay constant f_a. For instance, the coupling action with gluons is given by, as said above,

$$\mathcal{L} = \left(\theta_{eff} - \frac{\phi_{axion}}{f_a}\right) \frac{1}{16\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu a}. \tag{5.6}$$

However, since the simplest model does, unfortunately, not mean the most correct, researches for this axion ended with negative results. It seems that reliable models are the "invisible axion models"[2] which suggest a lighter mass and a weaker coupling.

Moreover, taking, as it is required, quantum mechanical effects into account, the kinetic term of the Lagrangian is somehow altered by the axion field. In fact, QCD non-perturbative effect induces, at QCD transition[3], a potential for ϕ_{axion} whose minimum is located at $\phi_{axion} = \theta_{eff} f_a$ cancelling θ_{eff} and, in the same way, solving the strong CP problem in a similar fashion. The principal difference lies in the fact that the axion is a "pseudo-NG"

[1] We would like to mention that some other solutions to this problem have been proposed. However, we here aim to describe the axion and its theoretical origin, which one exists only in this model.

[2] Once again we mention the existence of different models. Moreover, the axion interaction with photons and nucleons is model dependant even if the action is still inversely proportional to f_a.

[3] This transition occurs at $T \sim \Lambda_{QCD} \sim 150$ MeV.

boson and so is massive.

We can say the mass of the axion is generated by the potential that makes θ going to zero. This mass is given by[4]

$$m_a = 0.62\,\mathrm{eV}\,\frac{10^7\mathrm{GeV}}{f_a}. \tag{5.7}$$

The strong CP problem as well as its solution, or at least the solution we are interested in, have been exposed. The role it cosmologically plays could be non-negligible according its mass and, in particular, its mass contribution to the total energy density.

5.2 Axion as Cosmological Actor

Nowadays bounds on axion contribution to the matter budget are expected to be small. Since axion is not well known, it does not enable us to find an accurate mass nor energy density value. Here after, the different constraints in different physical research fields are exposed. The Fig. (5.1) illustrates and abstracts what follows.

The cosmological history of the axion began at the Peccei-Quinn symmetry breaking scale, i.e. at $T \sim f_a$. There are different scenarios capable to describe the evolution of axions.

First, if f_a is smaller than 10^8 GeV, axions can find thermal equilibrium and their two-photon decay

$$a \to \gamma + \gamma \tag{5.8}$$

can contribute the the Extragalactic Background Light (EBL) [6] via γ-emission. In such a case, they are called "thermal axions".

However, if the symmetry is broken at a scale higher than 10^8 GeV, axions never find thermal equilibrium because their interaction rate is much bigger than the age of the Universe. In that case, we can make one more distinction. If the symmetry breaking occurs before or after the inflation, i.e. if they undergo a post-inflationary reheating phase or not, the model can differ.

5.2.1 Invisible Axions

The most popular scenario assumes that axions are Cold Dark Matter particles. As a matter of fact, they would have been produced non-thermally and are so "invisible". According to the time of their apparition in the history, inflation, if it exists, can have an influence on their evolution.

Actually, if the $U(1)_{PQ}$ symmetry breaking precedes inflation, the "misalignment mechanism" can be considered. Just after the breaking of the $U(1)_{PQ}$ symmetry, the initial angle

[4]See [27] for more details.

θ is different from zero. When the QCD transition takes place, the potential that forces θ to go to zero is created so that this angle starts relaxing [27]. Namely, θ was "misaligned".

This mechanism leads to an energy density proportional to

$$\Omega_a h^2 \propto 1.9 \times 3^{\pm 1} \left(\frac{1\,\mu\text{eV}}{m_a}\right)^{1.175}. \tag{5.9}$$

The exact relation includes a function taking the misalignement angle into account. The incertitude on the initial angle does not allow real prediction for the mass of the axions [28].

Nevertheless, some inflationary models, which implications are beyond the scope of this dissertation, put a limit from which axions could be Cold Dark Matter. This limit is given by, see Fig. (5.1),

$$m_a < 10^{-3}\,\text{eV}. \tag{5.10}$$

On the other hand, if the symmetry breaking occurs after the inflation, string scenarios[5] take place. Afterwards, axions quickly become non-relativistic when they acquire mass at the QCD transition. They are so part of the Cold Dark Matter. The axion energy density is also inversely proportional to $m_a^{1.175}$ as in inflationary models. Some more parameters are taken into account. Finally, considering a wide window in which axions could contribute to the total energy density , i.e.

$$0.1 < \Omega_a < 1, \tag{5.11}$$

axion masses lies between [29]

$$6\,\mu\text{eV} < m_a < 2.5\,\text{meV}, \tag{5.12}$$

where it is important to notice that those values strongly depend on the "priors" or cosmological parameters chosen.

Particle physics laboratory experiments lead to the following bound on f_a value [27, 1]

$$f_a > 10^4\,\text{GeV}, \tag{5.13}$$

so that

$$m_a < 1\,\text{keV}. \tag{5.14}$$

Astrophysical constraints are obtained through analysis of star plasma, where the main axion production is supposed to be. Actually, low-mass weakly-interacting particles are produced in hot plasmas. Even if they are still hypothetical particles, they can be consider like an "energy-loss channel" for those stars [29]. Those constraints seems more stringent than the particle physics ones.

[5]The reference [29] mentions the Battye and Shellar model. Nevertheless, they also mention a slight difference between this model and the Silkivie *et al.* string model. We refer to [29] for further explanation.

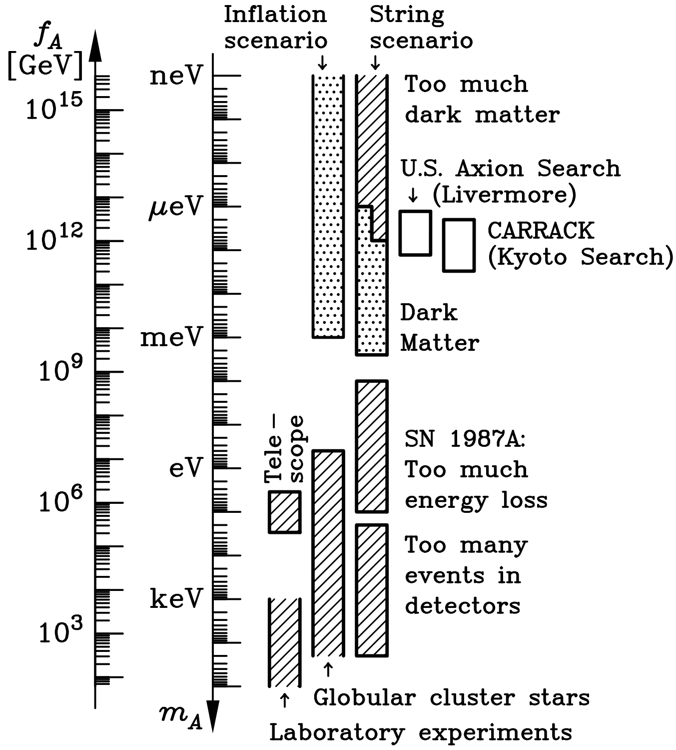


Figure 5.1: Exclusion regions (hatched) for the axion mass m_a and the energy scale at which the Peccei-Quinn symmetry is broken, i.e. f_a. The dotted regions indicate where the axion could possibly be part of the Dark Matter [29].

A strong bound is put by the Supernova SN 1987 data. The explosion of this Supernova is one of the main astrophysical source of data concerning neutrinos. Here, the neutrino analysis constraint the coupling of the axion with nucleons. The energy-loss channels which are not taken by neutrinos could be "invisible channels", e.g. axions[6]. Namely,

$$f_a > 6 \times 10^8 \,\text{GeV} \tag{5.15}$$

and an axion mass upper bound is, in this case, given by

$$m_a < 0.01 \,\text{eV}. \tag{5.16}$$

This value of f_a is kept as lower bound. We are now searching for an upper bound. The condition given by Eq. (2.7) simply restricts the decay constant by considering axion as the main Dark Matter component. This condition on the upper bound clearly imposes that we would work within the ΛCDM cosmological model. We find [1]

$$f_a \leq 10^{12} \text{GeV}. \tag{5.17}$$

In summary, we are left with the variation window for the "invisible axions",

$$6 \times 10^8 \,\text{GeV} \;\leq\; f_a \;\leq\; 10^{12} \,\text{GeV}. \tag{5.18}$$

[6]The description of those channels is beyond the scope of this dissertation.

More precisely, we could take into account that inflationary models do not impose lower bound on axion mass. In this way, astrophysical, laboratory or theoretical arguments show

$$m_a < 10^{-2}\,\mathrm{eV}. \tag{5.19}$$

The mass range where axions would contribute to the Cold Dark Matter is fairly uncertain but probably is somewhere in the range $1\mu\mathrm{eV} < m_a < 1$ meV [32].

At present, laboratory experiments as well as astrophysical researches assume axion is part of the halo Dark Matter. The mass of those "invisible" axions, and that is our conclusion, is not known. And as a matter of fact it is difficult to detect them.

Galactic Dark Matter axions could however be discovered in the mass range about 1 μeV. For instance, the microwave cavity experiments, which detect axions through their conversion into photons in external electromagnetic field, i.e. the Primakoff conversion[30].

5.2.2 Thermal Axions

Invisible axions are quite difficult to observe as they do not decay. In fact, the above-mentioned invisible axions do not decay or, at least, decay too slowly, i.e. their decay timescale is much longer than the age of the Universe, to leave any trace in the extragalactic background light (EBL).

The Supernova SN 1987a could lead to another stringent limit on axion mass. For a larger interaction strength[7] , i.e. a smaller f_a than implied by Eq. (5.15), axions are no longer able to compete with neutrino-loss channels. In fact, axions with such a sufficient interaction strength with baryons could produce a detectable signal in water Čerenkov detector. In particular, such detectors which registered the SN 1987a neutrino signal would have recorded too many events.

Axions with a mass

$$m_a \gtrsim 10\,\mathrm{eV} \tag{5.20}$$

are ruled out, see Fig. (5.1).

A narrow intermediate range of couplings lies between both decay-channel and Čerenkov constraints. Actually, for f_a around 10^6 GeV (see Fig. (5.1)), axions could be Dark Matter, [6, 32].

Overduin and Wesson [6] give, in their paper, the following window,

$$2.2\,\mathrm{eV} \leq m_a \leq 10\,\mathrm{eV}, \tag{5.21}$$

referring to Turner's paper [38]. Axions in such a mass range are called "multi-eV window axions".

Those particles would have been produced thermally in the early Universe as their decay constant is smaller than 10^8 GeV. They are so "less cold" than the invisible axions and are sometimes referred as Hot Dark Matter candidates even if the frontier between Hot and Cold Dark Matter is not quite clear.

[7] Interaction strength with baryons is model dependent. The different axions models are beyond the scope of this dissertation.

Since axions have reached thermal equilibrium, statistical mechanics laws can be used. Therefore, given the definition of the number of degrees of freedom g, we are able to find the following axion present energy density [6]

$$\begin{aligned}\Omega_a &= \frac{n_a\, m_a}{\rho_c} \\ &= \frac{5.2\times 10^{-3}}{h^2}\, m_a.\end{aligned} \tag{5.22}$$

According to Eq. (5.21) and Eq. (2.9), axion contribution to Dark Matter is comprised

$$0.0216 \leq \Omega_a \leq 0.0981, \tag{5.23}$$

using "our" prior conventions.

Even if they are not able to provide all the missing mass required in the ΛCDM model, axions could nevertheless suffice in low-density models, as mentioned by Overduin.

Moreover, his review [6] restricts this interval by bolometric arguments. Therefore the axion participation would be

$$0.0216 \lesssim \Omega_a \lesssim 0.0363. \tag{5.24}$$

This density is comparable to the energy density of baryonic Dark Matter.

Raffelt [32] also mention the possibility for axions to be thermally produced. Assuming the same theoretical constraints on decay-channels and Čerenkov detectors, he gives a lower bound on thermal axion mass of,

$$m_a < 1.05\,\mathrm{eV}, \tag{5.25}$$

or, equivalently,

$$f_a > 5.7\times 10^6\,\mathrm{GeV}. \tag{5.26}$$

As already mentioned, the interaction strength with ordinary matter is model dependent. Therefore a difference between Overduin & Wesson and Raffelt's values can be explained by a difference in the model as well as prior choice. Moreover, if axions decouple when $T \sim \Lambda_{QCD} \sim 150$MeV, axion mass cannot exceed 1-2 eV. Otherwise free-streaming axions could not fit with LSS theories in the same scheme than the neutrinos [32].

Telescope experiments investigate in the eV-window[8]. For instance, the CERN Axion Solar Telescope (CAST) which has first restricted its researches in the mass range $m_a \lesssim 0.02$ eV will soon extend the sensitivity to axion masses up to 1 eV [32].

5.2.3 In Summary...

Experiments are still not conclusive on axion mass. Nevertheless, until no argument excludes axion as Cold Dark Matter candidate, we have to consider its participation to the energy density as plausible.

[8]We refer to [30] for further explanations on the experiments driven at present. Using the 2-photon decay or the Primakoff conversion, experiments seems to limit the coupling strength for a given mass.

However, they could be Hot Dark Matter as well. What makes axions being different from other particles, e.g. neutrinos, is the fact that they only reach thermal equilibrium if their decay constant is bigger than the age of the Universe. In other terms, they do not need to drop out from equilibrium to become non-relativistic.

Thermal axion contribution seems to be so weak that the motivations as CDM are to be removed. Overduin & Wesson conclude that, if axions are to provide a significant portion of Dark Matter, then axions must have rest mass in the "invisible" range where they do not contribute significantly to the light of the night sky [6].

Whatever are their mass range, axions contribution, as weak as it could be, has to be taken into account.

The more we go, the more we learn that a main dark component is missing or is not.

Chapter 6

The Weakly Interacting Massive Particles

Under the generic name of WIMPs, i.e. weakly interacting massive particles, are brought together "exotic" particles. Indeed massive neutrinos and axions could easily be grabbed by "widely beyond the Standard Model" theories and so could also be incorporated in this category.

Nevertheless, under this acronym is hidden the deep meaning of our expectations. Dark Matter candidates should obviously be weakly interacting and, moreover, they should be massive, in fact, more massive than baryons. This argument seems however assumed even if possible influence on the total energy density could be provided without a huge mass, e.g. the dependance of axion decay constant on m_a^{-1}.

The WIMP is supposed to be a supersymmetric particle. Supersymmetry aims to unify all the interactions in one unique symmetry which, of course, would have been broken in the very early Universe.

In this section, we will overview some background of supersymmetry in order to introduce the WIMP candidates. Afterwards, the theoretical thoughts and the experimental limits will be exposed.

6.1 A Short Overview of Supersymmetry

Supersymmetry is a symmetry which is invariant under boson-fermion exchange. Indeed the Standard Model distinguishes between bosons, which deal with internal symmetries in mediating interactions, and fermions, i.e. "matter" particles, which deal with space-time symmetries.

So that we can sympathize with *her*, from now, we will call *her* by *her* cute nickname, Susy.

The Susy generator Q acts in modifying the spin of the initial state, i.e.

$$\begin{aligned} \bar{Q}|\,\text{boson}\rangle &= |\,\text{fermion}\rangle, \\ Q|\,\text{fermion}\rangle &= |\,\text{boson}\rangle. \end{aligned} \tag{6.1}$$

Since this Q operator is nilpotent, by applying twice this operator one goes back to the initial state. But this state is translated with respect to the initial state. This translation is done not in a simple euclidean space but in a superspace, namely an euclidean space plus two variables. Susy tends to adapt a Poincaré group, i.e. Lorentz transformations plus translations, to a "super-Poincaré" group[1] in order to incorporate boson/fermion symmetry with a space-time generator.

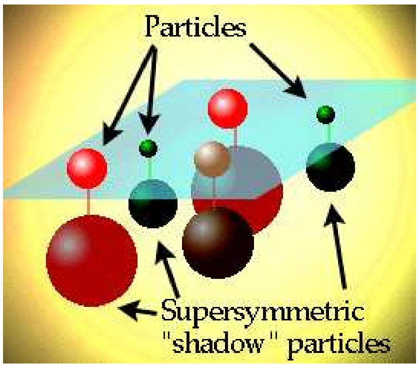


Figure 6.1: Particles and their superpartners in the MSSM [33].

However, no particle in Nature seems to correspond to a fermionic (resp. scalar) counterparts, i.e. having the same quantum numbers, of a given scalar (i.e. fermionic) particle. Namely, it means this symmetry implies the existence of additional particles, i.e. supersymmetric partners, each of them corresponding to a Standard Model particle. The particle content is simply doubled.

Such particles are not observed. Thus they must have a rest mass beyond our present detection abilities and so are more massive than their Standard Model counterparts. In a Minimal SuperSymmetric Model[2] (MSSM), the supersymmetry is slightly broken, i.e. Susy is not an exact symmetry of our Universe, in order to explain the asymmetry of the mass between particles and the so-called superpartners, see Fig. (6.1). Nevertheless, as an extension of the Standard Model of particle physics, MSSM is still conserved under the same gauge group, i.e. $SU(3)_C \times SU(2)_L \times U(1)$.

Here after Tab. (6.1), a recapitulative table of the "Standard Model" particles (see Tab. (3.1)), and their supersymmetric counterparts. The superpartners are denoted by a tilde and are renamed *s*fermions for the scalar counterparts of the fermions, and bos*ino*s, for the fermionic counterparts of the bosons.

[1]This is, of course, beyond our scope.
[2]Also called Minimal Susy Standard Model.

Particles			Superpartners		
	Symbol	Name		Symbol	Name
	e, μ, τ	electron-like leptons		$\tilde{e}$, $\tilde{\mu}$, $\tilde{\tau}$	selectrons, ...
Fermions	ν_e, ν_μ, ν_τ	neutrinos	Bosons	$\tilde{\nu}_e$, $\tilde{\nu}_\mu$, $\tilde{\nu}_\tau$	sneutrinos
	u, d, s, c, t, b	quarks		$\tilde{u}$, $\tilde{d}$, $\tilde{s}$, $\tilde{c}$, $\tilde{t}$, $\tilde{b}$	squarks

Table 6.1: The particle content of the Minimal Supersymmetric Standard Model.

The Susy equivalent of Tab. (3.3), describing the gauge bosons in the Standard Model, is given by the Tab. (6.2).

The Standard Model as it has been described in Chapter 3 implies the existence of the Higgs boson. Its supersymmetric extension we are shortly describing here requires two more higgses, which counterparts are called higgsinos and are denoted by $\tilde{h}_1$ and $\tilde{h}_2$. This additional Higgs field is required in order to give mass to both up- and down-type quarks before the electroweak breaking.

Particles			Superpartners		
	Symbol	Name		Symbol	Name
	γ	photon		$\tilde{\gamma}$	photino
Bosons	Z^0, $W^\pm$	weak gauge bosons	Fermions	$\tilde{Z}^0$, $\tilde{W}^\pm$	zino, winos
	g^a	gluons		$\tilde{g^a}$	gluinos
	g	graviton		$\tilde{g}$	gravitinos

Table 6.2: The gauge bosons counterparts in the Minimal Supersymmetric Standard Model.

Before Susy breaks down, i.e. around $T \sim 1$ TeV, superpartners were in thermal equilibrium. They could decay among themsleves leaving lighter ones. The Lightest SuperParticle (LSP) is the neutral Susy particle which can no longer decay into lighter ones.

This LSP is predicted to be stable, because MSSM conserves a new symmetry called R-parity. R-parity has been introduced in order to avoid terms not able to go back to Standard Model fields without violating common quantum numbers. Moreover, among other things, this R-parity preserves the proton of decaying.

All the Standard Model particles have a R-parity $R = +1$ and all the superpartners have $R = -1$. Therefore, in order to conserve this parity, superpartners are always created by pairs, or at least we must have an even number of sparticles. Moreover, the LSP can only decay through pair annihilation into photons, what makes it being a good candidate for Dark Matter.

6.2 The Lightest SuperParticle as Cosmological Actor

The first candidate for this Lightest Superparticle was the sneutrino. However, it seems experiments exclude this candidate [6, 35]. Axinos and gravitinos have also been proposed and ruled out as well [6, 35]. Selectrons and photinos were thought to be plausible candidates too [6].

At present, the LSP is most likely to be a "neutralino" $\tilde{\chi}$. The neutralinos are linear combinations of Susy counterparts of gauge bosons, i.e. a linear combination of the photino $\tilde{\gamma}$, the zino $\tilde{Z}$ and neutral higgsinos $\tilde{h}_1^0$ and $\tilde{h}_2^0$.

As usual, we wish to find the energy density of the neutralino, or any other LSP candidate. Of the same fashion than those we explicitly made in Appendix A, but with sofisticated definition of the number of degrees of freedom, the calculations of the neutralino relic density show LSPs would be a good candidate. In fact, the value of the LSP relic density would simply make Susy particles the favoured Dark Matter candidates. Namely, we have [1]

$$\Omega_{WIMP} \sim \frac{7 \times 10^{-27}\,\mathrm{cm}^3\,\mathrm{s}^{-1}}{\langle \sigma_{ann} v \rangle}, \tag{6.2}$$

with $\langle \sigma_{ann} v \rangle$ the average of annihilation cross section times the relative velocity. Since further calculations show that neutralinos could have an annihilation cross section in the range we are interested in, neutralinos are good Dark Matter candidates.

The mass eigenstate of the four neutralinos depends on the linear combination of states they are made of. In fact, a neutralino can be mostly "photino" or mostly "zino",... The Lightest Superparticle is therefore the lightest of such a state and its mass is placed, by accelerator experiment limits, at

$$m_{\tilde{\chi}} > 46\,\mathrm{GeV}. \tag{6.3}$$

The upper bound is put in the same fashion that we imposed the axion mass bound. We forbid the neutralino to overclose the Universe, what can occur if their mass is too large. The age of the Universe therefore implies an upper bound of

$$m_{\tilde{\chi}} < 3200\,\mathrm{GeV}. \tag{6.4}$$

However, non-minimal Susy theories constraint this value to

$$m_{\tilde{\chi}} \lesssim 600\,\mathrm{GeV}. \tag{6.5}$$

But simple considerations can help us to know them a little bit more. First, Susy WIMPs contribute to the cosmic background radiation by pair annihilation to photons in the MSSM. However, due to the stability of the LSP, this process takes place slowly via intermediate loops of charged particles. Other contributions of WIMPs to the background are possible in non-minimal Susy theories. Results of such studies are, for instance, in the review of Overduin and Wesson [6]. If we consider only the minimal decays, i.e. pair annihilation, this review shows that the cosmological density of WIMPs in galactic dark-matter halos is about

$$\Omega_{haloWIMPs} = (0.07 \pm 0.04)h. \tag{6.6}$$

In such a case, WIMPs could not make up the lack of mass without any other (Cold-)Dark Matter sources.

Second, WIMPs will form not only a background density in the Universe, but also will cluster gravitationally with ordinary stars in the galactic halo since the interaction they principally interact through is gravity. Therefore, they are supposed to be present in our own galaxy and a fortiori in our solar system. Driving direct detections on Earth is thus plausible. Moreover, since they are trapped in the same gravitational potential than the Sun, they are expected to move at the same velocity than that of the Sun.

In Chapter 1, it has already been mentioned that the amount of Dark Matter which is expected at [1, 35], for the position of the Sun,

$$\rho_{dark} \sim 0.3\,\mathrm{GeV\ cm}^{-3}. \tag{6.7}$$

For information, if WIMP mass were about 100 GeV, we should be surrounded by about 3×10^{-3} WIMPs per cm^3 [1].

In order to have a better understanding of Susy researches, let us have a look at the experiments driven at present. Moreover, this short review of experiments can serve as a promising conclusion of this Chapter.

6.3 WIMP Detection

The first difficulty faced in WIMPs detection lies in the simple fact that their existence is based on the assumption we cannot see them because they are too massive.

Therefore, laboratory direct experiments, in particular accelerator experiments, cannot provide us with more information for now. The two clues given above, i.e. contribution of WIMPs to the cosmic background plus presence in our galactic halo, can help us to detect them indeed.

We can distinguish two kinds of detection, namely "direct" and "indirect" detection. The first category aims to directly measure the neutralino interactions in laboratory while the second category, i.e. indirect researches, tries to detect the product of neutralino annihilation.

The motion of WIMPs is expected to be at a velocity of about 220 km/s, i.e. velocity of the Sun around the center of the galaxy. The interaction with ordinary particles is therefore made through elastic scattering.

Thus, the main principle of direct detection is to look for interaction of WIMPs with ordinary matter through the recoil of nuclei.

Such detections are made in measuring ionization in solids or crystals, e.g. germanium. However, the sensitivity of such detections, due to the importance of the "noise" produced by radioactivity, cosmic rays and so on, requires extremely good background discrimination. Therefore, such sensitive experiments are built in the deep underground.

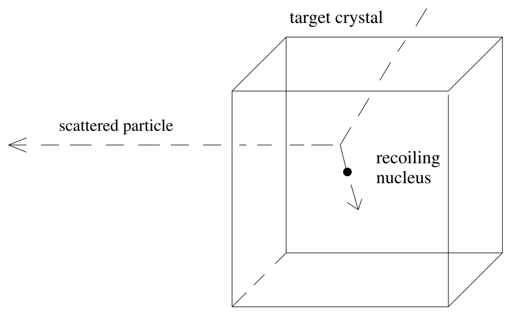


Figure 6.2: Illustration of elastic scattering of a Dark Matter particle with a nucleus [1].

More than 20 direct Dark Matter detection experiments are either now operating or are currently in development.

- EDELWEISS- Experience pour DEtecter Les WIMPs En Site Souterrain

  Located in the Alpes, this experiment is one of the many experiments driven using scintillation in crystals. Difficulties lie in the separation of WIMP signal from background. See, for instance [39].

- DAMA- particle DArk MAtter searches with highly radiopure scintillators at Gran Sasso

  Some experiments therefore attempt to separate the Dark Matter signal from the background by looking for an annual modulation in their rate. According to the fact the Sun's motion around the center of the galaxy is not in the same plane than the Earth's motion around the Sun, a seasonal modulation in the WIMP flux is expected. In fact, the flux will be larger in june when the velocity of the Earth is addes to the velocity of the Sun, see Fig. (6.3).

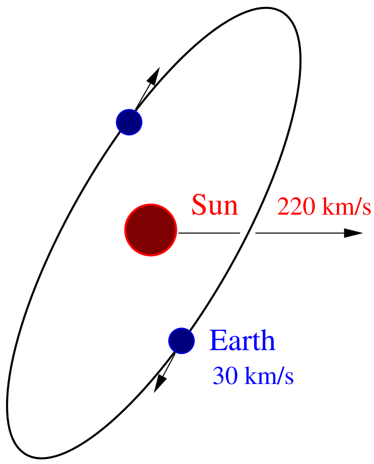


Figure 6.3: Principle of DAMA experiment: the seasonal variation of Earth and Sun's motions could lead to varition in the WIMPs flux [1].

This experiment seems to be really promising. Actually, a modulation in the signal is effectively found.This signal is consistent with a WIMP of a mass around 30-200 GeV. See for instance [40].

- CDMS, ZEPPLIN-I...

  Presently, the best detection limits come from CDMS, EDELWEISS and ZEPPLIN-I experiments. However, the sensitivity of those experiments does not enable us to explore a wide range of the Susy parameter space. Future generation detectors are expected to give important insights into the nature of Dark Matter.

Indirect detections experiments are also developed. Their principle is based on detection of decay products of WIMPs. Namely, the radiation produced in Dark Matter annihilation is observed. The flux of such radiation is proportional to the annihilation rate Γ_{ann}. This rate depends on the square of Dark Matter density, i.e. $\Gamma_{ann} \propto \rho^2_{dark}$. However, such experiments would need a good verification of all other possible phenomena their observations could result of.

High-Energy neutrinos from the Sun and Earth

- AMANDA- Antarctic Muon And Neutrino Detector Array

  A 1 km^3 ice cube in Antarctica plays the role of detector. Mainly developed for researches on neutrinos, this experiment would be indirectly able to prove the existence of WIMP through their decay products. In the present case, neutrinos with an energy about 10 GeV, resulting from WIMP annihilation in the center of the Sun, should be observed. In fact, a neutralino plus an anti-neutralino could decay into a baryon plus an anti-baryon. Those baryons should themselves decay, among others, into leptons, e.g. a muon μ plus its neutrino ν_μ.

- ANTARES

  Of the same fashion, we mention ANTARES, located in the Mediterranean.

- ...

Fluxes of γ-ray

Analysis of Dark Matter annihilation can also be done through observations in dense Dark Matter regions, e.g. our galactic center, Sun, Earth. Those regions where WIMPs would have accumulate are called amplifiers.

Fluxes of γ-ray from the center of the galaxy could thus teach us about Dark Matter WIMPs. See, for instance, [6, 35].

- EGRET
- MAGIC
- GLAST
- ...

Conclusion

> *The first principle is that you must not fool yourself and you are the easiest person to fool.*
>
> *Richard Feynman*

Even though the word conclusion is clearly not adapted here, the last things I could still write aspire not to abstract neither to finish the topic of this dissertation. Actually the readers could think, as I do, we are left wanting more: Dark Matter problem is not solved yet but clues are left indeed.

First, the cosmological point of view has been exposed. Most of the cosmologists favour the ΛCDM or Concordance Model. The WMAP report [24] is simply concluded by noticing cosmology has now a Standard Model after the fashion of particle physics. Few parameters therefore dictate the evolution of this model, and, by extension, the evolution of the Universe. Here philosophy supplants physics. Can scientists reduce the Universe to several parameter values?

Still concerning the Concordance Model and, more precisely, its assumptions on matter component, Hot Dark Matter is quite excluded. However, the Chapter 4 tells us that, according to their rest mass, neutrino energy density should be highly non-negligible. As a matter of fact, diverging models take a neutrino contribution into account, e.g. Mixed Dark Matter models. The Large Scale Structure formation theories nonetheless brake those models. Further researches shall answer the question whether the hundred neutrinos per cm^3 we are surrounded by have any cosmological role.

Present day cosmological researches are fixed on Cold Dark Matter candidates, which are still begging for an identification. Axions, even if not ruled out, are not considered promising for now. Neutralinos and other Susy particles are widely favoured. Dark Matter should be multi-component.

However, lots of particle physics candidates or models, as relevant, as critic as they could be, have not been mentioned. Light candidates [43], Warm Dark Matter models, sterile neutrinos [44], light scalar Dark Matter, Little Higgs models, Kaluza-Klein states, Superheavy Dark Matter, Q-balls, self-interacting Dark Matter, brane world Dark Matter, ... are of those.

Some critical points to the present knowledge on Dark Matter were raised. The frontier between the “cold “ and “hot” property of a particle is not clear. Actually the mass has long been the only argument, if not taken into account, at least kept in mind. Indeed the velocity of the particle has also an impact on the relic density and the Large Scale Structure formation. However it has simply been indirectly included through the qualificative (non-)relativistic dubbing particles according to their mass. A further classification of the properties could probably ressurect some points of view [42].

On the edge between two fields of physics, the new born study of the Dark Matter problem can hardly find conventions in language or communication among the concerned scientists. Namely, particle physics contribution is obviously based on the framework cosmologists and astrophysicists bring. And vice versa. Therefore, lots of assumptions which sometimes are not relevant could have been made. Astrophysics, as a phenomenological science, and particle physics, as a theoretical science, should both be checked with respect to the other. In another way, Dark Matter problem is begging for new models, cosmological as much as theoretical (dixit Ofer Lahav).

Finally, we mention the analogy drown by the same Ofer Lahav. The Dark Matter puzzle is somehow playing the role epicycles had. We cannot deny the data fit even if we do not know in what exactly. Epicycles just force theorists searching for further explanations.

Complicated and structurated scenarios are waiting, tower of Babel, to collapse taken over from clever ideas.

Appendix A

Statistical Mechanics and the Early Universe

In this appendix, the early Universe thermodynamics will be briefly exposed. The thermodynamic tools enable us to describe a radiation-dominated Universe, i.e. the "hot thermal Universe". In fact, at very high temperatures, particles, even massive, are part of the thermal bath.

Thermal equilibrium is established according to the rate of particle interactions relative to the expansion rate of the Universe $H(t)$ (see Eq. (1.24)). The minimal condition for equilibrium requires that the interaction rate should be larger than the expansion rate of the Universe.

The energy density for a given type of particle i, in equilibrium, associated to a perfect gas, is given by

$$\rho_i = \int E_i dn_{q_i}, \tag{A.1}$$

with $E_i^2 = m_i^2 + q_i^2$ and the density of states dn_{q_i} is given by

$$dn_{q_i} = \frac{g_i}{2\pi^2} \frac{q_i^2}{exp[(E_{q_i} - \mu_i)/T_i] \pm 1} dq_i, \tag{A.2}$$

which corresponds to a Fermi-Dirac distribution with the $+$ sign and to a Bose-Einstein distribution with the $-$ sign. The number of degrees of freedom is included through g_i: for instance, g_i could be the number of quantum states of spin ($g_i = 2s+1$) or of helicity ($g_\gamma = 2$, i.e. two polarisation states for the photon). The chemical potential μ_i is often associated with baryon number.

In order to compute those thermodynamical quantities for the radiation density, we assume the following results without demonstration.

Given the zêta Riemann function $\zeta(x)$, we have

$$\begin{aligned}\int_0^{+\infty} \frac{z^2}{e^z-1} &= 2\zeta(3),\\ \int_0^{+\infty} \frac{z^2}{e^z+1} &= \frac{3}{2}\zeta(3),\\ \int_0^{+\infty} \frac{z^3}{e^z-1} &= \frac{\pi^2}{15},\\ \int_0^{+\infty} \frac{z^3}{e^z+1} &= \frac{7\pi^2}{120}.\end{aligned} \tag{A.3}$$

Its particular value in $x = 3$ will be often used below $\zeta(3) = 1.202$.

The equations introduced above consider the case of a perfect gas of particles which do not interact among themselves. At high temperatures, as those governing at early times, we have $T >> m_i$. We can so neglect the mass in $E_i^2 = m_i^2 + q_i^2$.

For a photon gas, it is straightforward to find[1]

$$\begin{aligned}\rho_\gamma &= 2\frac{\pi^2}{30}T_\gamma^4,\\ n_\gamma &= \frac{2\zeta(3)}{\pi^2}T_\gamma^3,\end{aligned} \tag{A.4}$$

because photons are bosons.

For the neutrinos, fermionic particles, we find

$$\begin{aligned}\rho_\nu &= \frac{7}{8}2\,3\frac{\pi^2}{30}T_\nu^4,\\ n_{\nu_i} &= \frac{3}{2}\frac{\zeta(3)}{\pi^2}T_\nu^3,\end{aligned} \tag{A.5}$$

where we have explicitly shown the g_i decomposition in the first relation. Indeed, the 7/8 comes from the integral for a Fermi-Dirac distribution, the 2 stands for the 2 spin states and the 3 for the three generations. The second relation is for each neutrino species.

When the rate of the weak interaction becomes smaller than the Universe expansion rate, the "weak decoupling" occurs: the neutrinos drop out of equilibrium.The weak decoupling only concerns neutrinos. Indeed, unlike the other leptons, they do not interact through the electromagnetic interaction.

The temperature at which this transition is done is found by equaling H to Γ_{weak}, which leads to [9] $T_{decoupling} \sim 1$ MeV, with 1K corresponding to $(1.160445)^{-1} \times 10^{-4}$eV or 8.61738×10^{-9} MeV.

Neutrinos energy has simply been "redshifted" until now. Their present day temperature can easily be computed from entropy conservation of the Universe, see [5]. In fact, the $e^- - e^+$ annihilations reheat the photons without influencing the relics neutrinos. The temperature of the photons before and after the reheating is in a ratio of $(11/4)^{1/3}$ while the neutrinos temperature remains constant.

[1]Theses relations are given in 'natural' units.

Knowing that[2]

$$T_{CMB} = 2.725 \pm 0.002\,\mathrm{K}, \tag{A.6}$$

we find

$$\begin{aligned} T_\nu &= \left(\frac{4}{11}\right)^{1/3} \\ &\sim 1.953\,\mathrm{K}. \end{aligned} \tag{A.7}$$

From this relation, we can derive the number density of neutrinos as a function of photons number density. In fact, Eq. (A.4) and Eq. (A.5) lead to the following ratio

$$\frac{n_\nu}{n_\gamma} = \frac{3}{11}. \tag{A.8}$$

Re-expressing Eq. (A.4) in other units conventions

$$n_\gamma = \frac{2\zeta(3)}{\pi^2}\left(\frac{kT_\gamma}{\hbar c}\right)^3, \tag{A.9}$$

we easily compute the present number density of photons [3]

$$n_\gamma = 412.5 \text{ photons/cm}^3. \tag{A.12}$$

We can straightforwardly check the energy density of the CMB given by Eq. (2.6) by using Eq. (A.4), the CMB temperature value and the critical energy density,

$$\Omega_r h^2 = 2.48 \times 10^{-5}. \tag{A.13}$$

Therefore, with Eq. (A.8), we find the present neutrino number density

$$n_\nu = 112.5 \text{ neutrinos/cm}^3. \tag{A.14}$$

The energy density of the relativistic massless neutrinos should be

$$\Omega_\nu h^2 = 1.71 \times 10^{-5}. \tag{A.15}$$

[2] See [9] which refers to COBE data set.

[3] The constant values are the following:

$$k = 8.617 \times 10^{-11}\,\mathrm{MeV/K}, \tag{A.10}$$

for the Boltzmann constant;

$$\hbar c = 197.3 \text{ MeV fm}, \tag{A.11}$$

where 1 fm corresponds to 10^{-13} cm.

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Bibliography

[1] S. Khalil and C. Muñoz, "The enigma of the dark matter," Contemp. Phys. **43** (2002) 51 [arXiv:hep-ph/0110122].

[2] Y. Sofue, "Accurate Rotation Curves and Distribution of Dark Matter in Galaxies," arXiv:astro-ph/9906224.

[3] J. Lesgourgues, "An overview of cosmology," arXiv:astro-ph/0409426.

[4] J. Surdej, " General Astrophysics", notes de cours de première licence en Sciences Physiques, ULg, 2003.

[5] Y. de Rop, " Relativité générale", notes de cours de seconde licence en Sciences Physiques, ULg, 2004.

[6] J. M. Overduin and P. S. Wesson, "Dark matter and background light," Phys. Rept. **402** (2004) 267 [arXiv:astro-ph/0407207].

[7] http://cosmicweb.uchicago.edu/filaments.html

[8] S. Eidelman *et al.* [Particle Data Group], "Review of particle physics," Phys. Lett. B **592** (2004) 1.

[9] K. A. Olive and J. A. Peacock, "Big-Bang cosmology," Phys. Lett. B **592** (2004) 191

[10] B. D. Fields and S. Sarkar, "Big-Bang nucleosynthesis," Phys. Lett. B **592** (2004) 202.

[11] O. Lahav and A. R. Liddle, "The cosmological parameters," Phys. Lett. B **592** (2004) 206. arXiv:astro-ph/0406681.

[12] M. Drees and G. Gerbier, "Dark matter," Phys. Lett. B **592** (2004) 216.

[13] D. Spergel, "Particle Dark Matter," arXiv:astro-ph/9603026.

[14] R. L. Oldershaw, "A review of mass estimates for galactic dark matter objects," arXiv:astro-ph/0002363.

[15] F. Vissani, "What is the standard model of elementary particles and why we have to modify it," arXiv:hep-ph/0007040.

[16] J. L. Rosner, "Overview of the standard model," arXiv:hep-ph/9411396.

[17] M. Kaku, "Quantum field theory: A Modern introduction," New York, USA: Oxford Univ. Pr. (1993) 785 p.

[18] L. Valentin, “Noyaux et Particules: Modèles et Symétries,” Paris, France: Hermann, Amsterdam, Netherlands: North-holland (1989) 300p.

[19] I.J.R Aitchison and A.J.G. Hey, “ Gauge Theories in Particles Physics” (second edition), Institute of Physics Publishing, Bristol, 1996.

[20] S. F. King, “Cosmological implications of neutrino mass,” arXiv:hep-ph/0210089.

[21] D. P. Roy, “Neutrino mass and oscillation: An introductory review,” Pramana **54** (2000) 3 [arXiv:hep-ph/9903506].

[22] S. M. Bilenky, “The history of neutrino oscillations,” arXiv:hep-ph/0410090.

[23] U. Mosel, “Fields, Symmetries, and Quarks,(second, revised and enlarged edition)” Berlin Heidelberg, Germany: Springer (1999) 309 p.

[24] D. N. Spergel *et al.* [WMAP Collaboration], “First Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Determination of Cosmological Parameters,” Astrophys. J. Suppl. **148** (2003) 175 [arXiv:astro-ph/0302209].

[25] O. Elgaroy *et al.*, “A new limit on the total neutrino mass from the 2dF galaxy redshift survey,” Phys. Rev. Lett. **89** (2002) 061301 [arXiv:astro-ph/0204152].

[26] O. Elgaroy and O. Lahav, “Neutrino masses from cosmological probes,” New J. Phys. **7** (2005) 61 [arXiv:hep-ph/0412075].

[27] E. Masso, “Axions,” arXiv:hep-ph/0312064.

[28] H. Murayama, “Axions and Other Very Light Bosons, Theory,” in [8].

[29] G. Raffelt, “Axions and Other Very Light Bosons, Astrophysical Constraints,” in [8].

[30] C. Hagmann, K. v. Bibber and L. J. Rosenberg, “Axions and Other Very Light Bosons, Experimental Limits,” in [8].

[31] C. Lorcé, “Brisures de symétries et génération de masse,” Mémoire présenté en vue de l’obtention de grade de licencié en sciences physiques, 2003-2004, ULg.

[32] G. G. Raffelt, “Axions: Recent searches and new limits,” arXiv:hep-ph/0504152.

[33] D. I. Kazakov, “Beyond the standard model,” arXiv:hep-ph/0411064.

[34] H. E. Haber, “Supersymmetry, Theory,”in [8]

[35] G. Bertone, D. Hooper and J. Silk, “Particle dark matter: Evidence, candidates and constraints,” Phys. Rept. **405** (2005) 279 [arXiv:hep-ph/0404175].

[36] N. Fornengo, "Candidates for non-baryonic dark matter," Nucl. Phys. Proc. Suppl. **110** (2002) 26 [arXiv:hep-ph/0201156].

Further looks and explanations can be found in the following references.

[37] O. Elgaroy and O. Lahav, "The role of priors in deriving upper limits on neutrino masses from the 2dFGRS and WMAP," JCAP **0304** (2003) 004 [arXiv:astro-ph/0303089].

[38] M. S. Turner, "Axions From Sn1987a," Phys. Rev. Lett. **60** (1988) 1797.

[39] L. Chabert [EDELWEISS Collaboration], "Latest results from the EDELWEISS experiment," Eur. Phys. J. C **33** (2004) S965.

[40] P. Gondolo and G. Gelmini, "Compatibility of DAMA dark matter detection with other searches," arXiv:hep-ph/0504010.

[41] A. Blanchard, J. G. Bartlett and M. Douspis, "Cosmological implications from the observed properties of CMB," Comptes Rendus Physique **4** (2003) 909 [arXiv:astro-ph/0402297].

[42] C. Boehm and R. Schaeffer, "Constraints on dark matter interactions from structure formation: Damping lengths," arXiv:astro-ph/0410591.

[43] C. Boehm and Y. Ascasibar, "More evidence in favour of light dark matter particles?," Phys. Rev. D **70** (2004) 115013 [arXiv:hep-ph/0408213].

[44] M. Viel, J. Lesgourgues, M. G. Haehnelt, S. Matarrese and A. Riotto, "Constraining warm dark matter candidates including sterile neutrinos and light gravitinos with WMAP and the Lyman-alpha forest," Phys. Rev. D **71** (2005) 063534 [arXiv:astro-ph/0501562].

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