Jet quenching parameter $\hat{q}$ in the stochastic QCD vacuum

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MARIE CURIE ACTIONS

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Contents

Introduction

A hard collision yields an energetic parton, which traverses the medium of the size $L_{\parallel}$.

The average energy loss of the parton due to gluon radiation (R. Baier, Yu. Dokshitzer, A. Mueller, S. Peigne, D. Schiff, '96):

$$\Delta E = \frac{\alpha_s}{8} C_R \hat{q} L_{\parallel}^2.$$

Due to the non-Abelian Landau-Pomeranchuk-Migdal interference effect, $\Delta E \propto L_{\parallel}^2 \Rightarrow$ radiative parton energy loss $\gg$ collisional energy loss (due to whom $\Delta E \propto L_{\parallel}$).

The jet quenching parameter $\hat{q}$ is $\langle p_{\perp}^2 \rangle$ transferred from the medium to the parton per distance travelled.

Introduction

For a dilute plasma,

$$\langle p_\perp^2 \rangle \propto T^2, \quad \lambda \sim \frac{1}{n\sigma_t},$$

where $n \sim T^3$ is the particle-number density, σ_t is the Coulomb transport cross section:

$$\sigma_t = \int d\sigma_{\rm Coul}(1-\cos\theta) \sim g^4 \int_{(gT)^2} \frac{d^2 p_\perp}{p_\perp^4}\frac{p_\perp^2}{T^2} \sim \frac{g^4}{T^2}\ln\frac{1}{g} \Rightarrow$$

$$\hat{q}_{\rm pQCD} \sim \frac{\langle p_\perp^2 \rangle}{\lambda} \sim \alpha_s^2 N_c^2 T^3 \ln\frac{1}{g}.$$

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The calculation of $\hat{q}$ at strong coupling was done for $\mathcal{N} = 4$ SYM by H. Liu, K. Rajagopal, and U.A. Wiedemann, '06:

$$\hat{q}_{\rm SYM} = \frac{\pi^{3/2}\Gamma(3/4)}{\Gamma(5/4)}\sqrt{\lambda_{\rm 't\,Hooft}}\,T^3$$

in the large-N_c and large-$\lambda_{\rm 't\,Hooft}$ limits.

Introduction

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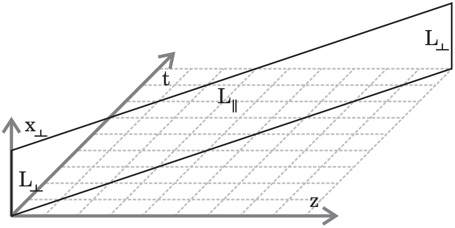
A motivation for the present research: To calculate $\hat{q}$ in some model which, like real QCD, does have conformal anomaly $\Rightarrow$ the Stochastic Vacuum Model.

Introduction

The jet quenching parameter $\hat{q}$ can be obtained from the expectation value of the Wilson loop in Minkowski space-time:

$$\left\langle \mathrm{Re}\, W^{\mathrm{Mink}}_{L_\parallel \times L_\perp} \right\rangle = \exp\left(-\frac{\hat{q}}{4\sqrt{2}} L_\parallel L_\perp^2 \right).$$



A counterintuitive property is the exponential fall-off instead of $\mathrm{e}^{i(\ldots)}$.

The exponential fall-off is a consequence of Landau damping of soft modes in the plasma.

The local formulation of the SVM and the light-cone Wilson loop

In the Euclidean space-time at $T = 0$

$$\left\langle W(C) \right\rangle \simeq \mathrm{tr}\ \exp\left[-\frac{1}{2!}\frac{g^2}{4}\int_{\Sigma(C)} d\sigma_{\mu\nu}(x)\int_{\Sigma(C)} d\sigma_{\lambda\rho}(x')\times \right.$$

$$\left. \times \left\langle F_{\mu\nu}(x)\Phi_{xx'}F_{\lambda\rho}(x')\Phi_{x'x}\right\rangle \right].$$

In the Fock-Schwinger gauge, where $\Phi_{x'x} = \hat{1}_R$, the SVM suggests the parametrization

$$\left\langle F^a_{\mu\nu}(x)F^b_{\lambda\rho}(x')\right\rangle = \delta^{ab}\frac{\left\langle F^2\right\rangle}{12(N_c^2-1)}\left\{\kappa(\delta_{\mu\lambda}\delta_{\nu\rho} - \delta_{\mu\rho}\delta_{\nu\lambda})D(u^2) + \right.$$

$$\left. + \frac{1-\kappa}{2}\left[\partial_\mu\left(u_\lambda\delta_{\nu\rho} - u_\rho\delta_{\nu\lambda}\right) + \partial_\nu\left(u_\rho\delta_{\mu\lambda} - u_\lambda\delta_{\mu\rho}\right)\right]D_1(u^2)\right\},$$

where $u = x - x'$, $D(0) = D_1(0) = 1$.

The local formulation of the SVM and the light-cone Wilson loop

High-energy scattering data yield $\kappa = 0.74$. We will disregard the function D_1 altogether, by fixing $\kappa = 1$.

Lattice simulations (Pisa lattice group, '92-'03) yield

$$D(u^2) = \mathrm{e}^{-\mu|u|},$$

where $\mu = 894\,\mathrm{MeV}$ is the inverse vacuum correlation length.

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$$\left\langle W(C) \right\rangle \simeq$$

$$\simeq \exp\left[-\frac{C_R}{48(N_c^2-1)} \cdot g^2 \left\langle F^2 \right\rangle \int d^4x \int d^4y \Sigma_{\mu\nu}(x) \mathrm{e}^{-\mu|x-y|} \Sigma_{\mu\nu}(y)\right],$$

where $\Sigma_{\mu\nu}(x) = \int_{\Sigma(C)} d\sigma_{\mu\nu}(w(\xi))\delta(x - w(\xi))$.

The local formulation of the SVM and the light-cone Wilson loop

For $C = R \times \mathcal{T}$ with $\mathcal{T} \gg R \Rightarrow$ an area law with

$$\sigma_{\rm fund} = \frac{\pi C_F}{12(N_c^2-1)} \frac{g^2 \left\langle F^2 \right\rangle}{\mu^2}.$$

For $N_c = 3$, $\sigma_{\rm fund} = (440\,{\rm MeV})^2 \Rightarrow$

$$g^2 \left\langle F^2 \right\rangle \simeq 3.55\,{\rm GeV}^4.$$

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The SVM suggests a representation of $\left\langle W(C) \right\rangle$ in terms of an effective local field theory of the field strength tensor $F^a_{\mu\nu}$:

$$\left\langle W(C) \right\rangle = {\rm tr} \int \mathcal{D}F^a_{\mu\nu} {\rm e}^{-S_{\rm Eucl}[F]}$$

with the action

$$S_{\rm Eucl}[F] = \frac{1}{2} \int d^4x \left[F^a_{\mu\nu} \mathcal{K}(x) F^a_{\mu\nu} + iF^a_{\mu\nu} t^a \Sigma_{\mu\nu} \right].$$

The local formulation of the SVM and the light-cone Wilson loop

$$\mathcal{K}^{-1}(x) = \frac{g^2 \left\langle F^2 \right\rangle}{6(N_c^2 - 1)} \mathrm{e}^{-\mu|x|} \Rightarrow \mathcal{K}(x) = \frac{N_c^2 - 1}{2\pi^2} \frac{\mu^4}{g^2 \left\langle F^2 \right\rangle} \left(1 - \frac{\partial^2}{\mu^2}\right)^{5/2}.$$

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At $T > 0$, the correlator $\left\langle F^a_{\mu\nu}(x) F^b_{\lambda\rho}(x') \right\rangle$ splits into

$$\left\langle E^a_i(x) E^b_k(x') \right\rangle, \quad \left\langle B^a_i(x) B^b_k(x') \right\rangle, \quad \left\langle E^a_i(x) B^b_k(x') \right\rangle.$$

The local formulation of the SVM and the light-cone Wilson loop

At $T > T_c = 270\,\mathrm{MeV}$:

- $\left\langle E_i^a(x) E_k^b(x') \right\rangle$ vanishes due to the deconfinement;
- $\left\langle E_i^a(x) B_k^b(x') \right\rangle \ll \left\langle B_i^a(x) B_k^b(x') \right\rangle$ (A. Di Giacomo et al., '97);
- $\left\langle B_i^a(x) B_k^b(x') \right\rangle = \frac{\langle F^2 \rangle}{12(N_c^2-1)} \delta^{ab} \delta_{ik} \mathrm{e}^{-\mu(T)|u|}$.

Due to the x_4-periodicity, the contour $C = L_{\|} \times L_{\perp}$ effectively splits into pieces (strips), whose extensions along the 3rd and the 4th axes are $\beta \equiv 1/T$.

The local formulation of the SVM and the light-cone Wilson loop

For the strip closest to the origin,

$$\left\langle W^{\mathrm{Eucl}}_{1-\mathrm{strip}} \right\rangle = \mathrm{tr} \int \mathcal{D}B_2^a \, \mathrm{e}^{-S_{\mathrm{Eucl}}[B]} =$$

$$= \exp\left[-\frac{C_R}{4} \int d^4x \int d^4y \Sigma_{13}(x) \mathcal{K}^{-1}(x-y) \Sigma_{13}(y)\right],$$

where

$$S_{\mathrm{Eucl}}[B] = \int d^4x \left(B_2^a \mathcal{K} B_2^a + i B_2^a t^a \Sigma_{13}\right),$$

$w_\mu(\xi_1, \xi_2) = \beta \xi_1 t_\mu + L_\perp \xi_2 r_\mu$, $t_\mu = (0,0,1,1)$, $r_\mu = (1,0,0,0)$, $\xi_{1,2} \in [0,1]$.

The local formulation of the SVM and the light-cone Wilson loop

Accounting for interactions between different strips

$$\chi_k = \frac{C_R}{4} \int d\sigma_{13}(w) \int d\sigma_{13}(w') \mathcal{K}^{-1}(w - w'),$$

where $w'_\mu = w_\mu(\xi'_1, \xi'_2) + (0, 0, \beta k, 0)_\mu$

$\Rightarrow$ the overall contribution to the Wilson-loop average:

$$-\ln \left\langle W^{\text{Eucl}}_{L_\parallel \times L_\perp} \right\rangle = \sum_{i=0}^{n-1} \sum_{k=0}^{i} \chi_k = \sum_{k=0}^{n-1} (n-k)\chi_k,$$

where

$$n \equiv k_{\max} = \frac{L_\parallel}{\beta\sqrt{2}}$$

is the full number of strips.

Evaluation of $\hat{q}$ through the Landau damping

In Minkowski space-time

$$\left\langle \mathrm{Re}\, W^{\mathrm{Mink}}_{1-\mathrm{strip}} \right\rangle = \mathrm{tr}\,\mathrm{Re}\int \mathcal{D}B_2^a\, \mathrm{e}^{-S_{\mathrm{Mink}}[B]} =$$

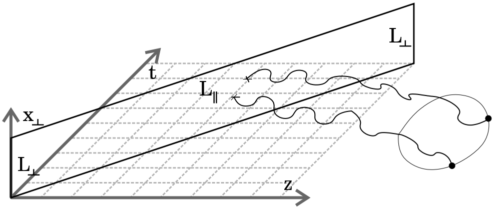
$$= \mathrm{Re}\, \exp\left[i\frac{C_R}{4}\int d^4x \int d^4y \Sigma_{13}(x)\mathcal{K}^{-1}(x-y)\Sigma_{13}(y)\right],$$

where

$$S_{\mathrm{Mink}}[B] = i\int d^4x\, (B_2^a \mathcal{K} B_2^a + B_2^a t^a \Sigma_{13})\,.$$

The exponential fall-off only appears when we account for the scattering partners in the medium, which are on-shell thermal gluons ⇒ these thermal gluons polarize the stochastic vacuum of soft background gluons.

Evaluation of $\hat{q}$ through the Landau damping



$$S_{\rm Mink}[B] \to i \int d^4x \left\{ B_2^a \left[\mathcal{K}(x) - i\mathcal{P}(x) \right] B_2^a + B_2^a t^a \Sigma_{13} \right\},$$

where

$$\mathcal{P}(p) = -\frac{M^2(T)}{p^2}, \quad M^2(T) \equiv \frac{\pi\zeta(1-\zeta^2)}{4} m_D^2(T),$$

$$\zeta \equiv \frac{\omega}{|\mathbf{p}|}, \quad m_D(T) = g(T)T\sqrt{N_c/3}$$

(cf. V.V. Klimov, '81, H.A. Weldon, '82).

Evaluation of $\hat{q}$ through the Landau damping

$$\left\langle \mathrm{Re}\, W^{\mathrm{Mink}}_{1-\mathrm{strip}} \right\rangle \equiv \mathrm{e}^{-\chi_0} = \exp\left[-\frac{C_R M^2(T)}{4} \int d\sigma_{13}(w) \int d\sigma_{13}(w') \times \right.$$

$$\left. \times \int \frac{d^4p}{(2\pi)^4} \mathrm{e}^{ip(w-w')} \frac{p^2}{p^4 \mathcal{K}^2(p) + M^4(T)} \right].$$

$$\hat{q} = -\frac{4\sqrt{2}}{L_\parallel L_\perp^2} \ln \left\langle \mathrm{Re}\, W^{\mathrm{Mink}}_{L_\parallel \times L_\perp} \right\rangle = \frac{4}{\beta L_\perp^2} \frac{1}{n} \sum_{k=0}^{n-1} (n-k)\chi_k,$$

where χ_k describes correlations between the strips separated from each other by the distance βk.

Evaluation of $\hat{q}$ through the Landau damping

The final result before the numerical evaluation:

$$\hat{q} = \frac{C_R \mathcal{N}(T) M^2(T)}{4\pi^2 n} \times$$

$$\times \sum_{k=0}^{n-1} (n-k) \int_{-1}^{1} \frac{dx}{\sqrt{k^2+2kx+2x^2}} \int_0^\infty \frac{dp p^4 J_1\left(p\beta\sqrt{k^2+2kx+2x^2}\right)}{p^4(p^2+\mu^2(T))^5 + \mathcal{N}(T) M^4(T)},$$

where

$$\mathcal{N}(T) \equiv \left(\frac{2\pi^2 \mu(T) g^2 \left\langle (F_{\mu\nu}^a)^2 \right\rangle_T}{N_c^2 - 1}\right)^2.$$

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where

$$\mathcal{N}(T) \equiv \left(\frac{2\pi^2 \mu(T) g^2 \left\langle (F_{\mu\nu}^a)^2 \right\rangle_T}{N_c^2 - 1}\right)^2.$$

$T_{\mathrm{d.r.}} \simeq 2T_c$ is the temperature of dimensional reduction (F. Karsch et al., '93-'96):

$$g^2 \left\langle (F_{\mu\nu}^a)^2 \right\rangle_T \propto T^4, \quad \mu(T) \propto T \ \text{ at } \ T > T_{\mathrm{d.r.}}.$$

Evaluation of $\hat{q}$ through the Landau damping

Two choices of parameters:

I. Inspired by the low-temperature lattice data (A. Di Giacomo et al.): $\mu(T) = 0.894\,\mathrm{MeV}$ until $T_{\mathrm{d.r.}}$;

$$g^2 \left\langle (F^a_{\mu\nu})^2 \right\rangle_T = g^2 \left\langle (F^a_{\mu\nu})^2 \right\rangle \coth\left(\frac{\mu}{2T}\right)$$

is also nearly constant up to $T_{\mathrm{d.r.}}$ (N.O. Agasian, '03); $g(m_D, T) = 2.5$ (H.-J. Pirner et al., '07).

II. Inspired by the high-temperature lattice data (F. Karsch et al.): Perturbative 1-loop $g(T) = \left(\frac{11}{8\pi^2} \ln \frac{T}{\Lambda}\right)^{-1/2}$, where $\Lambda \simeq 0.104\,T_c$; $\mu(T) = 1.04 g^2(T) T$; $\mu(T_c) = 0.894\,\mathrm{MeV}$;

$$g^2 \left\langle (F^a_{\mu\nu})^2 \right\rangle_T = \frac{72}{\pi} \mu^2(T) \sigma(T),$$

where $\sigma(T) = [c g^2(T) T]^2$ with $c = 0.566$.

Evaluation of $\hat{q}$ through the Landau damping

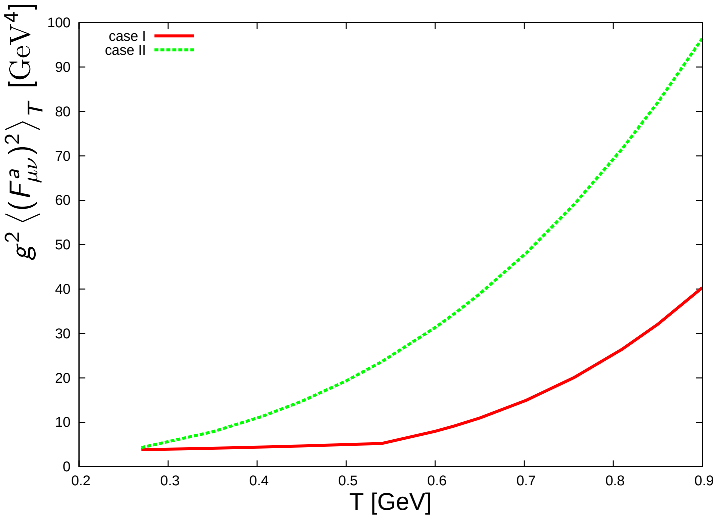

Figure: The chromo-magnetic condensate as a function of temperature at $T_c \leq T \leq 900\,\mathrm{MeV}$ in cases I and II.

Evaluation of $\hat{q}$ through the Landau damping

Calculating $\hat{q}$ numerically for both cases I and II at $n = 10 \gg 1$ and $\zeta = 0.1 \ll 1$ and subtracting $\hat{q}(T_c)$, since in the hadronic phase $\hat{q} \sim 0.01\,\mathrm{GeV}^2/\mathrm{fm}$ only $\Rightarrow$ both calculations of $\hat{q}$ follow a temperature dependence $\propto T^3$:

$$\hat{q}(T)^{\mathrm{fit}}_{\mathrm{case\,I}} = 0.16(T/T_c)^3\,\mathrm{GeV}^2/\mathrm{fm}, \quad \hat{q}(T)^{\mathrm{fit}}_{\mathrm{case\,II}} = 0.26(T/T_c)^3\,\mathrm{GeV}^2/\mathrm{fm}.$$

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One can prove that:

- indeed

$$\hat{q} \propto T^3 \quad \mathrm{at} \quad T \gtrsim T_{\rm d.r.}.$$

- an increase of the number of strips leads to a small increase of $\hat{q}(T)$:

$$\hat{q}(900\,\mathrm{MeV})^{(n=10)}_{\rm case\,I} = 1.26\,\mathrm{GeV}^2/\mathrm{fm}, \quad \hat{q}(900\,\mathrm{MeV})^{(n=50)}_{\rm case\,I} = 1.39\,\mathrm{GeV}^2/\mathrm{fm};$$

$$\hat{q}(900\,\mathrm{MeV})^{(n=10)}_{\rm case\,II} = 1.78\,\mathrm{GeV}^2/\mathrm{fm}, \quad \hat{q}(900\,\mathrm{MeV})^{(n=50)}_{\rm case\,II} = 1.98\,\mathrm{GeV}^2/\mathrm{fm}.$$

Evaluation of $\hat{q}$ through the Landau damping

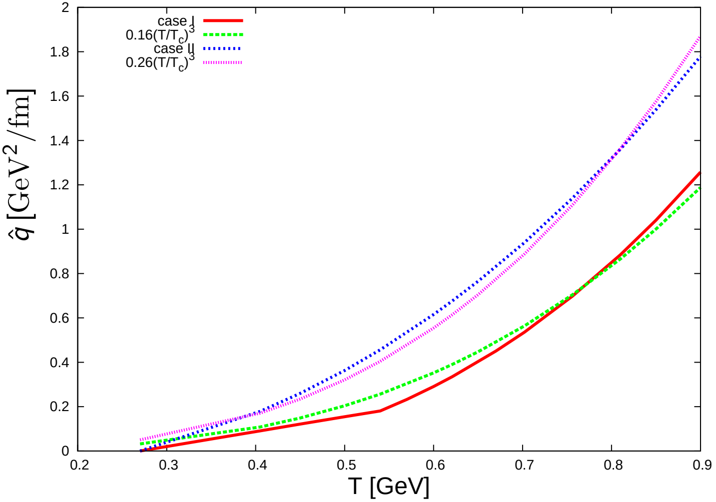

Figure: The jet quenching parameter $\hat{q}(T)$ in cases I and II, and the fitting curves $\sim T^3$ in both cases.

Summary

- The jet quenching parameter $\hat{q}$ is evaluated in SU(3) YM theory in the leading approximation $\propto (g^2 \left\langle (F^a_{\mu\nu})^2 \right\rangle_T)^2$. We have used a two-component model of the gluon plasma: the chromo-magnetic condensate describes the soft component of the plasma ("epoxy", G.E. Brown et al.); the hard thermal loop effective theory describes the hard component of the plasma (with $|p| \gtrsim \mu(T)$). Jet quenching originates from Landau damping of soft gluons by the on-shell hard thermal gluons.

- Numerically, our results are somewhat larger than in pQCD, $\hat{q}_{\rm pQCD} = 1.1 \div 1.4\,{\rm GeV}^2/{\rm fm}$ and closer to the nonperturbative result $\hat{q}_{\rm np} = 1.0 \div 1.9\,{\rm GeV}^2/{\rm fm}$ (B. Müller et al., '07).
However, $\hat{q}_{\rm np} \propto (g^2 \left\langle (F^a_{\mu\nu})^2 \right\rangle_T)^1$ and not $(g^2 \left\langle (F^a_{\mu\nu})^2 \right\rangle_T)^2$ as the total cross section.