Pion-Photon Transition Distribution Amplitudes

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Aurore Courtoy

Departamento de Física Teórica
Universidad de Valencia

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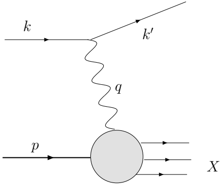
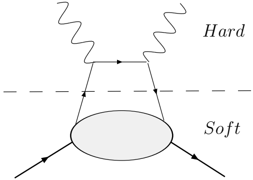
Deep Inelastic Scattering



⇓

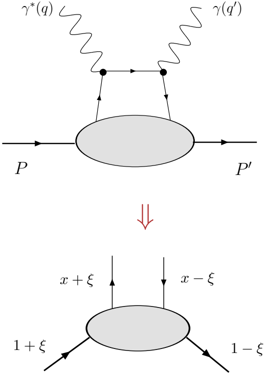


- Bjorken Limit: $x \equiv \frac{Q^2}{2p.q}$
- $d\sigma \propto l^{\mu\nu} W_{\mu\nu}$
- $W_{\mu\nu} = ?$

- Factorization
- $d\sigma = \Sigma_q \int dx f_q(x)\, d\sigma^{\text{parton}} \left(\frac{y}{x}\right)$
- $f_q(x)$
  $\Rightarrow$ Parton Distribution Functions
- Nonperturbative Objects

Deeply Virtual Compton Scattering

[Ji, Phys. Rev. Lett. **78** (1997) 610] & [Radyushkin, Phys. Lett. B **380** (1996) 417]



- Nondiagonal in momentum

  (Small) Momentum Transfer $t = (P' - P)^2$

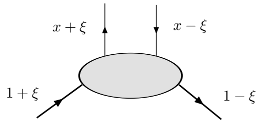
  Longitudinal Momentum asymmetry

  $\Rightarrow$ Skewness variable $\xi = \frac{(P-P')^+}{(P+P')^+}$

- Generalized Parton Distributions

  $\Rightarrow f(x, \xi, t)$

Properties of GPDs



Generalized Parton Distributions

$\Rightarrow f(x,\xi,t)$

- Sum Rule

  First moment of GPDs $\Rightarrow$ Form Factor

  $\int_{-1}^{1} dx\, H(x,\xi,t) = F_1(t)$

- Polynomiality

  Higher moments of GPDs $\Rightarrow$ Polynomials in ξ

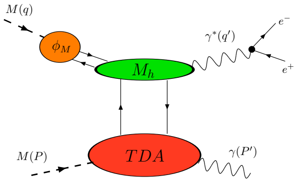
  $\int_{-1}^{1} dx x^{n-1} H(x,\xi,t) = \sum_{i=0}^{[\frac{n-1}{2}]} A_{n,2i}(t)(-2\xi)^{2i} + \mathrm{Mod}(n+1,2)\; C_n(t)(-2\xi)^n$

Hadron Structure

1. Deep Inelastic Scattering
   - ⇒Parton Distribution Functions
   - ⇒ Diagonal in momenta and particle states.
2. Deeply Virtual Compton Scattering
   - ⇒ Generalized Parton Distributions
   - ⇒ Diagonal in particle states and Nondiagonal in momenta.
3. Hadron-Photon Transitions
   - Nondiagonal in momenta AND Nondiagonal in particle states
   - New Distribution Functions

   ⇒ [B. Pire and L. Szymanowski, Phys. Rev. D **71** (2005) 111501]

TDA's Framework



- Meson Annihilation into 2 Photons

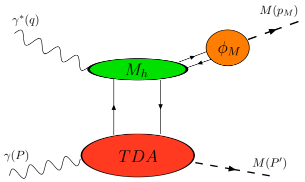
  $\bar{M}M \rightarrow \gamma^*\gamma$

- Backward VCS

  $\gamma^* M \rightarrow \gamma M$

Factorization

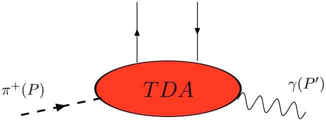
$$\mathcal{M}(Q^2, \xi, t) = \int dxdz\, \Phi_M(z)\ M_h(z, x, \xi)\ \mathrm{TDA}(x, \xi, t)$$



- Meson Pair Production

  $\gamma^*\gamma \rightarrow \bar{M}M$

π-γ Transition



Soft part

$\Rightarrow$ Transition Distribution Amplitude

Mesonic case $\Rightarrow \pi^+ \to \gamma$ or $\gamma \to \pi^-$

Constraint on ξ: $\frac{t}{2m_\pi^2 - t} < \xi < 1$

Vector and Axial-vector currents

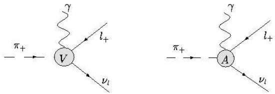
Lorentz Structure for the matrix element

Structure for $\pi^+ \to l^+ \nu_l \gamma$

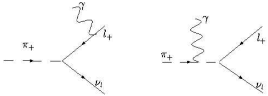
$\pi^+ \to l^+ \nu_l \gamma$

$$\mathcal{M}(\pi^+(P) \to l(s)\,\nu(r)\,\gamma(P')) = \mathcal{M}_{SD} + \mathcal{M}_{IB}$$

- $\mathcal{M}_{SD} \Rightarrow F_V(t)$ and $F_A(t)$



- $\mathcal{M}_{IB} \Rightarrow$ Inner Bremsstrahlung of Lepton and Pion



[M. Moreno, Phys. Rev. D **16** (1977) 720]

The lepton Bremsstrahlung does not contribute to the hadronic currents

- $$\mathcal{M} \propto l^\mu \Big\{ \langle \gamma(P')|\bar{q}(0)\gamma_\mu \tau^- q(0)|\pi^+(P)\rangle \; - \langle \gamma(P')|\bar{q}(0)\gamma_\mu \gamma_5 \tau^- q(0)|\pi^+(P)\rangle \Big\}$$
$$-\varepsilon^\nu \; l_{\mu\nu}\, i\sqrt{2}\, f_\pi \; P^\mu$$

Definition of the π-γ TDAs

- $\langle\gamma(P)|\bar{q}(0)\gamma_\mu\tau^- q(0)|\pi^+(P')\rangle = -ie\varepsilon^\nu\epsilon_{\mu\nu\rho\sigma}P'^\rho P^\sigma F_V(t)$
- $\langle\gamma(P)|\bar{q}(0)\gamma_\mu\gamma_5\tau^- q(0)|\pi^+(P')\rangle = e\varepsilon^\nu(P'_\mu P_\nu - g_{\mu\nu}P'.P)\ F_A(t)$
$$+\,e\varepsilon^\nu\left[(P'-P)_\mu P_\nu\frac{2\sqrt{2}f_\pi}{m_\pi^2-t}-\sqrt{2}f_\pi g_{\mu\nu}\right]$$
- From the hadronic currents to the parton distribution amplitudes

  Fourier Transform of Matrix element of operators at a light-like separation

  - Transition Form Factors $F_V(t)$ and $F_A(t)$ $\Rightarrow$ $V(x,\xi,t)$ and $A(x,\xi,t)$
  - f_π $\Rightarrow$ Pion Distribution Amplitude $\phi(x)$
- To leading twist

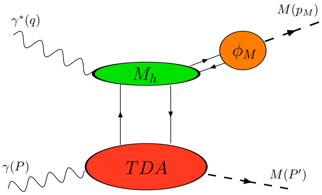
$$\int\frac{dz^-}{2\pi}e^{ixp^+z^-}\langle\gamma(P')|\bar{q}\left(-\frac{z}{2}\right)\gamma_\mu\,\tau^- q\left(\frac{z}{2}\right)|\pi^+(P)\rangle|_{z^+=z^\perp=0}=\frac{i}{p^+}e\varepsilon^\nu\epsilon_{\mu\nu\rho\sigma}p^\rho\Delta^\sigma\frac{V(x,\xi,t)}{\sqrt{2}f_\pi}$$

$$\int\frac{dz^-}{2\pi}e^{ixp^+z^-}\langle\gamma(P')|\bar{q}\left(-\frac{z}{2}\right)\gamma_\mu n^\mu\gamma_5\,\tau^- q\left(\frac{z}{2}\right)|\pi^+(P)\rangle\,|_{z^+=z^\perp=0}=\frac{1}{p^+}e\,(\vec{\varepsilon}^\perp.\vec{\Delta}^\perp)\,\frac{A(x,\xi,t)}{\sqrt{2}f_\pi}+\frac{1}{p^+}e\,(\varepsilon.\Delta)\sqrt{2}f_\pi\,\frac{2}{m_\pi^2-t}\,\phi\left(\frac{x+\xi}{2\xi}\right)$$

Back to Meson Pair Production

Cross Section Estimates

[Lansberg *et al.*, Phys. Rev. D **73** (2006) 074014]



$\rho_L^\pm \pi^\mp$ Production

- $e\gamma \to e\rho_L\pi \Rightarrow \gamma_L^*\gamma \to \rho_L^\pm\pi^\mp$
- Φ_{ρ_L}: vector DA
- Vector TDA enters the cross section

$\pi^\pm\pi^\mp$ Production

- $e\gamma \to e\pi\pi \Rightarrow \gamma_L^*\gamma \to \pi^\pm\pi^\mp$
- Φ_π: axial DA
- Axial TDA enters the cross section

Models

Distribution Functions as Functions of x, ξ, t

Experiments require Estimates for the cross sections

Nonperturbative Objects

Approaching QCD by Models, Effective theories,...

Pion $\rightarrow$ Chiral Models

Models

Distribution Functions as Functions of x, ξ, t

Experiments require Estimates for the cross sections

Nonperturbative Objects

Approaching QCD by Models, Effective theories,...

Pion $\longrightarrow$ Chiral Models

$\Rightarrow$ Nambu - Jona-Lasinio Model

Pion as Goldstone boson

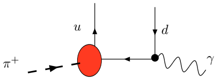
Pion as collective mode of fermions in the sense of Bethe-Salpeter

[S. P. Klevansky, Rev. Mod. Phys. **64** (1992) 649.]

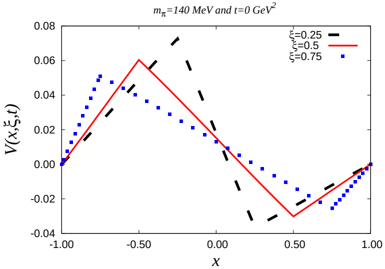
[A.C. & S. Noguera, Phys. Rev. D **76** (2007) 094026]

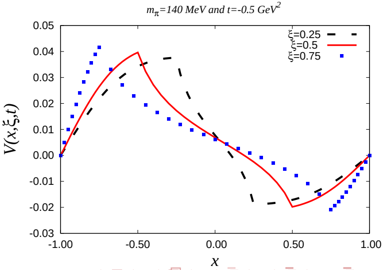
Vector TDA

$$\int \frac{dz^-}{2\pi} e^{ixp^+z^-} \langle \gamma(P')|\bar{q}\left(-\frac{z}{2}\right) \gamma_\mu \tau^- q\left(\frac{z}{2}\right)|\pi^+(P)\rangle|_{z^+=z^\perp=0} = \frac{i}{p^+} e\varepsilon^\nu \epsilon_{\mu\nu\rho\sigma} p^\rho \Delta^\sigma \frac{V^{\pi^+}(x,\xi,t)}{\sqrt{2}f_\pi}$$



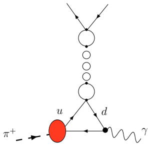
- Isospin Relations
  - $V^{\pi^+}(x,\xi,t) = Q_d\, v^{\pi^+}_{u\to d}(x,\xi,t) + Q_u\, v^{\pi^+}_{\bar{d}\to\bar{u}}(x,\xi,t)$
  - $v^{\pi^+}_{\bar{d}\to\bar{u}}(x,\xi,t) = v^{\pi^+}_{u\to d}(-x,\xi,t)$
- Even functions of ξ



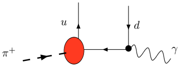


Axial TDA

$$\int \frac{dz^-}{2\pi} e^{ixp^+z^-} \langle\gamma(P')|\bar{q}\left(-\frac{z}{2}\right) \gamma_\mu n^\mu \gamma_5 \tau^- q\left(\frac{z}{2}\right)|\pi^+(P)\rangle\,|_{z^+=z^\perp=0} = \frac{1}{p^+} e\,(\vec{\varepsilon}^\perp.\vec{\Delta}^\perp)\,\frac{A^{\pi^+}(x,\xi,t)}{\sqrt{2}f_\pi} + \frac{1}{p^+} e\,(\varepsilon.\Delta)\,\sqrt{2}f_\pi\,\frac{2}{m_\pi^2-t}\,\phi\left(\frac{x+\xi}{2\xi}\right)$$



- Quark-antiquark with pion quantum numbers
  - Related to Pion DA
  - ⊃ Pion Pole

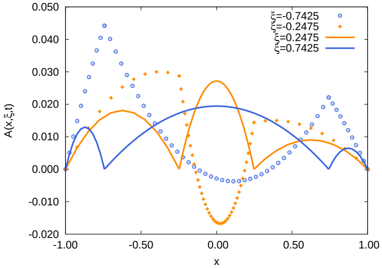


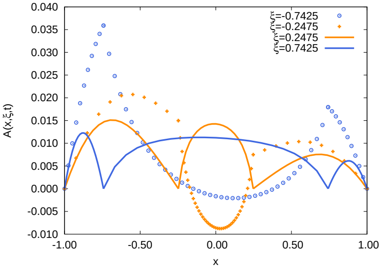
- Isospin Relations
  - $A^{\pi^+}(x,\xi,t) = Q_d\, a^{\pi^+}_{u\to d}(x,\xi,t) + Q_u\, a^{\pi^+}_{d\to\bar{u}}(x,\xi,t)$
  - $a^{\pi^+}_{d\to\bar{u}}(x,\xi,t) = -a^{\pi^+}_{u\to d}(-x,\xi,t)$

Axial TDA II

2 distinct behaviors according to the sign of ξ:

(a) $t = 0$ GeV^2 and (b) $t = -0.5$ GeV^2





Results

Vector TDA

Sum Rule

- $\int_{-1}^{1} dx\, V^{\pi^+}(x,\xi,t) = \frac{\sqrt{2}f_\pi}{m_\pi} F_V^{\pi^+}(t)$
- Our result: $F_V^{\pi^+}(0) = 0.0242$

In agreement with PDG: $F_V(0) = 0.017 \pm 0.008$

Polynomiality

$$\int_{-1}^{1} dx\, x^{n-1}\, V^{\pi^+}(x,\xi,t) = \sum_{i=0}^{n-1} C_{n,i}(t)\, \xi^i$$

- Numerically verified
- Chiral Limit $\Rightarrow i \to 2i$

Axial TDA

Sum Rule

- $\int_{-1}^{1} dx\, A^{\pi^+}(x,\xi,t) = \frac{\sqrt{2}f_\pi}{m_\pi} F_A^{\pi^+}(t)$
- Our result: $F_A^{\pi^+}(0) = 0.0239$

Twice the value of the PDG:

$F_A(0) = 0.0115 \pm 0.0005$

Polynomiality

$$\int_{-1}^{1} dx\, x^{n-1}\, A^{\pi^+}(x,\xi,t) = \sum_{i=0}^{n-1} C'_{n,i}(t)\, \xi^i$$

- Numerically verified

Conclusions

$V(x, \xi, t)$ and $A(x, \xi, t)$ (General Arguments)

- Introduction of an additional term $\longrightarrow$ Pion Pole contribution
- Hence $\longrightarrow$ Sum Rule recovered
- Isospin Relations :

$$V^{\pi^+}(x, \xi, t) = -\frac{1}{2}\, V^{\pi^+}(-x, \xi, t) \quad \& \quad A^{\pi^+}(x, \xi, t) = \frac{1}{2}\, A^{\pi^+}(-x, \xi, t)$$

for $|\xi| < x < 1$.

Conclusions

Calculation of the pion-photon TDAs in the NJL model

Covariant Bethe-Salpeter approach

GPDs' features extended to TDAs

- Support $[-1, 1] \longrightarrow$ OK
- Sum Rules recovered $\longrightarrow$ OK
- Polynomiality recovered $\longrightarrow$ OK
    - $V(x, \xi, t)$:
      Chiral Limit $\Rightarrow$ Polynomials $\supset$ even powers in ξ
    - $A(x, \xi, t)$:
      General Expression for the entire axial matrix element

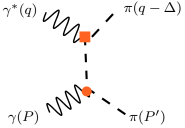
The end.

In progress . . .

Introduction of additional term $\longrightarrow$ Pion Pole contribution

$\Rightarrow$ Contribution to the cross section for meson pair production:

$$\mathcal{M}_{\pi\pi} = \mathcal{M}_{TDA} + \mathcal{M}_{\text{pion pole}}$$



Comparison I

[W. Broniowski and E. R. Arriola, arXiv:hep-ph/0701243]
& [B. C. Tiburzi, Phys. Rev. D **72** (2005) 094001]

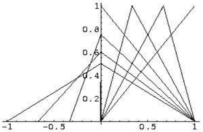
- Need for the pion pole contribution highlighted in both works
- Value of the axial form factor: twice the PDG value in both works
- Double Distributions: polynomiality recovered by definition
  This work: first study of the polynamiality property of TDAs

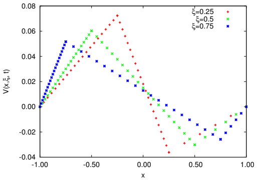
Comparison II

Shape of the TDAs: Discontinuity of the 1st derivative at $x = \pm\xi$ in both

- B& RA: similar to ours for the vectorial case

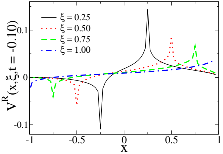
  So is the χ limit

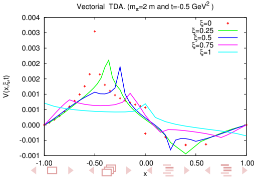




- T: rather peaked TDAs at $x = \pm\xi$

  Weak binding $\rightarrow$ ok peaks BUT not at $x = \pm\xi$





Outlook

- Cross section estimates
  - ⇒ Study of the pion pole contribution
- Evolution of TDAs
- $N\bar{N} \longrightarrow \gamma^\star \pi$

[Lansberg, Pire and Szymanowski, Phys. Rev. D **75** (2007) 074004]

Operators with 3 quark fields