Twistor techniques in QCD

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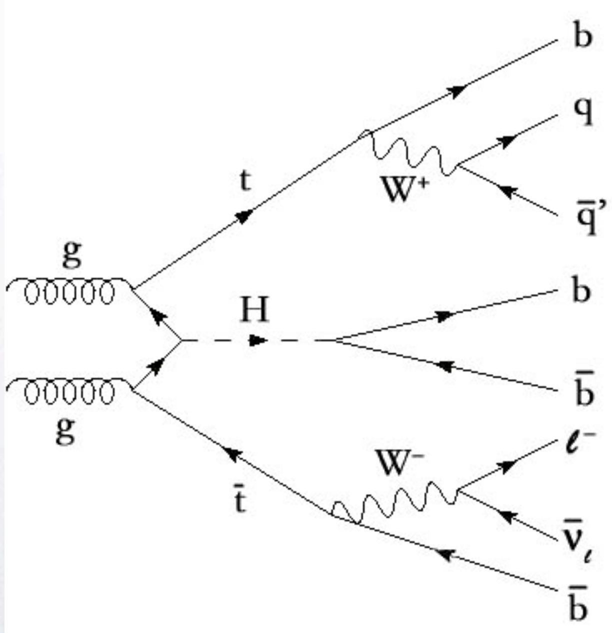
- Motivation
- Calculation of QCD amplitudes
  - Color decomposition
  - Spinor-helicity formalism
- Twistor techniques
  - Tree-level techniques
  - One-loop techniques
- Conclusion

Motivation

- Most of the events observed at the LHC will be QCD events.

- Many interesting signatures correspond to a multijet final state, e.g.

  - $t\,\bar{t}\,H \to \ell\nu + 6$ jets, **8 jets**
  - $p\,p \to \tilde{q}\,\tilde{q} \to (q\,\tilde{\chi}^0)\,(q\,\tilde{\chi}^0) \to (q\,cds)\,(q\,cds)$
    $p\,p \to$ **8 jets**



Motivation

- The background to the interesting LHC signatures is in very often given by high-multiplicity QCD events.
- Precision measurements require theory predictions beyond leading order.

$$\sigma = \underbrace{\alpha_s^n\,\sigma_0}_{\text{LO}} + \underbrace{\alpha_s^{n+1}\,\sigma_1}_{\text{NLO}} + \mathcal{O}(\alpha_s^{n+2})$$

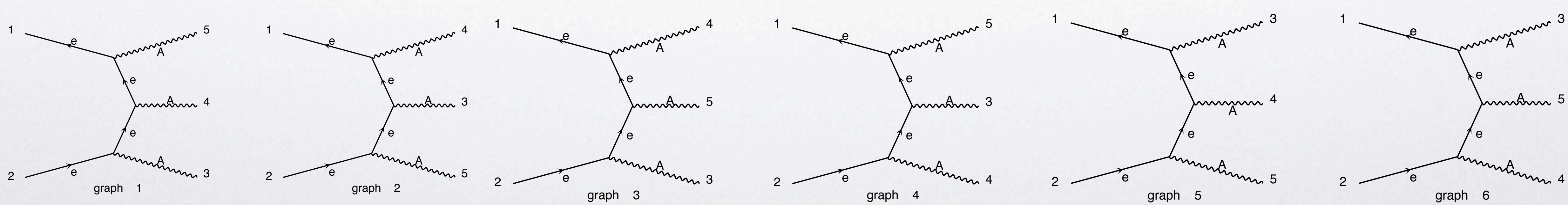
Feynman diagrams

- Standard technique for the calculation of amplitudes in quantum field theory

Feynman diagrams

- Example: $e^+ e^- \rightarrow \gamma \gamma \gamma$

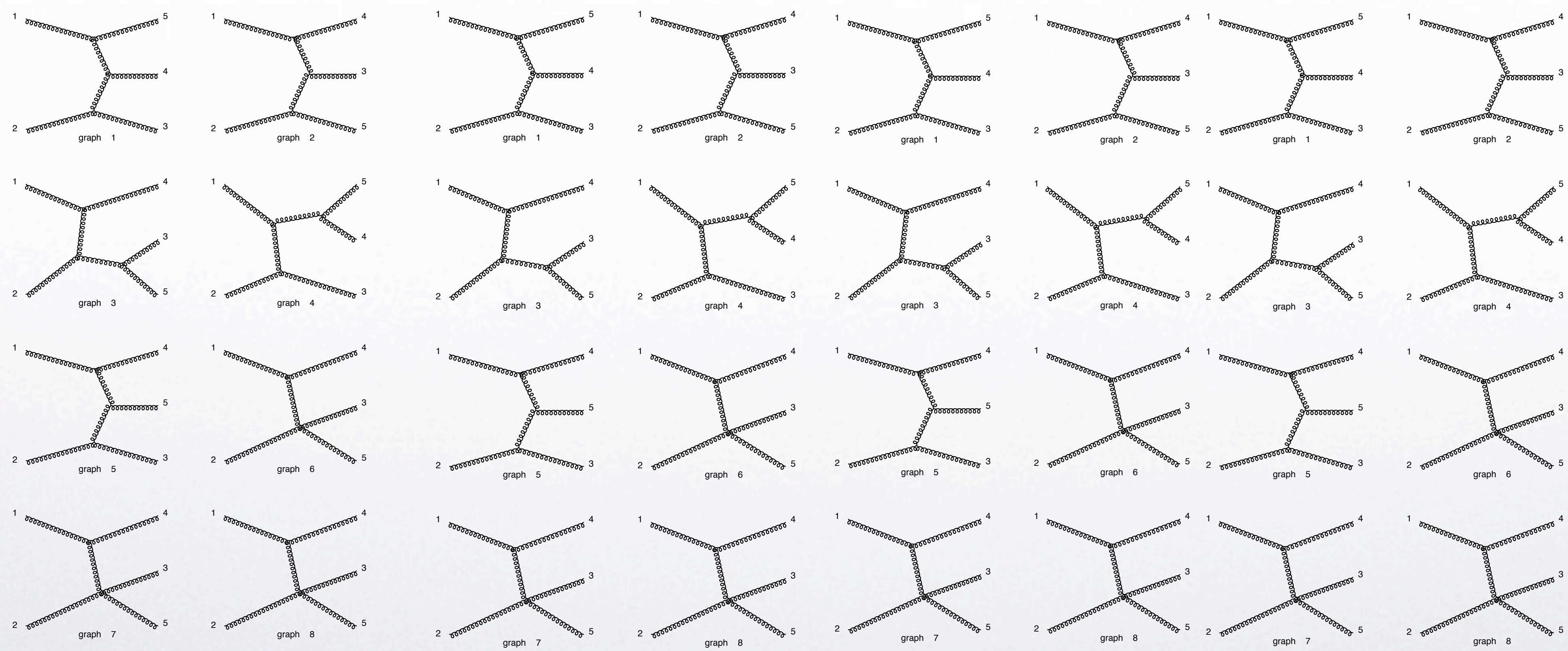
6 diagrams at tree level.



Feynman diagrams

- Example of a QCD process $g\,g \rightarrow g\,g\,g$

  45 Feynman diagrams at tree level.



...

Feynman diagrams

- The number of Feynman diagrams is growing extremely fast with the number of external particles.
  - 6 external gluons: 510
  - 7 external gluons: 5040
  - 8 external gluons: 40320
  - …
- The reason for this fast growth in complexity is color...

Color decomposition

- Solution: Factor out the color information!

$$\mathcal{A}_n^{\text{tree}}(\{k_i,\lambda_i,a_i\}) = g^{n-2} \sum_{\sigma\in S_n/Z_n} \mathrm{Tr}\left(T^{a_{\sigma(1)}}\cdots T^{a_{\sigma(n)}}\right)\, A_n^{\text{tree}}(\sigma(1^{\lambda_1}),\ldots,\sigma(n^{\lambda_n}))$$

- The color-ordered amplitudes
  - correspond to a specific color-ordering.
  - depend only on the momenta and helicities of the gluons. (but not on color!)
- This construction can be easily extended to quarks as well as beyond tree-level.

Spinor-helicity formalism

- As the color-ordered amplitudes depend only on the momenta and helicities, they can be easily calculated using **spinor-helicity formalism**.
- Example: The color-ordered amplitude $g\,g \to g\,g\,g$ for a specific helicity assignment:

$$A_5^{tree}(1^-, 2^-, 3^+, 4^+, 5^+) = \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 51 \rangle}$$

where $\langle ij \rangle \sim \sqrt{(p_i + p_j)^2}$.

Spinor-helicity formalism

- The color-ordered amplitudes can be calculated via:
  - Color-ordered Feynman rules.
  - Recursive techniques (Berends-Giele recursion)
  - The recently introduced twistor techniques...

Twistors in QCD

- In 1967, R. Penrose introduces a new mathematical object called a twistor.
- In 2003, E. Witten conjectures a duality between QCD and a certain type of string theory based on Pensore's twistor space. [hep-th/0312171]
- Main message:

„Be complex, but stay on-shell!“

Twistors in QCD

- Tree-level:
  - BCFW recursion
  - CSW formalism
- One-loop:
  - Generalized unitarity

BCFW recursion

- First conjectured by Britto, Cachazo and Feng, proven analytically by Witten. [hep-th/0412308, hep-th/0501052]

- Main idea: QCD amplitudes can be build recursively from smaller, on-shell amplitudes.

$$A_n(1^+, 2, \ldots, n^-) = \sum_{k=2}^{n-2} A_{k+1}\left(\hat{1}, 2, \ldots, k, -\hat{P}_{1,k}^{-h}\right) \frac{1}{P_{1,k}^2} A_{n-k+1}\left(\hat{P}_{1,k}^{h}, k+1, \ldots, \hat{n}\right)$$

BCFW recursion

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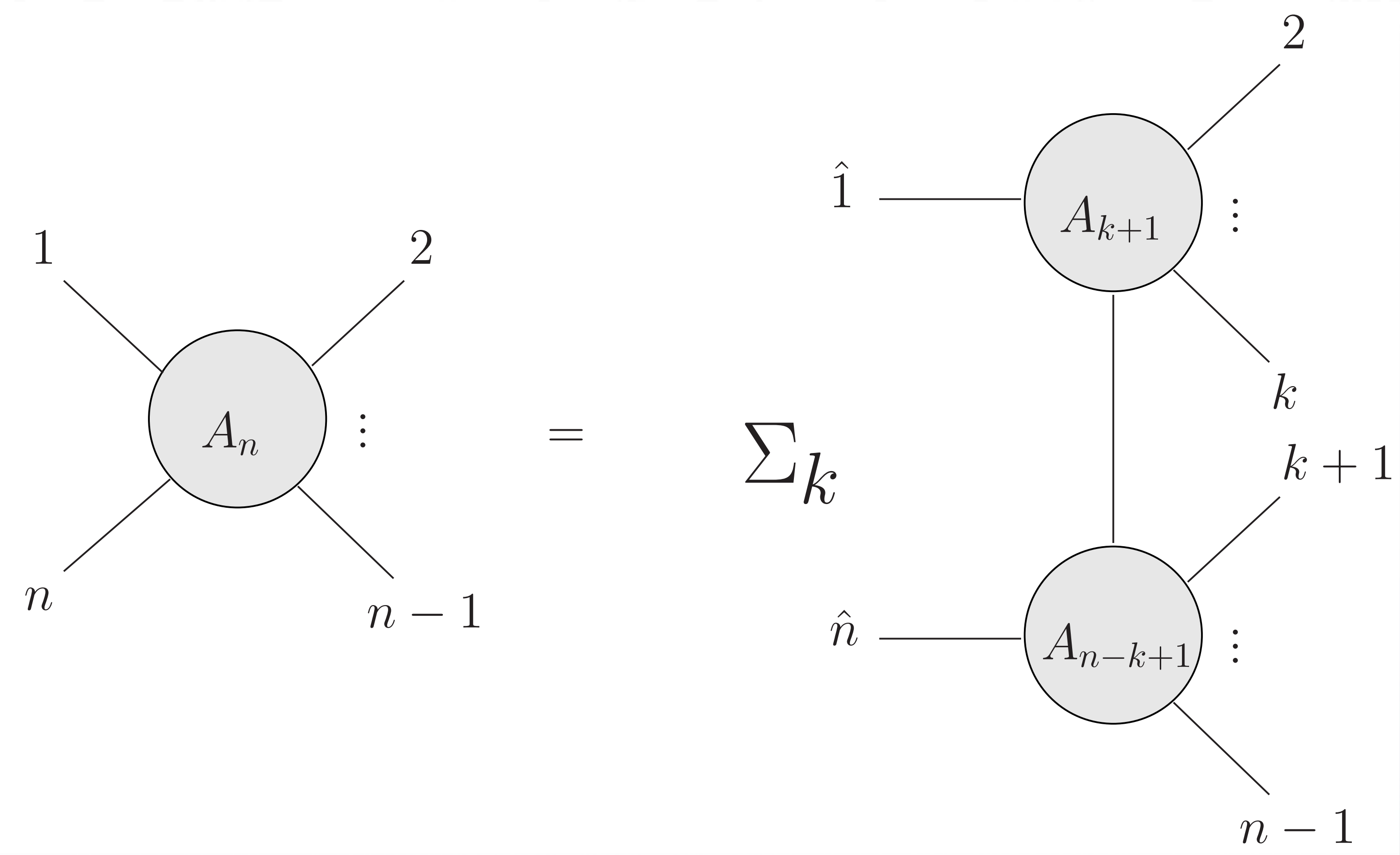

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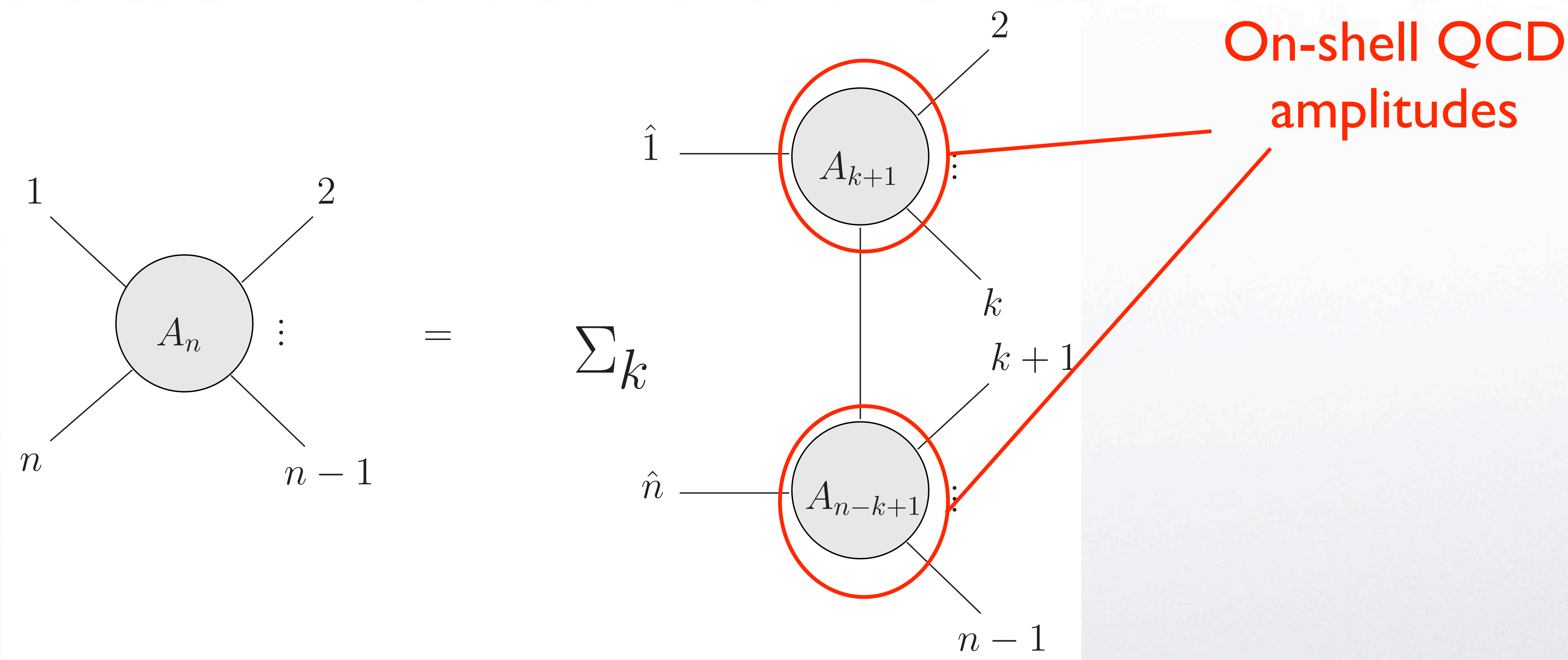

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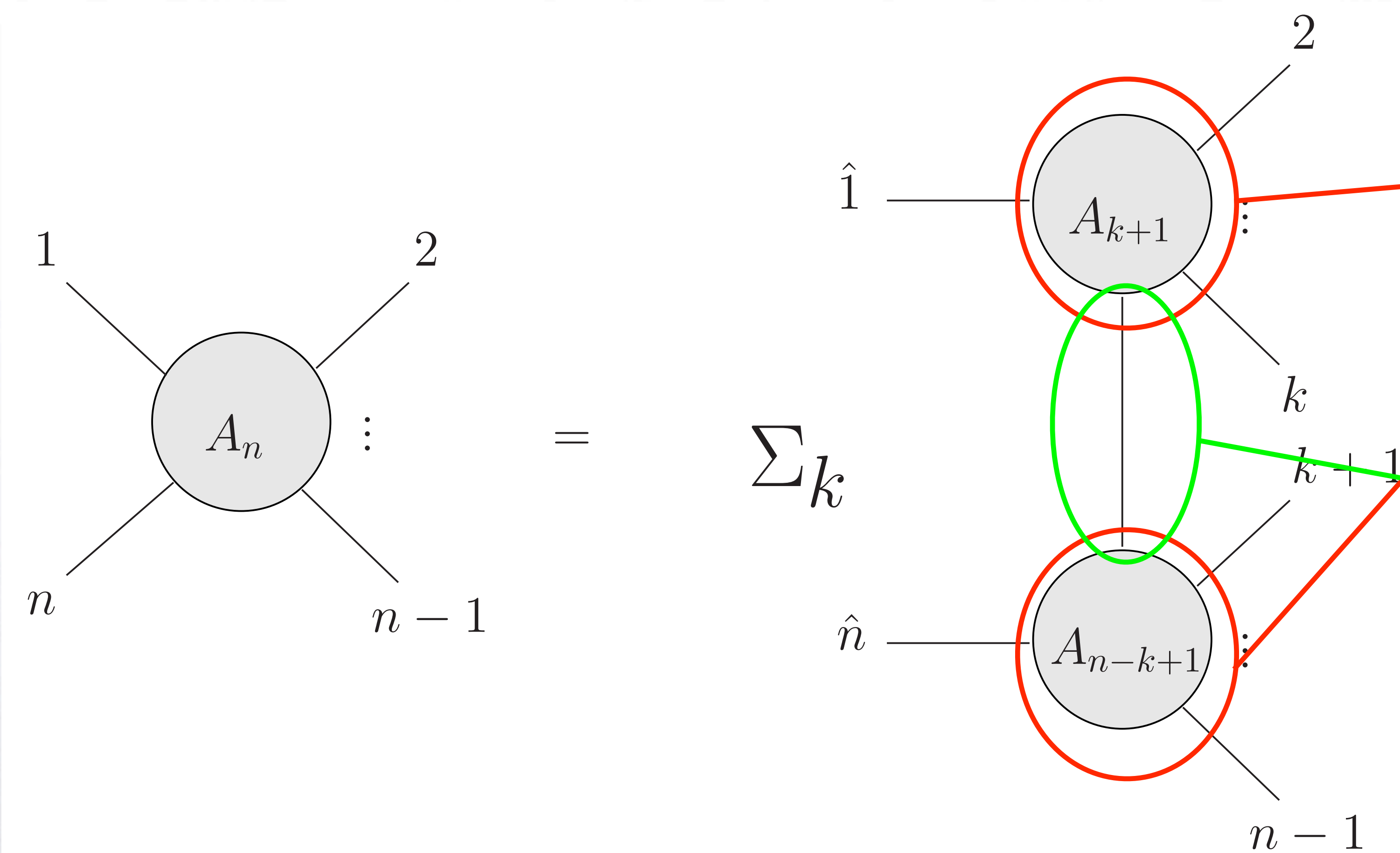


On-shell QCD amplitudes

Complex on-shell momentum

BCFW recursion

- Can be generalized to quarks. [Schwinn, Weinzierl, hep-ph/0703021]
- Does not only hold at the level of the color-ordered amplitudes, but can be generalized to the full color-dressed QCD amplitudes. [CD, Hoesche, Maltoni, hep-ph/0607057]
- Holds also for QED amplitudes [Ozeren, Stirling, hep-th/0509063]
- Very compact results for specific helicity configurations valid for an arbitrary number of gluons.

BCFW recursion

- Example of a 7 gluon amplitude from the BCFW recursion

$$A(1^-,2^-,3^-,4^+,5^+,6^+,7^+) =$$

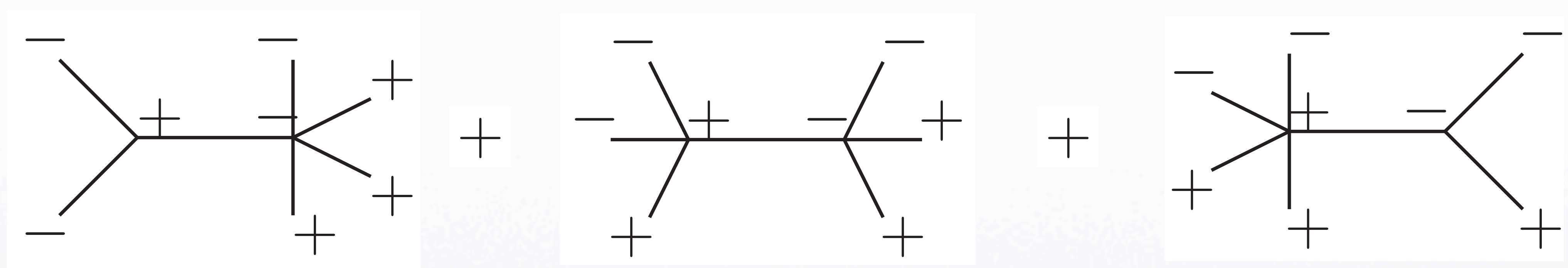
$$\frac{\langle 1|2+3|4]^3}{t_2^{[3]}\langle 5\ 6\rangle\langle 6\ 7\rangle\langle 7\ 1\rangle[2\ 3][3\ 4]\langle 5|4+3|2]}$$

$$-\frac{1}{\langle 3\ 4\rangle\langle 4\ 5\rangle\langle 6|7+1|2]}\left(\frac{\langle 3|(4+5)(6+7)|1\rangle^3}{t_3^{[3]}t_6^{[3]}\langle 6\ 7\rangle\langle 7\ 1\rangle\langle 5|4+3|2]}+\frac{\langle 3|2+1|7]^3}{t_7^{[3]}\langle 6\ 5\rangle[7\ 1][1\ 2]}\right)$$

CSW formalism

- Any tree-level amplitude can be written as a sum of **CSW diagrams** [Cachazo, Svrcek, Witten, hep-th/0403042]

$$A_6(1^-, 2^-, 3^-, 4^+, 5^+, 6^+) =$$



- 3 CSW diagrams vs. 510 Feynman diagrams.

CSW formalism

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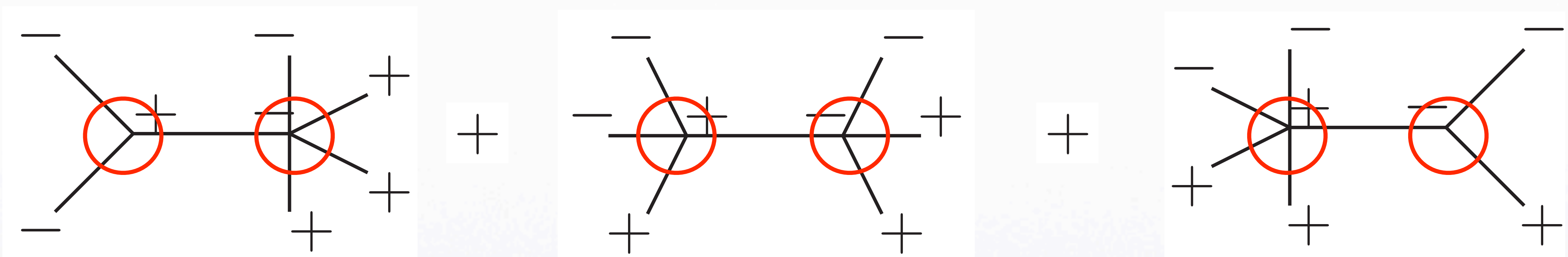

- The vertices are on-shell QCD amplitudes (for a specific helicity assignment) of the form

$$\frac{\langle 1P \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 4P \rangle \langle P1 \rangle}$$

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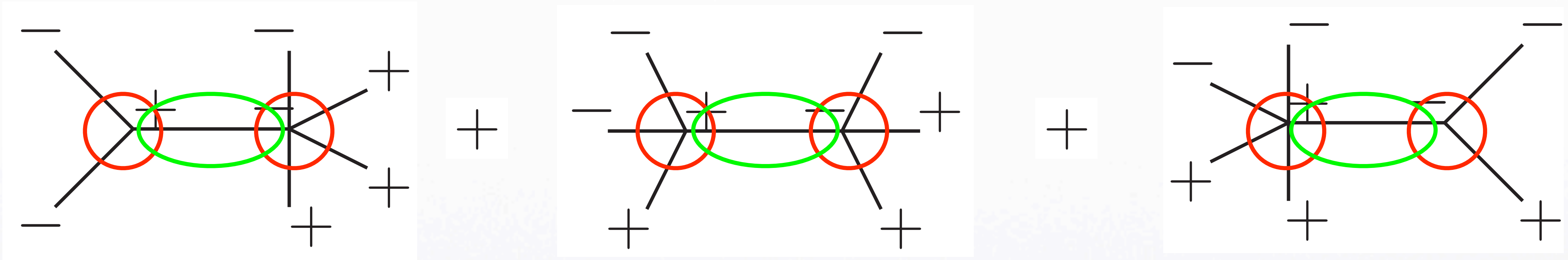

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$$\frac{\langle 1P\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 4P\rangle\langle P1\rangle}$$

Complex on-shell momentum

CSW formalism

- Can be generalized to quarks, and scalars.

[Georgiou, Khoze, hep-th/0404072;
Badger, Glover, Khoze, hep-th/0412275]

- Very easy diagrammatic technique, giving very compact results.

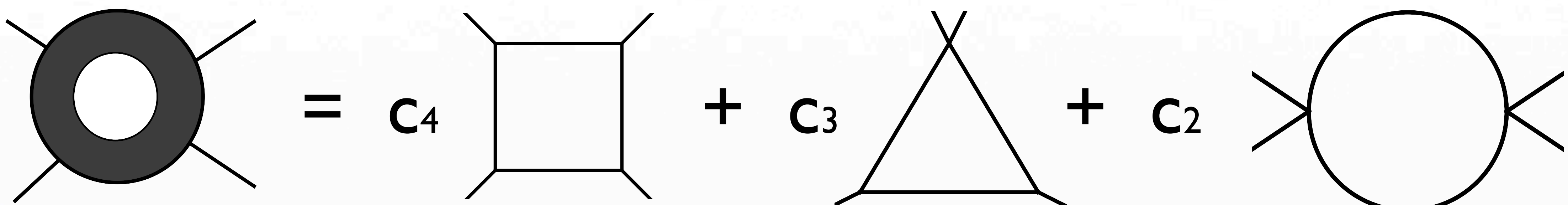
- Suitable not only for the calculation of amplitudes, but also for the calculation of
  - splitting functions
  - antenna functions

[Birthwright, Glover, Khoze, Marquard, hep-ph/0503063]

[CD, Maltoni]

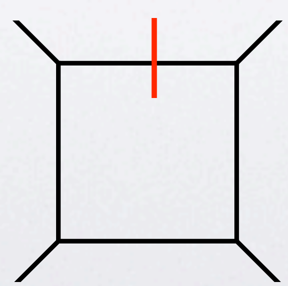
Generalized unitarity

- All one-loop QCD amplitudes can be reduced to a sum of boxes, triangles and bubbles:



we just need to calculate these coefficients!

- If we cut one line (*i.e.* put on-shell one loop propagator), unitarity tells us how the amplitude behaves:



[Bern, Dixon, Dunbar, Kosower, hep-ph/0943226]

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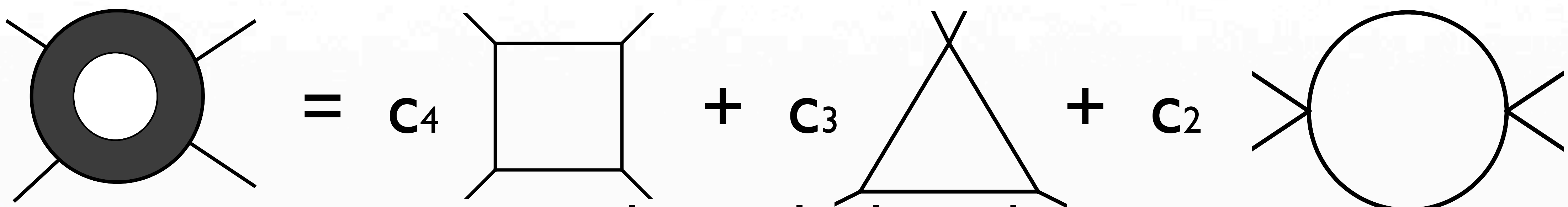
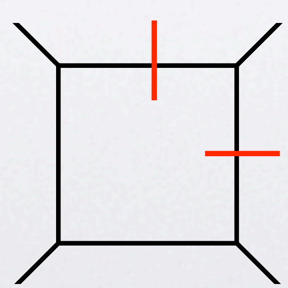


we just need to calculate these coefficients!

- If we cut a second line... the result is zero!

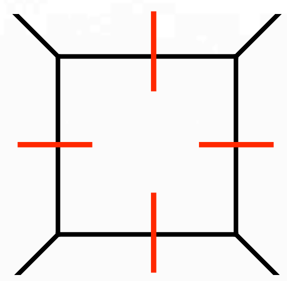


= 0 !

But this assumes the momenta to be real...

Generalized unitarity

- In complex momenta, more than one unitarity cut is allowed:



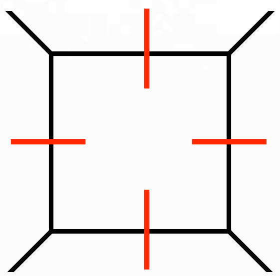
$\neq 0!$

Generalized unitarity

- In complex momenta, more than one unitarity cut is allowed:

[Britto, Cachazo, Feng, hep-th/0412103;
Anastasiou, Britto, Feng, Kunzt, Mastrolia, hep-ph/0607011;
Ossola, Papadopulos, Pittau, hep-ph/0802.1876]

c_4



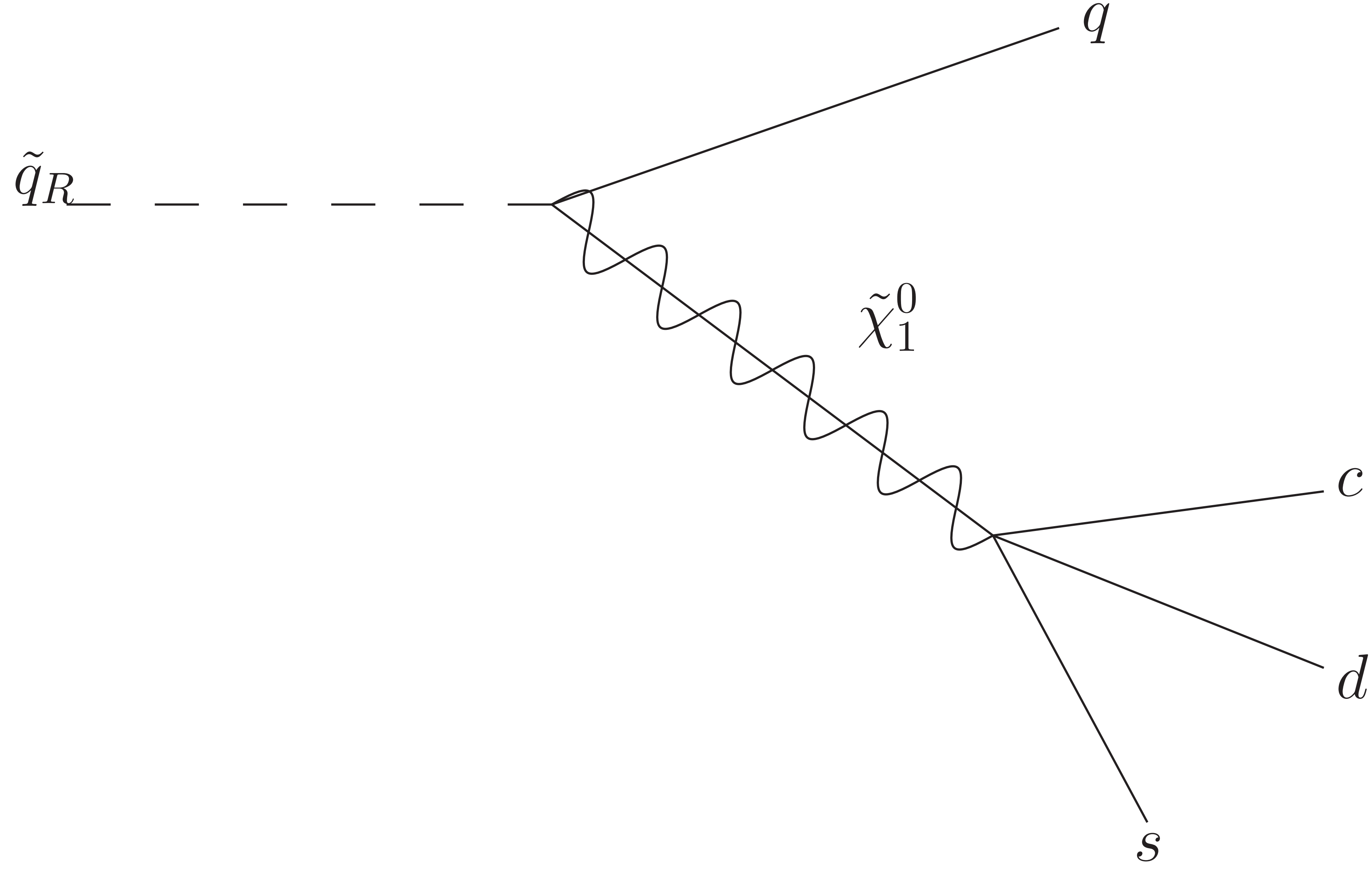
$$= \mathcal{A}^{tree}\mathcal{A}^{tree}\mathcal{A}^{tree}\mathcal{A}^{tree}$$

- The tree amplitudes can be calculate using other techniques (CSW, BCFW, ...).

- This fixes completely the coefficient c_4.

Conclusion

- Feynman diagrams are not the most efficient technique to calculate gauge theory amplitudes.
- Color-ordered amplitudes can be very easily and efficiently calculated using the twistor techniques, both at tree-level and at one-loop level.
- The results obtained from these techniques have in general very simple and compact analytic expressions.


$\tilde{q}_R$
q
$\tilde{\chi}_1^0$
c
d
s

CSW formalism

- Par exemple pour $gg \to ggg$, on a

Digarammes de Feynman	Diagrammes CSW

CSW formalism

- Par exemple pour $g\,g \to g\,g\,g$, on a

Digarammes de Feynman	Diagrammes CSW
45	

CSW formalism

- Par exemple pour $g\,g \rightarrow g\,g\,g$, on a

Digarammes de Feynman	Diagrammes CSW
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