Di-Photon Generalized Distribution Amplitude

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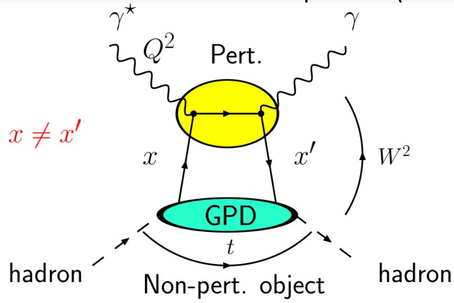
Deep Virtual Compton Scattering

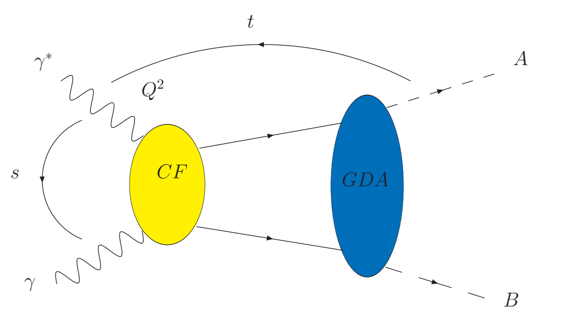
Exclusive Process and Non-Forward Amplitude ($-t \ll s = W^2$)



$$\text{Amplitude} \quad = \quad \text{Coefficient Function (perturbative)}$$
$$\otimes \quad \text{Generalized Parton Distribution (non-perturbative)}$$

If hadron = photon, cf S. Friot, B. Pire, L. Szymanowski *Phy. Lett. B 647 (2007)*

Crossed Process : $s \ll -t$



$$\begin{aligned} \text{Amplitude} \quad &= \quad \text{Coefficient Function (perturbative)} \\ &\otimes \quad \textbf{G}\text{eneralized } \textbf{D}\text{istribution } \textbf{A}\text{mplitude} \end{aligned}$$

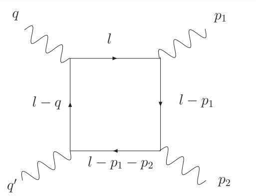
- A, B = Hadrons : Non-perturbative GDA
- A, B = Photons : Perturbative GDA (at leading order)

Motivation

- Prove the **factorization of the process** $\gamma^*(q)\gamma(p_1) \to \gamma(q')\gamma(p_2)$ with the diphoton GDA at lowest order in α_{em}.
- Calculate the leading (anomalous) **diphoton GDA** at Born order.
- Study its **QCD evolution equations**.

$\gamma^*(q)\gamma(p_1) \to \gamma(q')\gamma(p_2)$ process

Two-photon production in Virtual Compton scattering on a photon target (+ crossed diagrams) :



$$
\begin{aligned}
q &= p - \frac{Q^2}{2}N, \quad q' = \frac{Q^2}{2}N \\
p_1 &= \zeta p, \quad p_2 = \bar{\zeta} p \\
\text{loop momentum} \quad &: \quad l^\mu = z p^\mu + \beta n^\mu + l_T
\end{aligned}
$$

with the two light-cone vectors p, n ($N = \frac{n}{p \cdot n}$).
$z \in [-\infty, \ +\infty]$ will be interpreted as usual parton momentum fraction $\rightarrow z \in [0, \ 1]$

Tensorial Decomposition

- Amplitude of the $\gamma^*(q)\gamma(p_1) \to \gamma(q')\gamma(p_2)$ process :

$$A(\zeta,\ Q^2) = \epsilon_\mu \epsilon'_\nu \epsilon_{1\alpha}^{*} \epsilon_{2\beta}^{*} T^{\mu\nu\alpha\beta}(\zeta,\ Q^2),$$

  with transverse photon polarizations $\epsilon(q)$, $\epsilon'(q')$, $\epsilon_1(p_1)$ and $\epsilon_2(p_2)$.
- Tensorial decomposition at threshold $(s = 0)$:

$$\begin{aligned} T^{\mu\nu\alpha\beta}(\zeta,\ Q^2) &= \frac{1}{4} g_T^{\mu\nu} g_T^{\alpha\beta} \mathbf{W_1}(\zeta,\ \mathbf{Q^2}) \\ &+ \frac{1}{8}\left(g_T^{\mu\alpha} g_T^{\nu\beta} + g_T^{\nu\alpha} g_T^{\mu\beta} - g_T^{\mu\nu} g_T^{\alpha\beta}\right) \mathbf{W_2}(\zeta,\ \mathbf{Q^2}) \\ &+ \frac{1}{4}\left(g_T^{\mu\alpha} g_T^{\nu\beta} - g_T^{\mu\beta} g_T^{\alpha\nu}\right) \mathbf{W_3}(\zeta,\ \mathbf{Q^2}). \end{aligned}$$

Above threshold, more tensorial structures contribute.

Results and Interpretation

Each 6 diagrams UV divergent but the sum UV finite. Before integration over z, we get (m : quark's mass and IR regulator) :

$$\begin{aligned} W_1 \; &= \; -\log \frac{\mathbf{Q^2}}{\mathbf{m^2}} \sum_q \frac{e_q^4 N_C}{2\pi^2} \int_0^1 dz(2z-1) \Bigg[\frac{2z-\zeta}{z\bar{\zeta}} \theta(z-\zeta) \\ &+ \; \frac{2z-1-\zeta}{\bar{z}\zeta}\theta(\zeta - z) + \frac{2z-\bar{\zeta}}{z\zeta}\theta(z-\bar{\zeta}) + \frac{2z-1-\bar{\zeta}}{\bar{z}\bar{\zeta}}\theta(\bar{\zeta}-z) \Bigg] \end{aligned}$$

Interpretation through QCD factorization within the parton model :

$$W_1(\zeta,\ Q^2) = \sum_q \int_0^1 dz \;\, C_V^q(z) \Phi_1^q(z,\zeta,Q^2)$$

with the usual Born order coefficient function : $C_V^q = e_q^2 \left(\frac{1}{z} - \frac{1}{\bar{z}}\right)$

The diphoton GDA $\Phi_1^q(z,\ \zeta)$

We can directly calculate the **diphoton GDA** with

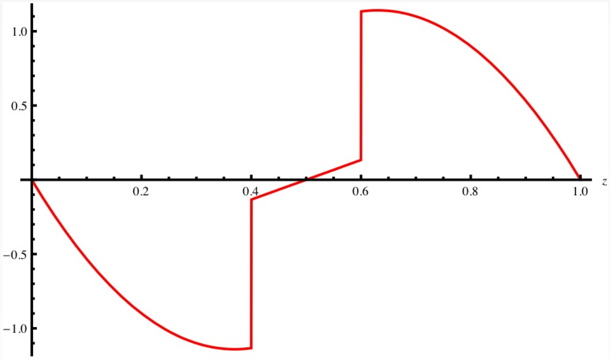
$$\begin{aligned} F^q(z,\ \zeta) &= \int \frac{dy}{2\pi} e^{i(2z-1)\frac{y}{2}} \langle \gamma(p_1)\gamma(p_2)|\bar{q}(-\frac{y}{2}N)\gamma.Nq(\frac{y}{2}N)|0\rangle \\ &= \frac{1}{2} g_\perp^{\mu\nu} \epsilon_\mu^*(p_1)\epsilon_\nu^*(p_2) \mathbf{\Phi_1^q(z,\zeta,0)} \end{aligned}$$

F^q UV divergent : use of dimensional regularization and renormalization procedure.

$$\begin{aligned} \Phi_{1_R}^q(z,\zeta,Q^2) &= \frac{N_C\, e_q^2}{2\pi^2} \log \frac{Q^2}{m^2} \Big[\frac{\bar{z}(2z-\zeta)}{\bar{\zeta}} \theta(z-\zeta) + \frac{\bar{z}(2z-\bar{\zeta})}{\zeta} \theta(z - \cdots \\ &+ \frac{z(2z-1-\zeta)}{\zeta} \theta(\zeta - z) + \frac{z(2z-1-\bar{\zeta})}{\bar{\zeta}} \theta(\bar{\zeta} - z) \Big] \end{aligned}$$

$\Rightarrow$ Factorization is proven

$$\frac{\Phi_{1_R}(z,\ \zeta{=}0.4)}{\frac{N_C\, e_q^2}{2\pi^2}\log\frac{Q^2}{m^2}}:$$



- Only *anomalous* part of GDAs (logarithmic factors i.e. $\propto \log\frac{Q^2}{m^2}$)
- Discontinuities of $\Phi_1(z,\ \zeta)$ at $z = \zeta$ and $z = 1 - \zeta$
- But still property of polynomiality for these GDAs.

Evolution equation (for Non-Singlet (in flavor) part)

Switching on QCD $\Rightarrow$ **inhomogenous** ERBL evolution equation :

$$Q^2 \frac{d}{dQ^2}\Phi_+^{NS}(z,\zeta,Q^2) = (\mathbf{e_q^2 - e_{q'}^2})\mathbf{f_1}(\mathbf{z},\zeta) + \int_0^1 du \; V_{NS}(u,z,Q^2)\Phi_+^{NS}(u,\zeta,Q^2)$$

with $f_1(z,\ \zeta) = \frac{\Phi_1(z,\ \zeta,\ Q^2)}{\log \frac{Q^2}{m^2}}$ and the usual QCD kernel

$$V_{NS}(u,z,Q^2) = \frac{\alpha_s(Q^2)}{2\pi} C_F \left[\frac{u}{z}\theta(z-u)\left(1+\frac{1}{z-u}\right) + \frac{\bar{u}}{\bar{z}}\theta(u-z)\left(1+\frac{1}{u-z}\right)\right]_+$$

cf De Witt et al Phys.Rev D 19 (1979) for inhom-DGLAP equation

Solution for large Q^2

$$\Phi_+^{NS}(z,\zeta,Q^2) \simeq (e_q^2 - e_{q'}^2)\log\frac{\mathbf{Q^2}}{\mathbf{m^2}} z(1-z) \times \sum_{p\ impair}^{\infty} \frac{f_p(\zeta)}{1 + \mathbf{6}\frac{\boldsymbol{\gamma_{qq}(p)}}{\mathbf{33-2N_f}}} C_p^{(3/2)}(2z-1)$$

where $\gamma_{qq}(p)$ are the usual anomalous dimensions

$$\gamma_{qq}(p) = C_F\left(\frac{1}{2} - \frac{1}{(p+1)(p+2)} + 2\sum_{k=2}^{p+1}\frac{1}{k}\right)$$

and

$$f_p(\zeta) = \frac{(p+1)(p+2)}{4(2p+3)}\int_0^1 dz\ z\bar{z} f_1(z,\zeta) C_p^{(3/2)}(2z-1).$$

Conclusion

- Factorization of the process $\gamma^*(q)\gamma(p_1) \to \gamma(q')\gamma(p_2)$ with only quark contribution to the GDA : interpretation of the final state $\gamma\gamma$ within the parton model.
- Domination of the anomalous part of GDAs at large Q^2.
- Modification of the QED behaviour of Φ_1^q by strong interaction (effect of the inhom-ERBL evolution equation).

THANK YOU

Photonic correlators

$$F^\gamma = \int \frac{dy}{2\pi} e^{i(2z-1)\frac{y}{2}} \langle \gamma(p_1)\gamma(p_2)| F^{N\mu}(-\frac{y}{2}N) F_\mu^N(\frac{y}{2}N)|0\rangle$$

$$\tilde{F}^\gamma = \int \frac{dy}{2\pi} e^{i(2z-1)\frac{y}{2}} \langle \gamma(p_1)\gamma(p_2)| F^{N\mu}(-\frac{y}{2}N) \tilde{F}_\mu^N(\frac{y}{2}N)|0\rangle,$$

where $F^{N\mu} = N_\nu F^{\nu\mu}$.

Mixing with quark correlators but these one begin at order α_{em}^0.

Results for quark correlators

UV divergences $\rightarrow$ dimensional regularization ($d = 4 + 2\epsilon$).
With $\frac{1}{\hat{\epsilon}} = \frac{1}{\epsilon} + \gamma_E - \log 4\pi$, we have for the

$$F^q = -\frac{N_C\, e_q^2}{4\pi^2} g_T^{\mu\nu} \epsilon_\mu^*(p_1)\epsilon_\nu^*(p_2) \left[\frac{1}{\hat{\epsilon}} + \log m^2\right] F(z,\zeta)\,,$$

with

$$\begin{aligned} F(z,\zeta) &= \frac{\bar{z}(2z-\zeta)}{\bar{\zeta}}\theta(z-\zeta) + \frac{\bar{z}(2z-\bar{\zeta})}{\zeta}\theta(z-\bar{\zeta}) \\ &+ \frac{z(2z-1-\zeta)}{\zeta}\theta(\zeta - z) + \frac{z(2z-1-\bar{\zeta})}{\bar{\zeta}}\theta(\bar{\zeta}-z) \end{aligned}$$

Renormalization procedure

Renormalization of correlators with mixing between quark and photonic correlators :

$$\left(\begin{array}{c} O^q \\ O^\gamma \end{array}\right)_R = \left(\begin{array}{cc} Z_{qq} & Z_{q\gamma} \\ Z_{\gamma q} & Z_{\gamma\gamma} \end{array}\right) \left(\begin{array}{c} O^q \\ O^\gamma \end{array}\right)$$

with

$$\begin{aligned} (O^q, O^\gamma) &= (\bar{q}(-\frac{y}{2}N)\gamma.Nq(\frac{y}{2}N),\ F^{N\mu}(-\frac{y}{2}N)F^N_\mu(\frac{y}{2}N)) \\ or &\quad (\bar{q}(-\frac{y}{2}N)\gamma.N\gamma^5 q(\frac{y}{2}N),\ F^{N\mu}(-\frac{y}{2}N)\tilde{F}^N_\mu(\frac{y}{2}N)) \end{aligned}$$

Renormalized matrix elements

- Matrix element of the renormalized quark-quark correlator :

$$\begin{aligned} <\gamma(p_1)\gamma(p_2)|O_R^q|0> &= Z_{qq} <\gamma(p_1)\gamma(p_2)|O^q|0> \\ &+ Z_{q\gamma} <\gamma(p_1)\gamma(p_2)|O^\gamma|0> \end{aligned}$$

with $Z_{qq} = 1 + \mathcal{O}\left(\frac{e^2}{\hat{\epsilon}}\right)$.

- Procedure

$$\begin{aligned} & <\gamma(p_1)\gamma(p_2)|O^q|0> \longrightarrow UV\ divergent \\ & <\gamma(p_1)\gamma(p_2)|O^\gamma|0>\ UV\ finite\ and\ of\ order\ \alpha_{em}^0 \\ \Rightarrow\quad & absorption\ of\ the\ divergence\ into\ Z_{q\gamma} \end{aligned}$$

- Renormalization condition

$$<\gamma(p_1)\gamma(p_2)|O_R^q|0> = 0 \quad at \quad M_R = m.$$

Renormalized GDAs

- As we want to use QCD factorization formula, we identify the renormalization scale M_R with M_F.

- So, the renormalized GDAs are

$$
\begin{aligned}
\Phi_1^q(z,\zeta,0) &= -\frac{N_C\, e_q^2}{2\pi^2} \log \frac{m^2}{M_F^2} F(z,\zeta) \\
\Phi_3^q(z,\zeta,0) &= -\frac{N_C\, e_q^2}{2\pi^2} \log \frac{m^2}{M_F^2} \tilde{F}(z,\zeta)
\end{aligned}
$$

Factorization and interpretation

Quark contribution at threshold :

$$W_1^q = \int_0^1 dz C_V^q(z) \Phi_1^q(z, \zeta, 0) \ , \qquad W_3^q = \int_0^1 dz C_A^q(z) \Phi_3^q(z, \zeta, 0) \ ,$$

with the Born order coefficient functions :

$$C_V^q = e_q^2 \left(\frac{1}{z} - \frac{1}{\bar{z}} \right) , \qquad C_A^q = e_q^2 \left(\frac{1}{z} + \frac{1}{\bar{z}} \right)$$

Factorization and interpretation

- Contribution of photonic operator : convolution of a coefficient function (α_{em}^2 and at scale M_F^2) with matrix elements of photonic correlators O^γ (α_{em}^0).
  $\Rightarrow$ same expressions as W_1 and W_3 with m replaced by M_F.
- Rewrite the integrand in W_1 as :

$$\mathcal{F}(z,\zeta)\ \log\frac{m^2}{Q^2} = \mathcal{F}(z,\zeta)\ \log\frac{m^2}{M_F^2} + \mathcal{F}(z,\zeta)\ \log\frac{M_F^2}{Q^2}$$

  with the first term $\rightarrow$ quark GDA of the photon
  and the second term $\rightarrow$ photon GDA of the photon.
- Choice $M_F^2 = Q^2$ $\rightarrow$ scaterring amplitude written solely in terms of quark correlator.
  $\Rightarrow$ Interpretation of this process within the parton model

C-even sector

2 real photons in the final state $\Rightarrow$ study of the C-even (singlet) sector of the GDAs.
Contribution of these combinations of operators :

$$\frac{1}{2}(O^q(x_1,\ x_2) - O^q(x_2,\ x_1)) \qquad \frac{1}{2}(\tilde{O}^q(x_1,\ x_2) + \tilde{O}^q(x_2,\ x_1))$$

$\Rightarrow$ combinations for the GDAs Φ_i^q :

$$\begin{aligned} \Phi_+^q(z,\ \zeta,\ 0) &= \frac{1}{2}(\Phi_1^q(z,\ \zeta,\ 0) - \Phi_1^q(\bar{z},\ \zeta,\ 0)) \\ \tilde{\Phi}_+^q(z,\ \zeta,\ 0) &= \frac{1}{2}(\Phi_3^q(z,\ \zeta,\ 0) + \Phi_3^q(\bar{z},\ \zeta,\ 0)) \end{aligned}$$

Flavor singlet sector

Flavor decomposition into :

- singlet sector

$$\Phi_+^S \propto \sum_{q=1}^{N_f} \Phi_+^q \qquad \tilde{\Phi}_+^S \propto \sum_{q=1}^{N_f} \tilde{\Phi}_+^q$$

- non-singlet sector

$$\Phi_+^{NS} = \Phi_+^q - \Phi_+^{q'} \qquad \tilde{\Phi}_+^{NS} = \tilde{\Phi}_+^q - \tilde{\Phi}_+^{q'}$$

For simplicity (no mixing with gluons), study of evolution for the case of non-singlet and vector GDA Φ_+^{NS}.