Baryon Shape

Lorcé Cédric
C.Lorce@ulg.ac.be

IFPA, AGO Department
University of Liège, Belgium

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Outline

Motivation

Nucleon is a very complicated system

Large Q^2: almost free *light* quarks and gluons

Low Q^2: strongly bound *heavy* quarks

1. Questions about "internal" structure:
   - How many quarks?
   - Which types of quarks?
   - Distribution?
   - Angular momentum?
   - ...
2. Questions about "external" structure:
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   - Spherical shape?
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Quadrupolar Distortions

Quadrupole moment

$$\mathcal{Q}_{ij} = \int \mathrm{d}^3x \, \frac{3x_ix_j - \mathbf{x}^2\delta_{ij}}{\mathbf{x}^2} \, \rho(\mathbf{x})$$

Spherical ($\mathcal{Q}_{33} = 0$) Prolate ($\mathcal{Q}_{33} < 0$) Oblate ($\mathcal{Q}_{33} > 0$)

Experimental side

Nucleon no spectroscopic quadrupole moment $\mathcal{Q} = 0$ (intrinsic $\mathcal{Q}_0$?)

Delta too short lifetime ($\sim 10^{-23}$ s) to be measured

$\gamma N \to \Delta$ transition form factors:

- magnetic dipole $M1$
- electric quadrupole $E2$
- Coulomb quadrupole $C2$

Chiral Quark-Soliton Model

Approximation:

- Gluon field is integrated out

χQSM Lagrangian

$$\mathcal{L}_{\chi QSM} = \bar{\psi}(i\gamma^\mu\partial_\mu - MU^{\gamma_5})\psi$$

is chiral invariant thanks to the chiral field U^{γ_5}

[Diakonov & Petrov, Phys.Lett. B493 (2000) 169-174]

Large N_C logic: U^{γ_5} is a classical non-trivial relativistic mean field,
i.e. a soliton

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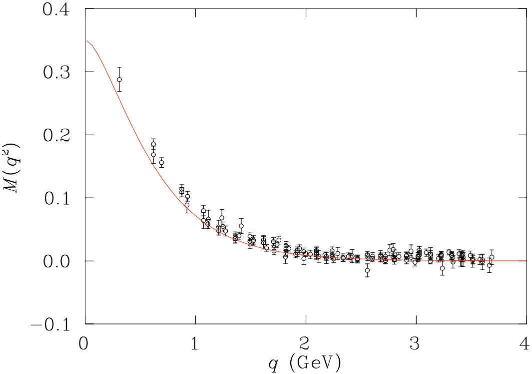

Solid line: model prediction

[Diakonov & Petrov, Nucl.Phys. B272 (1986) 457]

Points: Lattice data

[Bowman & *al.*, Nucl.Phys.Proc.Suppl. 128 (2004) 23]

Light-Cone Approach

Interacting theory vacuum contains indefinite number of $Q\bar{Q}$ pairs
$\Rightarrow$ no wave function

Physics does not depend on the way space-time is described

Light-cone description

$A^\mu = (A^+, \mathbf{A}^\perp, A^-)$ with $A^\pm = A^0 \pm A^3$

Consequences for *massive* particles:

- $k^+ > 0$
- $|\Omega_0\rangle = |0\rangle$
- concept of wave function well defined!

[Brodsky, Pauli & Pinsky, Phys.Rept. 301 (1998) 299-486]

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Light-Cone Wave Function

$SU(3)$ baryon wave function

$$|\Psi_B\rangle = \prod_1^{N_C} \int (\mathrm{d}\mathbf{p}) F(\mathbf{p}) a^\dagger(\mathbf{p}) \qquad \text{Valence part}$$
$$\times \exp\left(\int (\mathrm{d}\mathbf{p})(\mathrm{d}\mathbf{p}')\, a^\dagger(\mathbf{p}) W(\mathbf{p},\mathbf{p}') b^\dagger(\mathbf{p}')\right) |\Omega_0\rangle \qquad \text{Sea part}$$

with

- valence level wave function $F \Rightarrow h$ (s-wave) and j (p-wave)
- quark-antiquark pair wave function W
- $R, R^\dagger$ $SU(3)$ rotation matrices
- $B^*(R)$ baryon rotational wave function

[Diakonov & Petrov, Phys.Rev. D72 (2005) 074009]

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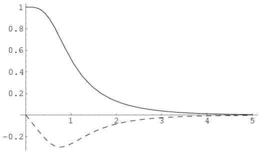
LCWF: Valence Part

One-particle Dirac Hamiltonian in mean field

$$\begin{cases} h' + h\,M\sin P - j(M\cos P + E_{lev}) = 0 \\ j' + 2j/r - j\,M\sin P - h(M\cos P - E_{lev}) = 0 \end{cases}$$



$E_{lev} \approx 200$ MeV for $M = 345$ MeV

Light-cone valence wave function

$$F^{j\sigma}(z,\mathbf{p}_\perp) = \sqrt{\frac{\mathcal{M}}{2\pi}}\left[\epsilon^{j\sigma}h(p) + (p_z\mathbf{1} + i\mathbf{p}_\perp \times \tau_\perp)^{\sigma}_{\sigma'}\epsilon^{j\sigma'}\frac{j(p)}{|\mathbf{p}|}\right]$$

with j isospin and σ spin indices, $p_z = z\mathcal{M} - E_{lev}$, z longitudinal momentum fraction

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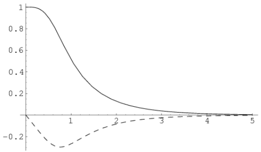

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LCWF: Sea Part

$W \Rightarrow$ approximate quark propagator in mean field $\sim \frac{1}{i\gamma^\mu\partial_\mu - MU^{\gamma_5}}$

Light-cone pair wave function

$$W^{j\sigma}_{j'\sigma'}(z,\mathbf{p}_\perp;z',\mathbf{p}'_\perp) = \frac{M\mathcal{M}}{2\pi Z}\Big\{\Sigma^j_{j'}(\mathbf{q})[M(z'-z)\tau_3+\mathbf{Q}_\perp\cdot\tau_\perp]^\sigma_{\sigma'}$$
$$+i\Pi^j_{j'}(\mathbf{q})[-M(z'+z)\mathbf{1}+i\mathbf{Q}_\perp\times\tau_\perp]^\sigma_{\sigma'}\Big\}$$

where $\mathbf{Q}_\perp = z\mathbf{p}'_\perp - z'\mathbf{p}_\perp$,
$Z = \mathcal{M}^2 zz'(z+z') + z(\mathbf{p}'^2_\perp + M^2) + z'(\mathbf{p}^2_\perp + M^2)$

LCWF: Rotational Part

$$T(B)^{f_1 f_2 f_3 f_4 j_5 \ldots}_{j_1 j_2 j_3 j_4 f_5 \ldots, k} = \int \mathrm{d}R\, B^*_k(R) R^{f_1}_{j_1} R^{f_2}_{j_2} R^{f_3}_{j_3} (R^{f_4}_{j_4} R^{\dagger j_5}_{f_5}) \ldots$$

Proton: $p^*_k(R) \propto \epsilon_{kl} R^{\dagger l}_1 R^3_3$

Proton 3Q rotational wave function

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Non-relativistic wave function: $F^{j\sigma}(p) \approx \epsilon^{j\sigma} h(p)$

Proton non-relativistic 3Q wave function

$$\Psi_{p,k} \propto \epsilon^{f_1 f_2} \epsilon^{\sigma_1 \sigma_2} \delta^{f_3}_1 \delta^{\sigma_3}_k h(p_1) h(p_2) h(p_3) + \text{cycl. perm. of } (1,2,3)$$

$\equiv SU(6)$ wave functions!

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Question:

Which part of baryons is actually made of $3Q$?

	$3Q$	$5Q$	$7Q$
B_8 & B_{10}	$\approx 72\%$	$\approx 21\%$	$\approx 7\%$
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$\gamma N \longrightarrow \Delta$ transition

$SU(6)$ symmetry $\quad \Rightarrow \quad E2 = 0$

5Q contribution

$\Rightarrow \quad E2 \propto (3K_{33} - K_{\pi\pi}) = \int \mathrm{d}^3q\, \frac{3q_z^2 - \mathbf{q}^2}{\mathbf{q}^2} f(\mathbf{q})$ quadrupole!

	$3Q$	$3Q + 5Q$	$Exp.$
$A_{3/2}\,(GeV^{-1/2})$	-0.296	-0.232	-0.250 ± 0.008
$A_{1/2}\,(GeV^{-1/2})$	-0.171	-0.129	-0.135 ± 0.006
$G_M^*\,(\mu_N)$	2.389	2.820	2.798
$G_E^*\,(\mu_N)$	0	0.026	0.046
$R_{EM} = -\frac{G_E^*}{G_M^*}$	0	-0.9%	-1.6%
$\Gamma_{p\Delta}\,(MeV)$	0.411	0.573	0.564

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Quadrupolar effect

Example proton $5Q$ (valence) contribution

$$\Delta u = \frac{6}{25}\left[453K^A_{\sigma\sigma} + 137K^A_{\pi\pi} + 14(3K^A_{33} - K^A_{\pi\pi})\right]$$

$$\delta u = \frac{6}{25}\left[453K^T_{\sigma\sigma} + 137K^T_{\pi\pi} - 7(3K^T_{33} - K^T_{\pi\pi})\right]$$

$$\Delta d = \frac{-4}{25}\left[159K^A_{\sigma\sigma} + 91K^A_{\pi\pi} - 38(3K^A_{33} - K^A_{\pi\pi})\right]$$

$$\delta d = \frac{-4}{25}\left[159K^T_{\sigma\sigma} + 91K^T_{\pi\pi} + 19(3K^T_{33} - K^T_{\pi\pi})\right]$$

$$\Delta s = \frac{-4}{25}\left[3K^A_{\sigma\sigma} + 17K^A_{\pi\pi} - 16(3K^A_{33} - K^A_{\pi\pi})\right]$$

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Conclusion

- Light-cone approach within a chiral model χQSM
- Explicit baryon wave functions
- Expansion in Fock space ($3Q$, $5Q$, ...)
- Explicit quadrupolar distortion
- *Ab initio* calculations

Problems ($\Rightarrow$ outlook)

- Theoretical errors?
- $SU(3)$ breaking?
- Valence level distortion due to the sea?

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Proton flavor composition

Vector charges "count" the number of quarks of a particular type

$$q_{tot} = q_{val} + q_{sea} \text{ with } q_{sea} = q_s - \bar{q}$$

Proton up to $7Q$ component

	q_{val}	q_s	$\bar{q}$	q_{tot}
u	1.923	0.202	0.125	2
d	1.017	0.128	0.145	1
s	0.060	0.028	0.088	0

Remarkable features:

- isospin-asymmetric sea $\bar{d} - \bar{u} = 0.019$ v.s. 0.118 ± 0.012
- asymmetric strange quark distribution $s(z) - \bar{s}(z) \neq 0$

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Proton axial charges

isovector $g_A^{(3)} = \Delta u - \Delta d$

octet $g_A^{(8)} = \frac{1}{\sqrt{3}}(\Delta u + \Delta d - 2\Delta s)$

flavor singlet $g_A^{(0)} = \Delta u + \Delta d + \Delta s$

	$g_A^{(3)}$	$g_A^{(8)}$	$g_A^{(0)}$
$NR\,3Q$	$\frac{5}{3} = 1.667$	$\frac{1}{\sqrt{3}} = 0.577$	1
$3Q$	$\frac{5}{3}\,\Phi^A = 1.435$	$\frac{1}{\sqrt{3}}\,\Phi^A = 0.497$	$\Phi^A = 0.861$
$3Q + 5Q$	1.241	0.444	0.787
Exp.	1.257 ± 0.003	0.34 ± 0.02	0.31 ± 0.07

Axial gluon anomaly $\Rightarrow$ $\Delta q(Q^2) = \Delta q - \alpha_S(Q^2)\Delta g(Q^2)$

Model scale $Q_0^2 \simeq 0.36$ GeV^2

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	$g_A^{(3)}$ $SU(2)$	$g_A^{(8)}$ $SU(3)$	$g_A^{(0)}(Q^2)$ $SU(3)$
$NR\,3Q$	$\frac{5}{3} = 1.667$	$\frac{1}{\sqrt{3}} = 0.577$	1
$3Q$	$\frac{5}{3}\,\Phi^A = 1.435$	$\frac{1}{\sqrt{3}}\,\Phi^A = 0.497$	$\Phi^A = 0.861$
$3Q + 5Q$	1.241	0.444	0.787
$Exp.$	1.257 ± 0.003	0.34 ± 0.02	0.31 ± 0.07

Axial gluon anomaly $\Rightarrow$ $\Delta q(Q^2) = \Delta q - \alpha_S(Q^2)\Delta g(Q^2)$
Model scale $Q_0^2 \approx 0.36$ GeV^2

Proton and Neutron Magnetic Moments

On the light cone: magnetic moment $\Leftrightarrow$ spin-flip matrix element

γ^+ conserves helicity $\Rightarrow$ relativistic process $h \leftrightarrow j$ and $j \leftrightarrow j$

$SU(6)$ symmetry $\quad \Rightarrow \quad \frac{\mu_p}{\mu_n} = -\frac{3}{2}$

	μ_p/μ_n	$\mu_p\,(\mu_N)$	$\mu_n\,(\mu_N)$
$3Q$	$-\frac{3}{2} = -1.5$	2.534	-1.689
$3Q+5Q$	-1.479	2.900	-1.961
Exp.	-1.460	2.793	-1.913

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Extra: Pentaquark Width

Why is it small?

Qualitative picture: pentaquark has *no* $3Q$ component

Generalized Goldberger-Treiman relation

$$g_{\Theta KN} = \frac{g_A^{\Theta\to KN}(M_\Theta + M_N)}{2F_K}$$

$M_\Theta = 1530$ MeV, $M_N = 940$ MeV and $F_K = 1.2F_\pi = 112$ MeV

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$\Gamma_\Theta = 2\,\frac{g^2_{\Theta KN}|\mathbf{p}|}{8\pi}\,\frac{(M_\Theta - M_N)^2 - m_K^2}{M_\Theta^2}$

kaon momentum $|\mathbf{p}| = 254$ MeV and kaon mass $m_K = 495$ MeV

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	$g_A^{\Theta\to KN}$	$g_{\Theta KN}$	Γ_Θ (MeV)	$\mathcal{N}^{(3)}(N)$
$NR\,5Q$	0.202	2.230	4.427	65%
$5Q$	0.114	1.592	2.256	77.7%
$5Q+7Q$	0.169	1.864	3.091	71.7%