The Wave Function of $[\mathbf{70}, 1^-]$ Baryons in the $1/N_c$ Expansion

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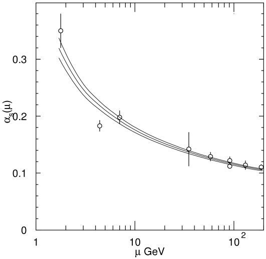
Outline

An introduction to large N_c QCD

- Impossible to solve QCD exactly
- No perturbative expansion of QCD at low energies with respect to g



- 't Hooft suggested to generalize QCD to N_c color (1974)
- $1/N_c$ should be the expansion parameter of QCD
- Witten power counting rules (1979)

The total wave function of baryons Ψ

$$\Psi = \psi_{lm}\chi\phi C$$

where ψ_{lm}, χ, ϕ and C are the space, spin, flavor and color part.

Baryons in large N_c QCD

$$\varepsilon_{i_1 i_2 i_3 \cdots i_{N_c}} q^{i_1} q^{i_2} q^{i_3} \cdots q^{i_{N_c}}$$

bound state of N_c quarks completely antisymmetric in color because baryons are colorless

Baryon mass grows with N_c

SU($2N_f$) is a good symmetry in $N_c \to \infty$ limit [1]

$$\mathrm{SU}(2N_f) \supset \mathrm{SU_S}(2) \times \mathrm{SU}_f(N_f)$$

Assume a SU(6) × O(3) symmetry at leading order

SU($2N_f$) operators

$$\begin{aligned} S^i &= q^\dagger (S^i \otimes \mathbb{1}) q \quad (3,1) \\ T^a &= q^\dagger (\mathbb{1} \otimes T^a) q \quad (1,\mathrm{adj}) \\ G^{ia} &= q^\dagger (S^i \otimes T^a) q \quad (3,\mathrm{adj}) \end{aligned}$$

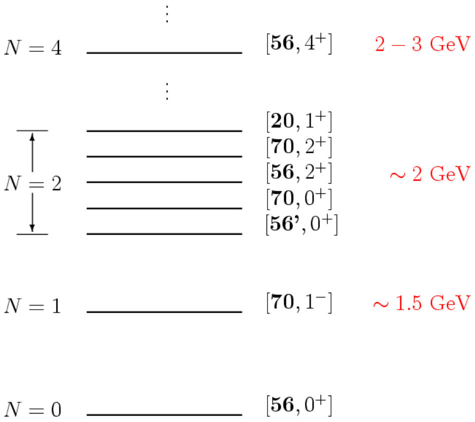
Mass operator

$$M = \sum_n \frac{1}{N_c^{n-1}} c_n O^n, \quad O^n = O_\ell^{(k)} \cdot O_{SF}^{(k)}$$

[1] R. Dashen and A. V. Manohar, Phys. Lett. **B315**, 425 (1993)

Baryon spectrum with a linear confinement and $N_c = 3$

Baryon spectrum (SU(6) notation: $[\mathbf{X}, l^P]$)



Quark-shell model configurations for N_c particles

- **The number of excited quarks** of large N_c baryons is **the same** as for $N_c = 3$
- **Suppression of the center of mass coordinate** in the baryon wave function for satisfying translational invariance

SU(6) multiplet	N	L	P	Harmonic oscillator configuration
56	0	0	+	$\|[N_c](0s)^{N_c}\rangle$
70	1	1	−	$\|[N_c-1,1](0s)^{N_c-1}(0p)\rangle$
56′	2	0	+	$\sqrt{\frac{N_c-1}{N_c}}\|[N_c](0s)^{N_c-1}(1s)\rangle - \sqrt{\frac{1}{N_c}}\|[N_c](0s)^{N_c-2}(0p)^2\rangle$
70	2	0	+	$\sqrt{\frac{1}{3}}\|[N_c-1,1](0s)^{N_c-1}(1s)\rangle + \sqrt{\frac{2}{3}}\|[N_c-1,1](0s)^{N_c-2}(0p)^2\rangle$
56	2	2	+	$\sqrt{\frac{N_c-1}{N_c}}\|[N_c](0s)^{N_c-1}(0d)\rangle - \sqrt{\frac{1}{N_c}}\|[N_c](0s)^{N_c-2}(0p)^2\rangle$
70	2	2	+	$\sqrt{\frac{1}{3}}\|[N_c-1,1](0s)^{N_c-1}(0d)\rangle + \sqrt{\frac{2}{3}}\|[N_c-1,1](0s)^{N_c-2}(0p)^2\rangle$

The $[\mathbf{70}, 1^-]$ baryons in the $1/N_c$ expansion: the approximate wave function [2]

- Standard procedure (based on Hartree approximation): **ground state core of $N_c - 1$ quarks** + **excited quark** (always the **last quark** of the wave function)
  Treat the core like a **ground state baryon**
- Orbital part: $s^{N_c-1}p$ **no antisymmetrization**
- Spin-flavor part **approximate**
- Introduce **excited quark** operators ℓ^i, s^i, t^a and g^{ia} + **core operators** S_c^i, T_c^a and G_c^{ia}.

[2] C. E. Carlson, C. D. Carone, J. L. Goity and R. F. Lebed, Phys. Rev. **D59**, 114008 (1999)

A totally symmetric orbital-spin-flavor state is given by

$$\Phi_S = \frac{1}{\sqrt{N_c - 1}} \sum_Y |[N_c-1,1]Y\rangle_O |[N_c-1,1]Y\rangle_{SF}$$

Illustration for $N_c = 5$ (center of mass motion removed)

$$\begin{array}{|c|c|c|c|c|}\hline 1&2&3&4&5\\\hline\end{array} = \frac{1}{\sqrt{4}}\left(\begin{array}{|c|c|c|c|}\hline \mathbf{1}&\mathbf{2}&\mathbf{3}&\mathbf{4}\\\hline \mathbf{5}\\\cline{1-1}\end{array}\ \begin{array}{|c|c|c|c|}\hline \mathbf{1}&\mathbf{2}&\mathbf{3}&\mathbf{4}\\\hline \mathbf{5}\\\cline{1-1}\end{array} + \begin{array}{|c|c|c|c|}\hline 1&2&3&5\\\hline 4\\\cline{1-1}\end{array}\ \begin{array}{|c|c|c|c|}\hline 1&2&3&5\\\hline 4\\\cline{1-1}\end{array} + \begin{array}{|c|c|c|c|}\hline 1&2&4&5\\\hline 3\\\cline{1-1}\end{array}\ \begin{array}{|c|c|c|c|}\hline 1&2&4&5\\\hline 3\\\cline{1-1}\end{array} + \begin{array}{|c|c|c|c|}\hline 1&3&4&5\\\hline 2\\\cline{1-1}\end{array}\ \begin{array}{|c|c|c|c|}\hline 1&3&4&5\\\hline 2\\\cline{1-1}\end{array} \right)$$

Young tableau	Young-Yamanouchi basis vectors of [41]
$\begin{array}{\|c\|c\|c\|c\|}\hline \mathbf{1}&\mathbf{2}&\mathbf{3}&\mathbf{4}\\\hline \mathbf{5}\\\cline{1-1}\end{array}$	$\frac{1}{\sqrt{20}}(\mathbf{4ssssp - sssps - sspss - spsss - pssss})$
$\begin{array}{\|c\|c\|c\|c\|}\hline 1&2&3&5\\\hline 4\\\cline{1-1}\end{array}$	$\frac{1}{\sqrt{12}}(3sssps - sspss - spsss - pssss)$
$\begin{array}{\|c\|c\|c\|c\|}\hline 1&2&4&5\\\hline 3\\\cline{1-1}\end{array}$	$\frac{1}{\sqrt{6}}(2sspss - spsss - pssss)$
$\begin{array}{\|c\|c\|c\|c\|}\hline 1&3&4&5\\\hline 2\\\cline{1-1}\end{array}$	$\frac{1}{\sqrt{2}}(spsss - pssss)$

The $[\mathbf{70}, 1^-]$ baryons in the $1/N_c$ expansion: the exact wave function

Two approaches:

1. Treat the multiplet **without splitting** the wave function [3]

$$|\ell S I I_3; J J_3\rangle = \sum_{m_\ell, S_3} \left(\begin{array}{cc|c} \ell & S & J \\ m_\ell & S_3 & J_3 \end{array} \right) |[N_c-1,1] S S_3 I I_3\rangle |[N_c-1,1] \ell m_\ell\rangle$$

2. **Split the wave function** into a core of $N_c - 1$ quarks + 1 quark [4]

   Two main differences:
   core not always in the ground state, **last quark not always excited**
   Present approach

[3] N. Matagne, Fl. Stancu, arXiv:hep-ph/0610099

[4] N. Matagne, Fl. Stancu, arXiv:0801.3575 [hep-ph], to be published in Phys. Rev. **D**

The exact decoupled wave function

$$|\ell_c \ell_q \ell S J J_3; I I_3\rangle_p = \sum_{m_c, m_q, m_\ell, S_3} \left(\begin{array}{cc|c} \ell_c & \ell_q & \ell \\ m_c & m_q & m_\ell \end{array} \right) \left(\begin{array}{cc|c} \ell & S & J \\ m_\ell & S_3 & J_3 \end{array} \right) |\ell_c m_c\rangle |\ell_q m_q\rangle \times |[N_c - 1, 1]p; S S_3; I I_3\rangle$$

where ℓ_c represents the **excitation of the core**

The **spin-flavor part** is

$$|[N_c - 1, 1]p; S S_3; I I_3\rangle = \sum_{\substack{p', p'', m_1, \\ m_2, i_1, i_2}} \mathbf{K([f']p'[f'']p''|[N_c - 1, 1]p)} \left(\begin{array}{cc|c} S_c & \frac{1}{2} & S \\ m_1 & m_2 & S_3 \end{array} \right) \left(\begin{array}{cc|c} I_c & \frac{1}{2} & I \\ i_1 & i_2 & I_3 \end{array} \right) |S_c m_1\rangle |1/2 m_2\rangle |I_c i_1\rangle |1/2 i_2\rangle$$

where $K([f']p'[f'']p''|[N_c - 1, 1]p)$ are isoscalar factors of S_{N_c} [5]

[5] N. Matagne, Fl. Stancu, Phys. Lett. **B631**, 7 (2005)

For illustration the case for $S = I = 1/2$

The second column with $p = 1$ is new

$[f']p'[f'']p''$	$[N_c-1,1]1$	$[N_c-1,1]2$
$\left[\frac{N_c+1}{2}, \frac{N_c-1}{2}\right] 1 \left[\frac{N_c+1}{2}, \frac{N_c-1}{2}\right] 1$	0	$-\sqrt{\frac{3(N_c-1)}{4N_c}}$
$\left[\frac{N_c+1}{2}, \frac{N_c-1}{2}\right] 2 \left[\frac{N_c+1}{2}, \frac{N_c-1}{2}\right] 2$	$\sqrt{\frac{N_c-3}{2(N_c-2)}}$	$\sqrt{\frac{N_c+3}{4N_c}}$
$\left[\frac{N_c+1}{2}, \frac{N_c-1}{2}\right] 2 \left[\frac{N_c+1}{2}, \frac{N_c-1}{2}\right] 1$	$-\frac{1}{2}\sqrt{\frac{N_c-1}{N_c-2}}$	0
$\left[\frac{N_c+1}{2}, \frac{N_c-1}{2}\right] 1 \left[\frac{N_c+1}{2}, \frac{N_c-1}{2}\right] 2$	$-\frac{1}{2}\sqrt{\frac{N_c-1}{N_c-2}}$	0

Similar Tables for the cases $S = 1/2, I = 3/2$ and $S = 3/2, I = 1/2$.

Matrix elements of the spin-spin and isospin-isospin operators with the exact and the approximate wave function

	$\langle s \cdot S_c \rangle$		$\langle S_c^2 \rangle$	
	approx. w.f.	exact w.f.	approx. w.f.	exact w.f.
$^2 8$	$-\frac{N_c+3}{4N_c}$	$-\frac{3(N_c-1)}{4N_c}$	$\frac{N_c+3}{2N_c}$	$\frac{3(N_c-1)}{2N_c}$
$^4 8$	$\frac{1}{2}$	$-\frac{3(N_c-5)}{4N_c}$	2	$\frac{3(3N_c-5)}{2N_c}$
$^2 10$	-1	$-\frac{3(N_c-1)}{4N_c}$	2	$\frac{3(N_c-1)}{2N_c}$

	$\langle t \cdot T_c \rangle$		$\langle T_c^2 \rangle$	
	approx. w.f.	exact w.f.	approx. w.f.	exact w.f.
$^2 8$	$-\frac{N_c+3}{4N_c}$	$-\frac{3(N_c-1)}{4N_c}$	$\frac{N_c+3}{2N_c}$	$\frac{3(N_c-1)}{2N_c}$
$^4 8$	-1	$-\frac{3(N_c-1)}{4N_c}$	2	$\frac{3(N_c-1)}{2N_c}$
$^2 10$	$\frac{1}{2}$	$-\frac{3(N_c-5)}{4N_c}$	2	$\frac{3(3N_c-5)}{2N_c}$

Results of the fits (nonstrange baryons)

- Only with the **7 resonances**: $^2N_{1/2}(1538 \pm 18)$, $^4N_{1/2}(1660 \pm 20)$, $^2N_{3/2}(1523 \pm 8)$, $^4N_{3/2}(1700 \pm 50)$, $^4N_{5/2}(1678 \pm 8)$, $^2\Delta_{1/2}(1645 \pm 30)$ and $^2\Delta_{3/2}(1720 \pm 50)$

Fit 1

Operator		Approx. w.f. (MeV)	Exact w.f. (MeV)
$O_1 = N_c \mathbb{1}$	$c_1 =$	211 ± 23	299 ± 20
$O_2 = \ell^i s^i$	$c_2 =$	3 ± 15	3 ± 15
$O_3 = \frac{1}{N_c} s^i S_c^i$	$c_3 =$	-1486 ± 141	-1096 ± 125
$O_4 = \frac{1}{N_c} S_c^i S_c^i$	$c_4 =$	1182 ± 74	1545 ± 122
$O_5 = \frac{1}{N_c} t^a T_c^a$	$c_5 =$	-1508 ± 149	417 ± 79
$\chi^2_{\rm dof}$		1.56	1.56

Fit 2

Operator		Approx. w.f. (MeV)	Exact w.f. (MeV)
$O_1 = N_c \mathbb{1}$	$c_1 =$	513 ± 4	519 ± 5
$O_2 = \ell^i s^i$	$c_2 =$	3 ± 15	3 ± 15
$O_3' = \frac{1}{N_c}\left(2 s^i S_c^i + S_c^i S_c^i + \frac{3}{4}\right)$	$c_3' =$	219 ± 19	150 ± 11
$O_5 = \frac{1}{N_c} t^a T_c^a$	$c_5 =$	417 ± 80	417 ± 80
χ^2_{dof}		1.04	1.04

Fit 3

Operator		Approx. w.f. (MeV)	Exact w.f. (MeV)
$O_1 = N_c \mathbb{1}$	$c_1 =$	516 ± 3	522 ± 3
$O_2 = \ell^i s^i$	$c_2 =$	3 ± 15	3 ± 15
$O_3 = \frac{1}{N_c} s^i S_c^i$	$c_3 =$	450 ± 33	450 ± 33
$O_5' = \frac{1}{N_c}\left(2 t^a T_c^a + T_c^a T_c^a + \frac{3}{4}\right)$	$c_5' =$	214 ± 28	139 ± 27
χ^2_{dof}		1.04	1.04

Fit 4

Operator		Approx. w.f. (MeV)	Exact w.f. (MeV)
$O_1 = N_c \mathbb{1}$	$c_1 =$	484 ± 4	484 ± 4
$O_2 = \ell^i s^i$	$c_2 =$	3 ± 15	3 ± 15
$O_3' = \frac{1}{N_c}\left(2s^i S_c^i + S_c^i S_c^i + \frac{3}{4}\right)$	$c_3' =$	150 ± 11	150 ± 11
$O_5' = \frac{1}{N_c}\left(2t^a T_c^a + T_c^a T_c^a + \frac{3}{4}\right)$	$c_5' =$	139 ± 27	139 ± 27
χ^2_{dof}		1.04	1.04

- With **7 resonances** and **2 mixing angles**

$$|N_J(\text{upper})\rangle = \cos\theta_J|^4N_J\rangle + \sin\theta_J|^2N_J\rangle$$
$$|N_J(\text{lower})\rangle = \cos\theta_J|^2N_J\rangle - \sin\theta_J|^4N_J\rangle$$

with $\theta_{1/2}^{exp} \approx -0.56$ rad and $\theta_{3/2}^{exp} \approx 0.10$ rad [6]

The **physical masses** become

$$M_J(\text{upper}) = M(^4N_J) + c_2\langle^4N_J|\ell\cdot s|^2N_J\rangle\tan\theta_J$$
$$M_J(\text{lower}) = M(^2N_J) - c_2\langle^4N_J|\ell\cdot s|^2N_J\rangle\tan\theta_J$$

[6] N. Isgur, Phys. Rev. **D62**, 054026 (2000)

Operator		Approx. w.f. (MeV)	Exact w.f. (MeV)
$O_1 = N_c \mathbb{1}$	$c_1 =$	484 ± 4	484 ± 4
$O_2 = \ell^i s^i$	$c_2 =$	-9 ± 15	-9 ± 15
$O_3' = \frac{1}{N_c}\left(2s^i S_c^i + S_c^i S_c^i + \frac{3}{4}\right)$	$c_3' =$	150 ± 11	150 ± 11
$O_5' = \frac{1}{N_c}\left(2t^a T_c^a + T_c^a T_c^a + \frac{3}{4}\right)$	$c_5' =$	139 ± 27	139 ± 27
χ^2_{dof}		1.05	1.05

Conclusions

- **Identical** $\chi^2_{\rm dof}$ and **rather similar results** for the dynamical coefficient with exact or approximate wave function
- Contribution of I^2 **as important as** S^2
- Separation of S^2 and I^2 into S_c^2, $S_c \cdot s$, I_c^2 and $I_c \cdot i$ **deteriorates the fit**
- Results **at variance** with that of Pirjol and Schat [7] (**claim that the inclusion of core and excited quark operators is necessary**)
- **Isoscalar factors** of S_{N_c} for irreductible representation $[N_c - 1, 1]$ **are derived**

[7] D. Pirjol, C. L. Schat, arXiv:0709.0714 [hep-ph]