TWO-PHOTON DECAYS OF PSEUDO-SCALAR QUARKONIA

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1 Introduction

• The physics of quarkonium decay seems to be better understood within the conventional framework of QCD. (For a recent review, see , e.g, N. Brambilla *et al*, hep-ph/0412158 ; P. Colangelo, F. De Fazio, R. Ferrandes, S. Nicotri, hep-ph/0609240 and other reviews quoted therein)

• However a recent estimation of the ratio of the two-photon width of the η_c' to that of the η_c by the CLEO collaboration (Asner et al) seems to contradict most of the existing theoretical predictions.

• CLEO gives $\Gamma_{\gamma\gamma}(\eta_c') = 1.3 \pm 0.6$ keV to be compared with the predicted values in the range 3.7 to 5.7 keV.

• The latest Belle measurement gives $\Gamma_{\gamma\gamma}(\eta_c') = 0.59 \pm 0.13 \pm 0.14$ keV, even smaller than the CLEO value.

• η_c' is the first radially-excited pseudoscalar charmonium 2^1S_0 with mass $M_{\eta_c'} = 3637.0 \pm 4.0\,\mathrm{MeV}$, width $\Gamma(\eta_{c'}) = 14.0 \pm 7.0\,\mathrm{MeV}$ (PDG-07) . Its first observation was done by Belle in $B \to K K_S K^- \pi^+$ decay and was further confirmed by BaBar.

• The nonrelativistic $\eta_c' \to \gamma\gamma$ decay rate differs from the decay rate for η_c only by the wave function at the origin. There exists also calculations using Bethe-Salpeter equation or relativistic quark model.

• The first excited state η_c' is more that $600\,\mathrm{MeV}$ above the η_c, the mass effects on the decay rate could be important. A better approach would be to use relativistic kinematics in the calculation of the decay rate.

• Recent relativistic calculations of two-photon decay rates for η_c , η_c' , η_b and η_b':

E. S. Ackleh and T. Barnes, (1991)

C. S. Kim, T. Lee and G. L. Wang, (2005)

M. R. Ahmady and R. R. Mendel, (1995)

K. T. Chao, H. W. Huang, J. H. Liu and J. Tang, (1997)

C. R. Munz, (1996)

D. Ebert, R. N. Faustov and V. O. Galkin, (2003)

S. B. Gerasimov and M. Majewski, (2005)

H. W. Crater, C. Y. Wong and P. Van Alstine, (2006)

O. Lakhina and E. S. Swanson, (2006)

S. Godfrey and N. Isgur, (1985)

N. Fabiano, (2003)

• This work :To derive an effective Lagrangian for the process $c\bar{c} \to \gamma\gamma$ by expanding the charm-quark propagator in powers of q^2/m_c^2 , with $q = p_c - p_{\bar{c}}$, and neglecting terms of $\mathcal{O}(q^2/m_c^2)$ terms.

• The two-photon decay amplitude for quarkonium is then given by the matrix element of the axial vector current $\bar{c}\gamma_\mu\gamma_5 c$ similar to the vector current $\bar{c}\gamma_\mu c$ for $J/\psi \to e^+e^-$.

• The matrix elements of the charmonium vector and axial vector currents computed with relativistic kinematic. For this purpose, relativistic spin projection operators are used (Kuhn, Kaplan and Safiani, 1979; Kuhn, Guberina, Peccei and Ruckl, 1979). This approach is similar to that used in Heavy Quark Effective Theory (HQET).

• This approach differs from the traditional one in the use of local operators for which the matrix elements could be measured or extracted from measured physical quantities, or computed from QCD sum rules (Novikov, Okun, Shifman, Vainshtein, Voloshin and Zakharov (1978); Reinders, Rubinstein , and Yazaki (1982); Dudek, Edwards and Richards (2006))

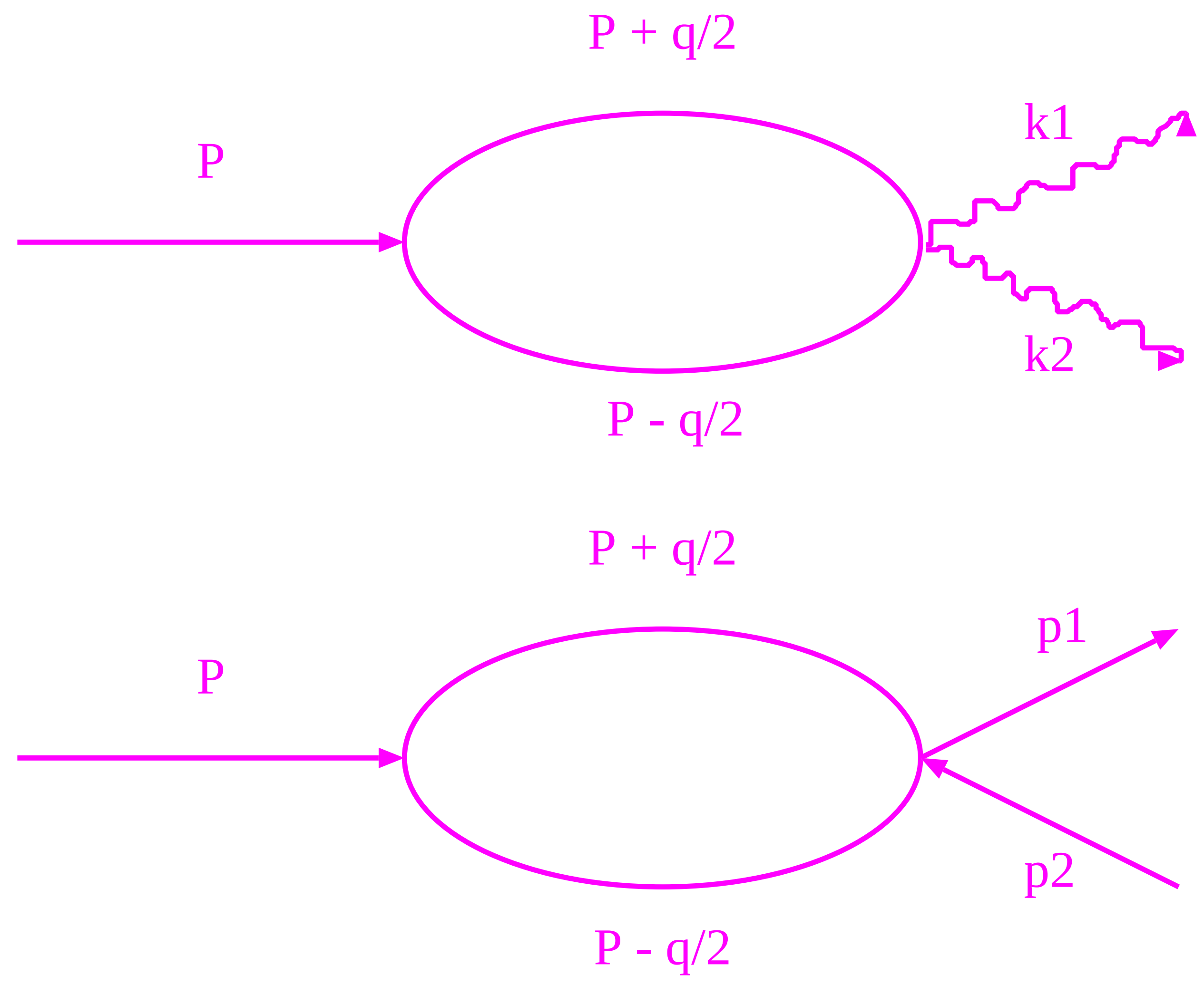

Figure 1: effective coupling between a $c\bar{c}$ and two-photon (upper) and a lepton pair

2 Effective Lagrangian for ${}^1S_0$ Decay into two-photon

• Decay of an S-wave Quarkonium into two photons and a dilepton pair $\ell\bar{\ell}$:

$$\begin{aligned}\mathcal{L}_{\rm eff}^{\gamma\gamma} &= -ic_1(\bar{c}\gamma_\sigma\gamma_5 c)\varepsilon_{\mu\nu\rho\sigma}F_{\mu\nu}A_\rho \\ \mathcal{L}_{\rm eff}^{\ell\bar{\ell}} &= -c_2(\bar{c}\gamma_\mu c)(\ell\gamma_\mu\bar{\ell})\end{aligned} \tag{1}$$

with

$$c_1 \simeq \frac{Q_c^2(4\pi\alpha_{\rm em})}{M_{\eta_c}^2 + b_{\eta_c}M_{\eta_c}}, \qquad c_2 = \frac{Q_c(4\pi\alpha_{\rm em})}{M_\psi^2} \tag{2}$$

The factor $1/(M_{\eta_c}^2 + b_{\eta_c}M_{\eta_c})$ in c_1 contains the binding-energy effects (the binding energy b is defined as $b = 2m_c - M$) and is obtained from the denominator of the charm-quark propagator (k_1, k_2 being the outgoing-photon momenta):

$$\frac{1}{[(k_1-k_2)^2/4 - m_c^2]} \tag{3}$$

- The decay amplitudes:

$$\mathcal{M}_{\ell\bar{\ell}} = Q_c(4\pi\alpha_{\rm em})\frac{f_\psi}{M_\psi}\varepsilon_\mu(\ell\gamma^\mu\bar{\ell})$$

$$\mathcal{M}_{\gamma\gamma} = -4iQ_c^2(4\pi\alpha_{\rm em})\frac{f_{\eta_c}}{M_{\eta_c}^2+b_{\eta_c}M_{\eta_c}}\varepsilon_{\mu\nu\rho\sigma}\varepsilon_1^\mu\varepsilon_2^\nu k_1^\rho k_2^\sigma \qquad (4)$$

$$\langle 0|\bar{c}\gamma_\mu c|\psi\rangle = f_\psi M_\psi \varepsilon^\mu, \quad \langle 0|\bar{c}\gamma_\mu\gamma_5 c|\eta_c\rangle = if_{\eta_c} \qquad (5)$$

from which the decay rates are:

$$\Gamma_{\ell\bar{\ell}}(\psi) = \frac{4\pi Q_c^2\alpha_{\rm em}^2 f_\psi^2}{3M_\psi}, \quad \Gamma_{\gamma\gamma}(\eta_c) = \frac{4\pi Q_c^4\alpha_{em}^2 f_{\eta_c}^2}{M_{\eta_c}}. \qquad (6)$$

- Recover the non-relativistic result by taking $M_\psi f_\psi^2 = 12|\psi(0)|^2$
- With NLO QCD radiative corrections:

$$\Gamma^{NLO}(^3S_1) = \Gamma^{LO}(^3S_1)\left(1-\frac{\alpha_s}{\pi}\frac{16}{3}\right) \qquad (7)$$

$$\Gamma^{NLO}(^1S_0) = \Gamma^{LO}(^1S_0)\left(1-\frac{\alpha_s}{\pi}\frac{(20-\pi^2)}{3}\right), \qquad (8)$$

3 Matrix elements of Local operators

• Matrix elements of local operators in a fermion-antifermion system with a given spin and angular momentum is given by:

$$\mathcal{A} = \int \frac{d^4 q}{(2\,\pi)^4}\,\mathrm{Tr}\,\mathcal{O}(q)\chi(P,q) \tag{9}$$

P is the total 4-momentum of the quarkonium system, q is the relative 4-momentum between the quark and anti-quark and $\chi(P,q)$ is the Bethe-Salpeter wave function. This expression is that of Kuhn, Kaplan and Safiani; Guberina, Kuhn, Peccei and Ruckl (1979).

• For a quarkonium system in a fixed total, orbital and spin angular momentum $\chi(P,q)$ is given by ($\mathbf{q}$ is the relative 3-momentum of the bound state)

$$\chi(P,q;J,J_z,L,S) = \sum_{M,S_z} 2\,\pi\delta(q^0 - \frac{\mathbf{q}^2}{2\,m})\psi_{LM}(\mathbf{q}) < LM;SS_z|JJ_z >$$

$$\times\sqrt{\frac{3}{m}}\sum_{s,\bar{s}} u(P/2+q,s)\bar{v}(P/2-q,\bar{s}) < \frac{1}{2}s;\frac{1}{2}\bar{s}|SS_z >$$

$$= \sum_{M,S_z} 2\,\pi\,\delta(q^0 - \frac{\mathbf{q}^2}{2\,m})\psi_{LM}(\mathbf{q})\mathcal{P}_{SS_z}(P,q) < LM;SS_z|JJ_z > \qquad (10)$$

The spin projection operators $\mathcal{P}_{SS_z}(P,q)$ are

$$\mathcal{P}_{0,0}(P,q) = \sqrt{\frac{3}{8m^3}}[-(\not{P}/2+\not{q})+m]\gamma_5\,[(\not{P}/2-\not{q})+m]$$

$$\mathcal{P}_{1,S_z}(P,q) = \sqrt{\frac{3}{8m^3}}[-(\not{P}/2+\not{q})+m]\not{\epsilon}(S_z)\,[(\not{P}/2-\not{q})+m] \qquad (11)$$

- For $S-$ wave quarkonium in a singlet $S=0$ and triplet $S=1$ state:

$$\mathcal{A}(^{2S+1}S_J) = \mathrm{Tr}\,(\mathcal{O}(0)\,\mathcal{P}_{J\,J_z}(P,0))\int\frac{d^3\,q}{(2\,\pi)^3}\,\psi_{00}(q) \qquad (12)$$

In this expression the q-dependence in the spin projection operator and in $\mathcal{O}$ has been dropped in the above expression and the integral in

Eq.(12) is the $S-$ state wave function at the origin (Guberina et al):

$$\int \frac{d^3 q}{(2\pi)^3} \psi_{00}(q) = \frac{1}{\sqrt{4\pi}} \mathcal{R}_0(0) \tag{13}$$

• Using Eq.(11) and Eq.(12) to compute the matrix elements $\langle 0|\bar{c}\gamma_\mu\gamma_5 c|P\rangle$ and $\langle 0|\bar{c}\gamma_\mu c|V\rangle$ for the singlet pseudo-scalar meson P and for the triplet vector meson V, we find, neglecting quadratic $O(q^2)$ terms.

$$\begin{aligned} f_P &= \sqrt{\frac{3}{32\pi m^3}} \mathcal{R}_0(0)\,(4\,m) \\ f_V &= \sqrt{\frac{3}{32\pi m^3}} \mathcal{R}_0(0)\,\frac{(M^2 + 4\,m^2)}{M} \end{aligned} \tag{14}$$

• f_P/f_V (from Eq.(14)) is only quadratic in the binding energy b, and is of the order $O(b^2/M^2)$. Thus the relation $f_P \simeq f_V$ is valid to a good approximation. It is expected that this relation holds also for excited state of charmonium and upsilon where the binding terms $O(b^2/M^2)$ can be neglected.

• A manifestation of heavy-quark spin symmetry(HQSS).

• HQSS and local operator expansion for $c\bar{c} \to \gamma\gamma$: Prediction of the two-photon decay rates of the singlet η_c and η_b.

• The ratio of the η_c two-photon width to J/ψ leptonic width:

$$R_{\eta_c} = \frac{\Gamma_{\gamma\gamma}(\eta_c)}{\Gamma_{\ell\bar{\ell}}(J/\psi)} = 3\,Q_c^2\,\frac{M_{J/\psi}}{M_{\eta_c}}\left(1 + \frac{\alpha_s}{\pi}\,\frac{(\pi^2 - 4)}{3}\right) \tag{15}$$

• HQSS prediction: With $f_{\eta_c} = f_{J/\psi}$ as shown above, $\Gamma_{\gamma\gamma}(\eta_c) = 7.46$ keV. With NLO QCD radiative corrections and $\alpha_s = 0.26$, $\Gamma_{\gamma\gamma}(\eta_c) = 9.66$ keV, somewhat higher than the world everage value $7.4 \pm 0.9 \pm 2.1$ keV.

• Thus the effective Lagrangian approach can successfully predict the η_c two-photon width in a simple, essentially model-independent manner.

4 HQSS predictions for $\Gamma_{\gamma\gamma}(\eta_c')$

- Extrapolating HQSS to $2S$ states, *i.e* $f_{\psi'} = f_{\eta_c'}$, and neglecting binding energy effects gives: $\Gamma_{\gamma\gamma}(\eta_c') = \Gamma_{\gamma\gamma}(\eta_c)\frac{f^2_{\psi'}}{f^2_{J/\psi}} = 3.45$ keV.
- The above predicted value is more than twice the evaluation by CLEO (1.3 ± 0.6 keV), but nearly in agreement with other theoretical calculations (Ackleh et al (3.7 keV), Kim et al (4.44 ± 0.48 keV), Ahmady et al ($5.7 \pm 0.5 \pm 0.6$ keV).
- Binding energy effects:

$$\Gamma_{\gamma\gamma}(\eta_c') = \Gamma_{\gamma\gamma}(\eta_c)\left(\frac{M^2_{\eta_c} + b_{\eta_c} M_{\eta_c}}{M^2_{\eta_c'} + b_{\eta_c'} M_{\eta_c'}}\right)^2 \frac{M^3_{\eta_c'}}{M^3_{\eta_c}}\left(\frac{\Gamma_{e^+e^-}(\psi')}{\Gamma_{e^+e^-}(J/\psi)}\frac{M_{\psi'}}{M_{J/\psi}}\right). \tag{16}$$

- For $M_{\eta_c} \simeq M_{J/\psi}$, $M_{\eta_c'} \simeq M_{\psi'}$, we have

$$\Gamma_{\gamma\gamma}(\eta_c') = \Gamma_{\gamma\gamma}(\eta_c)\left(\frac{1 + b_{\eta_c}/M_{\eta_c}}{1 + b_{\eta_c'}/M_{\eta_c'}}\right)^2 \left(\frac{\Gamma_{e^+e^-}(\psi')}{\Gamma_{e^+e^-}(J/\psi)}\right). \tag{17}$$

which gives

$$\Gamma_{\gamma\gamma}(\eta_c') = 4.1\,\mathrm{keV} \tag{18}$$

- Experiments and theoretical predictions:

$\Gamma_{\gamma\gamma}$	This paper	Ackleh	Kim	Ahmady	Münz	Chao	Ebert
η_c	$7.5-10$	4.8	7.14 ± 0.95	$11.8 \pm 0.8 \pm 0.6$	3.5 ± 0.4	5.5	5.5
η_c'	$3.5-4.5$	3.7	4.44 ± 0.48	$5.7 \pm 0.5 \pm 0.6$	1.38 ± 0.3	2.1	1.8

Table 1: Theoretical predictions for $\Gamma_{\gamma\gamma}(\eta_c)$ and $\Gamma_{\gamma\gamma}(\eta_c')$. (All values are in units of keV).

- The measured values are :

$$\begin{aligned}
\Gamma_{\gamma\gamma}(\eta_c) &= 7.7 \pm 0.4 \pm 0.5 \pm 2.2\,\mathrm{keV}, \quad \text{CLEO} \\
&= 7.48 \pm 0.5 \pm 0.73\,\mathrm{keV}, \quad \text{Belle} \\
\Gamma_{\gamma\gamma}(\eta_c') &= 1.3 \pm 0.6\,\mathrm{keV}, \quad \text{CLEO} \\
&= 0.59 \pm 0.13 \pm 0.14\,\mathrm{keV}, \quad \text{Belle}
\end{aligned} \tag{19}$$

- The latest Belle and BaBar values for the total widths are:

$$\begin{aligned}\Gamma(\eta_c) &= 36.6 \pm 1.5 \pm 2.0\,\mathrm{MeV}, \quad \text{Belle} \\ &= 34.3 \pm 2.3 \pm 0.9\,\mathrm{MeV}, \quad \text{BaBar}\end{aligned} \tag{20}$$

which is bigger than the CLEO and PDG-07 values:

$$\begin{aligned}\Gamma(\eta_c) &= 24.8 \pm 3.4 \pm 3.5\,\mathrm{MeV}, \quad \text{CLEO} \\ &= 25.5 \pm 3.4\,\mathrm{MeV}, \quad \mathrm{PDG}-07\end{aligned} \tag{21}$$

for $\eta_{c'}$, the Belle and BaBar measured values are

$$\begin{aligned}\Gamma(\eta_{c'}) &= 19.1 \pm 6.9 \pm 6.0\,\mathrm{MeV}, \quad \text{Belle} \\ &= 17.0 \pm 8.3 \pm 2.5\,\mathrm{MeV}, \quad \text{BaBar}\end{aligned} \tag{22}$$

$$\begin{aligned}\Gamma(\eta_{c'}) &= 6.3 \pm 12.4 \pm 4.0\,\mathrm{MeV}, \quad \text{CLEO} \\ &= 14.0 \pm 7.0\,\mathrm{MeV}, \quad \mathrm{PDG}-07\end{aligned} \tag{23}$$

- CLEO extraction of $\Gamma_{\gamma\gamma}(\eta_c')$

$$R(\eta_c'/\eta_c) = \frac{\Gamma_{\gamma\gamma}(\eta_c') \times \mathcal{B}(\eta_c' \to K_S K \pi)}{\Gamma_{\gamma\gamma}(\eta_c) \times \mathcal{B}(\eta_c \to K_S K \pi)} = 0.18 \pm 0.05 \pm 0.02 \qquad (24)$$

- To extract $\Gamma_{\gamma\gamma}(\eta_c')$ from the above data, CLEO assumes

$$\mathcal{B}(\eta_c' \to K_S K \pi) \approx \mathcal{B}(\eta_c \to K_S K \pi) \qquad (25)$$

and finds

$$\Gamma_{\gamma\gamma}(\eta_c') = 1.3 \pm 0.6\,\mathrm{keV} \qquad (26)$$

- Belle measurements of $B \to \eta_c K$ and $B \to \eta_c' K$

$$R(\eta_c' K/\eta_c K) = \frac{\mathcal{B}(B^0 \to \eta_c' K^0) \times \mathcal{B}(\eta_c' \to K_S K^+ \pi^-)}{\mathcal{B}(B^0 \to \eta_c K^0) \times \mathcal{B}(\eta_c \to K_S K^+ \pi^-)} = 0.38 \pm 0.12 \pm 0.05 \qquad (27)$$

- Using the approximate equality Eq.(25), one would obtain

$$\frac{\mathcal{B}(B^0 \to \eta_c' K^0)}{\mathcal{B}(B^0 \to \eta_c K^0)} \approx 0.4 \qquad (28)$$

which agrees more or less with the QCDF predicted value (Song *et al*, (2004)):

$$\frac{\mathcal{B}(B^0 \to \eta_c' K^0)}{\mathcal{B}(B^0 \to \eta_c K^0)} \approx 0.9 \times (\frac{f_{\eta_c'}}{f_{\eta_c}})^2 \approx 0.45 \tag{29}$$

• The extracted Belle value is close to the measured ratio obtained from the Babar measured values for the $B^+ \to \eta_c K^+$ and $B^+ \to \eta_c' K^+$ branching ratio (PDG, (2006))

$$\frac{\mathcal{B}(B^+ \to \eta_c' K^+)}{\mathcal{B}(B^+ \to \eta_c K^+)} = 0.38 \pm 0.25 \tag{30}$$

• This is expected since from $SU(2)$ flavor symmetry, one would expect the near equality between the ratios $\mathcal{B}(B^0 \to \eta_c' K^0)/\mathcal{B}(B^0 \to \eta_c K^0)$ and $\mathcal{B}(B^+ \to \eta_c' K^+)/\mathcal{B}(B^+ \to \eta_c K^+)$.

• Thus the assumption of the approximate equality between the $\eta_c' \to KK\pi$ and $\eta_c \to KK\pi$ branching ratio seems to be justified to some extent. This implies the small $\eta_c' \to \gamma\gamma$ decay rate quoted above.

• The good agreement with QCDF predictions for the measured ratio $\mathcal{B}(B^0 \to \eta_c' K^0)/\mathcal{B}(B^0 \to \eta_c K^0)$ and $\mathcal{B}(B^+ \to \eta_c' K^+)/\mathcal{B}(B^+ \to \eta_c K^+)$ at Belle and Babar suggests that $f_{\eta_c'}/f_{\eta_c} \approx f_{\psi'}/f_{J/\psi}$, hence the predicted value $\Gamma_{\gamma\gamma}(\eta_c') = 4.1\,\mathrm{keV}$.

• Comparing $R(\eta_c'/\eta_c)$ with $R(\eta_c' K/\eta_c K)$ and using the Belle data, we find

$$R(\eta_c'/\eta_c) \approx R(\eta_c' K/\eta_c K)/0.9 \tag{31}$$

which implies $R(\eta_c'/\eta_c) \approx 0.42 \pm 0.13 \pm 0.05$, twice bigger than the CLEO data.

• Two-photon branching ratio for η_c and $\eta_{c'}$ is given by

$$\mathcal{B}(\eta_c, \eta_{c'} \to \gamma\gamma) = \frac{9}{2}\, Q_c^4\, \frac{\alpha_{em}^2}{\alpha_s^2} \left(1 - 8.2\, \frac{\alpha_s}{\pi}\right) \tag{32}$$

with α_s evaluated at the appropriate scale.

• With $\alpha_s = 0.26$, Eq.(32) gives $\mathcal{B}(\eta_c \to \gamma\gamma) = 3.6 \times 10^{-4}$ to be compared with the measured value of $(2.8 \pm 0.9) \times 10^{-4}$(PDG), but this prediction is rather sensitive to α_s: with $\alpha_s = 0.28$, one would get

$\mathcal{B}(\eta_c \to \gamma\gamma) = 2.95 \times 10^{-4}$, in better agreement with experiment.

• From the measured η_c and $\eta_{c'}$ widths in Eq.(20) and Eq.(22), one gets $10.4\,\mathrm{keV}$ and $5.3\,\mathrm{keV}$ respectively for the η_c and $\eta_{c'}$ two-photon widths.

• Eq.(32) implies

$$\mathcal{B}(\eta_c \to \gamma\gamma) \approx \mathcal{B}(\eta_{c'} \to \gamma\gamma) \tag{33}$$

• This relation gives, from the measured $\mathcal{B}(\eta_c \to \gamma\gamma)$, $\Gamma(\eta_{c'} \to \gamma\gamma) = (4.1 \pm 2.3)\,\mathrm{keV}$, and $R(\eta_c'/\eta_c) \approx 0.40 \pm 0.20$ using the Belle, BaBar and CLEO data and the approximate equality Eq.(25).

• This result shows that it is difficult to understand the very small recent Belle measured $\eta_{c'}$ two-photon width.

• Lattice QCD(Dudek et al) result $f_{\eta_c'} \simeq f_{\psi'}/3$ could explain the very small new Belle measurement of $\Gamma_{\gamma\gamma}(\eta_c')$ but would make the total width of $\eta_{c'}$ much smaller than the measured value from CLEO, BaBar and Belle.

5 HQSS predictions for $\Gamma_{\gamma\gamma}(\eta_b)$ and $\Gamma_{\gamma\gamma}(\eta_b')$

• Since the b-quark mass is significantly higher than the c-quark mass, the effective Lagrangian and HQSS approach should work better for bottomonia decays to leptons and photons.

• Only a single candidate for the ground state (η_b) has been found by the Aleph collaboration, its mass was evaluated to be 9300 ± 28 MeV (Heister, PDG). Its predicted mass is $30 - 50$ MeV below the Υ mass.

• The η_b two-photon decay rate is given by:

$$R_{\eta_b} = \frac{\Gamma_{\gamma\gamma}(\eta_b)}{\Gamma_{\ell\bar{\ell}}(\Upsilon)} = 3\, Q_b^2\, \frac{M_\Upsilon}{M_{\eta_b}} \left(1 + \frac{\alpha_s}{\pi} \frac{(\pi^2 - 4)}{3}\right) \tag{34}$$

(neglecting the small b_{η_b}/M_{η_b} binding enery term.) This gives $\Gamma_{\gamma\gamma}(\eta_b) = 560\,\mathrm{eV}$ ($\alpha_s(M_\Upsilon) = 0.16$, $M_{\eta_b} = 9300$ MeV) .

• For η_b' and higher excited state, one has ($M_{\eta_b} \simeq M_\Upsilon$ and $M_{\eta_b'} \simeq M_{\Upsilon'}$)

$$\Gamma_{\gamma\gamma}(\eta_b') = \Gamma_{\gamma\gamma}(\eta_b) \left(\frac{1 + b_{\eta_b}/M_{\eta_b}}{1 + b_{\eta_b'}/M_{\eta_b'}} \right)^2 \left(\frac{\Gamma_{e^+e^-}(\Upsilon')}{\Gamma_{e^+e^-}(\Upsilon)} \right). \tag{35}$$

which gives $\Gamma_{\gamma\gamma}(\eta_b') = 250\,\mathrm{eV}$ and $\Gamma_{\gamma\gamma}(\eta_b'') = 187\,\mathrm{eV}$.

$\Gamma_{\gamma\gamma}$	This paper	Sch.	Lak.	Ack.	Kim	Ahm.	Mün.	Eb.	God.	Fab.	Pen.
η_b	560	460	230	170	384 ± 47	520	220 ± 40	350	214	466 ± 101	659 ± 92
η_b'	269	200	70	-	191 ± 25	-	110 ± 20	150	121	-	-
η_b''	208	-	40	-	-	-	84 ± 12	100	90.6	-	-

Table 2: Summary of theoretical predictions for $\Gamma_{\gamma\gamma}(\eta_b)$, $\Gamma_{\gamma\gamma}(\eta_b')$ and $\Gamma_{\gamma\gamma}(\eta_b'')$. (All values are in units of eV).

• Radiative corrections are cancelled up to corrections due to differences in the scales of α_s.

• Eq.(34) can be used to determine in a reliable way the value of α_s . The momentum scale at which α_s is to be evaluated here could be in principle be fixed with R_{η_b}

• Further check of consistency of the value for α_s by the branching ratio for $\eta_b \to \gamma\gamma$:

$$\frac{\Gamma_{\gamma\gamma}(\eta_b)}{\Gamma_{gg}(\eta_b)} = \frac{9}{2}\, Q_b^4\, \frac{\alpha_{em}^2}{\alpha_s^2} \left(1 - 7.8\, \frac{\alpha_s}{\pi}\right) \tag{36}$$

6 Conclusion

• Effective Lagrangian approach and HQSS can be used to compute quarkonium decays into lepton and photon with relativistic kinematic.

• The predicted $\eta_c' \to \gamma\gamma$ width is larger than the CLEO estimated value and the recent Belle measurement.

• Many relativistic calculations, could give a smaller value for $\eta_c' \to \gamma\gamma$ width but also produce smaller value for the $\eta_c \to \gamma\gamma$ width.

• Measurements of the two-photon widths for η_b and higher excited states could provide a test for HQSS and a determination of the α_s coupling constant at the scale around the Υ mass, as has been done with the Υ leptonic width in the past.

7 Acknowledgments

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