Can nuclear models be made predictive? Most nuclear models diverge when the are extrapolated beyond regions where they are fitted.

Nuclear matter a densities 2 to 3 times normal nuclear matter density.

Nuclei far from the stability line or embedded in a background of nuclear matter.

Get the relevant physics right before attempting to simplify codes or to set up *à priori* density functionals…

We discuss this issue in connection with the so-called “relativistic mean field theory”.

Nuclei and nuclear matter are systems composed of Dirac particles ψ which interact with chiral and vector fields. The system is described by the lagrangian:

$$L = \bar{\psi}\left(i\partial_\mu \gamma^\mu - g\varphi U + g_\omega \omega_\mu \gamma^\mu\right)\psi$$

$$+\frac{1}{4\nu} tr\left[\left(\partial_\mu \varphi\right)\left(\partial^\mu \varphi\right) + \varphi^2 \left(\partial_\mu U\right)\left(\partial^\mu U^\dagger\right) + b\varphi\left(U + U^\dagger\right)\right] - V\left(\varphi^2\right)$$

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\omega^2 \omega_\mu \omega^\mu$$

where φ is a scalar field and:

$$U = \exp\left(i\gamma_5 \tau_a \theta_a\right) \qquad \omega^\mu = \left(\phi, \vec{\omega}\right)$$

The chiral symmetry breaking term gives the pion a mass.

A primary concern is the spin-orbit interaction.

In the Dirac representation:

$$h = \frac{\vec{\alpha}\cdot\vec{\nabla}}{i} + \beta g\varphi - g_\omega\phi = \begin{pmatrix} g\varphi - g_\omega\phi & \frac{\vec{\sigma}\cdot\vec{\nabla}}{i} \\ \frac{\vec{\sigma}\cdot\vec{\nabla}}{i} & -g\varphi - g_\omega\phi \end{pmatrix}$$

Expand the matrix element of h between positive energy states to order $\frac{k^2}{m_0^2}$:

$$\left\langle \vec{k}\sigma \left| h_{eff} \right| \vec{k}'\sigma' \right\rangle = \left\langle (m_0)\,\vec{k}\sigma + |h|\,(m_0)\,\vec{k}'\sigma' + \right\rangle$$

$$h_{eff} = m_0 - \frac{\nabla^2}{2m_0}$$

$$+g(\varphi - \varphi_0) - g_\omega\phi + \frac{\nabla^2}{4m_0^2}\left(g(\varphi - \varphi_0) - g_\omega\phi\right) + \left(g(\varphi - \varphi_0) - g_\omega\phi\right) \frac{\;}{4}$$

$$+\frac{1}{4m_0^2}\vec{\nabla}\cdot(g\varphi + g_\omega\phi)\,\vec{\nabla} - \frac{1}{4m_0^2}\vec{l}.\vec{\sigma}\left[\frac{1}{r}\frac{d}{dr}(g\varphi + g_\omega\phi)\right]$$

The spin orbit interaction is caused by the field $(g\varphi + g_\omega\phi)$. In nuclei $g(\varphi - \varphi_0)$ almost cancells $-g_\omega\phi$.

For example, the Finelli,Kaiser,Vretenar,Weise group estimate the scalar and vector fields $g\varphi$ and $g_\omega \phi$ from QCD sum rule estimates. Using also the Gell-Mann Oakes Renner relation they estimate:

$$\frac{g\varphi - g\varphi_0}{g_\omega \phi} = -\frac{1}{4}\frac{\sigma_N}{m_u + m_d}\frac{\rho_S}{\rho} \simeq -1$$

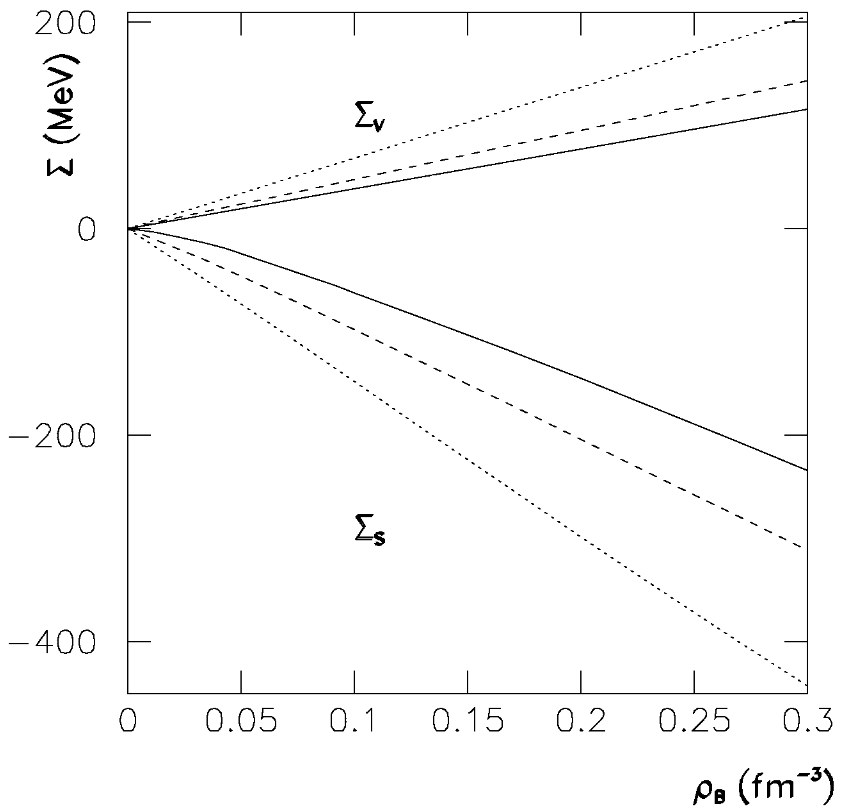
$$g\varphi - g\varphi_0 = m - m_0 = -\frac{\sigma_N m_0}{m_\pi^2 f_\pi^2}\rho_s \simeq -320 \ MeV$$

with $\sigma_N \simeq 45 \ MeV$ and $m_u + m_d \simeq 12 \ MeV$. There is an estimated 10% to 20% error in these estimates.

The Huguet,Caillon,Labarsouque group (nucl-th/0712.3661) use a quark-diquark model of the nucleon, based on a NJL lagrangian to estimate the change in mass of the nucleon due to the change in the quark condensate at finite density. They calculate the corresponding scalar and vector self-energies:

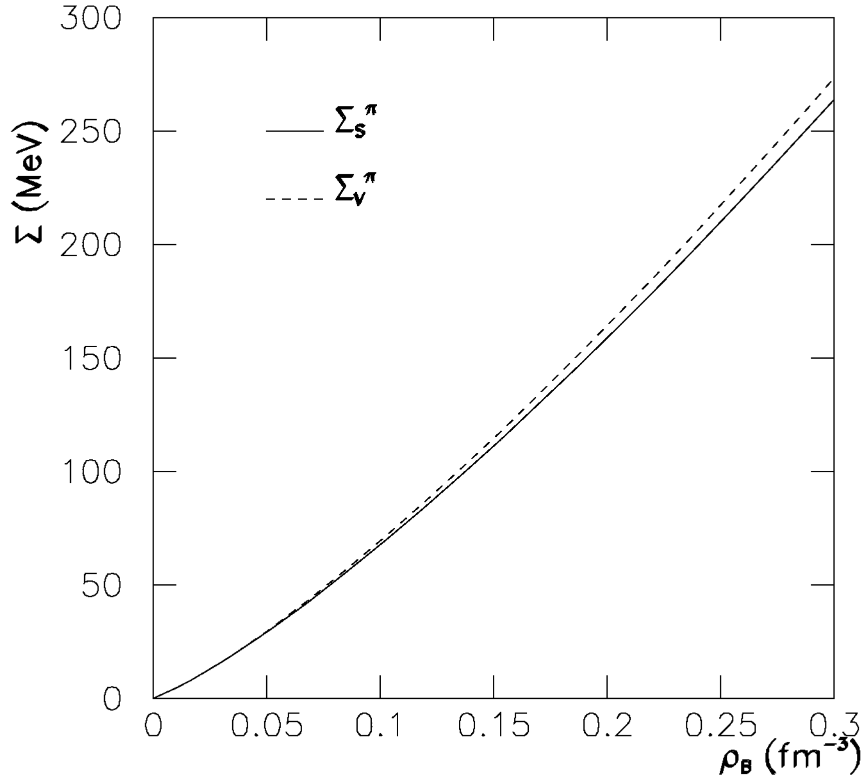
$$\Sigma_S = g\,(\varphi - \varphi_0) \qquad \Sigma_V = g_\omega \phi$$

Huguet,Caillon,Labarsouque (nucl-th/0712.3661)



- They then use chiral perturbation theory to calculate the exchange (Fock) term and the 2p-2h excitations produced by the pion, including excitation of the Δ
- Such terms also contribute to the scalar and vector self-energies of the nucleon.

Huguet,Caillon,Labarsouque (nucl-th/0712.3661



To simplify this SPA exposé, we neglect the ρ meson field, which acts in a $N \neq Z$ nuclei and we consider the case of a spherically symmetric closed shell nucleus. When nucleons couple to scalar and vector classical fields, the latter are are determined by the energy functional (similar to the original Walecka model):

$$E\left(\varphi,\phi\right) - E_0 = \sum_{\lambda \in F} \left\langle \lambda \left| \frac{\vec{\alpha}\cdot\vec{\nabla}}{i} + g\beta\varphi - g_\omega \phi \right| \lambda \right\rangle$$

$$+ \sum_{\lambda \in D} \left\langle \lambda \left| \frac{\vec{\alpha}\cdot\vec{\nabla}}{i} + g\beta\varphi - g_\omega \phi \right| \lambda \right\rangle$$

$$+\nu \sum_{\vec{k}} \sqrt{k^2 + g^2\varphi_0^2} + \rho_0 g_\omega \int d^3 r \phi$$

$$+ \int \left[\frac{1}{2}\left(\vec{\nabla}\varphi\right)^2 + V\left(\varphi^2\right) - V\left(\varphi_0^2\right) - \frac{1}{2}\left(\vec{\nabla}\phi\right)^2 - \frac{1}{2} m_\omega^2 \phi^2\right]$$

In nuclear matter, the energy per unit volume reduces to:

$$\frac{E(m)}{V} = \frac{\nu}{V} \sum_{k<k_F} \sqrt{k^2 + m^2} \quad (Fermi\ sea\ contribution)$$

$$-\frac{\nu}{V} \sum_{\vec{k}} \left(\sqrt{k^2 + m^2} - \sqrt{k^2 + m_0^2} \right) \quad (Dirac\ sea\ contribution)$$

$$+V\left(\frac{m^2}{g^2}\right) - V\left(\frac{m_0^2}{g^2}\right) + \frac{1}{2}\frac{g_\omega^2 \rho^2}{m_\omega^2} \quad (scalar\ and\ vector\ field\ contributions)$$

where $m = g\varphi$ and $m_0 = g\varphi_0$.

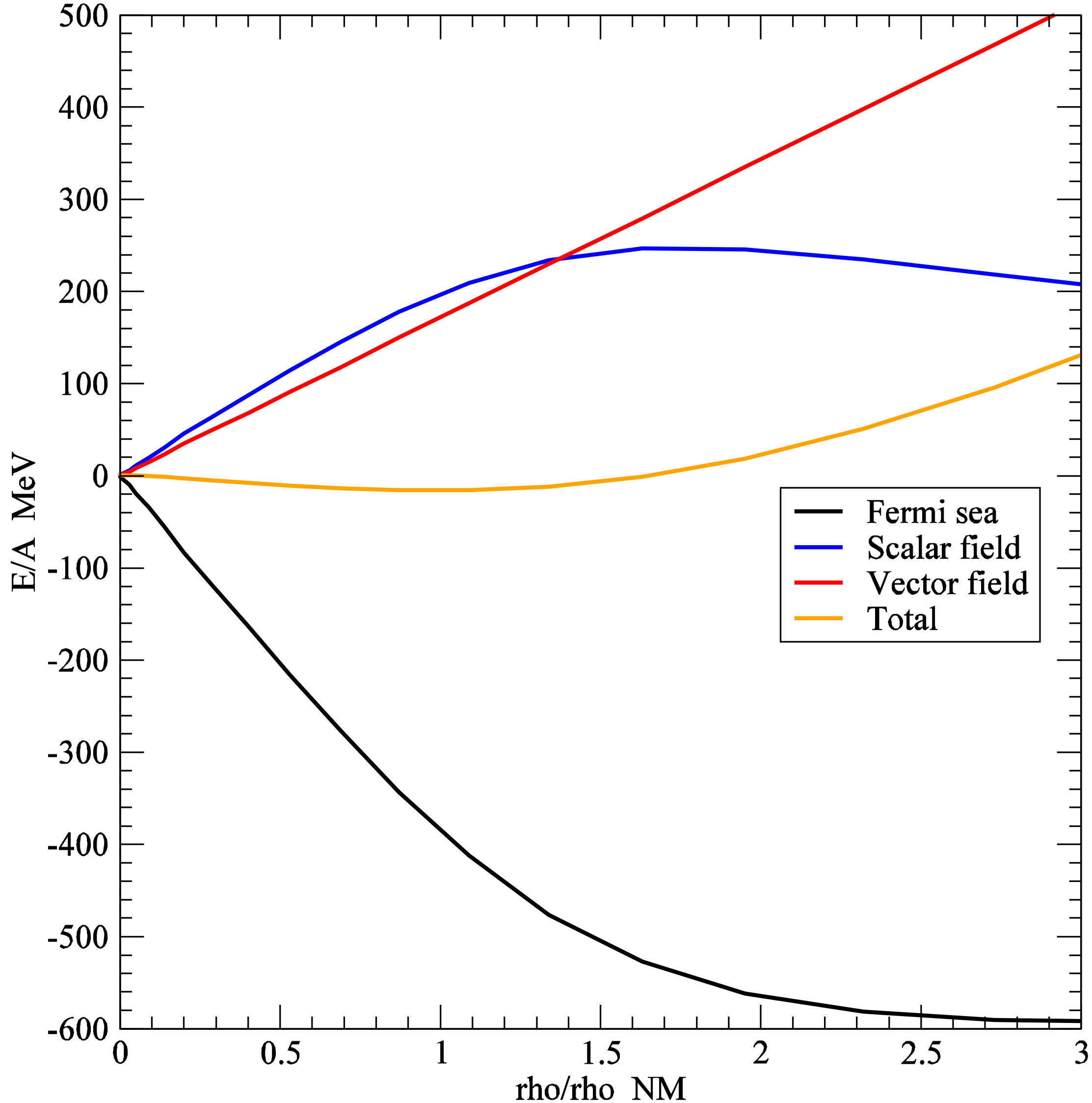
Usual approximations made in practically all applications so far:

- $$V\left(\frac{m^2}{g^2}\right) - V\left(\frac{m_0^2}{g^2}\right) \simeq \frac{1}{2}\frac{m_\sigma^2}{g^2}(m - m_0)^2$$

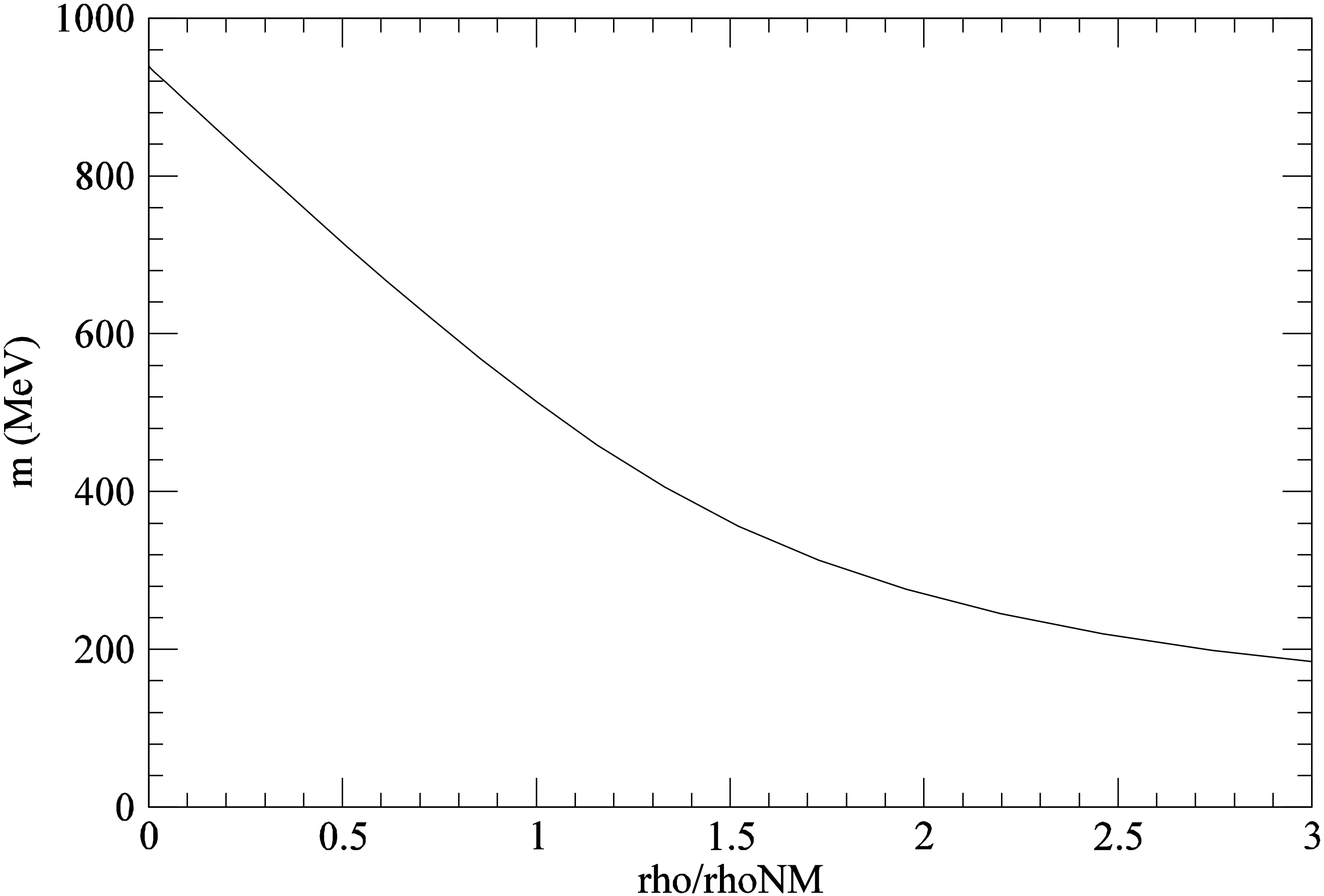
- no-Dirac sea approximation: simply omit the Dirac sea contribution.

Various contributions to E/A of symmetric nuclear matter.

(eovera3.8g)



Nucleon mass as a function of the density.



Problems concerning extrapolation to higher (or lower) densities:

- Very strong dependence of various contributions on the density: nuclear saturation can be considerably modified by small changes in the parameters of the model.

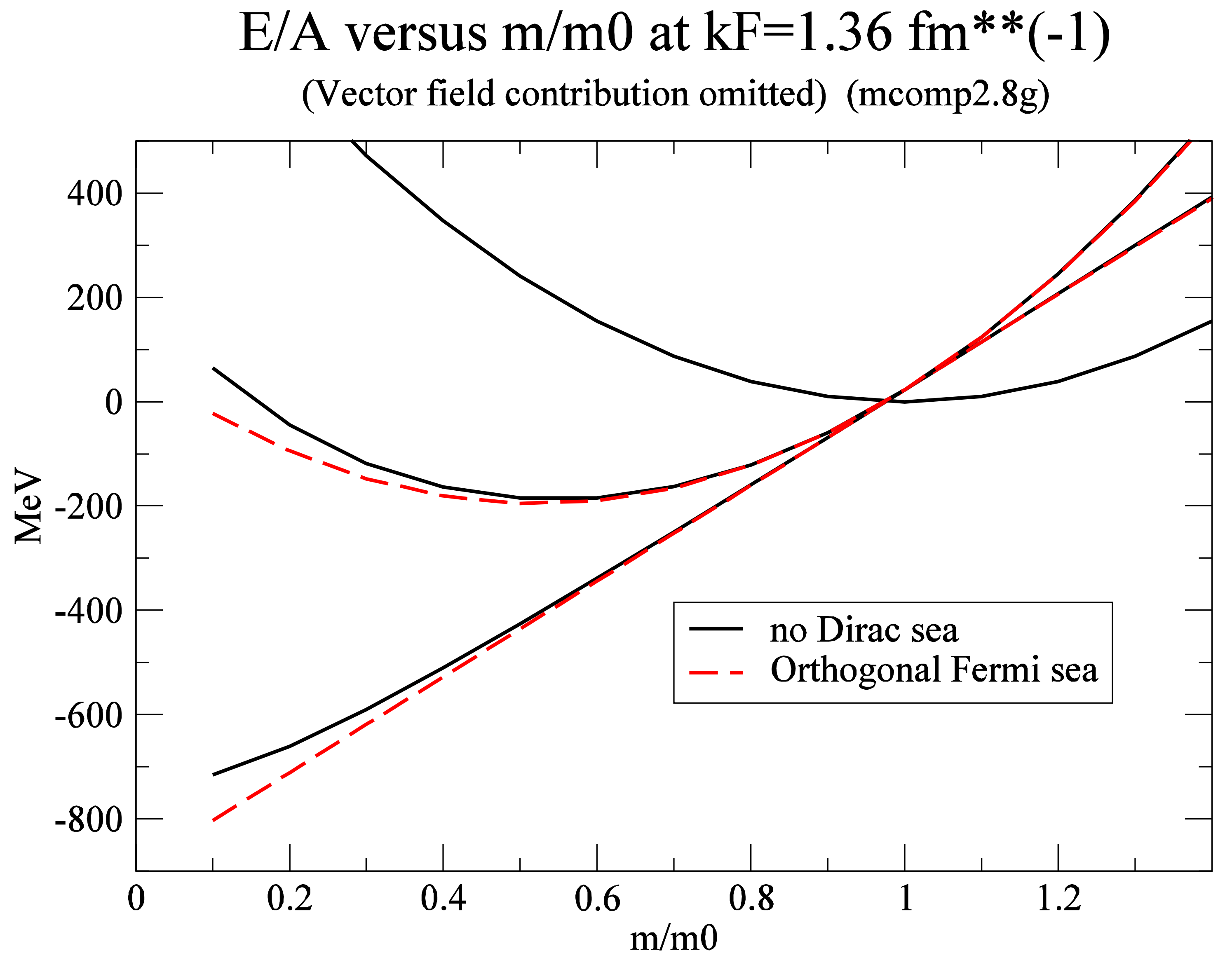
- The neglect of the Dirac sea contribution.

When the mass of a Dirac particle is reduced by about 40%, the overlap of a positive energy Dirac orbit $\left|+\vec{k}\sigma\tau\right\rangle$ with the corresponding negative energy obrbit $\left|-\vec{k}\sigma\tau\right\rangle$ is on the average 6% for $k = k_F = 1.36\ fm^{-1}$ at normal nuclear density and it reaches 17% at twice nuclear matter density. If the contribution of the Dirac sea is neglected, the Fermi sea orbits should be kept orthogonal to the neglected unperturbed Dirac sea orbits. The Dirac hamiltonian $h = \frac{\vec{\alpha}\cdot\vec{\nabla}}{i} + \beta m - g_\omega\phi$ should be replaced by the hamiltonian h_{eff} such that:

$$\left\langle \vec{k}\sigma \left| h_{eff} \right| \vec{k}\sigma \right\rangle$$

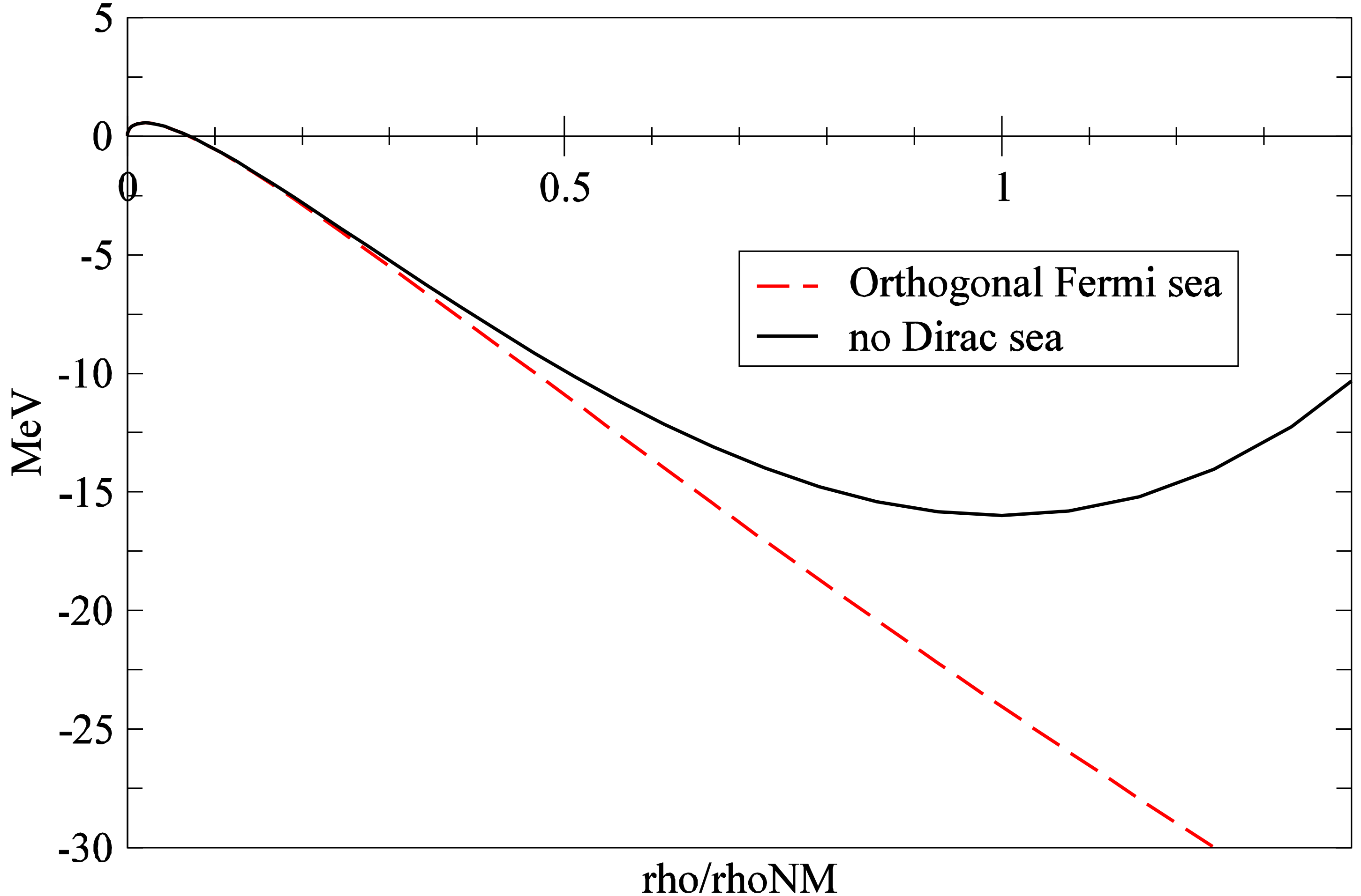
$$= \left\langle +(m_0)\,\vec{k}\sigma \left| \frac{\vec{\alpha}\cdot\vec{\nabla}}{i} + \beta m - g_\omega\phi \right| +(m_0)\,\vec{k}\sigma \right\rangle$$

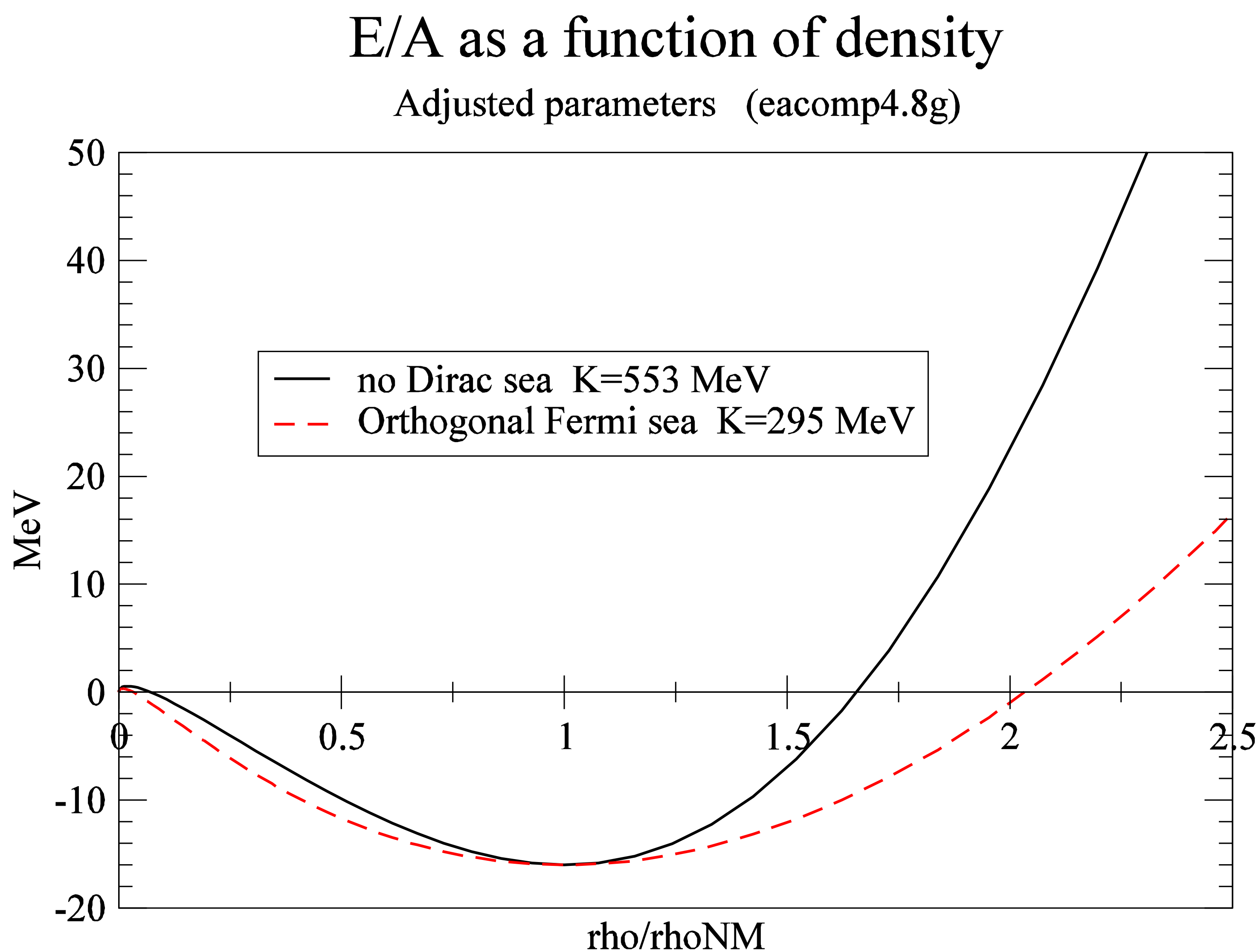
$$= \left[\sqrt{k^2+m_0^2} - g_\omega\phi + (m-m_0)\,\frac{m_0}{\sqrt{k^2+m_0^2}} \right]$$


E/A versus m/m0 at kF=1.36 fm**(-1)
(Vector field contribution omitted) (mcomp2.8g)
MeV
400
200
0
-200
-400
-600
-800
0
0.2
0.4
0.6
0.8
1
1.2
m/m0
no Dirac sea
Orthogonal Fermi sea

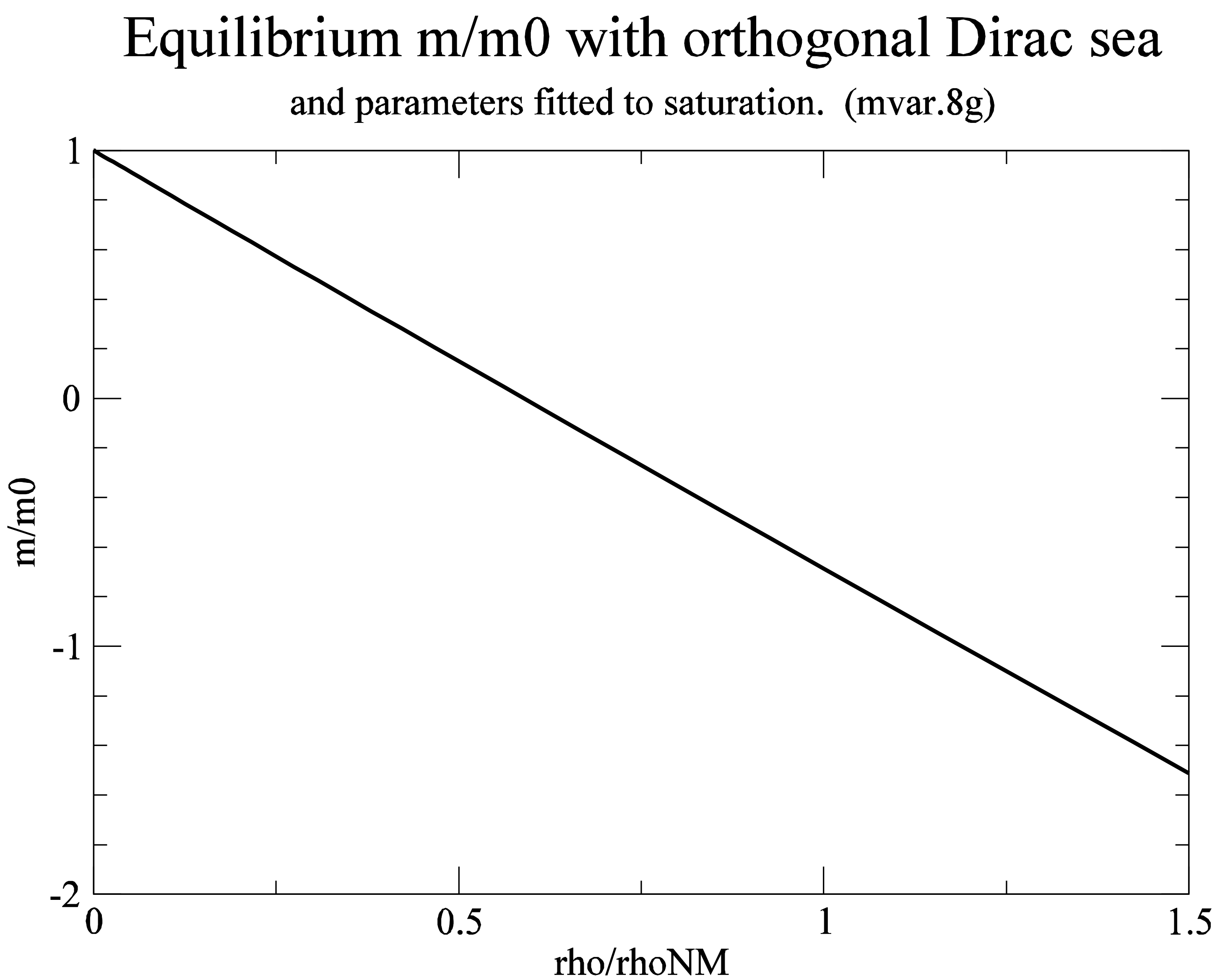
E/A in nuclear matter as a function of density

(eacomp2.8g)





E/A as a function of density
Adjusted parameters (eacomp4.8g)
MeV
rho/rhoNM
no Dirac sea K=553 MeV
Orthogonal Fermi sea K=295 MeV
50
40
30
20
10
0
-10
-20
0
0.5
1
1.5
2
2.5


Equilibrium m/m0 with orthogonal Dirac sea
and parameters fitted to saturation. (mvar.8g)
1
0
-1
-2
m/m0
0
0.5
1
1.5
rho/rhoNM