Diffusion and particle production in relativistic systems

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Topics

1. Introduction
2. Relativistic Diffusion Model for R(y;t) with three sources
3. Net protons dn/dy at SPS, RHIC and LHC energies: Au + Au, Pb + Pb. Linear diffusion model
4. Produced charged particles dn/dη at RHIC [and LHC]: Analytical solution of the nonlinear diffusion problem. Asymmetric and symmetric systems
5. Conclusion

1. Relativistic heavy-ion collisions

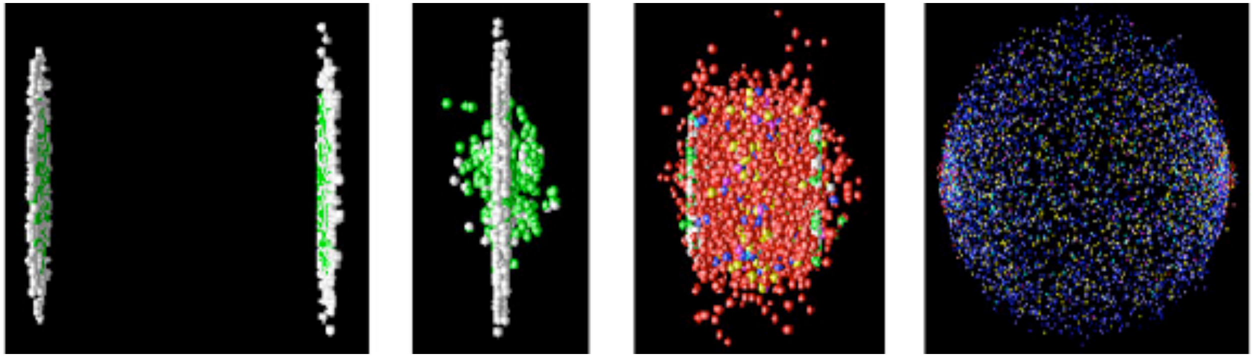


Fig. Courtesy U Frankfurt

In central collisions of Au-Au at $\sqrt{s}$=200 GeV/particle pair,
the partons in 14% of the incoming baryons are likely to be deconfined.
[cf. GW, Phys. Rev. C 69, 024906(2004)]

A large fraction of the produced particles is in
thermal equilibrium
[cf. GW et al., Annalen Phys. 15, 369 (2006)]

Diffusion: A well-studied problem in physics, biology, ..

- Lord Rayleigh, Phil. Mag. 32, 424 (1891):
  First use of a »Fokker-Planck« equation (FPE)
- A. Einstein, Ann. Phys. 17, 549 (1905); 19, 289 (1906)
  Brownian motion, FPE
- M.v. Smoluchowski, Ann. Phys. 21, 756 (1906) Brownian motion
- A.D. Fokker, Ann. Phys. 43, 810 (1914)
  (re-)introduction of the »FPE« into the literature
- M. Planck, Sitzber. Preuß. Akad. Wiss. p. 324 (1917)
  first derivation of the FPE

Review article on the history: N.G. van Kampen, Phys. Bl. 53, 1012 (1997)
(german language)

Diffusion in Relativistic Heavy-Ion collisions

- Due to the large number of produced particles (about 5000 charged hadrons in a central Au + Au collision at 200 GeV per nucleon pair), there is diffusion in momentum space.

- Associated with it is the corresponding diffusion in coordinate space.

- The general problem is 7-dim in (**x**,**p**,t).

- Choose proper relativistic variables to provide an analytical treatment that can directly be compared to data. In this work: (rapidity y, time t).

2. Relativistic Diffusion Model

In rapidity space y=0.5 ln ((E+p)/(E-p)) the model ist based on the generalized (nonlinear) Fokker-Planck equation for the distribution function R(y,t); for a single source:

$$\frac{\partial}{\partial t} R(y,t) = -\nabla_y \Big[J(y) R(y,t) \Big] + \nabla_y D_y(R(y,t)) \nabla_y R(y,t).$$

$$J(y) = (y_{eq} - y)/\tau_y \qquad D_y[R(y,t)] = D_y^p \cdot R(y,t)^\kappa$$

$$\nabla_y = \frac{\partial}{\partial y}$$

Drift term with relaxation time τ_y

Nonlinear diffusion term: $\kappa \neq 0$ in the high-density particle production phase

In a two-step model, the nonlinearity in the diffusion is taken into account only in the high-density particle-production phase. The linear evolution is valid for most of the interaction time of 5..10 fm/c.

Linear Diffusion Model

The linearized model for the distribution function R(y,t) can be solved analytically for a single source, and the 3 sources k=1,2,3 are then added incoherently

$$\frac{\partial}{\partial t} R(y,t) = -\nabla_y \Big[J(y) R(y,t) \Big] + D_y \nabla_y^2 R(y,t).$$

$$J(y) = (y_{eq} - y)/\tau_y$$

Drift term with relaxation time τ_y

Linear diffusion term, D_y=const.

Drift and diffusion coefficient are related through a dissipation-fluctuation theorem

Linear Model κ=0: GW, Z. Phys. A 355, 301 (1996); EPJ A5, 85(1999)
Phys. Lett. B 569, 67 (2003), 3 sources

Dissipation-fluctuation relation in y- space

The rapidity diffusion coefficient D_y is calculated from τ_y and the equilibrium temperature T in the weak-coupling limit as

$$D_y(\tau_y, T) = \frac{1}{2\pi\tau_y}\left[c(\sqrt{s}, T)m^2 T \times \left(1 + 2\frac{T}{m} + 2\left(\frac{T}{m}\right)^2\right)\right]^{-2} \exp\left(\frac{2m}{T}\right)$$

GW, Eur. Phys. Lett. 47, 30 (1999)

Note that $D \sim T|\tau_y$ as in the Einstein relation of Brownian motion.
The diffusion coefficient as obtained from this statistical consideration is further enhanced (D_y^{eff}) due to collective expansion.

Linear RDM in y- space

In a moments expansion and for δ–function initial conditions, the mean values become

$$< y_{1,2}(t) >= y_{eq}[1 - exp(-t/\tau_y)] \mp y_{max} \exp{(-t/\tau_y)}$$

$$<y_3(t)>=y_{eq}$$

and the variances are

$$\sigma^2_{1,2,eq}(t) = D_y^{1,2,eq}\tau_y[1 - \exp(-2t/\tau_y)]$$

The rapidity relaxation time τ_y determines the peak positions
The rapidity diffusion coefficient D_y determines the variances.

$D \sim T|\tau_y$ as in Brownian motion

Linear RDM in y-space

The equilibrium value y_{eq} that appears in the drift function $J(y) = -(y-y_{eq})/\tau_y$ (and in the mean value of the rapidity) is obtained from energy- and momentum conservation in the subsystem of participants as

$$y_{eq} := \frac{1}{2}\cdot\ln\left(\frac{m_1{}^t\cdot e^{y_b} + m_2{}^t\cdot e^{-y_b}}{m_2{}^t\cdot e^{y_b} + m_1{}^t\cdot e^{-y_b}}\right)$$

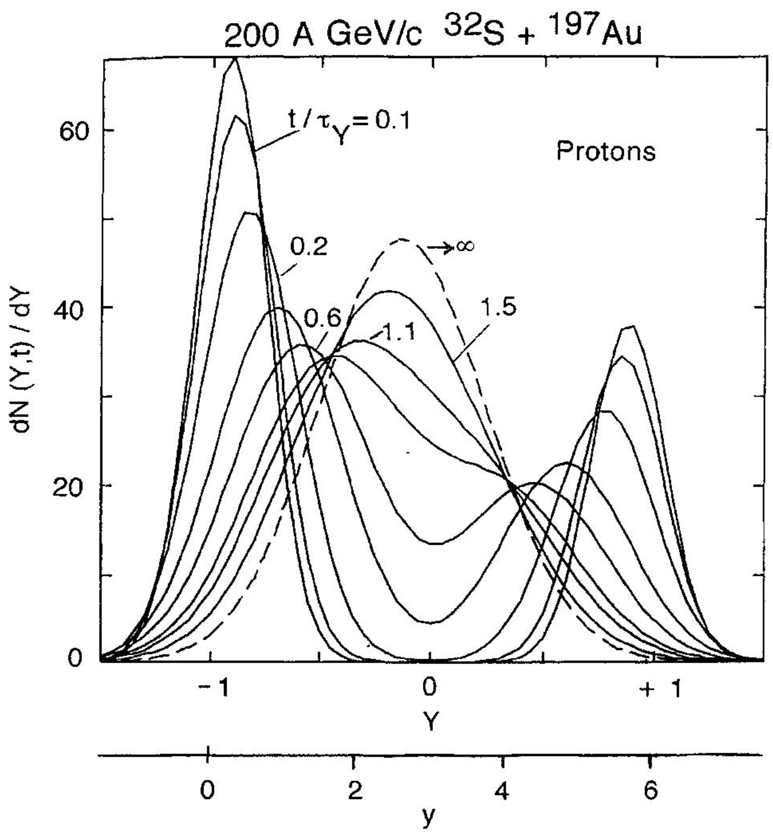
with the beam rapidity y_b, and transverse masses

$$m_k{}^t := \sqrt{m_k{}^2 + \left(p_k{}^t\right)^2}$$

m_k = participant masses (k=1,2)

y_{eq} = 0 for symmetric systems but ≠ 0 for asymmetric systems

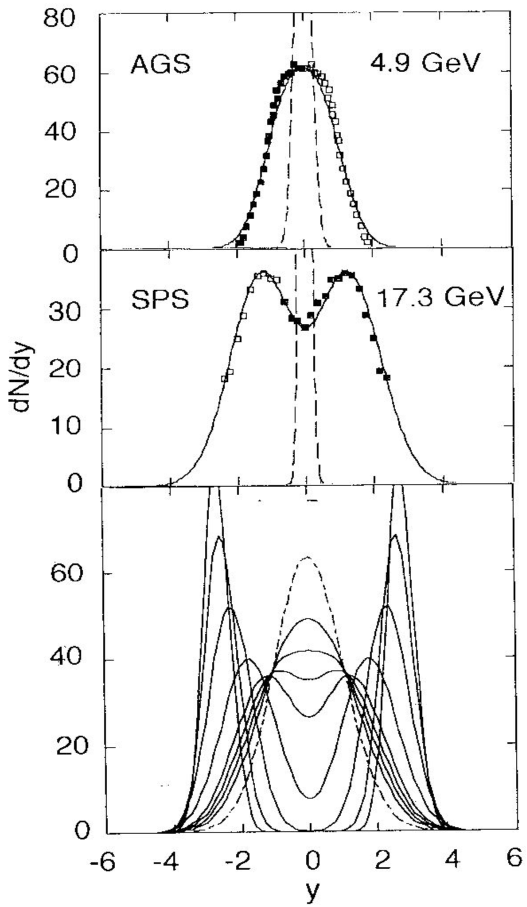
3. RDM: time evolution of the rapidity density for net protons - 2 sources



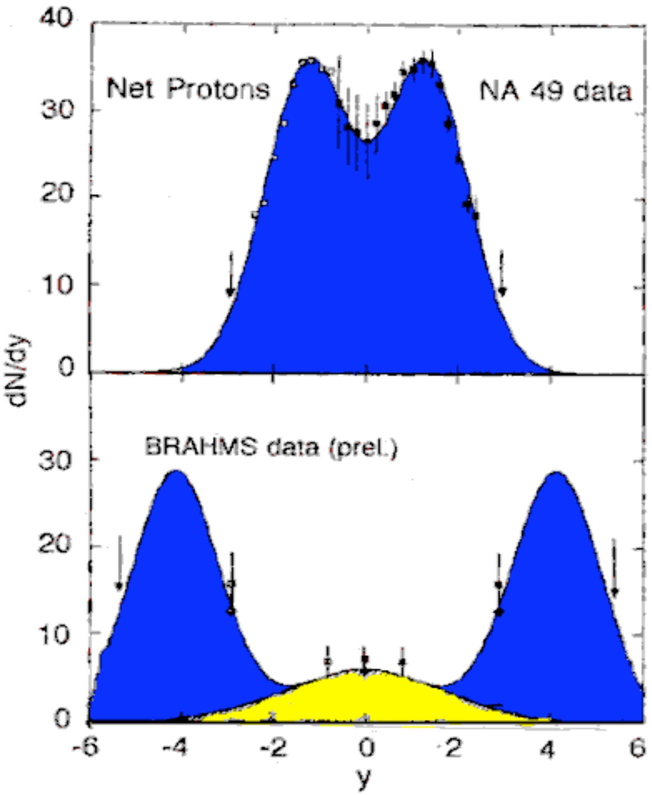
- Time evolution for 200 A GeV/c S+Au, linear model; δ–function initial conditions
- Selected weighted solutions of the transport eq. at various values of t/τ_y
- The stationary solution (Gaussian) is approached for t/τ_y »1

Central Collisions at AGS, SPS

- Rapidity density distributions evolve from bell-shape to double-hump as the energy increases from AGS (4.9 GeV) to SPS (17.3 GeV)
- Diffusion-model solutions are shown for SPS energies



Central Au+Au @ RHIC vs. SPS



- BRAHMS data at $\sqrt{s_{NN}}$=200 GeV for net protons
- Central 10% of the cross section
- Relativistic Diffusion Model for the nonequilibrium contributions

- 3rd source at midrapidity: "soft" physics not sufficient at RHIC, partonic processes are important

GW, PLB 569, 67 (2003) and Phys. Rev. C 69, 024906 (2004)

3-sources RDM for Au+Au: Net protons

- Incoherent superposition of nonequilibrium and equilibrium solutions of the transport equation yields a very satisfactory representation of the data
- The midrapidity source of the distribution function is likely to be of partonic origin. It corresponds to a rapidly expanding local thermal equilibrium distribution.

$$\frac{dN(y, t_{int})}{dy} = N_1 \cdot R_1(y, t_{int}) + N_2 \cdot R_2(y, t_{int}) + N_{eq} \cdot R_{eq}(y)$$

N_{eq}~ 56 baryons (22 protons)

$N_{1,2}$~169 baryons (68 protons)

GW, Phys. Lett. B 569, 67 (2003); Phys. Rev. C 69, 024906 (2004)

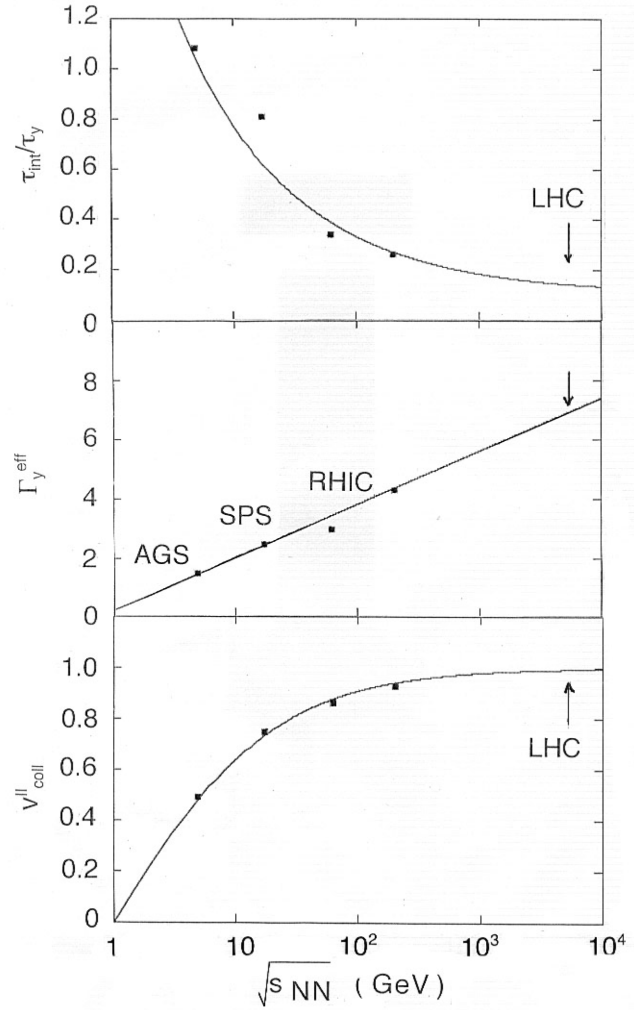
Heavy relativistic systems: energy dependence

Parameters for heavy relativistic systems at AGS, SPS and RHIC, and extrapolated to LHC energies.

- τ_{int}/τ_y : interaction time/rapidity relaxation time
- $\Gamma_y^{eff:}$ effective rapidity width coefficient

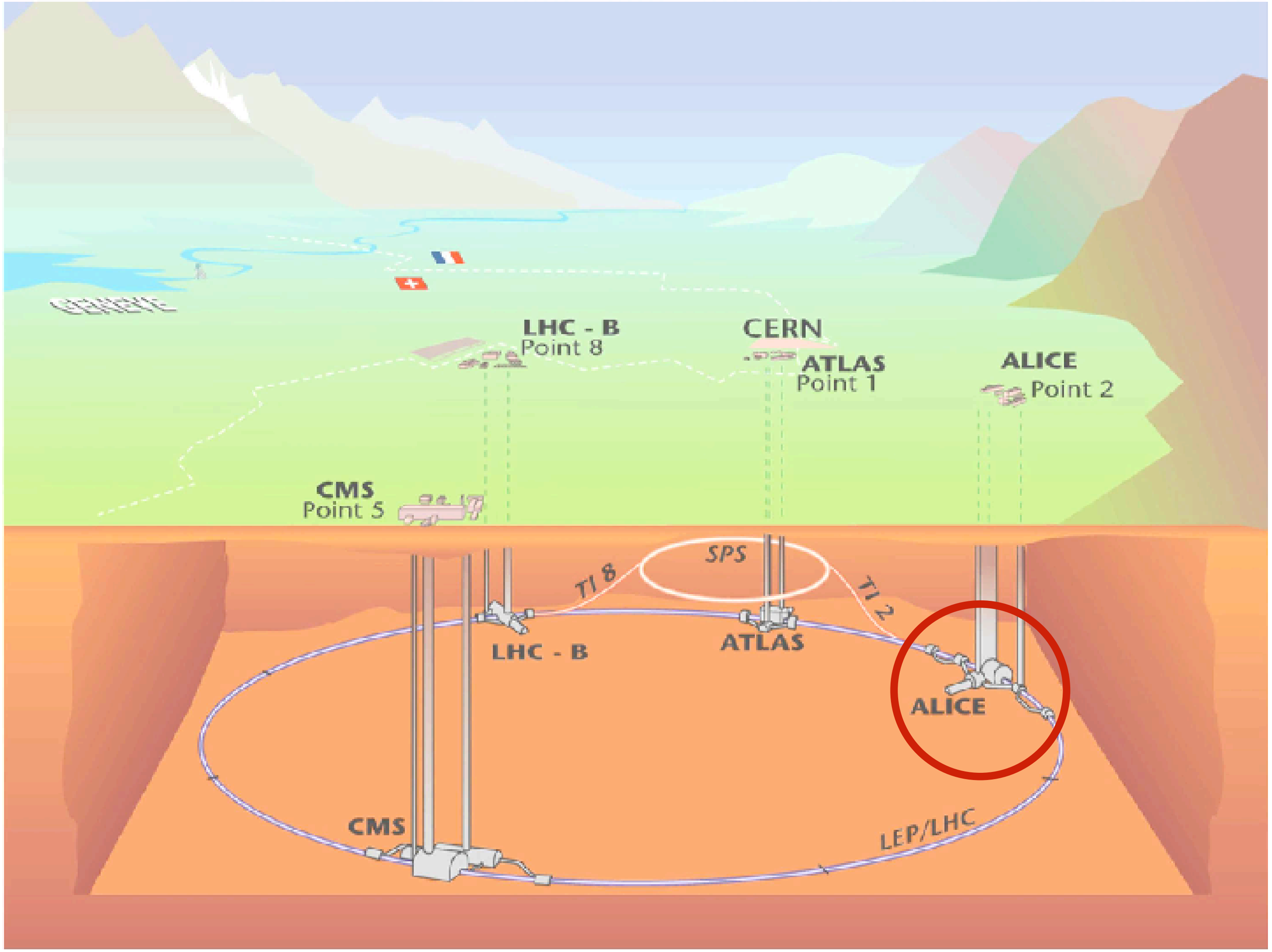
$$\Gamma_y^{eff} = (8 \cdot \ln 2 \cdot D_y^{eff} \cdot \tau_y)^{1/2}$$

- v_{coll} : the longitudinal expansion velocity (cf. GW, Europhys. Lett. 74, 29 (2006)).

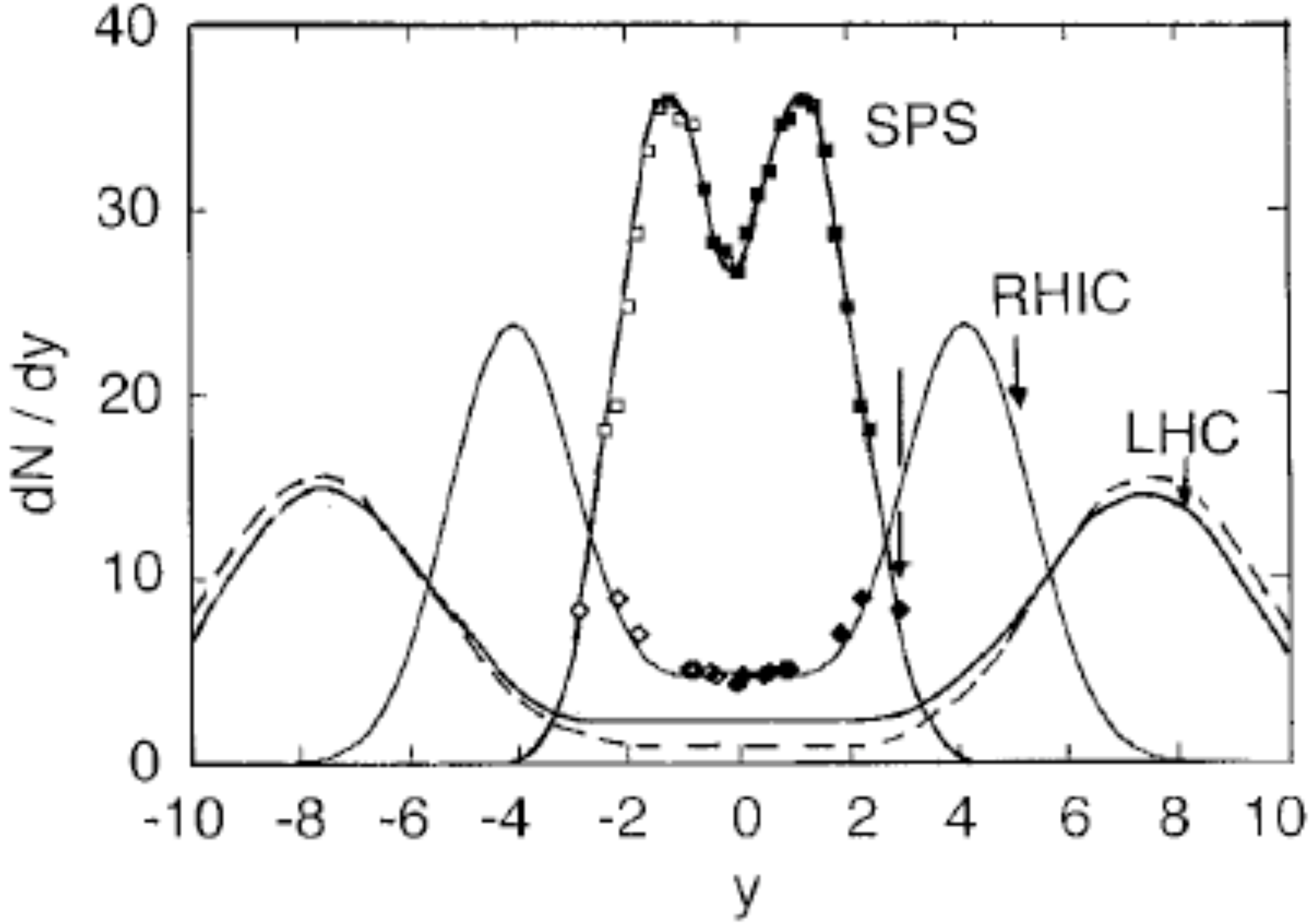


LHC @CERN

Pb + Pb at LHC: 5520 GeV/particle pair



Heavy relativistic systems: Net protons



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4. Particle production and diffusion

Two-step model: Nonlinear (density-dependent) diffusion during the initial high-density particle-production phase of < 1 fm/c, then transition to linear diffusion during the interaction time of 7-10 fm/c:

$$\frac{\partial}{\partial t} P(y,t) = D_y^p \nabla_y P(y,t)^\kappa \nabla_y P(y,t) \qquad t^* = t \cdot D_y^p$$

$$\frac{\partial}{\partial t^*} P(y,t^*) = \nabla_y P(y,t^*)^\kappa \nabla_y P(y,t^*)$$

Similarity solutions of the nonlinear equation:

$$P(y,t) = y^{2\lambda/\kappa(1+\lambda)} \Phi(\xi) \qquad \xi = y^{1/(1+\lambda)}/t^{1/2}$$

$$\lambda_1 = -\kappa/(\kappa+2)$$

$$\lambda_2 = -\kappa/(\kappa+1)$$

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Source solution (Hill&Hill 1990)

$$\frac{\Phi^{\kappa}\Phi'}{\xi} - \frac{2\Phi^{\kappa+1}}{(\kappa+2)\xi^2} + \frac{2\Phi}{(\kappa+2)^2} = C_1 \qquad \Phi' = \partial\Phi/\partial\xi$$

For κ=-1 and C_1 =0 (For κ=-1 instantaneous spread in rapidity space without a free boundary, as expected in particle production.)

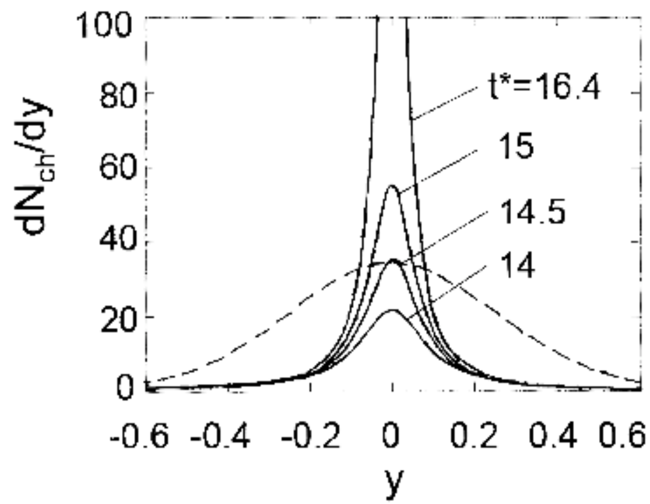
$$\frac{\Phi'}{\xi\Phi} - \frac{2}{\xi^2} + 2\Phi = 0$$

The solution becomes with $\xi = (y/t)^{1/2}$

$$P(y,t) = [C_2 y\xi^{-1} + y\xi^2/2]^{-1}$$

C_2 depends on particle number. This reduces to

$$P(y,t) = [C_2 t + y^2/(2t)]^{-1}$$



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Using the power-law result as an initial condition for the linear diffusion problem yields (Y.Mehtar-Tani &GW)

$$R(y,t) = \frac{C}{2a}\sqrt{\frac{2\pi}{\sigma_y^2}}\,\Re\Big[\exp\Big[-\frac{(y+iv)^2}{2\sigma_y^2}\Big]\operatorname{erfc}\Big(\frac{iy-v}{\sqrt{2\sigma_y^2}}\Big)\Big]$$

with

$$\sigma_y^2(t) = D_y^p \tau_y [1-\exp(-2t/\tau_y)] \qquad C = 2\tau_p D_y^p$$

$$v = a\exp(-t/\tau_y) \qquad a = \tau_p D_y^p \sqrt{2\exp(-\tau_p D_y^p)}$$

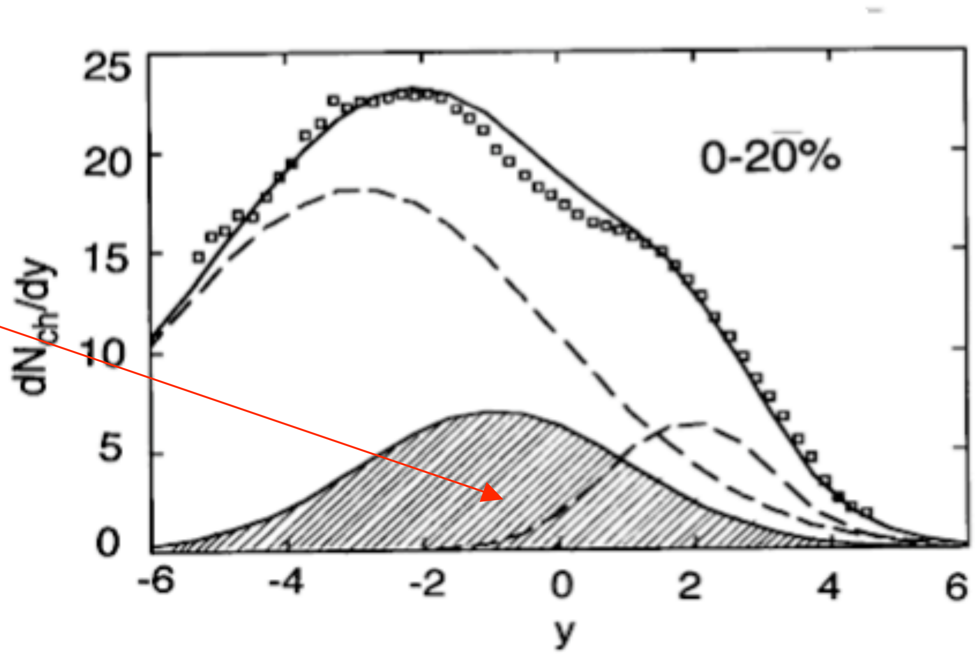
For $t \to \infty$ $\Re\Big[\operatorname{erfc}\Big(\frac{iy}{\sqrt{2\sigma_y^2}}\Big)\Big] = 1$ such that the Gaussian limit is attained

$$R(y, t\to\infty) = \sqrt{\frac{1}{2\pi\sigma_y^2}}\exp\Big[-\frac{y^2}{2\sigma_y^2}\Big]$$ which is a solution of the linear FPE

Compare the solution of the linear 3-sources problem with data.

d+Au 200 GeV charged hadrons: RDM-analysis

In the RDM-analysis, determine in particular the importance of the Moving equilibrium (gluonic) source



GW, TM, NS, MB, Phys. Lett. B 633, 38 (2006)

Data: B.B. Back et al., PHOBOS coll Phys. Rev. C72, 031901 (2005)

$\eta = -\ln(\tan(\theta/2)) \approx y$

d+Au 200 GeV charged hadrons: conversion to pseudorapidity space

$$\eta = -\ln[\tan(\theta/2)]$$

$$\frac{dN}{d\eta} = \frac{dN}{dy}\frac{dy}{d\eta} = \frac{p}{E}\frac{dN}{dy} = J(\eta, \langle m\rangle/\langle p_T\rangle)\frac{dN}{dy}$$

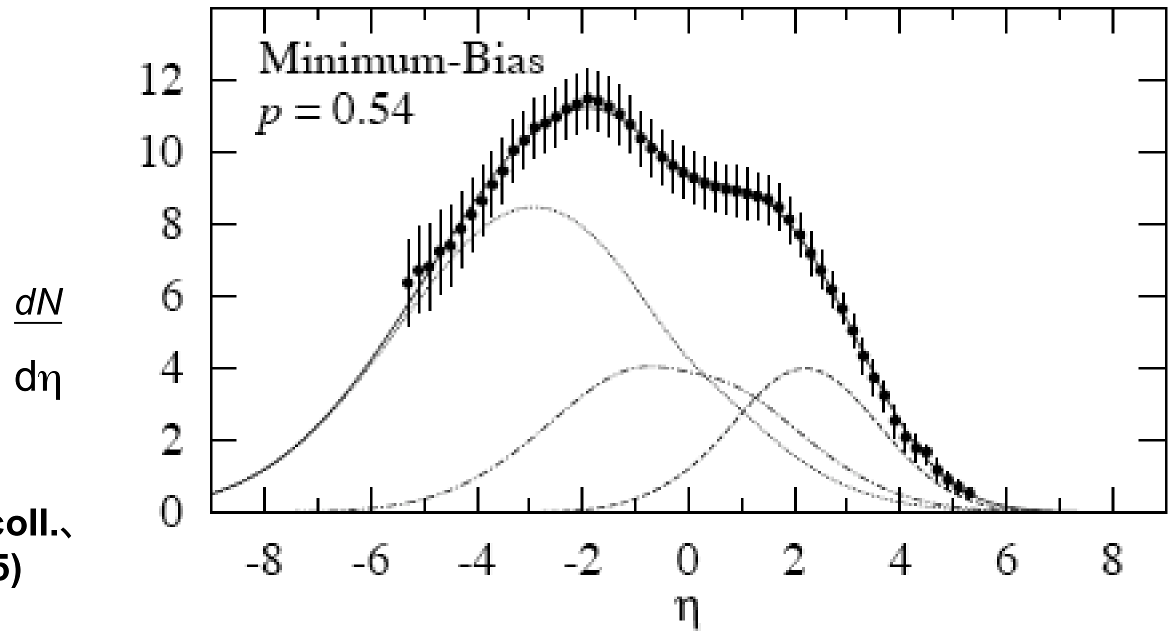
$$J(\eta, \langle m\rangle/\langle p_T\rangle) = \cosh(\eta)\cdot[1 + (\langle m\rangle/\langle p_T\rangle)^2 + \sinh^2(\eta)]^{-1/2}$$

Here we approximate the average mass $\langle m\rangle$ of produced charged hadrons in the central region by the pion mass m_π, and use a mean transverse momentum $\langle p_T\rangle$ = 0.4 GeV/c. In the Au-like region, the average mass is larger due to the participant protons, but since their number $Z_1 < 5.41$ is small compared to the number of produced charged hadrons in the d + Au system, the increase above the pion mass remains small: $\langle m\rangle \approx m_p \cdot Z_1/N^1_{\mathrm{ch}} + m_\pi \cdot (N^1_{\mathrm{ch}} - Z_1)/N^1_{\mathrm{ch}} \approx 0.17$ GeV. This increase turns out to have a negligeable effect on the results of the numerical optimization, where we use $\langle m\rangle/\langle p_T\rangle = 0.45$ for the Jacobian transformations in the three regions. For reasonable deviations of the mean transverse momentum from 0.4 GeV/c, the results remain consistent with the data within the experimental error bars.

d+Au 200 GeV charged hadrons: conversion to dN/dη

Minimum-bias distribution

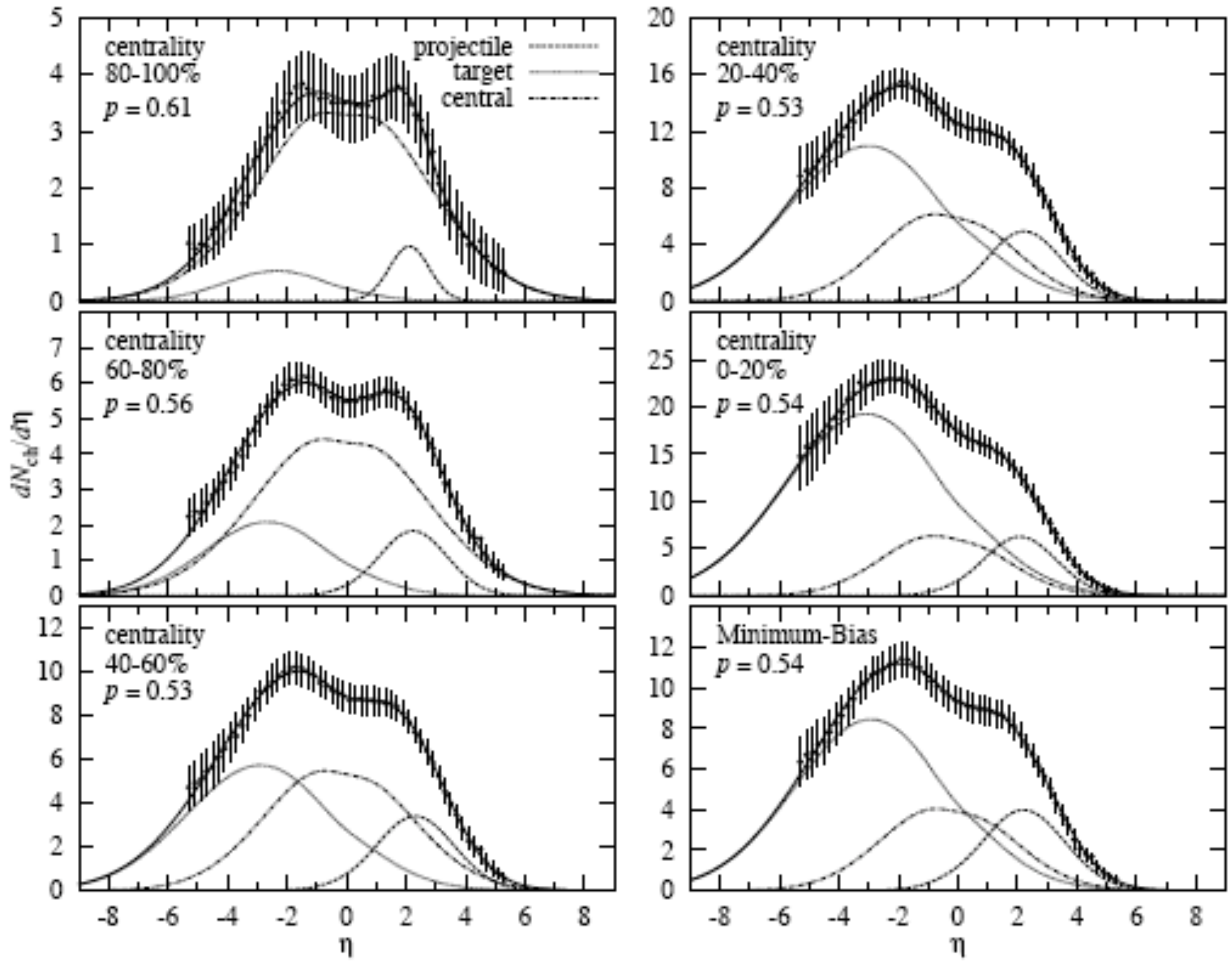
GW, TM, NS, MB, Annalen Phys. 15, 369 (2006)



Data: B.B. Back et al., PHOBOS coll., Phys. Rev. C72, 031901 (2005)

$\eta = -\ln(\tan(\theta/2)) \approx y$

200 GeV d+Au pseudorap. distributions in the RDM



GW, M.Biyajima, T.Mizoguchi, N.Suzuki, Annalen Phys. 15, 369 (2006)

Asymmetric systems are more sensitive to details of the nonequi-librium-statistical evolution

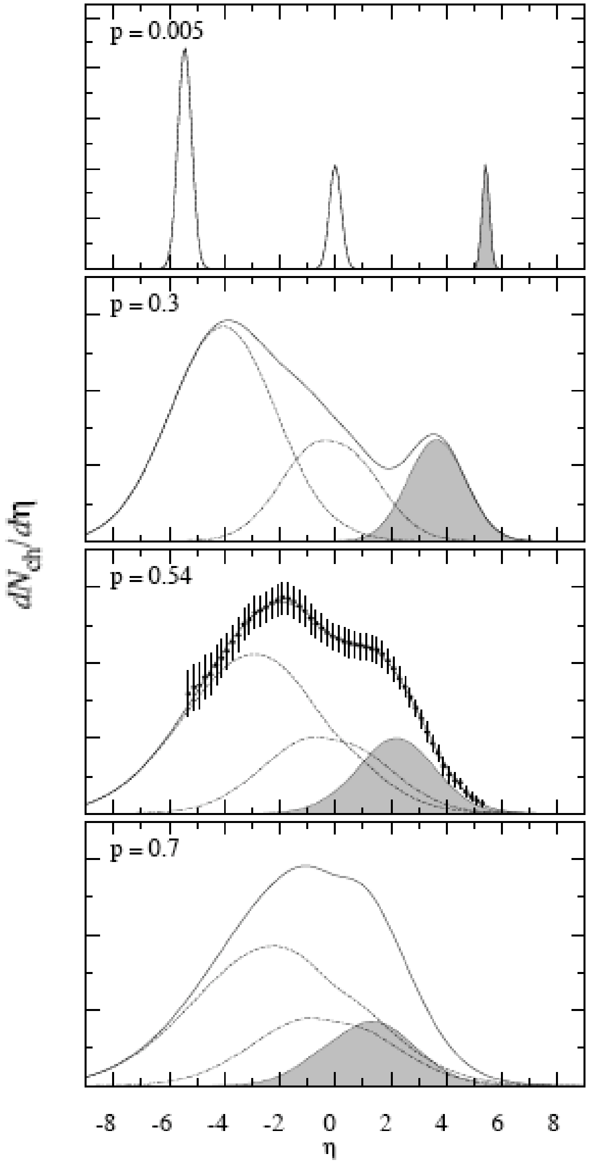
Data: PHOBOS coll. Phys. Rev. C72, 031901 (2005)

d+Au 200 GeV time development

- The mean values of all three η-distributions approach η_{eq} for large times

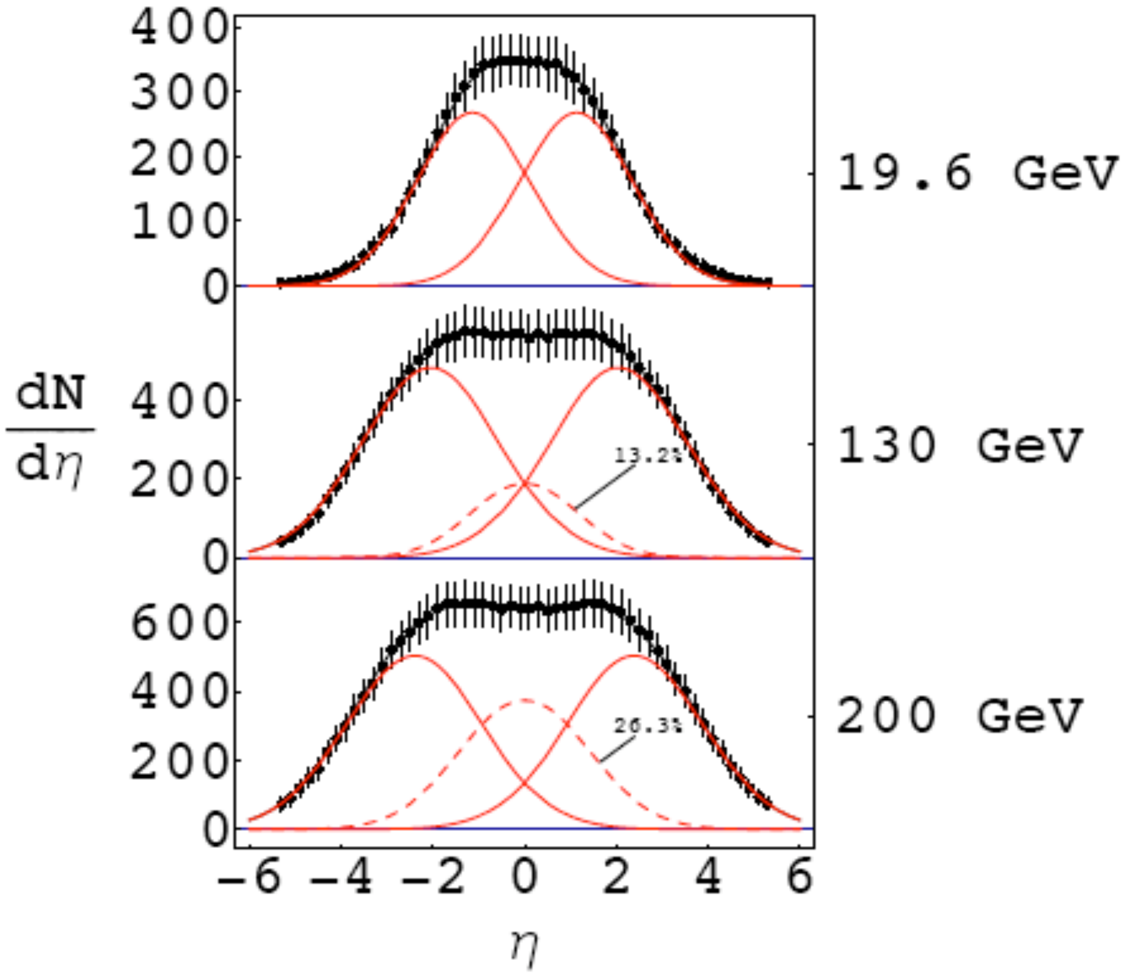
- The actual collision stops before full equilibrium is reached

GW, MB,TM (2007)



Produced particles in the 3-sources RDM:

Charged-hadron pseudorapidity (partial) distributions



Au + Au central collisions, 0-6%:

χ^2-minimization with respect to PHOBOS data (two sources only at the lowest energy).

The midrapidity source for produced hadrons tends to be more important as compared to net protons.

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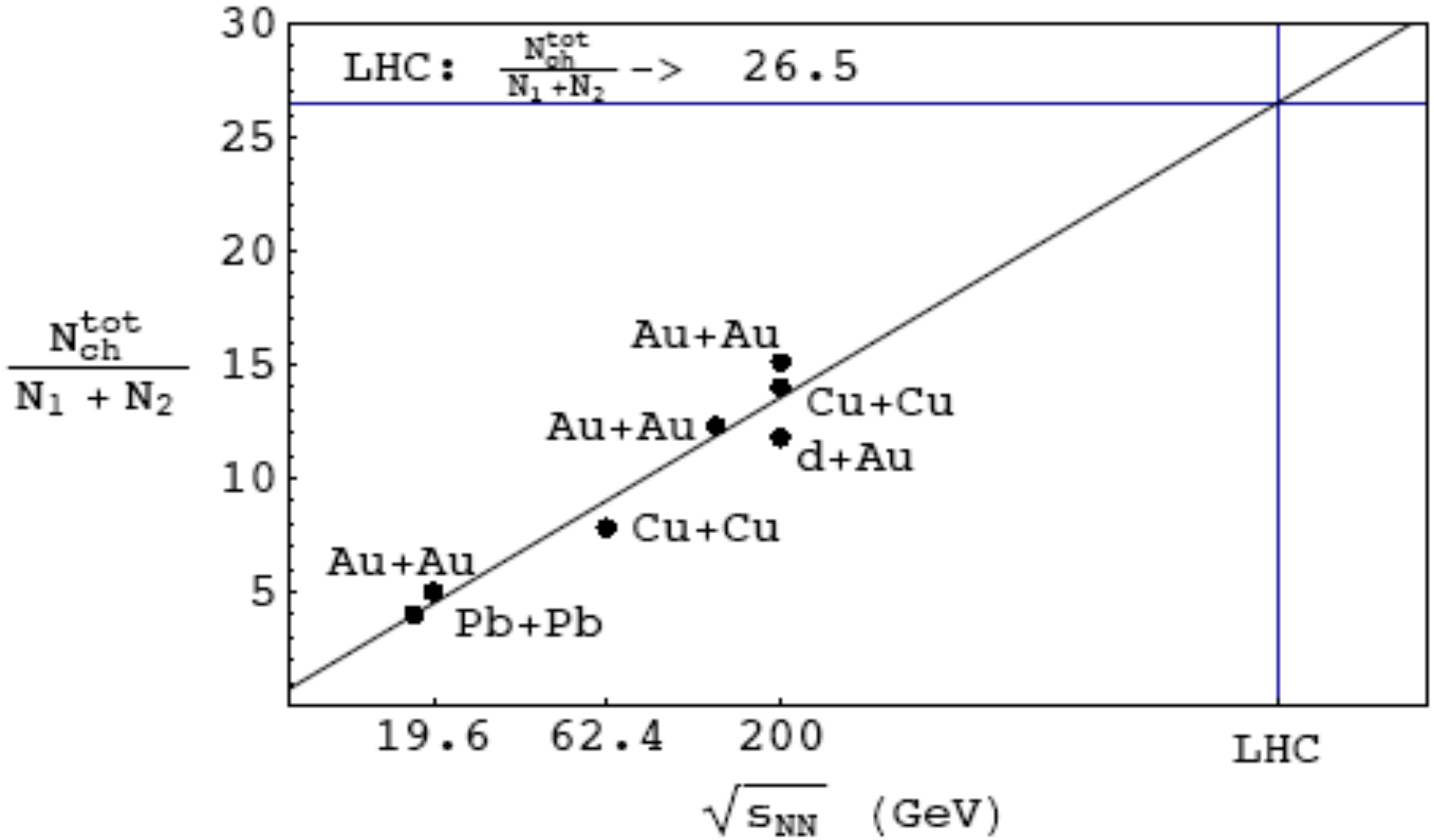
R. Kuiper and G. Wolschin, Annalen Phys. 16, 67 (2007)

Produced charged hadrons at LHC: extrapolating total yield per no. of participants



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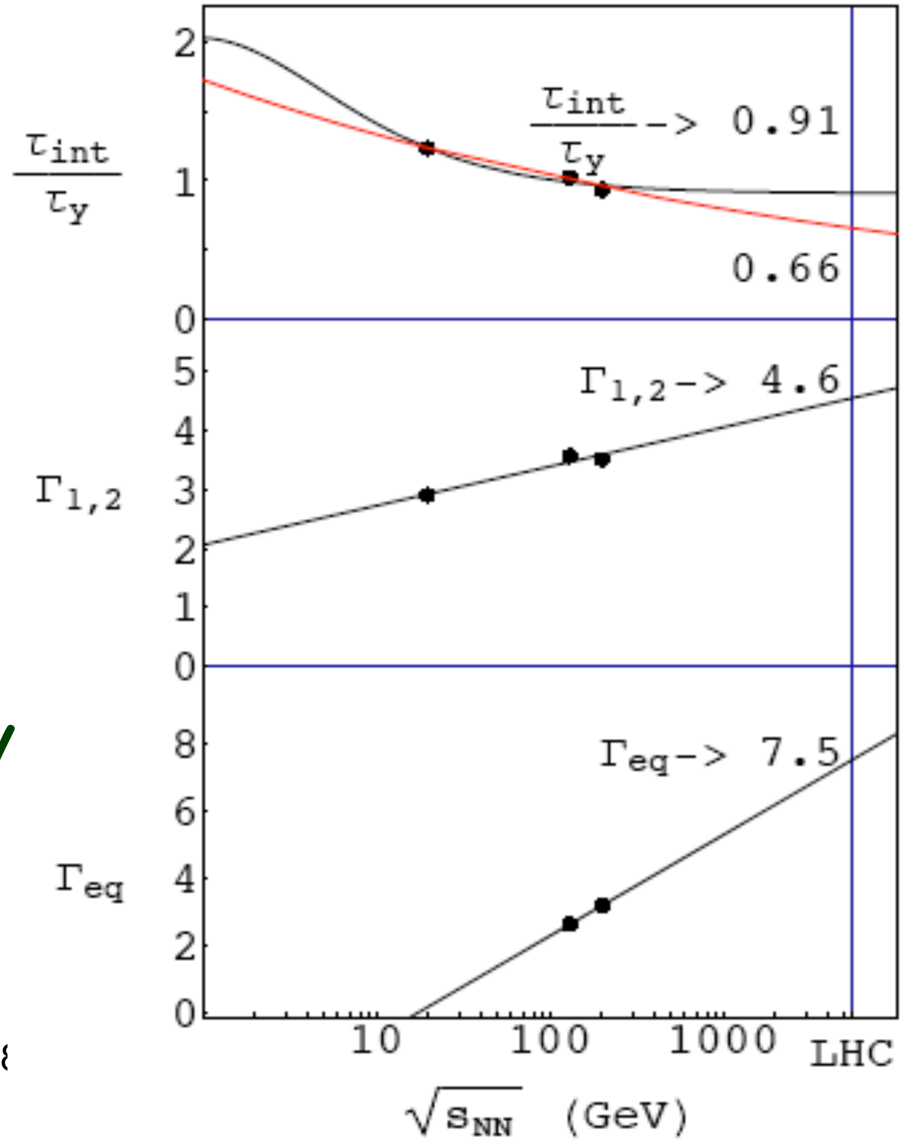
Produced charged hadrons at LHC: extrapolating the diffusion-model parameters

time parameter

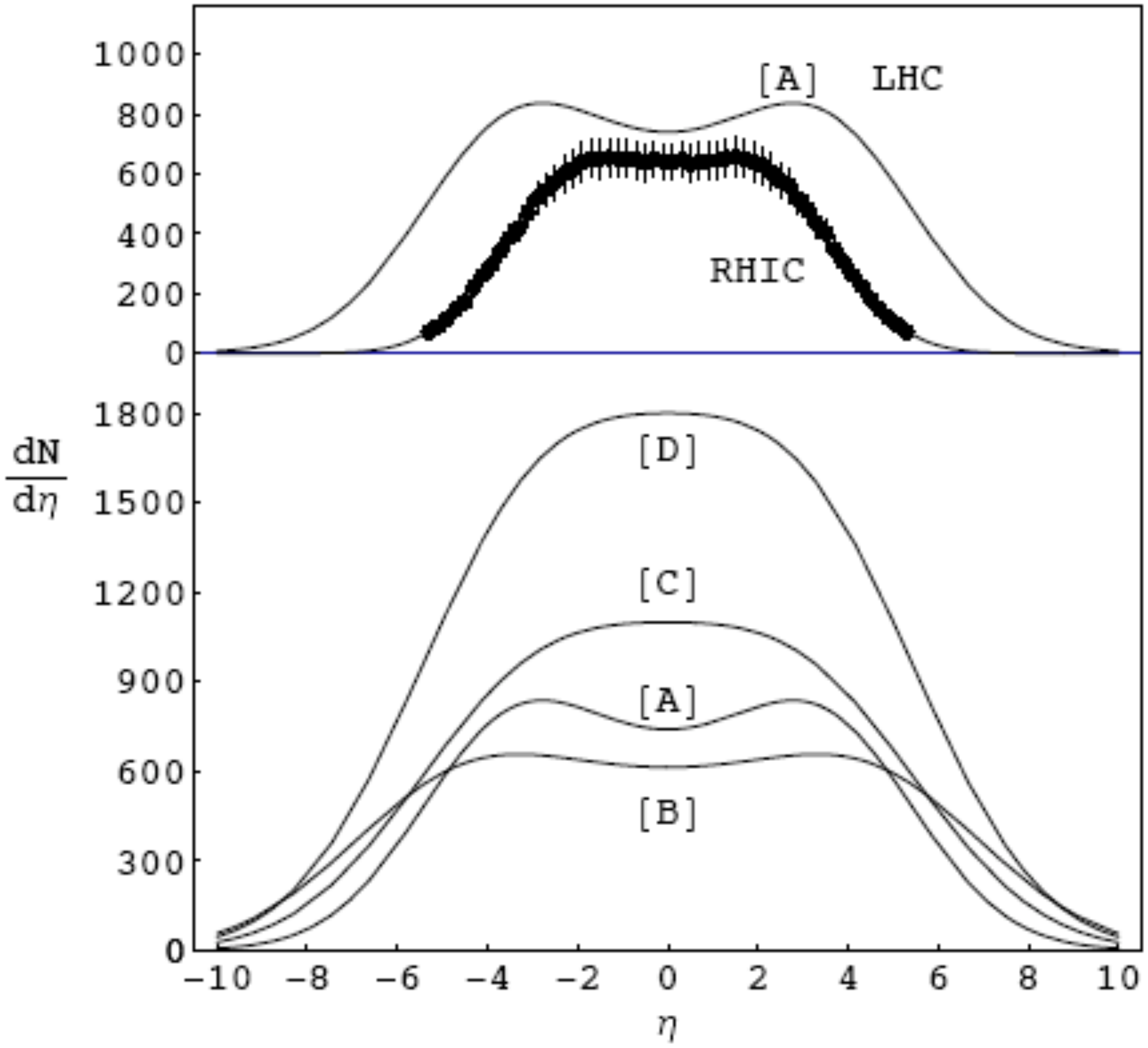
widths of the beam-like partial distributions

width of the midrapidity distribution

Central coll. 0-6%



Produced charged hadrons at LHC in the RDM



LHC: Pb+Pb @ 5.52 TeV central collisions, 0-6%

[A]: RDM-extrapolation (sinh-form for τ_{int}/τ_y)

[B]: RDM-extrapolation (exp-form for τ_{int}/τ_y)

[C]: log-extrapolation of dn/dη at η=0

[D]: saturation-model (*) extrap.of dn/dη at η=0

R. Kuiper and G.Wolschin, Ann. Phys. 16, 67 (2006)

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(*) K.Golec-Biernat and M. Wüsthoff, Phys. Rev. D 59, 014017 (1998); N.Armesto et al., PRL 94, 022002(05

5. Conclusion

- Net Protons: The Relativistic Diffusion Model with 3 sources describes rapidity distributions for net protons accurately.
  The locally equilibrated midrapidity region contains 13-14% of the net protons at RHIC energies: indirect evidence for deconfinement and longitudinal collective expansion.

- Charged Hadrons: The energy and centrality dependence of pseudorapidity distributions for produced charged hadrons is precisely described (symmetric&asymmetric systems).
  In central collisions, the thermalized central source may be the consequence of a locally equilibrated quark-gluon plasma.

- LHC: Rapidity distributions for net protons, and pseudorapidity spectra for produced charged hadrons are predicted for LHC.

Reconsider the 3-sources model in QCD, with a midrapidity source from gluon-gluon collisions, and two forward/backward sources that arise from the valence quarks.