Mesons in the quark phase diagram

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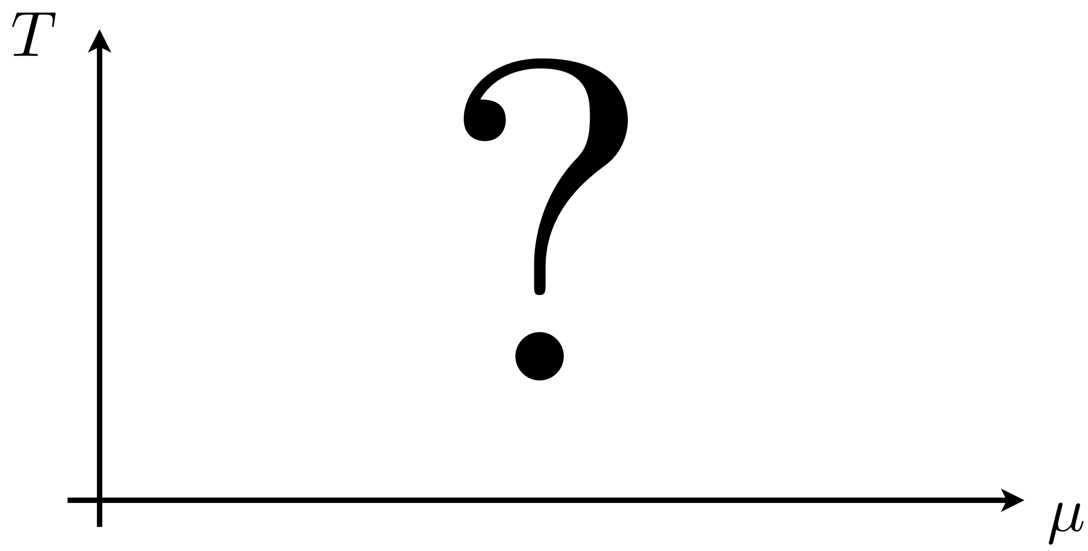
in collaboration with David Blaschke and Roberto Anglani

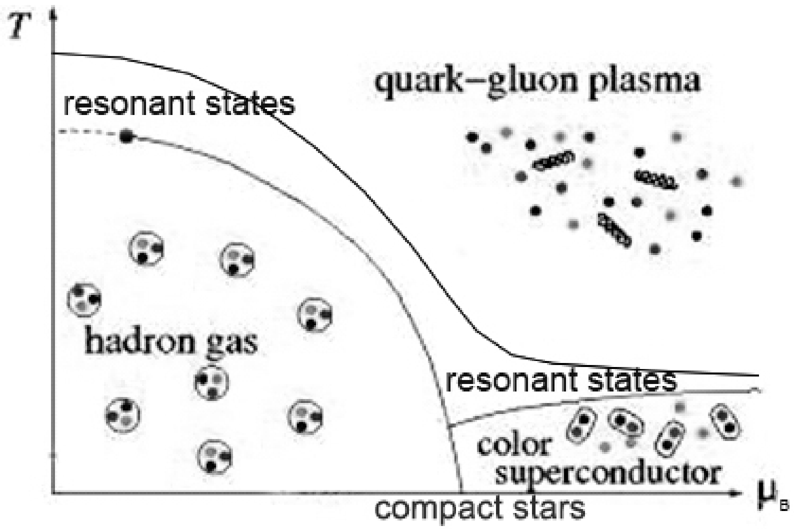
investigation of the
phase diagram of QCD
beyond mean field level
in NJL framework

one challenging problem of quantum chromodynamics is

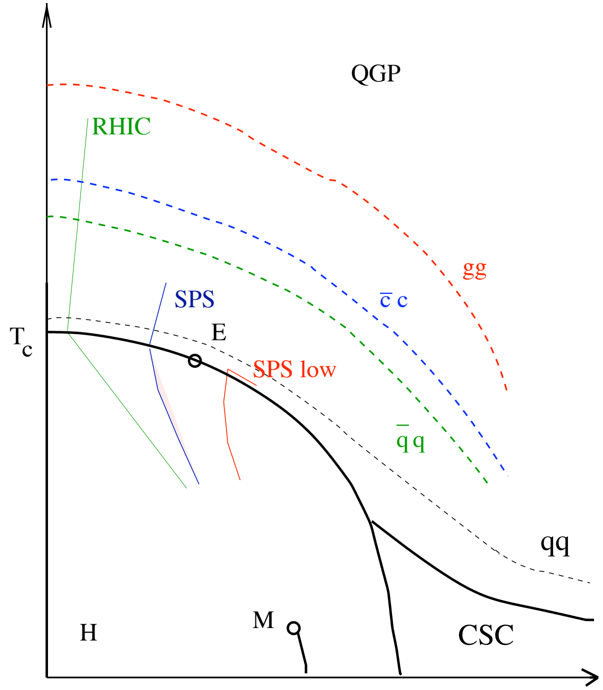
the study of phase diagram



1.1. what we know about phase diagram



Zhuang, P.F. et al. 0710.3634 [hep-ph]



Shuryak and Zahed hep-ph/0403127

2. the effective model

how to give a reliable description in the region around the critical values of chemical potential?

any perturbative expansion of QCD is completely unreliable and an exact description for the matter in this condition does not exist

we have to accept a good compromise.
an effective model:
the Nambu--Jona-Lasinio

The NJL model of QCD mimics the quark–quark interaction mediated by gluons with an effective point–like four fermion interaction

cons
absence of gluon in the Lagrangian; quarks are not confined; etc.

pro
a simple approach to the description of chiral symmetry breaking and phase transitions

2.2 the starting point: the NJL Lagrangian

For the description of hot, dense Fermi-systems, with strong short-range interactions we consider a Lagrangian with internal degrees of freedom (2-flavor, 3-color), with a current -current-type four-Fermion

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{qq} + \mathcal{L}_{q\bar{q}}$$

$$\mathcal{L}_0 = \bar{q}(i\partial\!\!\!/ - m_0 + \mu\gamma_0)q$$

$$\mathcal{L}_{q\bar{q}} = G_S\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\boldsymbol{\tau}q)^2\right]$$

$$\mathcal{L}_{qq} = G_D \sum_{A=2,5,7}\left[\bar{q}i\gamma_5 C\tau_2\lambda_A\bar{q}^T\right]\left[q^T iC\gamma_5\tau_2\lambda_A q\right]$$

$$q = \begin{pmatrix}\psi_1\\ \psi_2\\ \psi_3\\ \psi_4\end{pmatrix} \otimes \begin{pmatrix}u\\ d\end{pmatrix} \otimes \begin{pmatrix}r\\ g\\ b\end{pmatrix}$$

$\boldsymbol{\tau} = (\tau_1, \tau_2, \tau_3)$ $\quad C = i\gamma_2\gamma_0$

$m_{0,u} = m_{0,d} = m_0$
$\mu_u = \mu_d = \mu$

G_S scalar and pseudoscalar strenght coupling
G_D scalar diquark channel

2.3 the partition function

the partition function $\mathcal{Z} = \int [dq]\,[d\bar{q}]\exp\left[\int_0^\beta d\tau \int d^3x\, \mathcal{L}\right]$ $\Omega = -T\ln Z$

Hubbard–Stratonovich auxiliary fields

$$\mathcal{Z} = \int [dq]\,[d\bar{q}]\,[d\Delta_A]\,[d\Delta_A^*]\,[d\sigma]\,[d\boldsymbol{\pi}]\exp\left[\int_0^\beta d\tau \int d^3x\, \mathcal{L}\right]$$

$$\mathcal{L}_{\rm eff} = -\frac{\sigma^2+\pi^2}{4G_S} - \frac{\Delta_A^*\Delta_A}{4G_D} + \bar{q}(i\partial\!\!\!/ - m_0 + \mu\gamma_0)q - \bar{q}(\sigma + i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi})q + i\frac{\Delta_A^*}{2}q^T iC\gamma_5\tau_2\lambda_A q - i\frac{\Delta_A}{2}\bar{q}i\gamma_5 C\tau_2\lambda_A\bar{q}^T$$

Nambu–Gorkov formalism

$\Psi \equiv \frac{1}{\sqrt{2}}\begin{pmatrix} q \\ q^c \end{pmatrix}$ $\bar{\Psi} \equiv \frac{1}{\sqrt{2}}\begin{pmatrix} \bar{q} & \bar{q}^c \end{pmatrix}$ $q^c(x) \equiv C\bar{q}^T(x)$

$$\mathcal{Z} = \int [d\Delta_A]\,[d\Delta_A^*]\,[d\sigma]\,[d\boldsymbol{\pi}]\exp\left[\int_0^\beta d\tau \int d^3x \left(-\frac{\sigma^2+\boldsymbol{\pi}^2}{4G_S} - \frac{\Delta_A^*\Delta_A}{4G_D}\right)\right]\int [d\Psi]\,[d\bar{\Psi}]\exp\left[\int_0^\beta d\tau \int d^3x\, \bar{\Psi}\, S^{-1}\Psi\right]$$

$$S^{-1} \equiv \begin{pmatrix} i\partial\!\!\!/ + \mu\gamma_0 - m_0 - \sigma - i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi} & \Delta_A\gamma_5\tau_2\lambda_A \\ -\Delta_A^*\gamma_5\tau_2\lambda_A & i\partial\!\!\!/ - \mu\gamma_0 - m_0 - \sigma - i\gamma_5\boldsymbol{\tau}^t\cdot\boldsymbol{\pi} \end{pmatrix}$$

$$\mathcal{Z} = \int [d\Delta_A]\,[d\Delta_A^*]\,[d\sigma]\,[d\boldsymbol{\pi}]\left\{\exp\left[\int_0^\beta d\tau \int d^3x \left(-\frac{\sigma^2+\boldsymbol{\pi}^2}{4G_S} - \frac{\Delta_A^*\Delta_A}{4G_D}\right)\right]\exp\left[\mathrm{Tr}\left(\ln S^{-1}\right)\right]\right\}$$

3. the mean field approximation

how to calculate this?

$$\mathcal{Z} = \int [d\Delta_A]\,[d\Delta_A^*]\,[d\sigma]\,[d\boldsymbol{\pi}] \left\{ \exp\left[\int_0^\beta d\tau \int d^3x \left(-\frac{\sigma^2 + \boldsymbol{\pi}^2}{4G_S} - \frac{\Delta_A^* \Delta_A}{4G_D} \right) \right] \exp\left[\mathrm{Tr}\left(\ln S^{-1} \right) \right] \right\}$$

the mean field approximation (MFA)

decompose bosonic collective fields into a

homogeneous MF part + ~~fluctuation part~~

↓ order parameter: characterization of phase structure

↓ ~~correlations~~

$\Delta \rightarrow \Delta_{MF} + \delta \;\; \sigma \rightarrow \sigma_{MF} + \sigma$

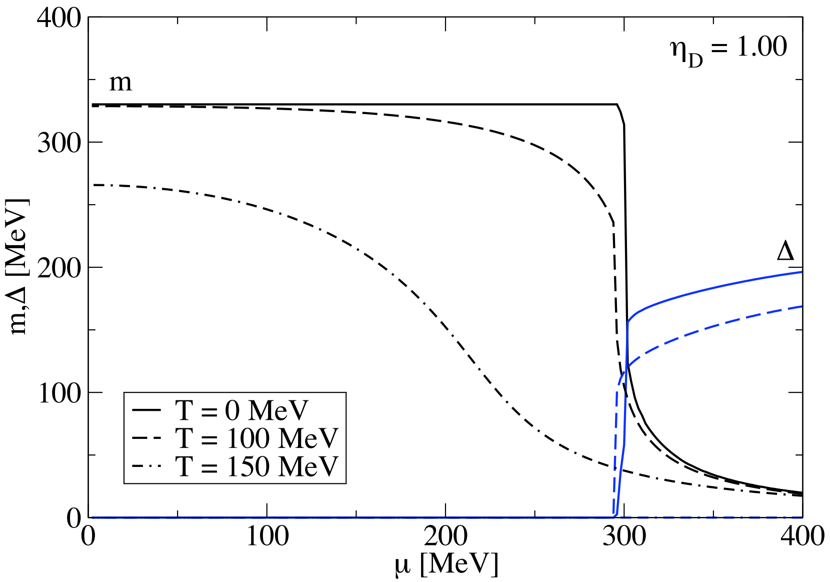
$$\frac{\partial \Omega}{\partial m} = \frac{\partial \Omega}{\partial \Delta} = 0 \qquad \mathcal{Z}_{MF} = \exp\left[\beta V \left(-\frac{\sigma_{MF}^2 + \boldsymbol{\pi}_{MF}^2}{4G_S} - \frac{\Delta_{MF}^* \Delta_{MF}}{4G_D} \right) \right] \exp\left[\mathrm{Tr}\left(\ln S_{MF}^{-1} \right) \right]$$

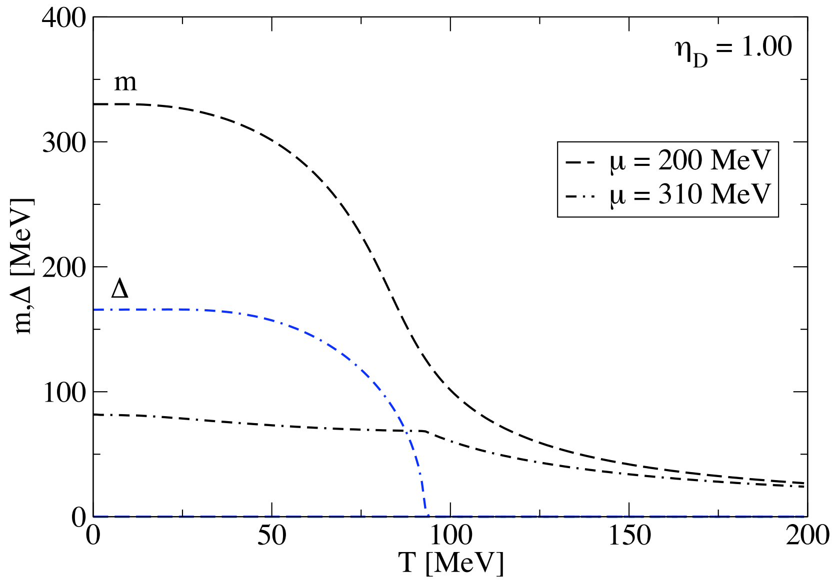
$$m - m_0 = 8G_S\, m \int \frac{d^3p}{(2\pi)^3} \frac{1}{E_\mathbf{p}} \left\{ \left[1 - 2n_F(E_\mathbf{p}^-)\right] \frac{\xi_\mathbf{p}^-}{E_\mathbf{p}^-} + \left[1 - 2n_F(E_\mathbf{p}^+)\right] \frac{\xi_\mathbf{p}^+}{E_\mathbf{p}^+} + n_F(-\xi_\mathbf{p}^+) - n_F(\xi_\mathbf{p}^-) \right\}$$

$$\Delta = 8G_D \int \frac{d^3p}{(2\pi)^3} \left[\frac{1 - 2n_F(E_\mathbf{p}^-)}{E_\mathbf{p}^-} + \frac{1 - 2n_F(E_\mathbf{p}^+)}{E_\mathbf{p}^+} \right]$$

$E_\mathbf{p}^\pm = \sqrt{\left(\xi_\mathbf{p}^\pm\right)^2 + \Delta^2}$ with $\xi_\mathbf{p}^\pm = E_\mathbf{p} \pm \mu$, $E_\mathbf{p} = \sqrt{m^2 + \mathbf{p}^2}$

3.1 results of MFA





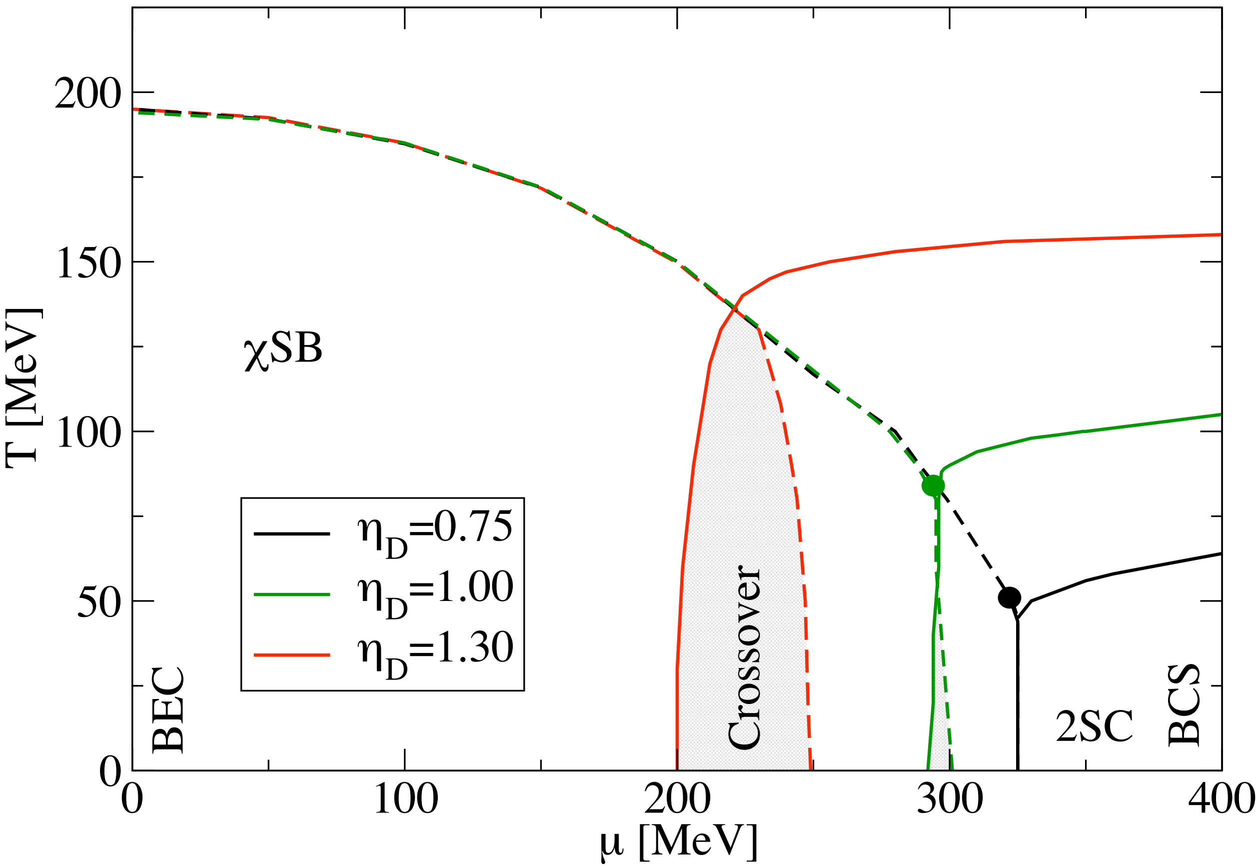
$\Lambda = 629.540$ MeV

$m_0 = 5.27697$ MeV

$G_S \Lambda^2 = 2.17576$

H. Grigorian, Phys. Part. Nucl. Lett. **4**, 223 (2007) [arXiv:hep-ph/0602238].

the phase diagram MF



4 what about fluctuations?

$$\mathcal{Z} = \int [d\Delta_A]\,[d\Delta_A^*]\,[d\sigma]\,[d\boldsymbol{\pi}] \left\{ \exp\left[\int_0^\beta d\tau \int d^3x \left(-\frac{\sigma^2 + \boldsymbol{\pi}^2}{4G_S} - \frac{\Delta_A^* \Delta_A}{4G_D} \right) \right] \exp\left[\mathrm{Tr}\left(\ln S^{-1} \right) \right] \right\}$$

$$S^{-1} = S_{MF}^{-1} + \Sigma$$

$$\Sigma \equiv \begin{pmatrix} -\sigma - i\gamma_5 \boldsymbol{\tau} \cdot \boldsymbol{\pi} & \delta_A \gamma_5 \tau_2 \lambda_A \\ -\delta_A^* \gamma_5 \tau_2 \lambda_A & -\sigma - i\gamma_5 \boldsymbol{\tau}^t \cdot \boldsymbol{\pi} \end{pmatrix}$$

$$\begin{aligned} \mathrm{Tr}[\ln(S^{-1})] &= \mathrm{Tr}[\ln(S_{MF}^{-1} + \Sigma)] \\ &= \mathrm{Tr}\{\ln[S_{MF}^{-1}(1 + S_{MF}\Sigma)]\} \\ &= \mathrm{Tr}\ln S_{MF}^{-1} + \mathrm{Tr}\ln[1 + S_{MF}\Sigma] \\ &= \mathrm{Tr}\ln S_{MF}^{-1} + \mathrm{Tr}[S_{MF}\Sigma - \frac{1}{2} S_{MF}\Sigma S_{MF}\Sigma + \dots] \end{aligned}$$

$$\mathrm{Tr}\left(S_{MF}\Sigma S_{MF}\Sigma\right) = (\boldsymbol{\pi}, \sigma, \delta_2^*, \delta_2, \delta_5^*, \delta_7^*) \begin{pmatrix} \Pi_{\pi\pi} & 0 & 0 & 0 & 0 & 0 \\ 0 & \Pi_{\sigma\sigma} & \Pi_{\sigma\delta_2} & \Pi_{\sigma\delta_2^*} & 0 & 0 \\ 0 & \Pi_{\delta_2^*\sigma} & \Pi_{\delta_2^*\delta_2} & \Pi_{\delta_2^*\delta_2^*} & 0 & 0 \\ 0 & \Pi_{\delta_2\sigma} & \Pi_{\delta_2\delta_2} & \Pi_{\delta_2\delta_2^*} & 0 & 0 \\ 0 & 0 & 0 & 0 & \Pi_{\delta_5^*\delta_5} & 0 \\ 0 & 0 & 0 & 0 & 0 & \Pi_{\delta_7^*\delta_7} \end{pmatrix} \begin{pmatrix} \boldsymbol{\pi} \\ \sigma \\ \delta_2 \\ \delta_2^* \\ \delta_5 \\ \delta_7 \end{pmatrix}$$

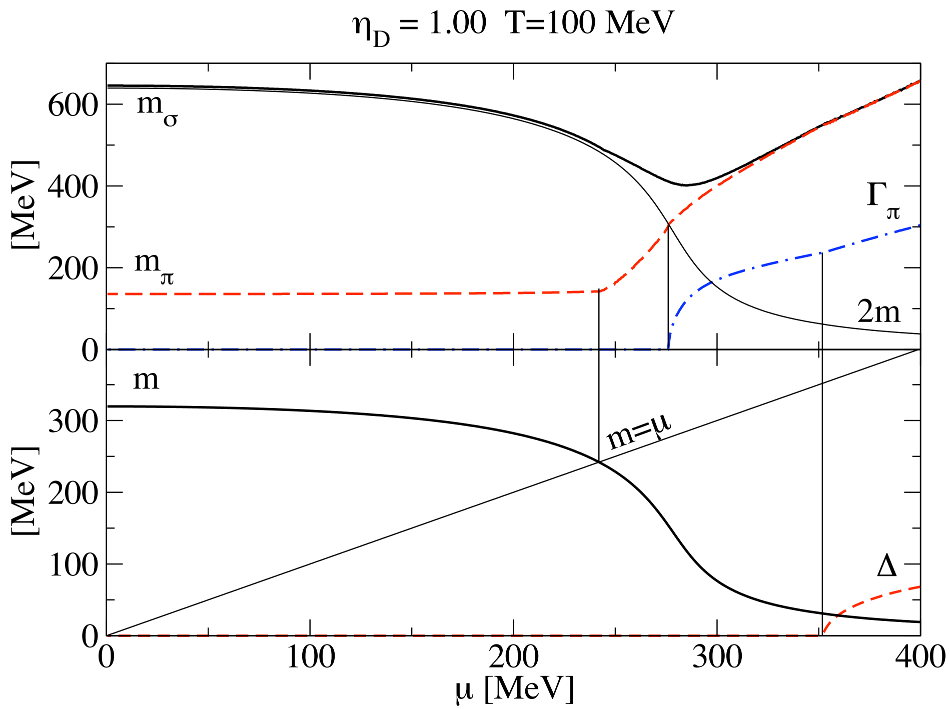
4.1 meson polarization functions and masses

$$\Pi_{\pi\pi}(q_0,\mathbf{q}) = 2\int\frac{d^3p}{(2\pi)^3}\sum_{s_p,s_k}\mathcal{T}_{-}^{+}(s_p,s_k)\left\{\frac{n_F(s_p\xi_{\mathbf{p}}^{s_p})-n_F(s_k\xi_{\mathbf{p+q}}^{s_k})}{q_0-s_k\xi_{\mathbf{p+q}}^{s_k}+s_p\xi_{\mathbf{p}}^{s_p}}-\frac{n_F(s_p\xi_{\mathbf{p}}^{s_p})-n_F(s_k\xi_{\mathbf{p+q}}^{s_k})}{q_0+s_k\xi_{\mathbf{p+q}}^{s_k}-s_p\xi_{\mathbf{p}}^{s_p}}\right.$$
$$\left.+\sum_{t_p,t_k}\frac{t_pt_k}{E_{\mathbf{p}}^{s_p}E_{\mathbf{p+q}}^{s_k}}\frac{n_F(t_pE_{\mathbf{p}}^{s_p})-n_F(t_kE_{\mathbf{p+q}}^{s_k})}{q_0-t_kE_{\mathbf{p+q}}^{s_k}+t_pE_{\mathbf{p}}^{s_p}}\left(t_pt_kE_{\mathbf{p}}^{s_p}E_{\mathbf{p+q}}^{s_k}+s_ps_k\xi_{\mathbf{p}}^{s_p}\xi_{\mathbf{p+q}}^{s_k}+|\Delta|^2\right)\right\}$$

$$\Pi_{\sigma\sigma}(q_0,\mathbf{q}) = 2\int\frac{d^3p}{(2\pi)^3}\sum_{s_p,s_k}\mathcal{T}_{-}^{-}(s_p,s_k)\left\{\frac{n_F(s_p\xi_{\mathbf{p}}^{s_p})-n_F(s_k\xi_{\mathbf{k}}^{s_k})}{q_0-s_k\xi_{\mathbf{k}}^{s_k}+s_p\xi_{\mathbf{p}}^{s_p}}-\frac{n_F(s_p\xi_{\mathbf{p}}^{s_p})-n_F(s_k\xi_{\mathbf{k}}^{s_k})}{q_0+s_k\xi_{\mathbf{k}}^{s_k}-s_p\xi_{\mathbf{p}}^{s_p}}\right.$$
$$\left.+\sum_{t_p,t_k}\frac{t_pt_k}{E_{\mathbf{p}}^{s_p}E_{\mathbf{k}}^{s_k}}\frac{n_F(t_pE_{\mathbf{p}}^{s_p})-n_F(t_kE_{\mathbf{k}}^{s_k})}{q_0-t_kE_{\mathbf{k}}^{s_k}+t_pE_{\mathbf{p}}^{s_p}}\left(t_pt_kE_{\mathbf{p}}^{s_p}E_{\mathbf{k}}^{s_k}+s_ps_k\xi_{\mathbf{p}}^{s_p}\xi_{\mathbf{k}}^{s_k}-|\Delta|^2\right)\right\}$$

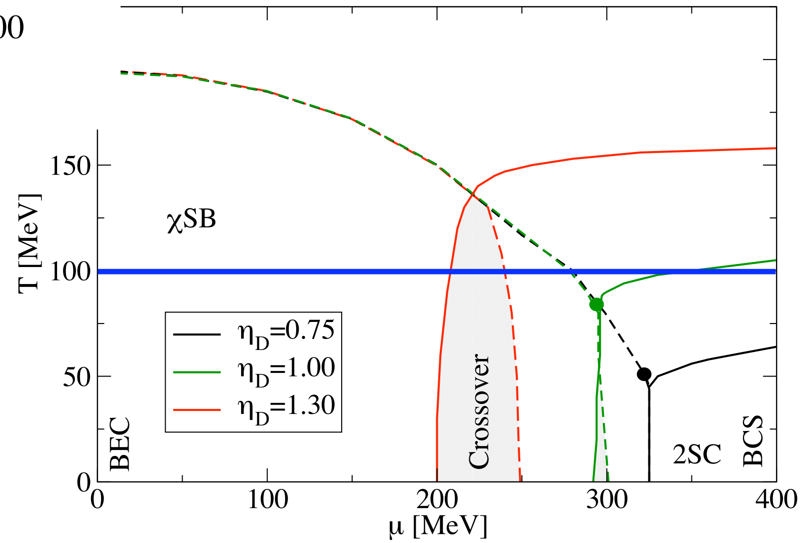
$$\mathcal{T}_{\pm}^{\mp}(s_p,s_k) = \left(1\pm s_ps_k\frac{\mathbf{p}\cdot\mathbf{k}\mp m^2}{E_{\mathbf{p}}E_{\mathbf{k}}}\right)$$

4.2 the pion mass

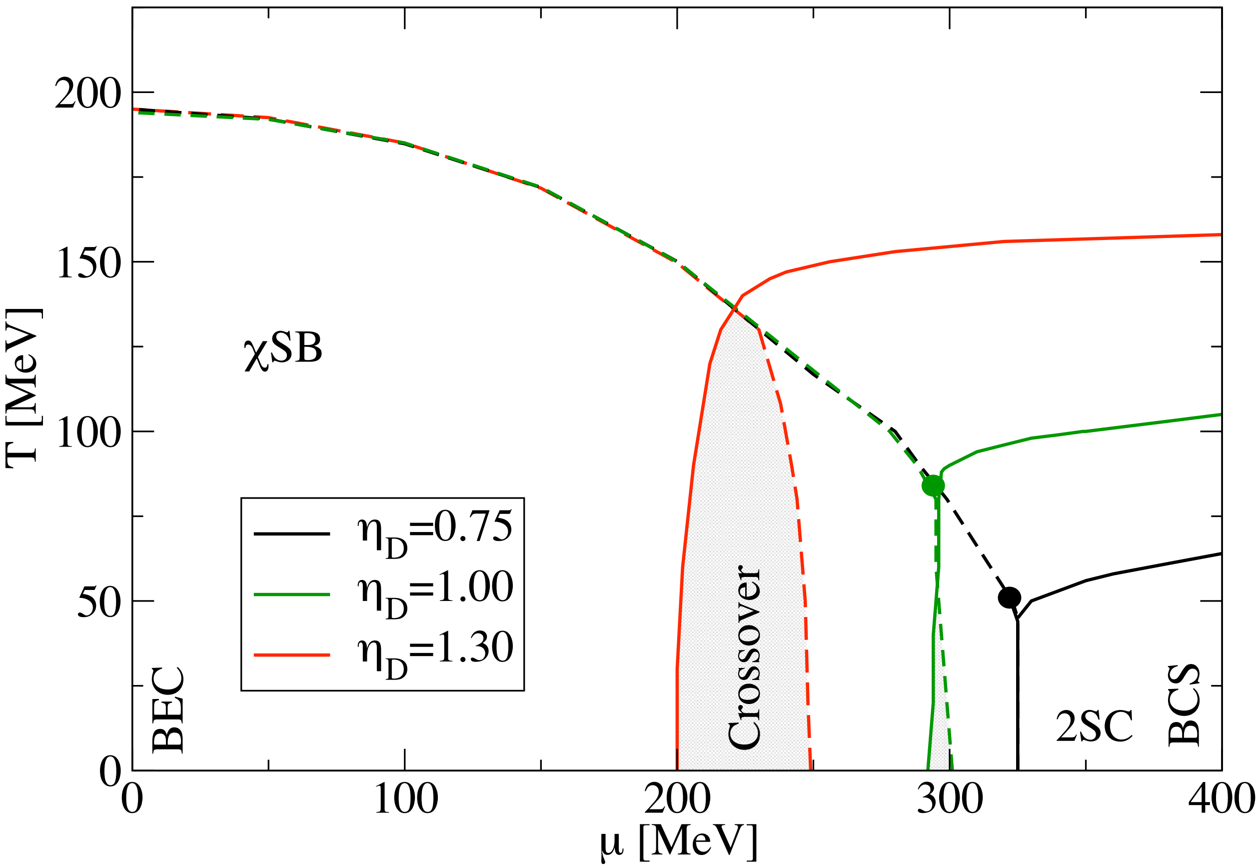


$$\Gamma_i = \frac{\mathrm{Im}\Pi_{ii}}{m_i}$$

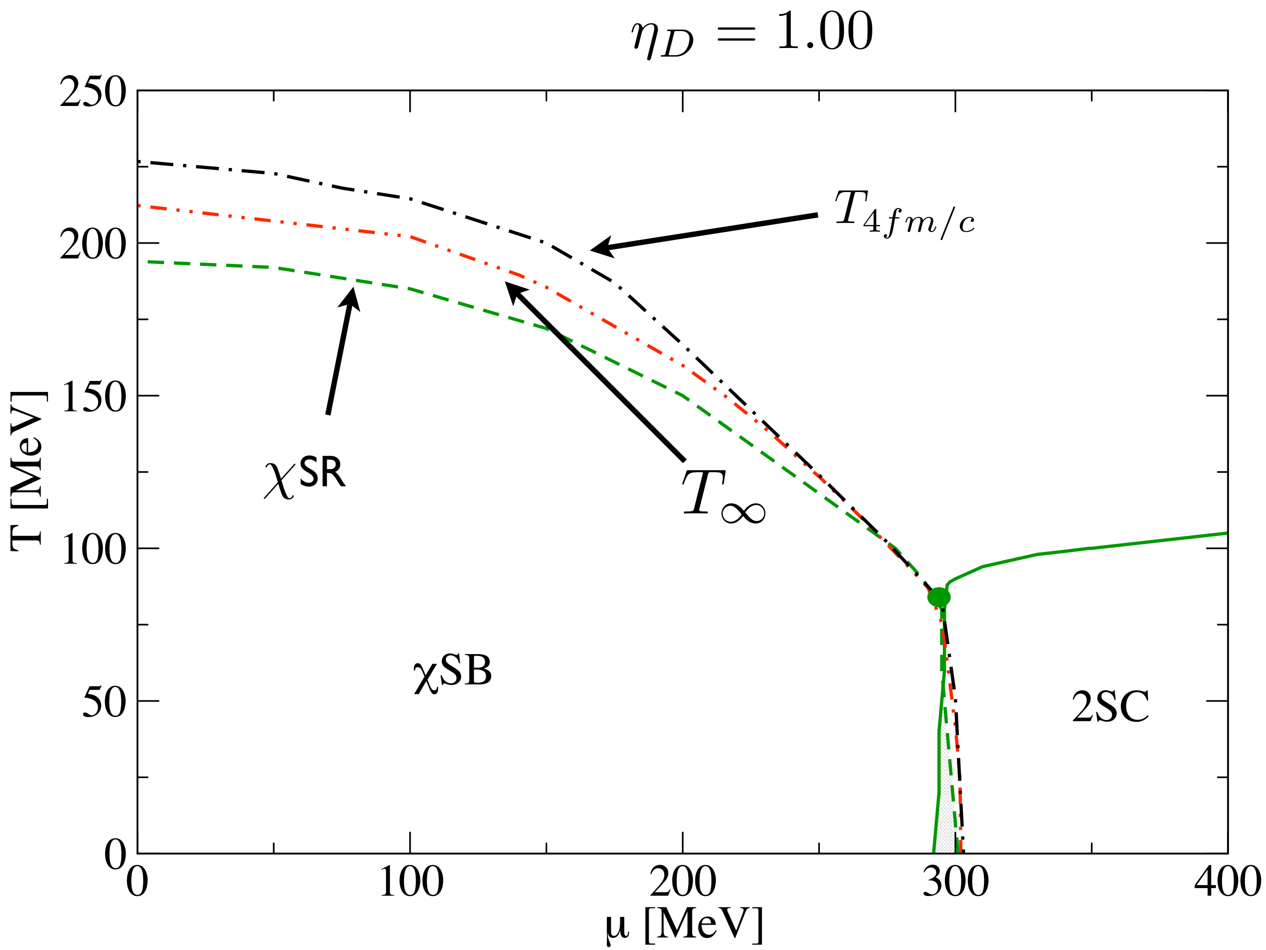
$$1 - 2G_S\Pi(q_0, \boldsymbol{q} = \boldsymbol{0}) = 0$$



the phase diagram MF

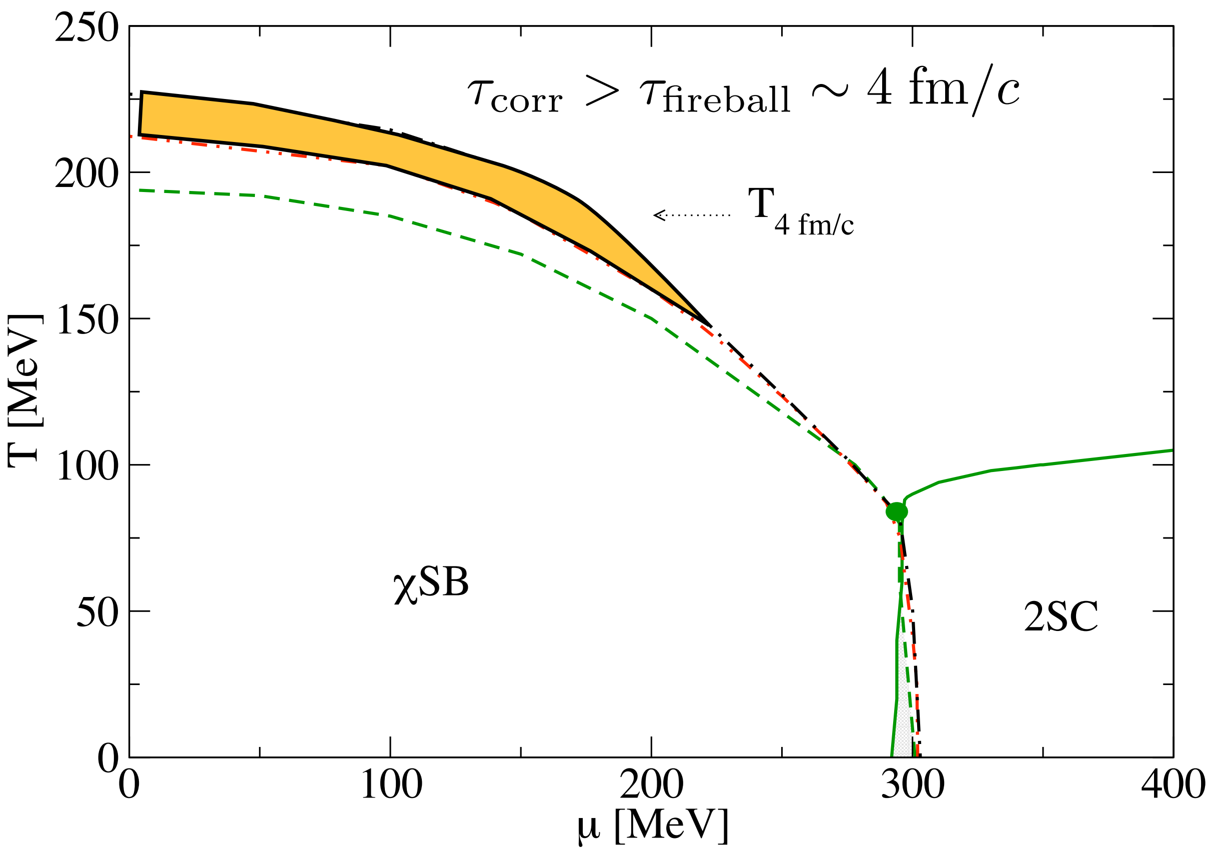


the phase diagram revisited



$T(\tau = \Gamma_\pi^{-1})$

RHIC phenomenology



QGP probed in RHIC is far to be a perfect liquid; an explanation: strong correlations in the plasma

Shuryak and Zahed (PRL, 2003)

Region dominated by strong correlated states with a lifetime > 4 fm/c

summary and outlook

fluctuations are included in Gaussian approximation beyond MF

some properties of mesons are studied
diquark calculations almost finished
new insight for phase diagram; important for HIC and CSs

color and electrical neutrality, beta-equilibrium to be implemented
investigation of BEC-BCS crossover (strong coupling)

acknowledgments

Thanks for your attention