High-Energy QCD
Saturation and fluctuation effects

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Outline

- Perturbative evolution in high-energy QCD:
  - Leading log approx.: Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation
  - Saturation effects: Balitsky-Kovchegov (BK) equation
  - Fluctuation effects: new evolution as a reaction-diffusion process

- Asymptotic solutions: BK equation
  - Equivalence with statistical physics
  - Asymptotic properties: saturation scale and geometric scaling

- Asymptotic solution: including fluctuations
  - Stochastic evolution
  - Consequences: diffusive scaling
  - Present physical picture of high-energy QCD

- Outlook
Saturation effects: basics

\[ Q = Q_s(Y) \]

How to obtain this in QCD?
Perturbative evolution in high-energy QCD
Bremsstrahlung:

\[ p \]

\[ k_z = xp \]

\[ x \ll 1 \]

Probability of emission

\[ dP \sim \alpha_s \frac{dk^2}{k^2} \frac{dx}{x} \]

In the small-\(x\) limit

\[ \int_x^1 \frac{dx_1}{x_1} \sim \alpha_s \log(1/x) \]

\[ n\text{-gluon emission} \rightarrow \alpha_s^n \log^n (1/x) \]
Consider a fast-moving $q\bar{q}$ dipole (Rapidity: $Y = \log(s)$) [Mueller,93]

- Probability $\bar{\alpha}K$ of emission
- Independent emissions in coordinate space (transverse plane)
Consider a fast-moving $q\bar{q}$ dipole (Rapidity: $Y = \log(s)$) of size $r$

- Probability $\bar{\alpha}K$ of emission
- Independent emissions in coordinate space (transverse plane)
- Large-$N_c$ approximation
How to observe this system?

\[ T(r, Y) \approx \alpha_s^2 n(r, Y) \]

Count the number of dipoles of a given size
Consider a small increase in rapidity

\[
\partial_Y T(x, y; Y) = T(x, z; Y) + T(z, y; Y) - T(x, y; Y)
\]
Consider a small increase in rapidity \( \Rightarrow \) splitting

\[
\partial_Y T(x, y; Y)
= \tilde{\alpha} \int d^2 z \frac{(x - y)^2}{(x - z)^2(z - y)^2} \left[ T(x, z; Y) + T(z, y; Y) - T(x, y; Y) \right]
\]

[Balitsky,Fadin,Kuraev,Lipatov,78]
The solution goes like

\[ T(Y) \sim e^{\omega Y} \quad \text{with} \quad \omega = 4\bar{\alpha} \log(2) \approx 0.5 \]

- Fast growth of the amplitude
- Intercept value too large
- Violation of the Froissart bound:
  \[ T(Y) \leq C \log^2(s) \quad T(r, b) \leq 1 \]
- + problem of diffusion in the infrared
Saturation effects

Multiple scattering

★ Proportional to $T^2$
★ important when $T \approx 1$

\[
\partial_Y \langle T(x, y; Y) \rangle
\]

\[
= \bar{\alpha} \int d^2z \frac{(x - y)^2}{(x - z)^2(z - y)^2} \left[ \langle T(x, z; Y) \rangle + \langle T(z, y; Y) \rangle - \langle T(x, y; Y) \rangle \right. \\
\left. - \langle T(x, z; Y)T(z, y; Y) \rangle \right]
\]

contains

\[
\partial \langle T(x, y; Y) \rangle \longrightarrow \langle T(x, z; Y)T(z, y; Y) \rangle
\]
In general: complete hierarchy

\[ \partial_Y \langle T^k \rangle \rightarrow \langle T^k \rangle, \langle T^{k+1} \rangle \]

Mean field approx.: \( \langle T_{xz} T_{zy} \rangle = \langle T_{xz} \rangle \langle T_{zy} \rangle \)

\[ \partial_Y \langle T_{xy} \rangle = \frac{\bar{\alpha}}{2\pi} \int d^2z \frac{(x - y)^2}{(x - z)^2(z - y)^2} [\langle T_{xz} \rangle + \langle T_{zy} \rangle - \langle T_{xy} \rangle - \langle T_{xz} \rangle \langle T_{zy} \rangle] \]

Simplest perturbative evolution equation satisfying unitarity constraint
Consider evolution of $\langle T^{(2)} \rangle$

$\partial_Y \langle T^{(2)} \rangle \propto \langle T^{(2)} \rangle$

[E. Iancu, D. Triantafyllopoulos, 05]
Also A. Mueller, S. Munier, A. Shoshi, S. Wong

Usual BFKL ladder
Consider evolution of $\langle T^{(2)} \rangle$ [E. Iancu, D. Triantafyllopoulos, 05] Also A. Mueller, S. Munier, A. Shoshi, S. Wong

Usual BFKL ladder

fan diagram $\rightarrow$ saturation effects

$\partial_Y \langle T^{(2)} \rangle \propto \langle T^{(2)} \rangle$

$\partial_Y \langle T^{(2)} \rangle \propto \langle T^{(3)} \rangle$
Consider evolution of $\langle T^{(2)} \rangle$

Usual BFKL ladder

fan diagram $\longrightarrow$ saturation effects

splitting $\longrightarrow$ fluctuations, pomeron loops

$\partial_Y \langle T^{(2)} \rangle \propto \langle T^{(2)} \rangle$

$\partial_Y \langle T^{(2)} \rangle \propto \langle T^{(3)} \rangle$

$\partial_Y \langle T^{(2)} \rangle \propto \langle T \rangle$
⇒ complicated hierarchy

\[ \partial_Y \langle T^{(2)}(x_1, y_1; x_2, y_2) \rangle \]

\[ = \frac{\bar{\alpha}}{2\pi} \int d^2 z \frac{(x_2 - y_2)^2}{(x_2 - z)^2(z - y_2)^2} \left[ \langle T^{(2)}(x_1, y_1; x_2, z) \rangle + \langle T^{(2)}(x_1, y_1; z, y_2) \rangle \right] \]

\[ - \langle T^{(2)}(x_1, y_1; x_2, y_2) \rangle - \langle T^{(3)}(x_1, y_1; x_2, z; z, y_2) \rangle + (1 \leftrightarrow 2) \]

\[ + \frac{\bar{\alpha}}{2\pi} \left( \frac{\alpha_s}{2\pi} \right)^2 \int_{uvz} M_{uvz} A_0(x_1 y_1 | uz) A_0(x_2 y_2 | vz) \nabla^2_u \nabla^2_v \langle T^{(1)}(u, v) \rangle \]

- **Saturation**: important when \( T^{(2)} \sim T^{(1)} \sim 1 \) i.e. near unitarity
- **Fluctuations**: important when \( T^{(2)} \sim \alpha_s^2 T^{(1)} \) or \( T \sim \alpha_s^2 \) i.e. dilute regime
Reaction-diffusion process

\[ A \xrightleftharpoons{\gamma \sigma} A + A \]

Master equation: \( P_n \equiv \text{proba to have } n \text{ particles} \)

\[
\partial_t P_n = \gamma (n - 1) P_{n-1} - \gamma nP_n + \sigma n(n + 1) P_{n+1} - \sigma n(n - 1) P_n
\]

Particle densities: we observe a subset of \( k \) particles

\[
\langle n^k \rangle \equiv \sum_{N=k}^{\infty} \frac{N!}{(N - k)!} P_N
\]
Reaction-diffusion process: \[ A \xrightleftharpoons{\gamma}{\sigma} A + A \]

Master equation: \[ P_n \equiv \text{proba to have } n \text{ particles} \]

\[
\partial_t P_n = \gamma (n-1) P_{n-1} - \gamma n P_n + \sigma n (n+1) P_{n+1} - \sigma n (n-1) P_n
\]

Evolution equation: \[ \langle n^k \rangle \equiv \text{particle density/correlators} \]

\[
\partial_t \langle n^k \rangle = \gamma k \langle n^k \rangle + \gamma k (k-1) \langle n^{k-1} \rangle - \sigma k (k+1) \langle n^{k+1} \rangle
\]

Scattering amplitude for this system off a target

\[ A(t) = \sum_{k=0}^{\infty} (-)^k \langle n^k \rangle_{t_0} \langle T^k \rangle_{t-t_0} \]

\( t_0 \)-independent \[ \Rightarrow \]

\[
\partial_t \langle T^k \rangle = \gamma \langle T^k \rangle - \gamma \langle T^{k+1} \rangle + \sigma \langle T^{k-1} \rangle
\]

BFKL\hspace{5em} \text{sat.} \hspace{5em} \text{fluct.}
For QCD particle = (effective) dipoles

Dipole splitting $\equiv$ BFKL kernel

$$\gamma \sim \bar{\alpha} \frac{(x - y)^2}{(x - z)^2(z - y)^2}$$

Effective dipole merging

$$\sigma(x_1y_1, x_2y_2 \rightarrow uv) \sim \bar{\alpha}\alpha_s^2 \nabla_u^2 \nabla_v^2 \left\{ M_{uvz} \log^2 \left[ \frac{(x_1 - u)^2(y_1 - z)^2}{(x_1 - z)^2(y_1 - u)^2} \right] \log^2 \left[ \frac{(x_2 - v)^2(y_2 - z)^2}{(x_2 - z)^2(y_2 - v)^2} \right] \right\}$$

Remarks:

- merging not always positive
- fluctuations = gluon-number fluctuations
- Can be obtained from projectile or target point of view
- Known at large $N_c$. 
Solutions

The BK equation
\[ b\text{-independent situation: momentum space } (L = \log(k^2/k_0^2)) \]

\[ \partial_{\bar{\alpha}Y} T(k) = \chi_{\text{BFKL}}(-\partial L)T(k) - T^2(k) \]

Diffusive approximation:

\[ \chi_{\text{BFKL}}(-\partial L) = \chi(\frac{1}{2}) + \frac{1}{2}(\partial L + \frac{1}{2})^2 \]

Time \( t = \bar{\alpha}Y \), Space \( x \approx \log(k^2) \), \( u \propto T \)

Fisher-Kolmogorov-Petrovsky-Piscounov (F-KPP)

\[ \partial_t u(x, t) = \partial^2_x u(x, t) + u(x, t) - u^2(x, t) \]
Asymptotic solution: traveling wave

\[ u(x, t) = u(x - v_c t) \]

Position: \[ X(t) = X_0 + v_c t \]
Mechanism: take only the linear part
\[ \partial_Y T = \chi(-\partial_L)T - T^2 \]

\[ T(k) = \int \frac{d\gamma}{2i\pi} T_0(\gamma) \exp(\chi(\gamma)Y - \gamma L) \]

⇒ Wave of slope \( \gamma \) travels at speed \( v = \frac{\chi(\gamma)}{\gamma} \)

The minimal speed is selected during evolution
**Geometric scaling**

Numerical simulations:

![Graph showing geometric scaling](image)

- $Y = 0$
- $Y = 5$
- $Y = 10$
- $Y = 15$
- $Y = 20$
- $Y = 25$

$\log(k^2)$

---

G. Soyez  
KEK, Tokyo, Japan, April 2006  
Fluctuations in High-Energy QCD – p. 20/35
Geometric scaling

Numerical simulations:

\[ T(k, Y) = T \left( \frac{k^2}{Q_s^2(Y)} \right) \approx \left( \frac{k^2}{Q_s^2(Y)} \right)^{-\gamma_c} \]

\[ Q_s^2(Y) \propto \exp^{v_c Y} \]
Can we extend this including the $b$ dependence

Go to momentum space

$$\tilde{T}(k, q) = \int d^2x \, d^2y \, e^{ik \cdot x} \, e^{i(q-k) \cdot y} \, \frac{T(x, y)}{(x-y)^2}$$

new form of the BK equation

$$\partial_Y \tilde{T}(k, q) = \frac{\bar{\alpha}}{\pi} \int \frac{d^2k'}{(k-k')^2} \left\{ \tilde{T}(k', q) - \frac{1}{4} \left[ \frac{k^2}{k'^2} + \frac{(q-k)^2}{(q-k')^2} \right] \tilde{T}(k, q) \right\}$$

$$- \frac{\bar{\alpha}}{2\pi} \int d^2k' \, \tilde{T}(k, k') \tilde{T}(k-k', q-k')$$

[C.Marquet, R.Peschanski, G.S., 05]
One can prove analytically that:

- formation of a traveling wave at large $p$ (or $k$)
- $q$ dependence: scales like a constant or linearly ($Y = 25$)

Predicts geometric scaling for $t$-dependent processes
Solutions

Fluctuation effects
no $b$-dependence + coarse-graining $\longrightarrow$ Langevin equation

\[ \partial_Y T(k, Y) = \bar{\alpha} K_{BFKL} \otimes T(k, Y) - \bar{\alpha} T^2(k, Y) + \bar{\alpha} \sqrt{\kappa \alpha_s^2 T(k, Y)} \nu(k, Y) \]

with $\langle \nu(k, Y) \rangle = 0$

$\langle \nu(k, Y) \nu(k', Y') \rangle = \frac{1}{\bar{\alpha}} \delta(Y - Y') k \delta(k - k')$

Diffusive approximation

\[ \partial_t u(x, t) = \partial_x^2 u + u - u^2 + \sqrt{2 \kappa u} \nu(x, t) \]

stochastic F-KPP
Decrease of the velocity/exponent of the saturation scale
For asymptotically small $\alpha_s$ (not true here)[A. Mueller, S. Munier, E. Brunet, B. Derrida]

$$v^* \xrightarrow{\alpha_s^2 \kappa \to 0} v_{BK} - \frac{\bar{\alpha} \pi^2 \gamma_c \chi''(\gamma_c)}{2 \log^2(\alpha_s^2 \kappa)}$$
Event properties (2/2)

- Dispersion of the events

\[ \Delta \log[Q_s^2(Y)] \approx \sqrt{D_{\text{diff}} \bar{\alpha} Y} \quad \text{with} \quad D_{\text{diff}} \sim \frac{1}{\log^3(\alpha_s^2 \kappa) \mid \kappa \to 0}. \]

- No important dispersion in early stages of the evolution!
- Clear effect of fluctuations
- Violations of geometric scaling (not in early stages)
- Agrees with predictions
• Dense regime: $\langle T^2 \rangle \approx \langle T \rangle^2$

• Dilute regime: $\langle T^2 \rangle \approx \langle T \rangle$ (pre-asymptotics!)
High-energy behaviour

Evolution with saturation & fluctuations $\equiv$

- superposition of unitary front (with geometric scaling)
- with a dispersion (yielding geometric scaling violations)

$$
\langle T(r, Y) \rangle = \int d\rho_s \ T_{\text{event}}(\rho - \rho_s) \ \frac{1}{\sqrt{\pi \sigma}} \ \exp\left( - \frac{(\rho_s - \bar{\rho}_s)^2}{\sigma^2} \right)
$$

with $\rho = \log(1/r^2)$, $\rho_s = \log(Q_s^2)$

$$
T_{\text{event}}(\rho - \rho_s) = \begin{cases} 
1 & r > Q_s \\
(rQ_s)\gamma & r < Q_s 
\end{cases}
$$
High-energy behaviour

\[ \log \left( \frac{r_0^2}{r^2} \right) \]

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### Strong noise limit

<table>
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<td>Diffusive scaling</td>
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<tr>
<td>( \langle T \rangle = f \left[ \log (k^2/Q_s^2) \right] )</td>
<td>( \langle T \rangle = f \left[ \log (k^2/Q_s^2) / \sqrt{DY} \right] )</td>
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<tr>
<td>( \langle T(k) \rangle = \langle T \rangle^k )</td>
<td>( \langle T(k) \rangle = \langle T \rangle )</td>
</tr>
</tbody>
</table>

At high-energy, amplitudes are dominated by hot-spots *i.e.* rare fluctuations at saturation

- true for strong fluctuations
- asymptotically true in general
Saturation fit:
\[
\langle T(r, Y) \rangle = \begin{cases} 
(r^2 Q_s^2) \gamma_c e^{-\frac{2 \log^2(r Q_s)}{c_Y}} & r < Q_s \\
1 - e^{-a-b \log^2(r Q_s)} & r > Q_s 
\end{cases}
\]
\[Q_s^2(Y) = \lambda Y, \quad \rho_s = \log(Q_s^2)\]

Saturation+fluctuations fit:
\[
\langle T(r, Y) \rangle = \int d\rho_s T(r, \rho_s) \frac{1}{\sqrt{\pi} \sigma} e^{-\frac{(\rho_s - \bar{\rho}_s)^2}{\sigma^2}}
\]
\[T(r, \rho_s) = \begin{cases} 
(r^2 Q_s^2) & r < Q_s \\
1 & r > Q_s 
\end{cases}
\]

colour transparency
**Describing $F_2$**

Saturation fit:

$$\langle T(r, Y) \rangle = (r^2 Q_s^2)^{\gamma_c} e^{-\frac{2 \log^2(r Q_s)}{c_Y}} \rightarrow r^2 Q_s^2$$

Saturation+fluctuations fit:

$$\langle T(r, Y) \rangle = \int d\rho_s T(r, \rho_s) \frac{1}{\sqrt{\pi} \sigma} e^{-\frac{(\rho_s - \overline{\rho}_s)^2}{\sigma^2}} \rightarrow \frac{1}{2} \text{erfc} \left( \frac{-\log(r Q_s^2)}{\sqrt{2DY}} \right)$$

![Graphs showing Geometric and Diffusive scaling](image)

$Y \rightarrow \infty$ Geometric scaling

$Y \rightarrow \infty$ Diffusive scaling
Both fits can describe the data for $x \leq 0.01$.
Effects of saturation

- Evolution equations for high-energy QCD
  Balitsky, Balitsky-Kovchegov (dipole), JIMWLK (CGC)
- Good knowledge of the asymptotic solutions
  Traveling waves → geometric scaling, saturation scale $\propto \exp(\bar{\alpha} v_c Y)$

Effects of fluctuations

- Known at large-$N_c$
- Consequences on saturation (e.g. geometric scaling violations)
  Diffusive scaling
- analytical solutions: $\alpha_s \ll 1$
- numerical solutions: coherent with statistical-physics analog
phenomenological tests:

- do we observe geometric scaling at nonzero momentum transfer?
- predictions for LHC? diffusive scaling at high-energy?
phenomenological tests:
- do we observe geometric scaling at nonzero momentum transfer?
- predictions for LHC? diffusive scaling at high-energy?

theoretical questions:
- importance of geometric scaling violations
- analytical predictions (pomeron loops, triple pomeron vertex)
- numerical simulations: include impact parameter
- beyond large-$N_c$