Stochastic high-energy QCD

Gregory Soyez

Based on : C. Marquet, Y. Hatta, E. Iancu, G.S., D. Triantafyllopoulos, N. P. A773 (2006) 95, [hep-ph/0601150]

C. Marquet, E. Iancu, G.S., hep-ph/0605174

C. Marquet, R. Peschanski, G.S., Phys. Rev. D73 (2006) 114005, [hep-ph/0512186]

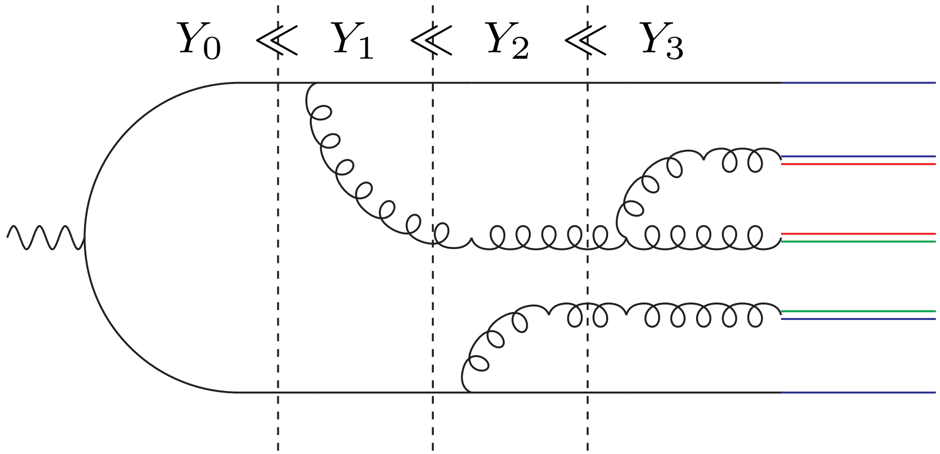
C. Marquet, G.S., B. Xiao, hep-ph/0606233

Outline

- Summary of High-energy evolution equations
  saturation and fluctuations
- Asymptotic solutions
  statistical physics and numerical results
- Physical picture of high-energy QCD
  - Stochastic saturation scale
  - Diffusive scaling and black spots
- The DIS data

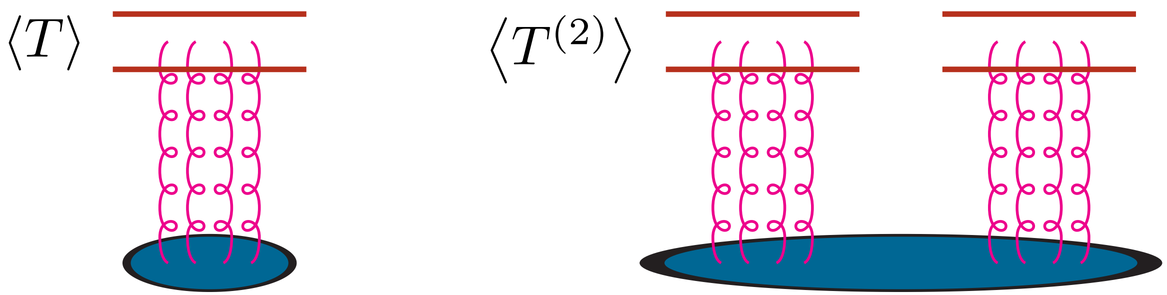
DSM-SPhT
Service de Physique Théorique

[Mueller, 93]



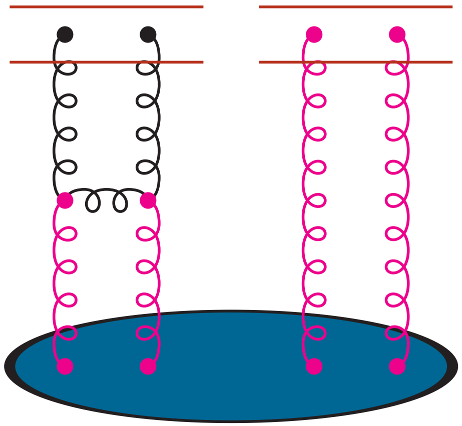
$n(r, Y)$ dipoles of size r

- High-energy: Bremsstrahlung of soft gluons
- Large-N_c: gluon at $\mathbf{z}$ = $q\bar{q}$ pair at $\mathbf{z}$
  $\Rightarrow$ gluon emission = dipole splitting
  $\Rightarrow$ set of dipoles with rapidity $Y = \log(1/x)$ and transverse coord. $(\mathbf{x}, \mathbf{y})$
- scattering amplitude & correlations



Evolution towards high energy

[Balitsky, Fadin, Kuraev, Lipatov; 74]



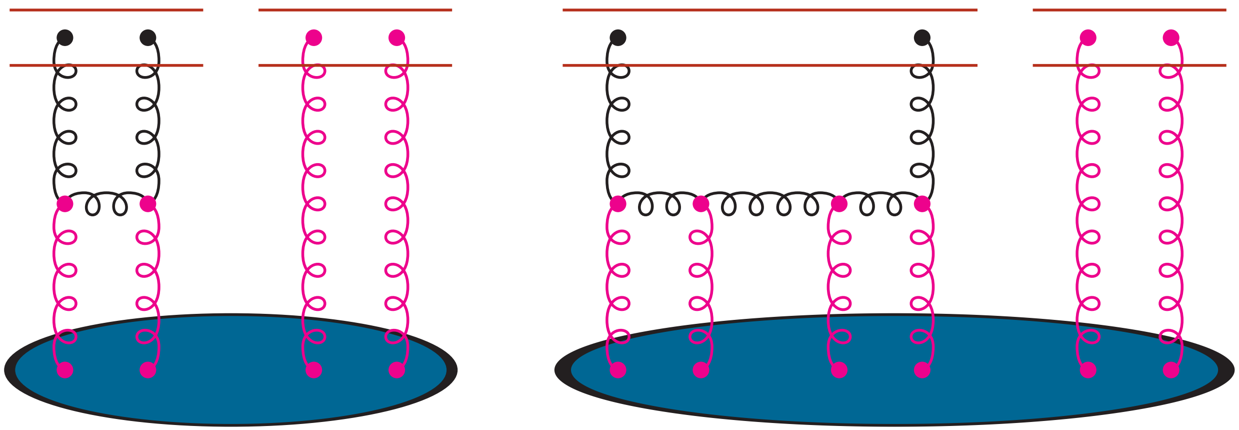
Ladder-type diagrams $\Rightarrow$ BFKL equation

$$\partial_Y \langle T_{\mathbf{xy}} \rangle = \frac{\bar{\alpha}}{2\pi} \int_{\mathbf{z}} \frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{z}-\mathbf{y})^2} \left[\langle T_{\mathbf{xz}} \rangle + \langle T_{\mathbf{zy}} \rangle - \langle T_{\mathbf{xy}} \rangle \right]$$

$$\partial_Y \left\langle T^{(2)}_{\mathbf{x}_1\mathbf{y}_1,\mathbf{x}_2\mathbf{y}_2} \right\rangle = \frac{\bar{\alpha}}{2\pi} \int_{\mathbf{z}} \frac{(\mathbf{x}_1-\mathbf{y}_1)^2}{(\mathbf{x}_1-\mathbf{z})^2(\mathbf{z}-\mathbf{y}_1)^2} \left[\left\langle T^{(2)}_{\mathbf{x}_1\mathbf{z}} \right\rangle + \left\langle T^{(2)}_{\mathbf{z}\mathbf{y}_1} \right\rangle - \left\langle T^{(2)}_{\mathbf{x}_1\mathbf{y}_1} \right\rangle \right] + (1 \leftrightarrow 2)$$

unitarity violations

[Balitsky 96; Kovchegov 99]



Unitarity corrections for $T \sim 1$: Add fan diagrams $\Rightarrow$ Balitsky equation

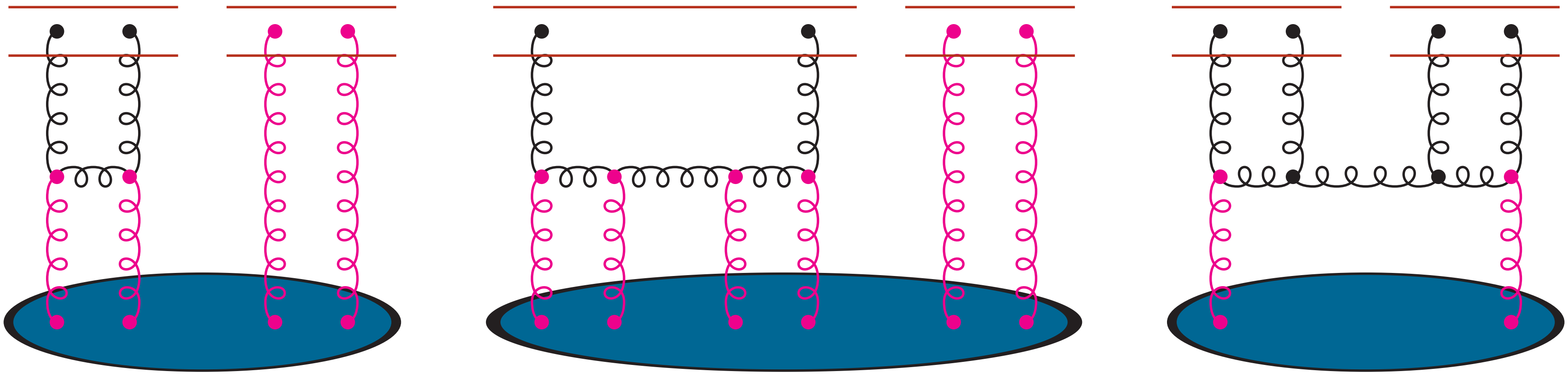
$$\partial_Y \langle T_{\mathbf{xy}} \rangle = \frac{\bar\alpha}{2\pi} \int_{\mathbf{z}} \frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{z}-\mathbf{y})^2} \left[\langle T_{\mathbf{xz}} \rangle + \langle T_{\mathbf{zy}} \rangle - \langle T_{\mathbf{xy}} \rangle - \left\langle T^{(2)}_{\mathbf{xz},\mathbf{zy}} \right\rangle \right]$$

$$\partial_Y \left\langle T^{(2)}_{\mathbf{x}_1\mathbf{y}_1} \right\rangle = \frac{\bar\alpha}{2\pi} \int_{\mathbf{z}} \frac{(\mathbf{x}_1-\mathbf{y}_1)^2}{(\mathbf{x}_1-\mathbf{z})^2(\mathbf{z}-\mathbf{y}_1)^2} \left[\left\langle T^{(2)}_{\mathbf{x}_1\mathbf{z}} \right\rangle + \left\langle T^{(2)}_{\mathbf{z}\mathbf{y}_1} \right\rangle - \left\langle T^{(2)}_{\mathbf{x}_1\mathbf{y}_1} \right\rangle - \left\langle T^{(3)}_{\mathbf{x}_1\mathbf{z},\mathbf{z}\mathbf{y}_1} \right\rangle \right]$$

- infinite hierarchy: Balitsky/JIMWLK
- Mean field $\left\langle T^{(2)}_{\mathbf{xz},\mathbf{zy}} \right\rangle = \langle T_{\mathbf{xz}} \rangle \langle T_{\mathbf{zy}} \rangle$: BK equation

Evolution towards high energy

[Iancu, Triantafyllopoulos; 05]



gluon-number fluctuations for $T \sim \alpha_s^2$: add fluctuations in the target

$$\partial_Y \left\langle T^{(2)}_{\mathbf{x}_1\mathbf{y}_1,\mathbf{x}_2\mathbf{y}_2} \right\rangle \Big|_{\text{fluct}} = \frac{1}{2}\frac{\bar{\alpha}}{2\pi}\left(\frac{\alpha_s}{2\pi}\right)^2 \int_{\mathbf{uvz}} \mathcal{M}_{\mathbf{uvz}} \mathcal{A}_0(1|\mathbf{uz})\mathcal{A}_0(2|\mathbf{zv}) \nabla^2_{\mathbf{u}} \nabla^2_{\mathbf{v}} \left\langle T_{\mathbf{uv}} \right\rangle$$

- equivalent to a reaction-diffusion problem ([Iancu, G.S., Triantafyllopoulos])
- projectile-target duality ([Kovner, Lublinsky])

BK vs. F-KPP equation

[S. Munier, R. Peschanski]

b-independent BK in momentum space

$$\boxed{\partial_{\bar\alpha Y} T(k) = \chi_{\text{BFKL}}(-\partial_{\log(k^2)}) T(k) - T^2(k)}$$

Diffusive approximation:

$\chi_{\text{BFKL}}(-\partial_{\log(k^2)})$ up to $\partial^2_{\log(k^2)}$

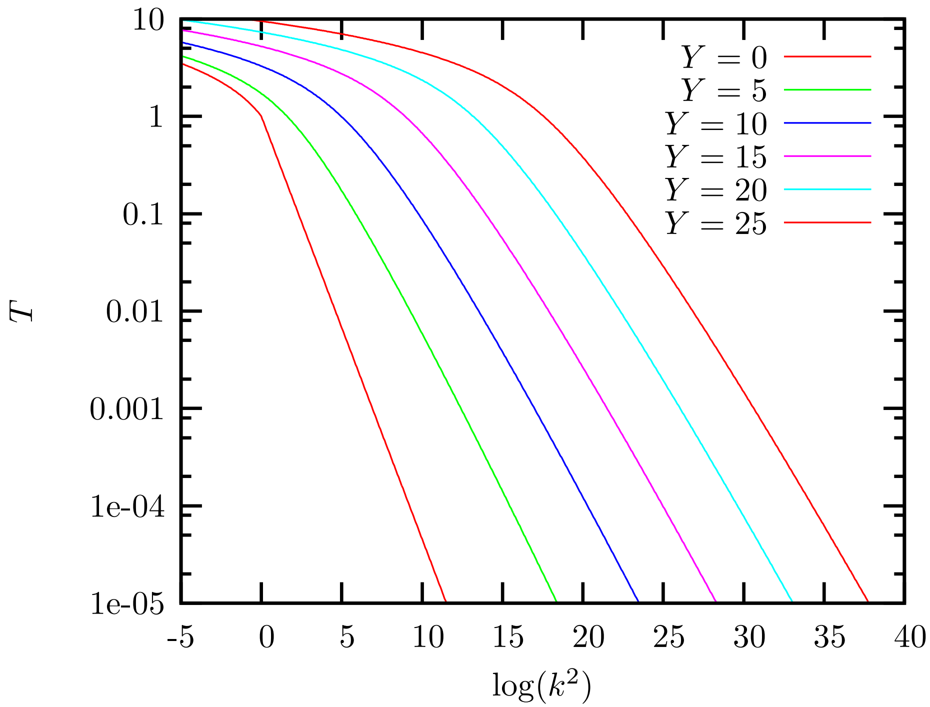
Time $t = \bar\alpha Y$, Space $x \approx \log(k^2)$, $u \propto T$

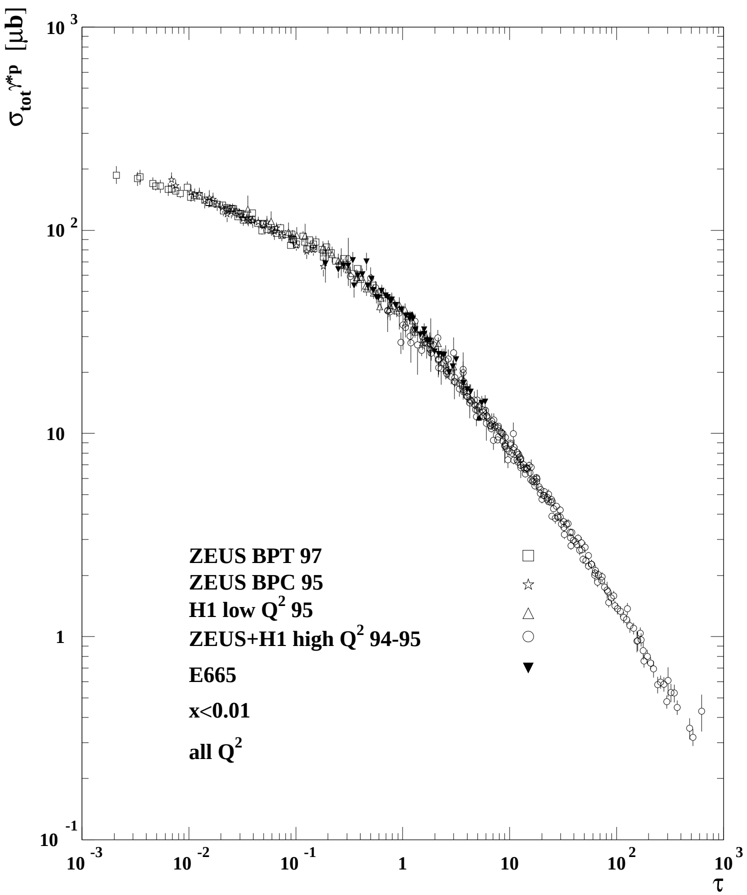
$$\boxed{\partial_t u(x,t) = \partial_x^2 u(x,t) + u(x,t) - u^2(x,t)}$$

Fisher-Kolmogorov-Petrovsky-Piscounov (F-KPP)

Prediction: ([Munier, Peschanski])

formation of a traveling-wave pattern





$$T(k,Y) = T(\log(k^2) - v_c Y) = T\left(\frac{k^2}{Q_s^2(Y)}\right) \qquad \text{with } Q_s^2 \sim \exp(v_c Y)$$

Geometric scaling (speed of the wave $\to$ energy dependence of Q_s^2)

[Golec-Biernat, Stasto, Kwiecinski]

With fluctuations

no b-dependence + local approximation for fluctuations (introduces a factor κ)

$\longrightarrow$ Langevin equation

$$\boxed{\partial_Y T(k,Y) = \bar\alpha K_{\text{BFKL}} \otimes T(k,Y) - \bar\alpha T^2(k,Y) + \bar\alpha\sqrt{\kappa\alpha_s^2 T(k,Y)}\,\nu(k,Y)}$$

with $\langle \nu(k,Y)\rangle = 0$

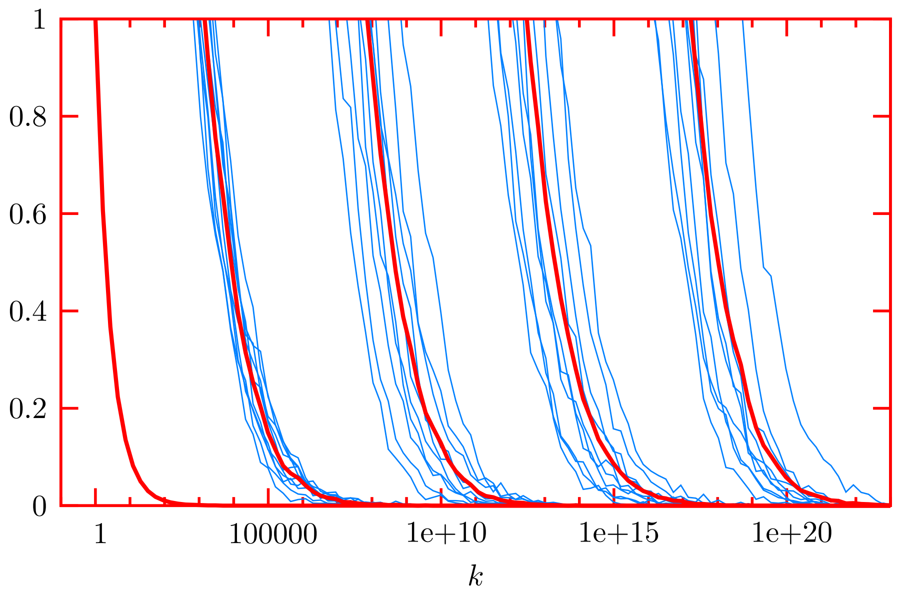
$$\langle \nu(k,Y)\nu(k',Y')\rangle = \frac{1}{\bar\alpha}\delta(Y-Y')\,k\delta(k-k')$$

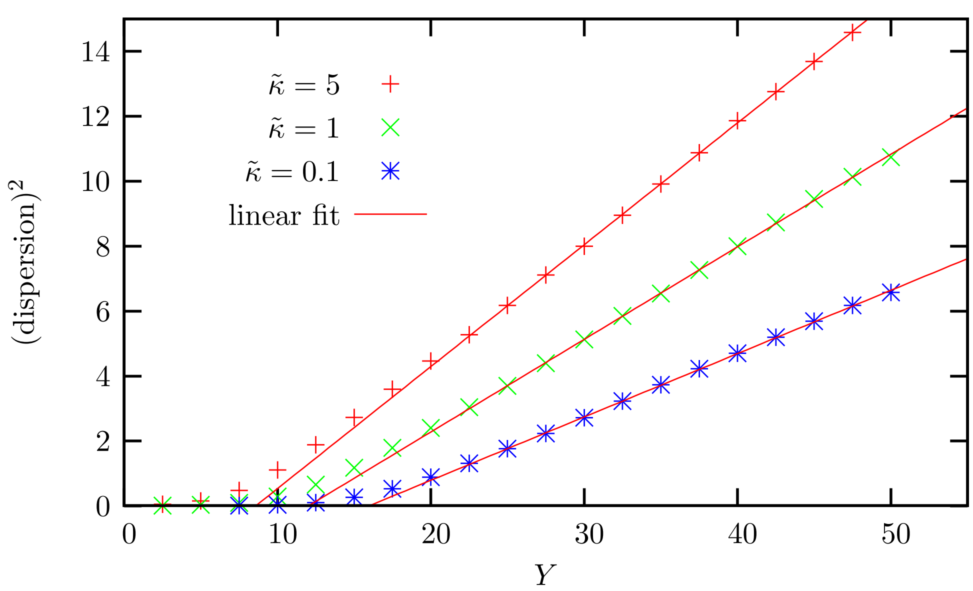
Diffusive approximation

$$\boxed{\partial_t u(x,t) = \partial_x^2 u + u - u^2 + \sqrt{2\kappa u}\,\nu(x,t)}$$

stochastic F-KPP

[G.S. 05]





- Dispersion of the travelling-wave events $\Rightarrow$ geometric scaling violations

$$\Delta \log[Q_s^2(Y)] \approx \sqrt{D_{\text{diff}}\bar{\alpha}Y}$$

- Stochastic saturation scale:

$$\rho_s = \log(Q_s^2) \quad \text{with probability } P(\rho_s)$$

Stochastic saturation scale

Cumulants for $P(\rho_s)$ (see Stephane Munier's talk):

$$\kappa_1 = \langle\rho_s\rangle = \bar{\alpha} v Y \;, \qquad \kappa_2 = \sigma^2 = \bar{\alpha} D Y \;, \qquad \kappa_n = \frac{3\gamma_c^2}{\pi^2}\frac{n!\zeta(n)}{\gamma_c^n}\sigma^2 \;,$$

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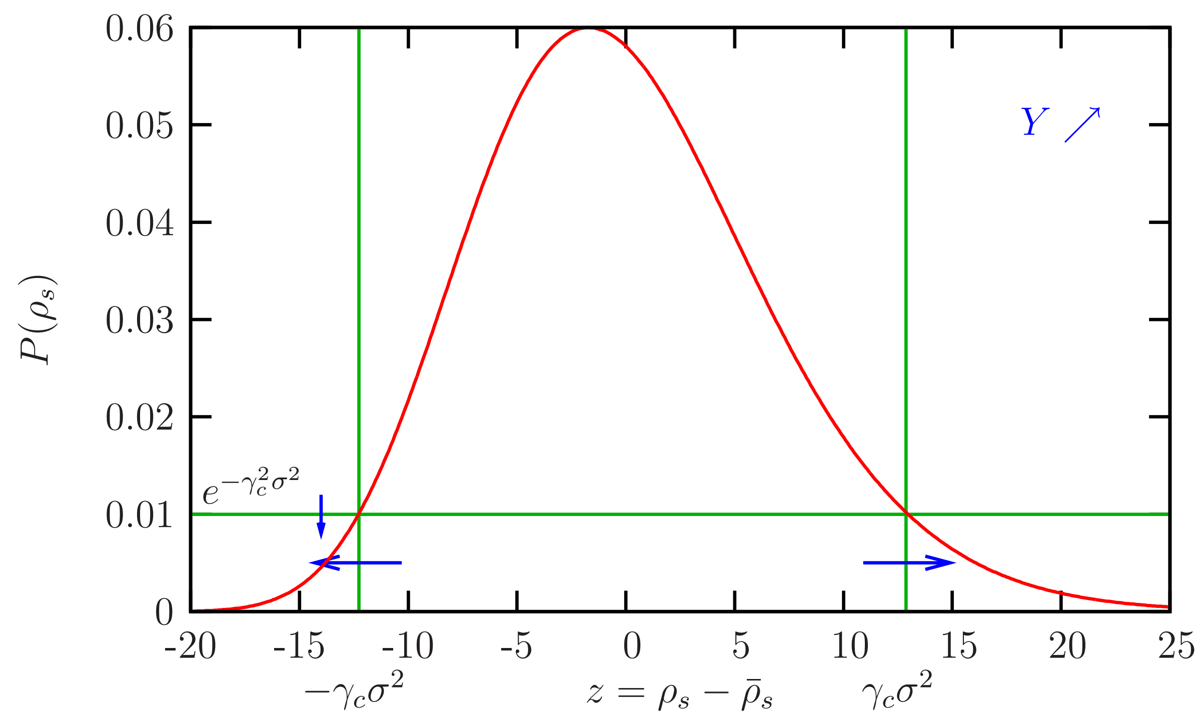
$$\kappa_1 = \langle \rho_s \rangle = \bar\alpha v Y\ , \qquad \kappa_2 = \sigma^2 = \bar\alpha D Y\ , \qquad \kappa_n = \frac{3\gamma_c^2}{\pi^2}\frac{n!\zeta(n)}{\gamma_c^n}\sigma^2\ ,$$

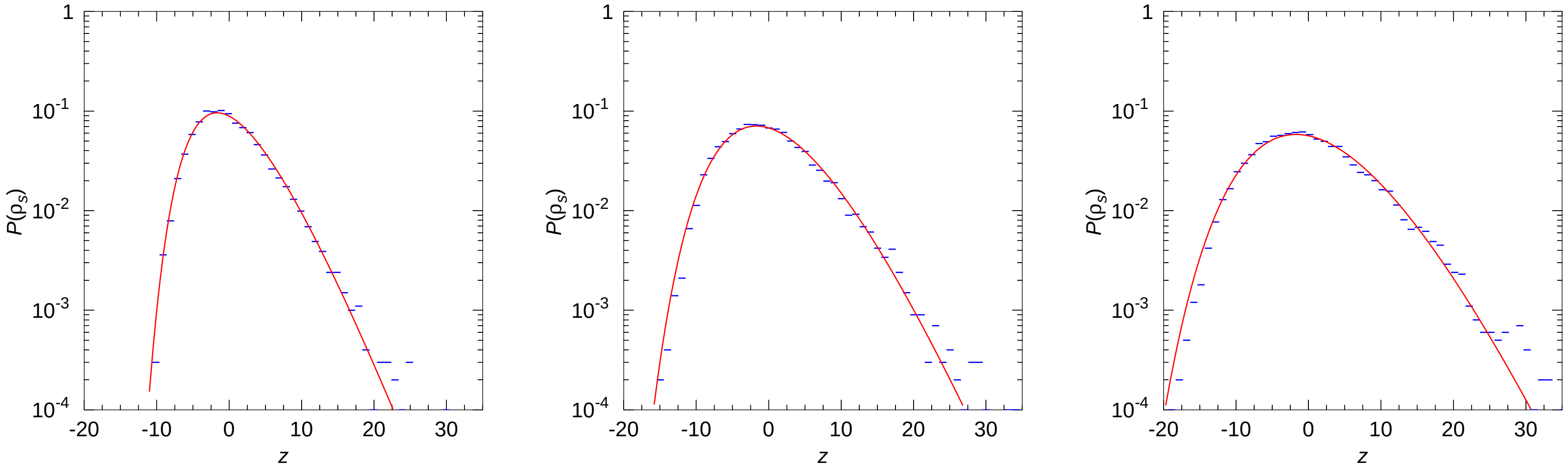
$$\Longrightarrow \quad P(\rho_s) = \int_{c-i\infty}^{c+i\infty}\frac{d\lambda}{2i\pi}\exp\left\{-\lambda(\rho_s-\bar\rho_s) - \frac{3\gamma_c\sigma^2}{\pi^2}\lambda\left[\gamma_E + \psi\left(1-\frac{\lambda}{\gamma_c}\right)\right]\right\}$$



[C. Marquet
G. S.
B. Xiao, 06]

Gumbel	Gaussian	exponential
$\exp[-\exp(-z)]$	$\exp(-z^2/\sigma^2)$	$\exp(-\gamma_c z)$

- Coherent with numerical simulations of the QCD Langevin equation



- For the computation of amplitudes:

$$P(\rho_s) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{(\rho_s - \bar{\rho}_s)^2}{2\sigma^2}\right]$$

is sufficient

High-energy behaviour

[Y. Hatta, E. Iancu, C. Marquet, G. S., D. Triantafyllopoulos, 06]

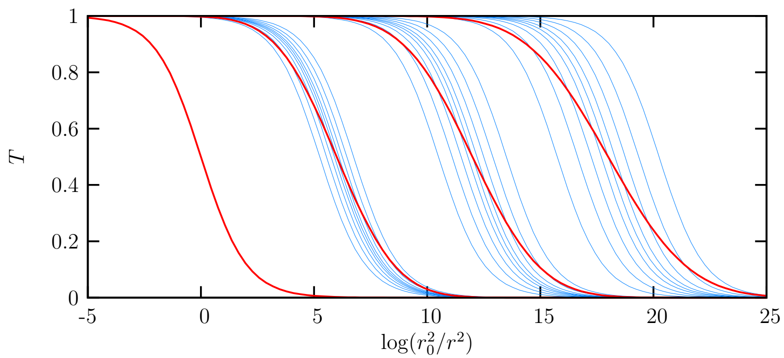
Evolution with saturation & fluctuations $\equiv$

- superposition of unitary front (with geometric scaling)
- with a dispersion (yielding geometric scaling violations)

$$\langle T(r,Y)\rangle = \int d\rho_s \, T_{\text{event}}(\rho-\rho_s)\; P(\rho_s)$$

with $\rho = \log(1/r^2)$, $\rho_s = \log(Q_s^2)$

$$T_{\text{event}}(\rho-\rho_s) = \begin{cases} 1 & r > 1/Q_s \qquad \text{saturation} \\ r^2 Q_s^2 & r < 1/Q_s \qquad \text{colour transparency} \end{cases}$$



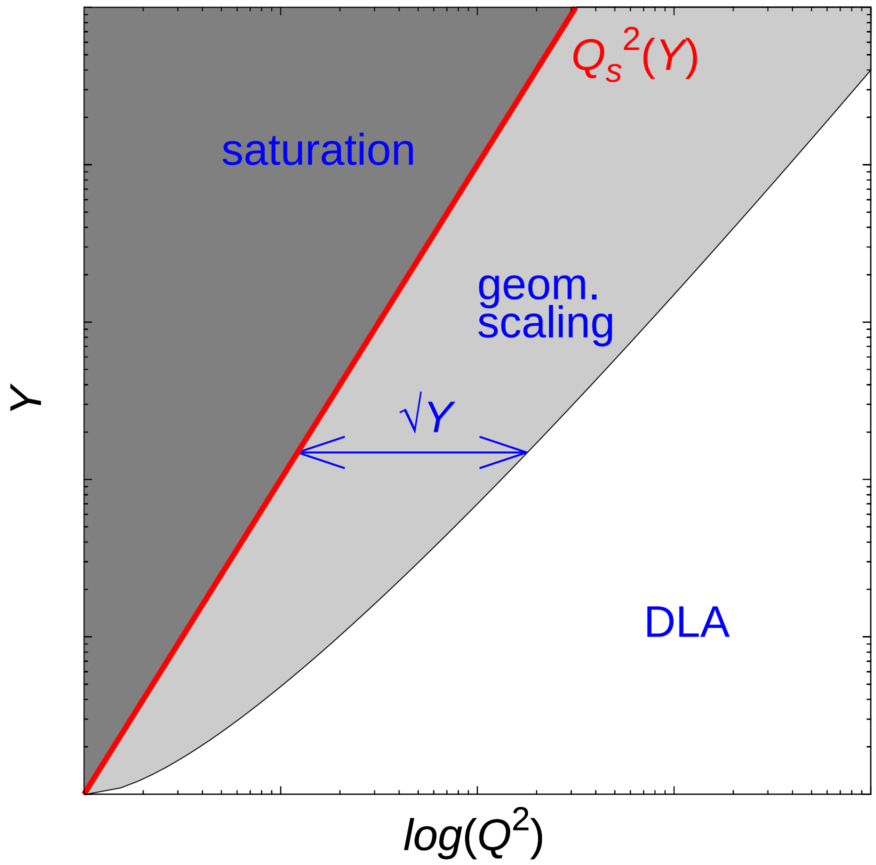
dispersion $\sim DY$

Energy:	Intermediate ($DY \ll 1$)	High energy ($DY \gg 1$)
Physics:	Mean field (BK)	Fluctuations
Amplitude:	Geometric scaling $\langle T\rangle = f\left[\log(k^2/Q_s^2)\right]$	Diffusive scaling $\langle T\rangle = f\left[\log(k^2/Q_s^2)/\sqrt{DY}\right]$

At high-energy, amplitudes are dominated by <u>black-spots</u>
i.e. rare fluctuations at saturation: $T = 1$ or 0

- $T_{\text{event}} = \Theta(Q_s - k) \qquad \Rightarrow \qquad \langle T^{(k)}\rangle \to \frac{1}{2}\text{erfc}\left[\log(k_{\text{max}}^2/Q_s^2)/\sqrt{DY}\right]$
- also obtained from strong-fluctuation limit

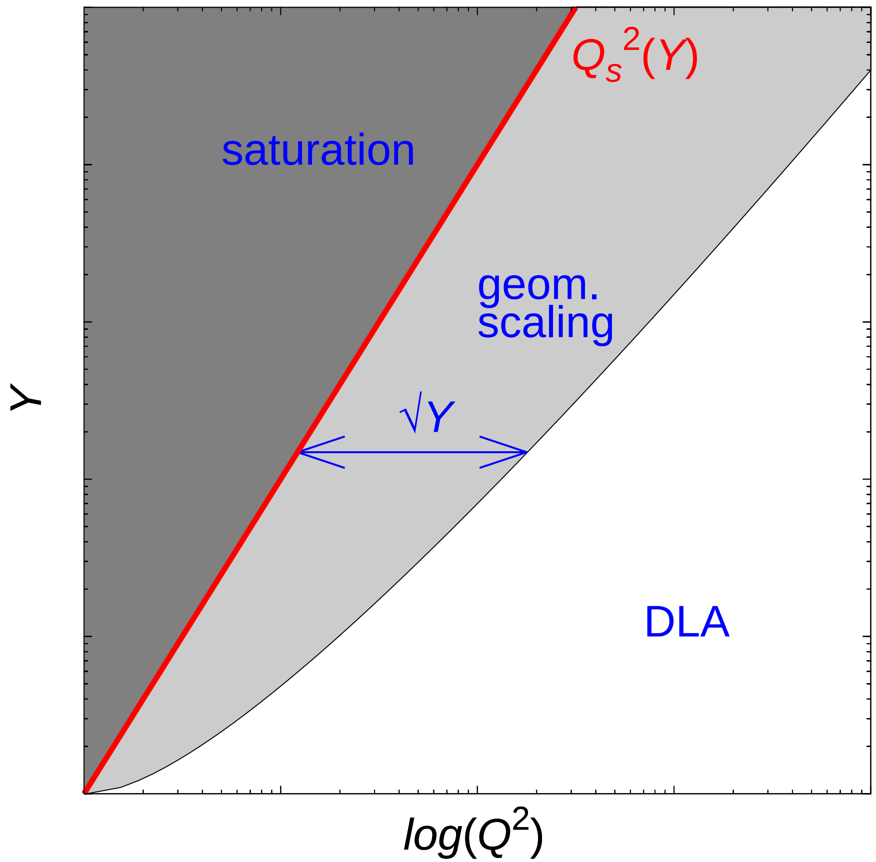
saturation (dispersion $\ll 1$):



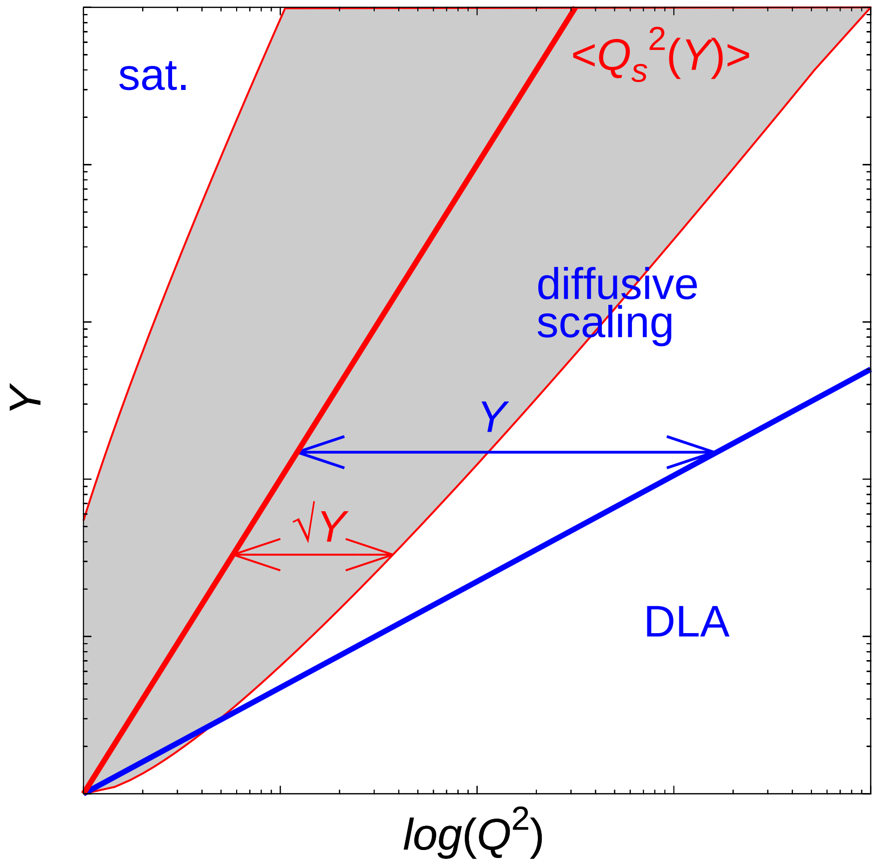
consequences of saturation even when $T \ll 1$

Phase space & scaling

saturation (dispersion $\ll 1$):



saturation+fluctuations (dispersion $\gg 1$):



consequences of saturation even when $T \ll 1$

Following fits to the F_2^p data ($x \le 0.01$):

Saturation fit: [Iancu, Itakura, Munier]

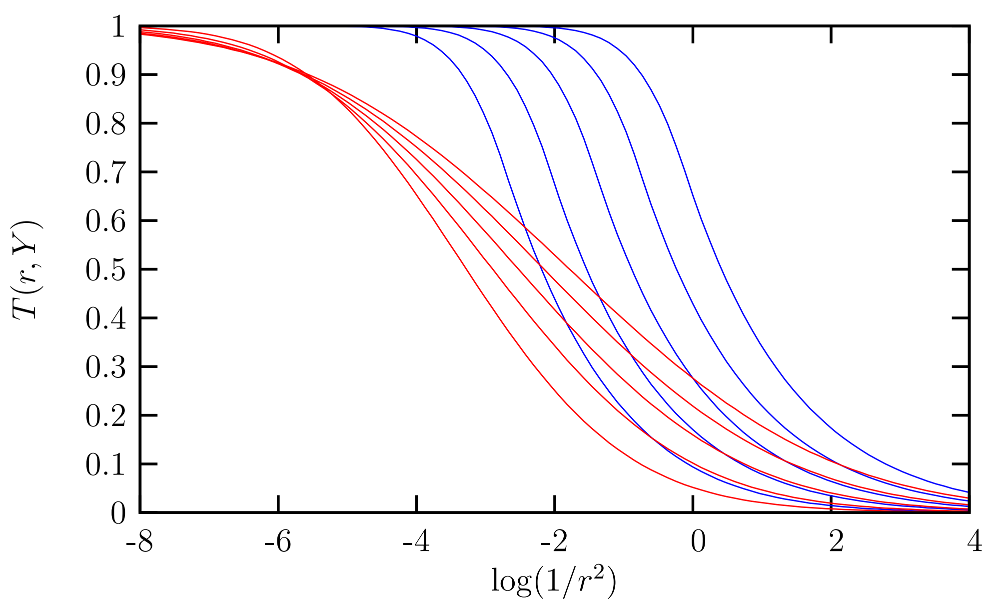
$$\langle T(r,Y)\rangle = \begin{cases} (r^2Q_s^2)^{\gamma_c} e^{-\frac{2\log^2(rQ_s)}{CY}} & r < 1/Q_s \\ 1 - e^{-a-b\log^2(rQ_s)} & r > 1/Q_s \end{cases}$$

$$Q_s^2(Y) = \lambda Y,\ \rho_s = \log(Q_s^2)$$

Saturation+fluctuations fit: [in preparation]

$$\langle T(r,Y)\rangle = \int d\rho_s\, T(r,\rho_s) \frac{1}{\sqrt{\pi}\sigma} e^{-\frac{(\rho_s-\bar{\rho}_s)^2}{\sigma^2}}$$

$$T(r,\rho_s) = \begin{cases} r^2Q_s^2 & r < 1/Q_s \\ 1 & r > 1/Q_s \end{cases}$$



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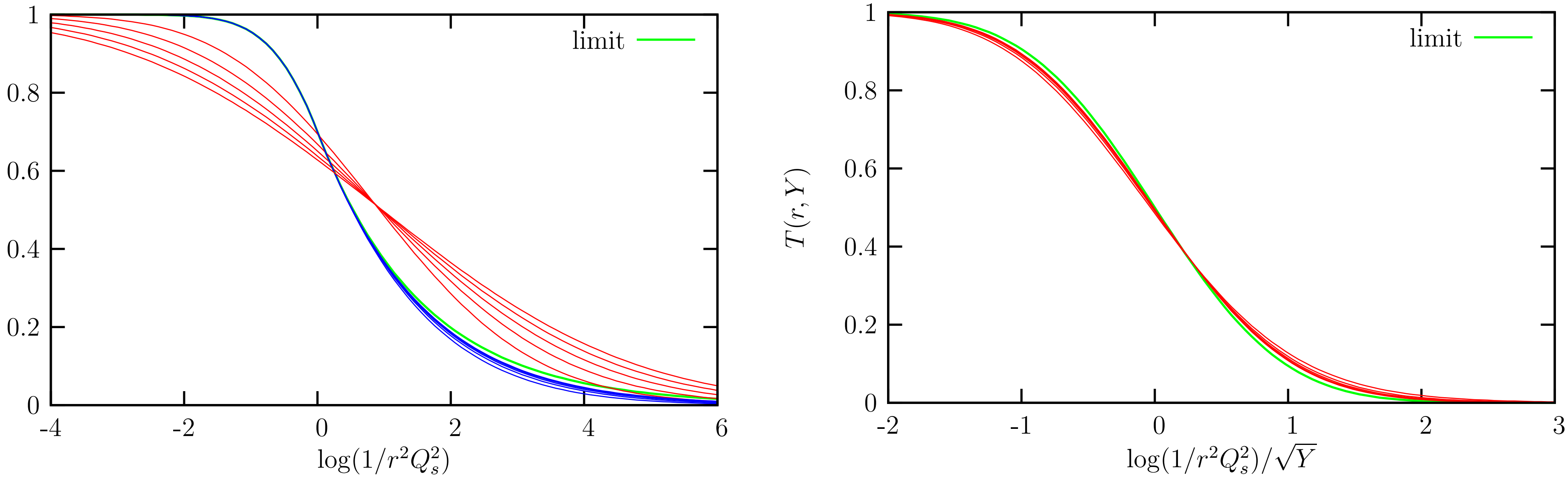
$$\langle T(r,Y)\rangle = (r^2Q_s^2)^{\gamma_c} e^{-\frac{2\log^2(rQ_s)}{CY}} \quad \to (r^2Q_s^2)^{\gamma_c}$$

Saturation+fluctuations fit: [in preparation]

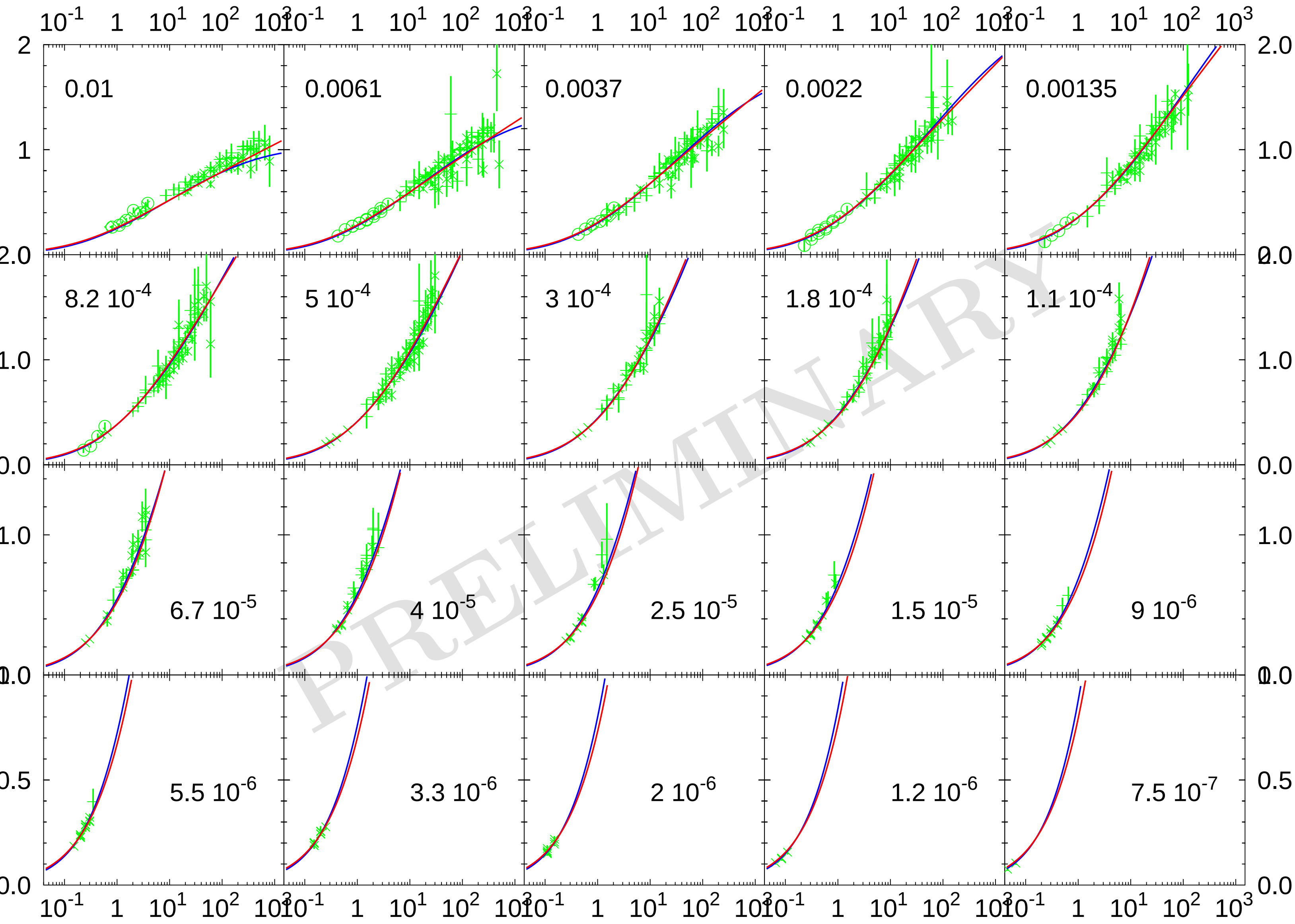
$$\langle T(r,Y)\rangle = \int d\rho_s\, T(r,\rho_s) \frac{1}{\sqrt{\pi}\sigma} e^{-\frac{(\rho_s-\bar\rho_s)^2}{\sigma^2}} \quad \to \frac{1}{2}\mathrm{erfc}\left(\frac{-\log(r^2\bar Q_s^2)}{\sqrt{2DY}}\right)$$



$\xrightarrow{Y\to\infty}$ Geometric scaling

$\xrightarrow{Y\to\infty}$ Diffusive scaling

Describing F_2



Both fits can describe the data for $x \leq 0.01$

Conclusion

- Fluctuations $\Rightarrow$ stochastic saturation scale
- At high energy,
  - Gaussian probability
  - $T_{\text{event}} = \Theta(\rho_s - \rho)$

  $\Rightarrow$ diffusive scaling
- Perspectives:
  - applications for diffraction (see talk by C. Marquet)
  - predictions for LHC: diffraction, forward jets (under study)
  - geometric scaling at non-zero momentum transfer (DVCS, ρ mesons)
  - include b-dependent fluctuations (under study)
  - beyond large-N_c (cfr. talk by Y. Hatta)