QCD saturation phenomenology at HERA

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Based on : C. Marquet, R. Peschanski, G. Soyez, Nucl.Phys.A**756** (2005) 399 [hep-ph/0502020]

C. Marquet, G. Soyez, Nucl.Phys. A**760** (2005) 208 [hep-ph/0504080]

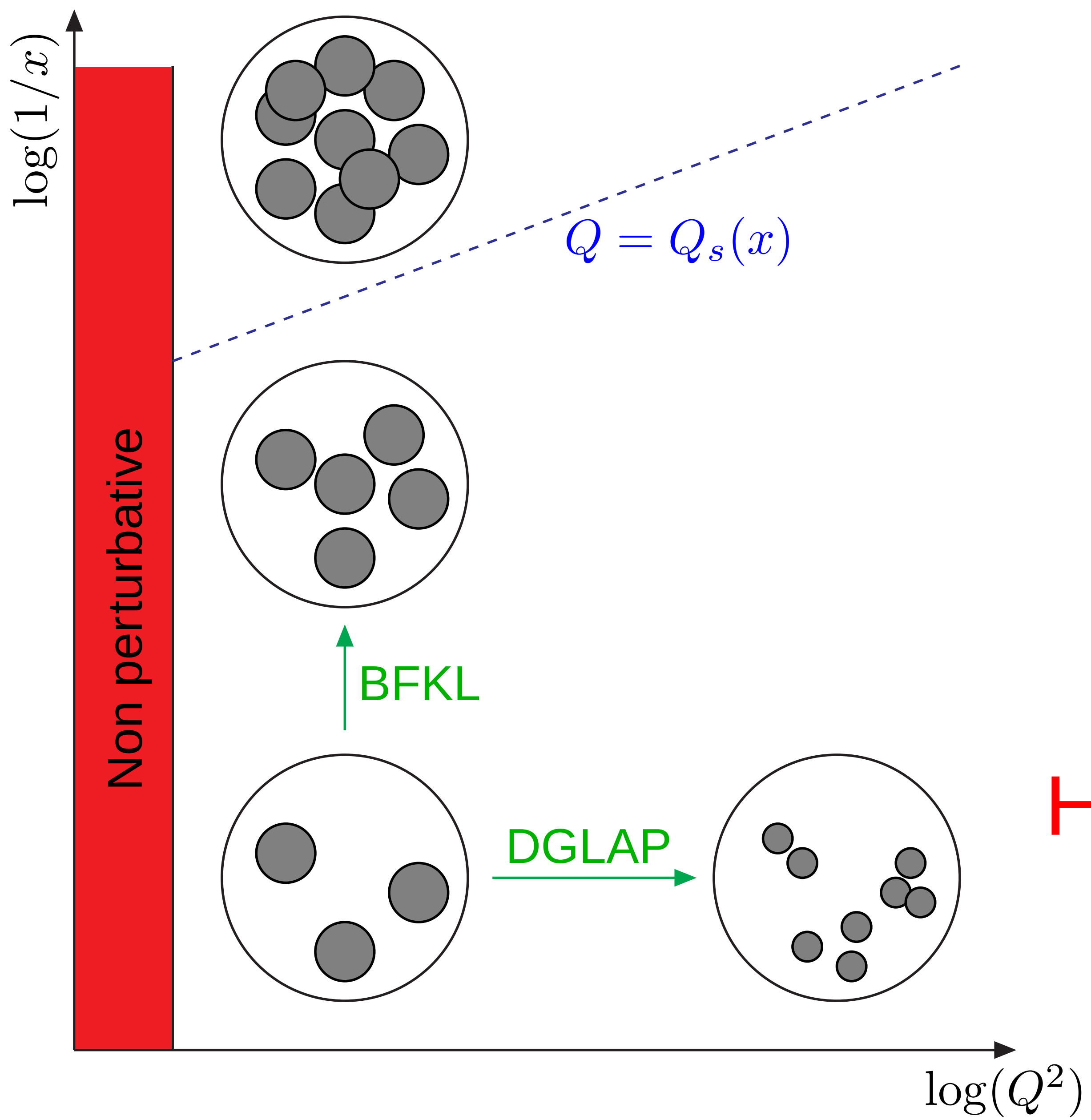
F. Gelis, R. Peschanski, G. Soyez, L. Schoeffel, hep-ph/0610435

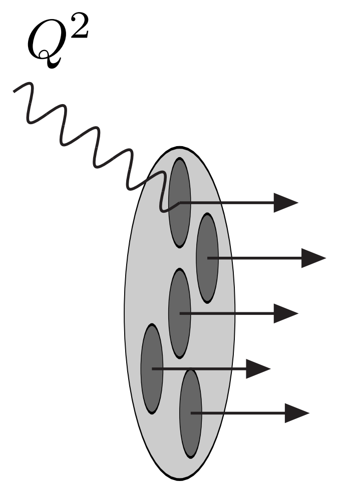
C. Marquet, R. Peschanski, G.S., in preparation

Outline

- Motivation: Bremsstrahlung & resummation:
- Perturbative evolution in high-energy QCD:
  - Dipole model and leading log approx.: BFKL equation
  - Unitarity/Saturation effects: Balitsky/JIMWLK and BK equation
- Asymptotic solutions: saturation $\Rightarrow$ geometric scaling
  - impact-parameter-independent BK: mechanism for geometric scaling
  - full BK equation: geometric scaling at nonzero momentum transfer
- Phenomenological consequences
  - Geometric scaling for F_2 (different approaches)
  - Geometric scaling in vector meson production and DVCS

Motivation: why saturation ?





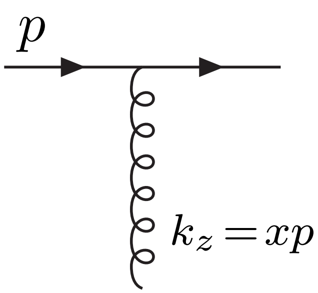
Size $\sim 1/Q$

Energy $\sim Q^2/x$

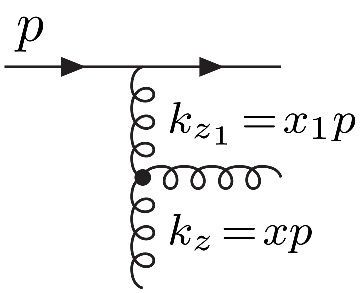
How to describe this in QCD ?

Motivation: why resummation ?

Bremsstrahlung:



$x \ll 1$



$x \ll x_1 \ll 1$

Probability of emission

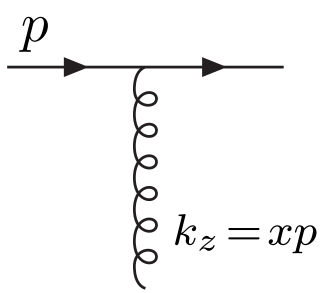
$$dP \sim \alpha_s \frac{dk_\perp^2}{k_\perp^2} \frac{dx}{x}$$

In the small-x limit

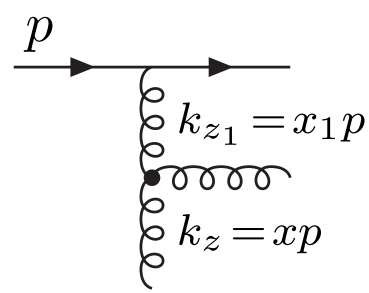
$$\int_x^1 \frac{dx_1}{x_1} \sim \alpha_s \log(1/x)$$

Motivation: why resummation ?

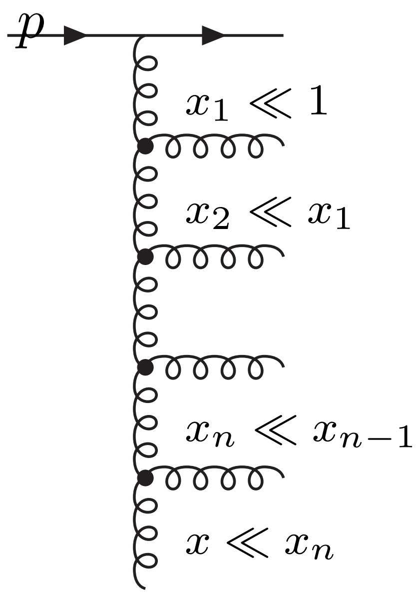
Bremsstrahlung:



$x \ll 1$



$x \ll x_1 \ll 1$



Probability of emission

$$dP \sim \alpha_s \frac{dk_\perp^2}{k_\perp^2} \frac{dx}{x}$$

In the small-x limit

$$\int_x^1 \frac{dx_n}{x_n} \dots \int_{x_2}^1 \frac{dx_1}{x_1} \sim \frac{1}{n!} \alpha_s^n \log^n(1/x)$$

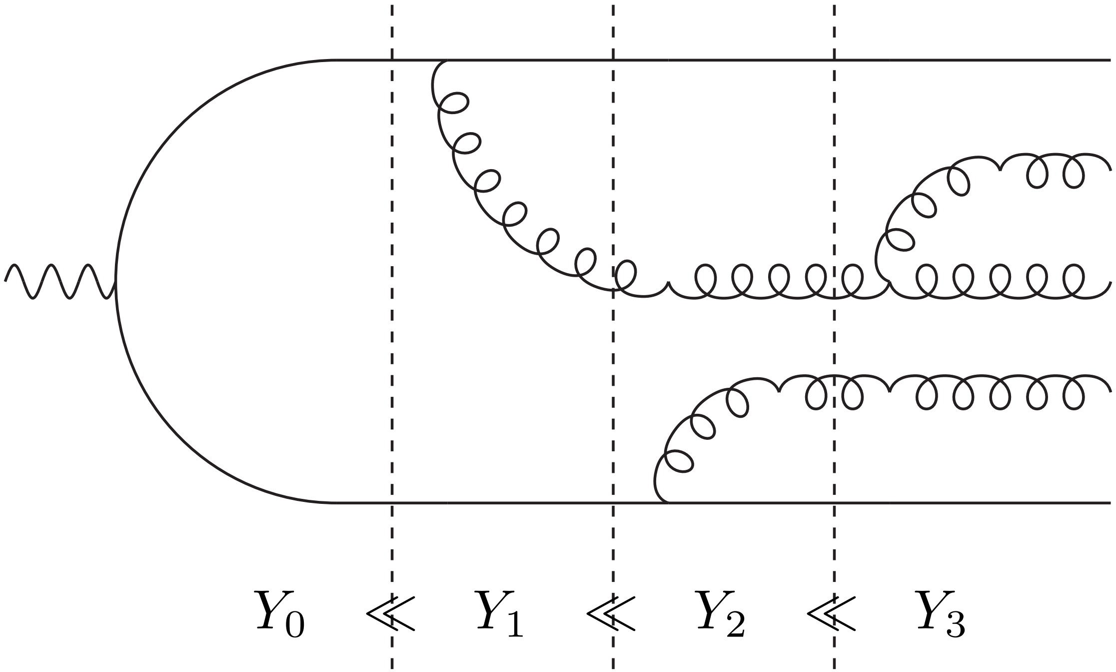
Same order when $\alpha_s \log(1/x) \sim 1$

Perturbative evolution in high-energy QCD

Dipole picture

Consider a fast-moving $q\bar{q}$ dipole (Rapidity: $Y = \log(s)$)

[Mueller,93]

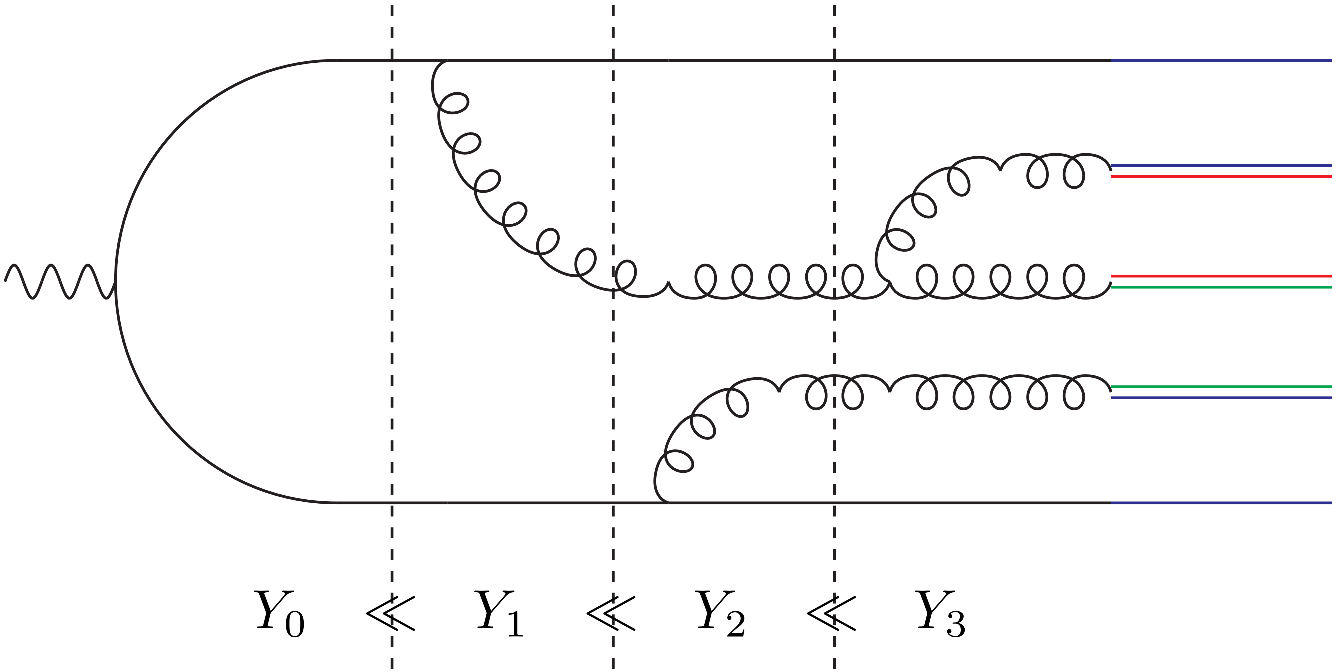


- Probability $\bar{\alpha}K$ of emission
- Independent emissions in coordinate space (transverse plane)

Dipole picture

Consider a fast-moving $q\bar{q}$ dipole (Rapidity: $Y = \log(s)$)

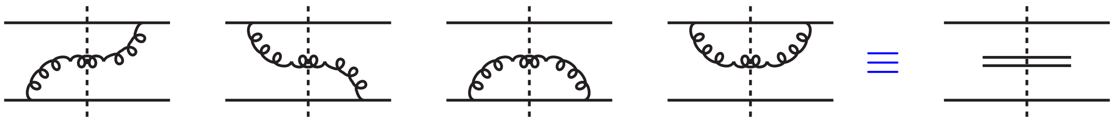
[Mueller,93]



$n(r, Y)$ dipoles

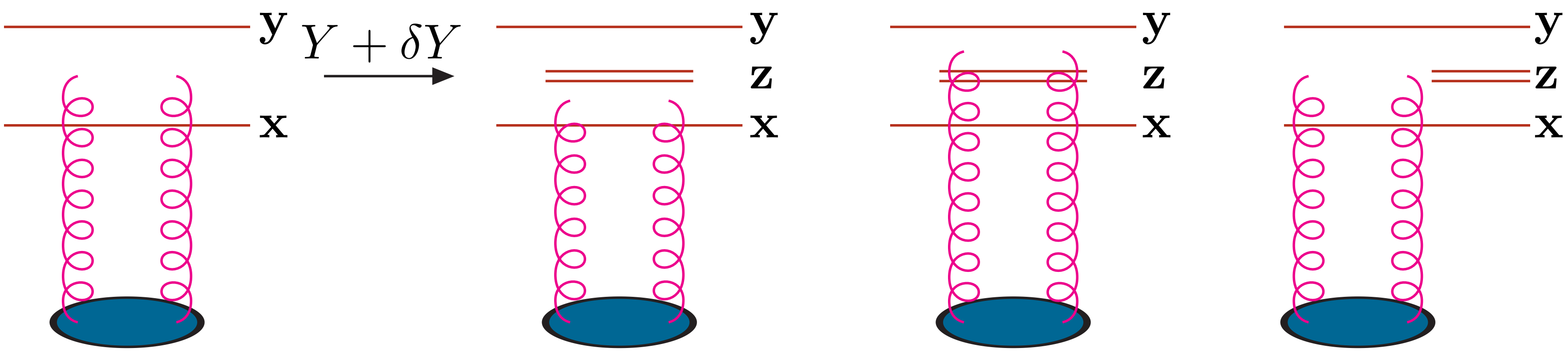
of size r

- Probability $\bar{\alpha}K$ of emission
- Independent emissions in coordinate space (transverse plane)
- Large-N_c approximation



BFKL evolution (1/2)

Consider a small increase in rapidity $\Rightarrow$ splitting



$$\partial_Y T(\mathbf{x},\mathbf{y};Y)$$
$$= \quad \bar{\alpha} \int \underbrace{d^2z \frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{z}-\mathbf{y})^2}}_{\text{Emission proba from pQCD}} \underbrace{\left[T(\mathbf{x},\mathbf{z};Y) + T(\mathbf{z},\mathbf{y};Y) - T(\mathbf{x},\mathbf{y};Y)\right]}_{\text{all possible interactions}}$$

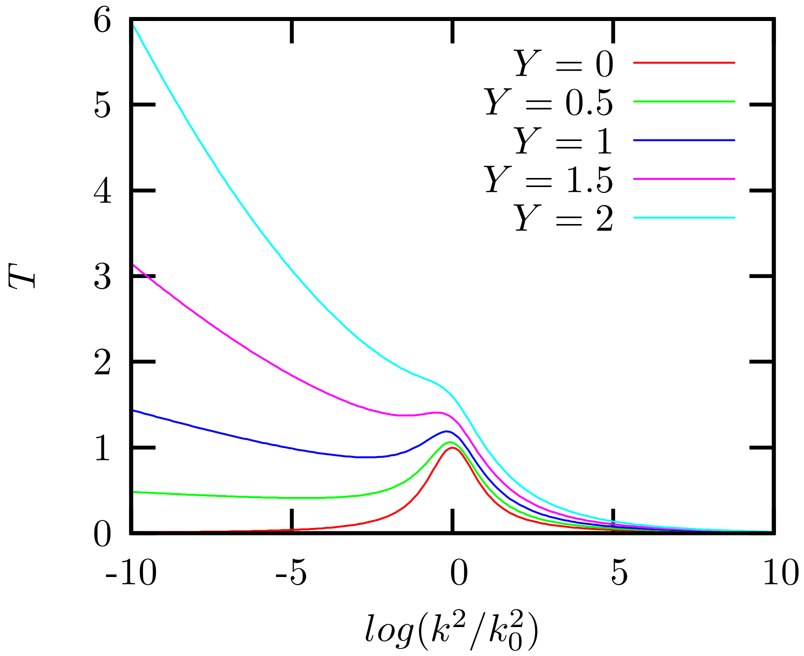
[Balitsky,Fadin,Kuraev,Lipatov,78]

BFKL evolution (2/2)

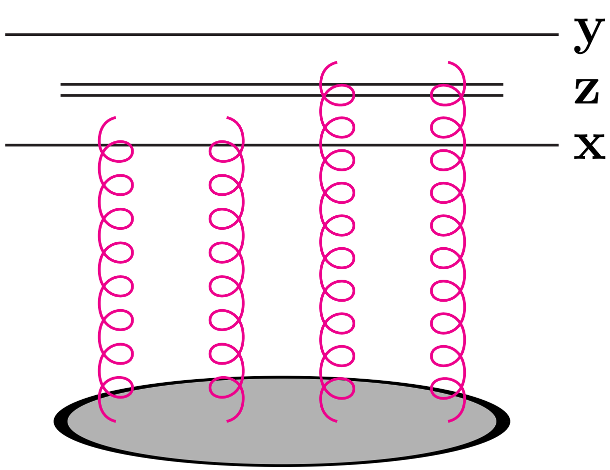
The solution goes like

$$T(Y) \sim e^{\omega Y} \qquad \text{with} \quad \omega = 4\bar{\alpha}\log(2) \approx 0.5$$

- Fast growth of the amplitude
- Intercept value too large
- Violation of the Froissart unitarity:
  $T(Y) \le C \log^2(s)$ $\qquad$ $T(r,b) \le 1$
- problem of diffusion in the infrared



Saturation effects



Multiple scattering

- $\star$ Proportional to T^2
- $\star$ important when $T \approx 1$

$\langle \cdot \rangle \equiv$ average over target field

$$\partial_Y \langle T(\mathbf{x},\mathbf{y};Y)\rangle$$
$$= \bar{\alpha} \int d^2z \, \frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{z}-\mathbf{y})^2} \left[\langle T(\mathbf{x},\mathbf{z};Y)\rangle + \langle T(\mathbf{z},\mathbf{y};Y)\rangle - \langle T(\mathbf{x},\mathbf{y};Y)\rangle \right.$$
$$\left. - \langle T(\mathbf{x},\mathbf{z};Y) T(\mathbf{z},\mathbf{y};Y)\rangle \right]$$

But

$\partial_Y \langle T(\mathbf{x},\mathbf{y};Y)\rangle$ contains a new object: $\langle T(\mathbf{x},\mathbf{z};Y) T(\mathbf{z},\mathbf{y};Y)\rangle$

Balitsky, Kovchegov and JIMWLK

In general: complete hierarchy [Balitsky, 96]

$$\partial_Y \langle T^k \rangle \longrightarrow \underbrace{\langle T^k \rangle}_{\text{BFKL}}, \underbrace{\langle T^{k+1} \rangle}_{\text{saturation}}$$

- Beyond large-N_c: the hierarchy involves quadrupoles, sextupoles, ...
- Balitsky hierarchy $\equiv$ JIMWKL eq. (Colour Glass Condensate formalism)

Mean field approx.: $\langle T_{\mathbf{xz}} T_{\mathbf{zy}} \rangle = \langle T_{\mathbf{xz}} \rangle \langle T_{\mathbf{zy}} \rangle$

$$\partial_Y \langle T_{\mathbf{xy}} \rangle = \frac{\bar{\alpha}}{2\pi} \int d^2 z \, \frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2 (\mathbf{z}-\mathbf{y})^2} \left[\langle T_{\mathbf{xz}} \rangle + \langle T_{\mathbf{zy}} \rangle - \langle T_{\mathbf{xy}} \rangle - \langle T_{\mathbf{xz}} \rangle \langle T_{\mathbf{zy}} \rangle \right]$$

[Balitsky 96,Kovchegov 99]

Simplest perturbative evolution equation satisfying unitarity constraint

Solutions

The BK equation

b-independent case

Case 1: BK equation with no impact parameter dependence

$$T_{\mathbf{xy}} \to T\left(\mathbf{r} = \mathbf{x} - \mathbf{y}, \mathbf{b} = \frac{\mathbf{x}+\mathbf{y}}{2}\right) \to T(r)$$

Introduce $L = \log(r_0^2/r^2)$ (or $L = \log(k^2/k_0^2)$ in momentum space)

Note:

- all arguments work for $T(r)$ or its Fourier transform $\tilde{T}(k)$
- for $\tilde{T}$, the non-linear term in the BK equation is simply $-\tilde{T}^2(k)$

Evolution mechanism

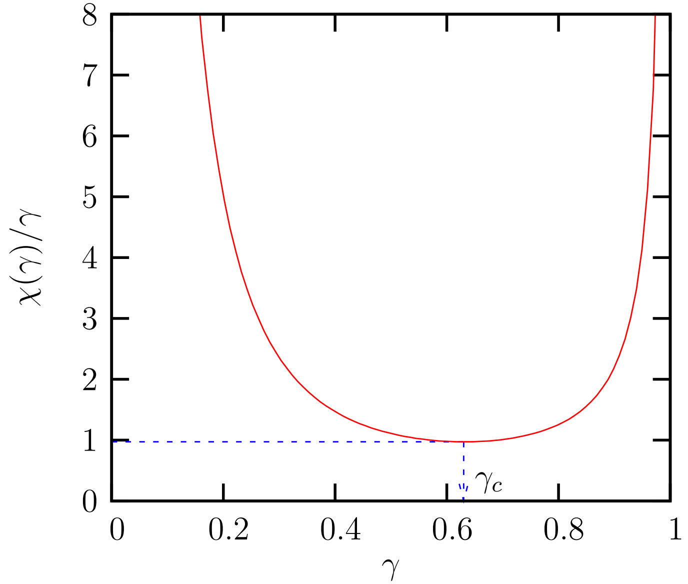
[S.Munier,R.Peschanski,03]

BK equation: $\partial_Y T = \underbrace{\chi(-\partial_L)T}_{\text{BFKL}} - T^2$

When $T \ll 1$ BFKL works: $\partial_Y T = \chi(-\partial_L)T$

Solution known:

$$\begin{aligned} T(k) &= \int \frac{d\gamma}{2i\pi} T_0(\gamma) \exp\left[\chi(\gamma)\bar{\alpha}Y - \gamma L\right] \\ &= \int \frac{d\gamma}{2i\pi} T_0(\gamma) \exp\left[-\gamma\left(L - \frac{\chi(\gamma)}{\gamma}\bar{\alpha}Y\right)\right] \end{aligned}$$



$\frac{\chi(\gamma)}{\gamma}$ min. when $\gamma = \gamma_c$

$\Rightarrow$ Wave of slope γ travels at speed $v = \chi(\gamma)/\gamma$

$$Y = Y_0$$

Evolution mechanism

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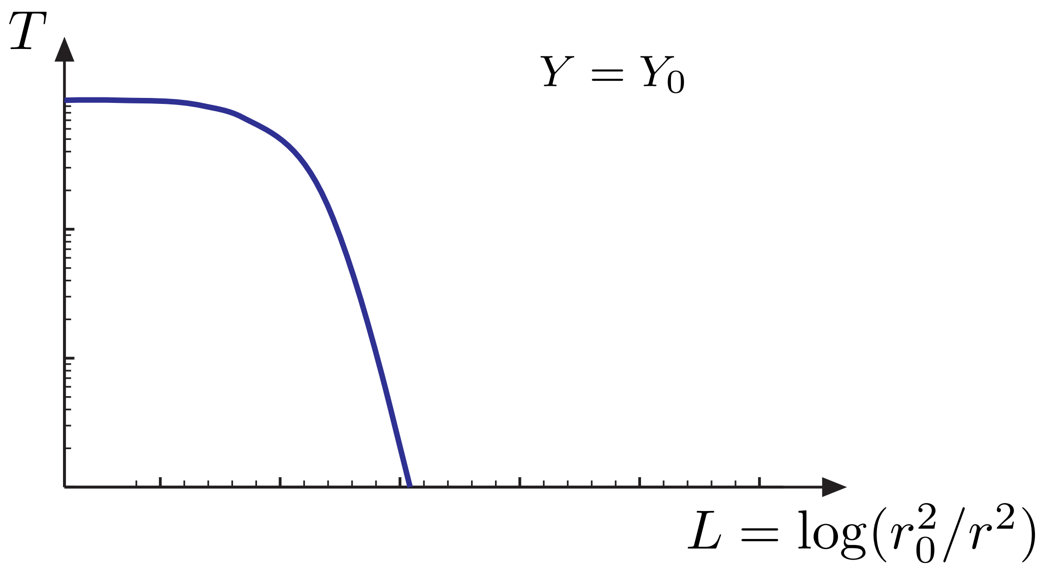
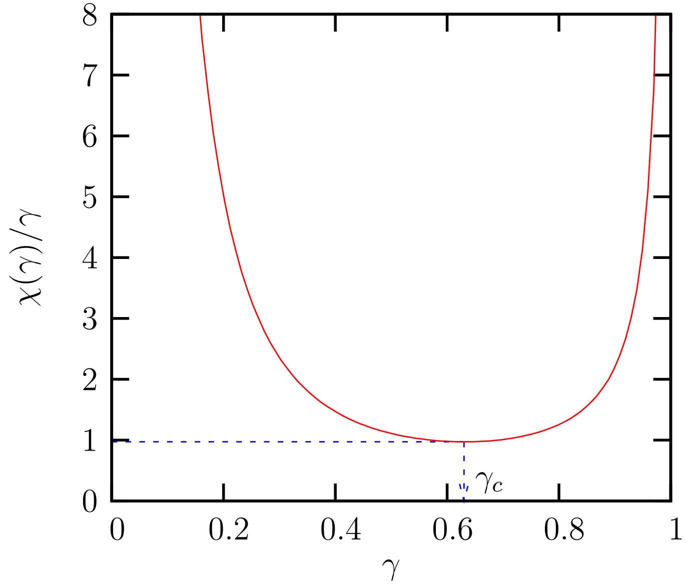


[S.Munier,R.Peschanski,03]



$\frac{\chi(\gamma)}{\gamma}$ min. when $\gamma = \gamma_c$

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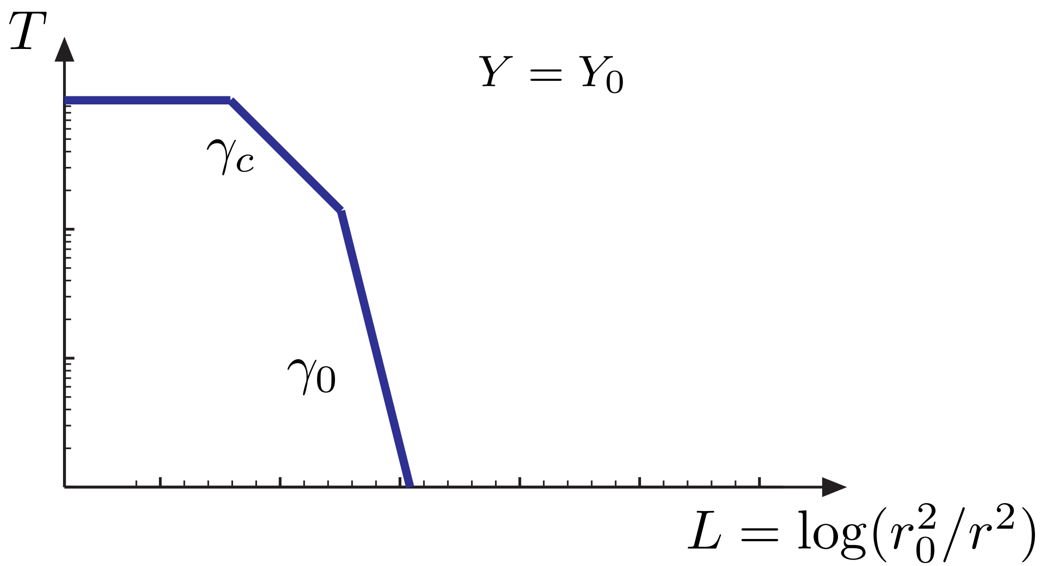

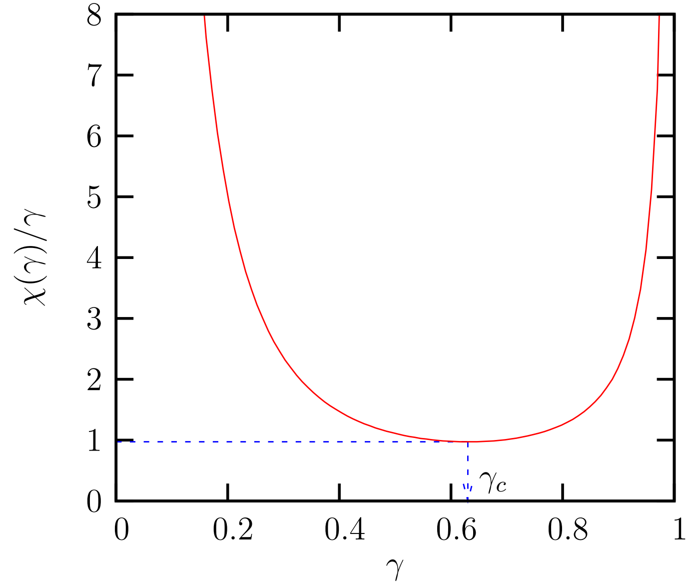

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[S.Munier,R.Peschanski,03]

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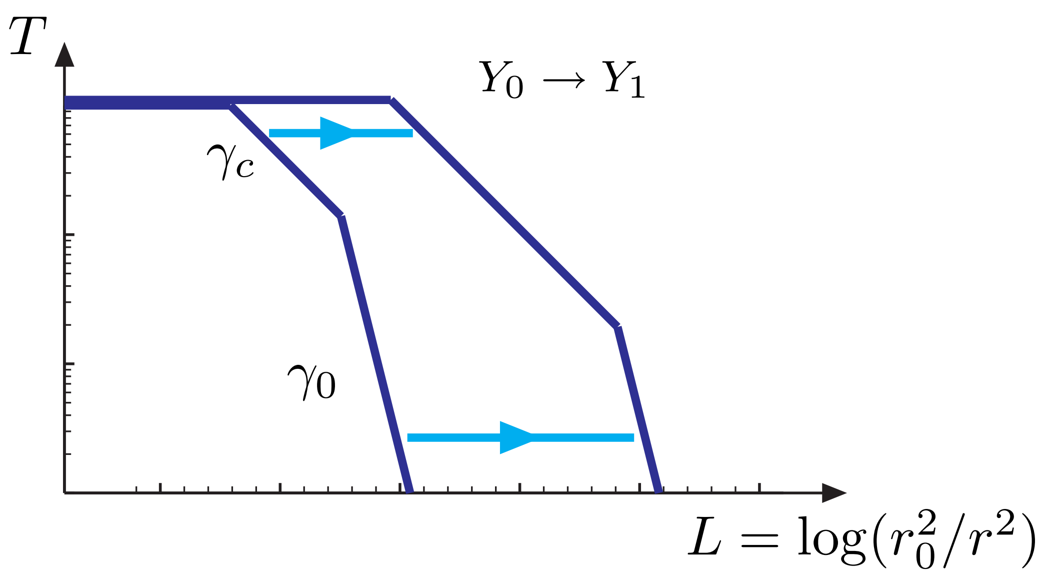

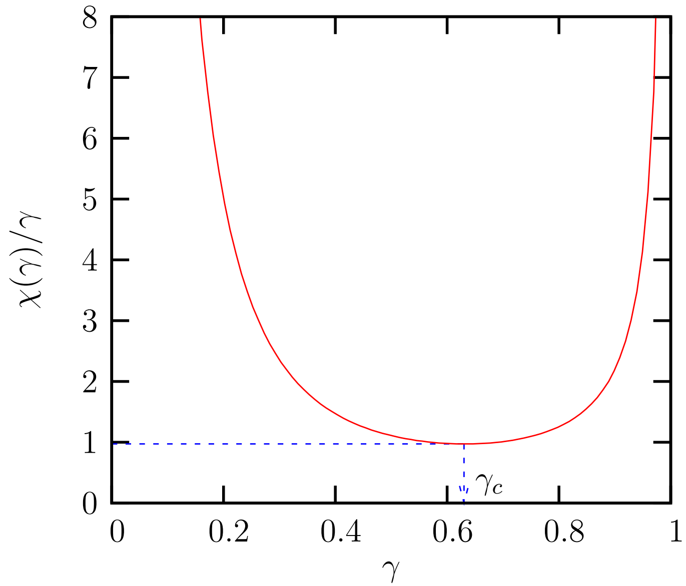

$\frac{\chi(\gamma)}{\gamma}$ min. when $\gamma = \gamma_c$

The minimal speed is selected during evoution

Geometric scaling

Consequence: geometric scaling ($Q_s \equiv$ saturation scale $\equiv$ front position)

$$T(r,Y) \quad = \quad T\left[rQ_s(Y)\right] \qquad \text{with} \quad Q_s^2(Y) = v_c\bar\alpha Y$$

$$\overset{rQ_s\ll 1}{=} \quad \underbrace{\left[r^2Q_s^2(Y)\right]^{\gamma_c}}_{\text{slope } \gamma_c} \quad \underbrace{\exp\left[\frac{\log^2(r^2Q_s^2)}{2\chi''(\gamma_c)\bar\alpha Y}\right]}_{\text{scaling window}}$$

$$|\log(r^2Q_s^2)| \lesssim \sqrt{\chi''(\gamma_c)\bar\alpha Y}$$

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- Generic arguments: exponential rise + saturation $\Rightarrow$ select γ_c
- Parameters fixed by linear kernel only
- Saturation effects even though $T \ll 1$

Geometric scaling

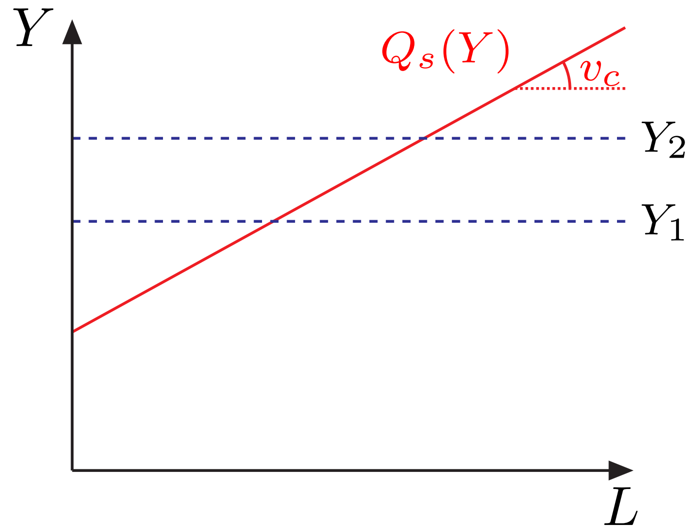
Consequence: geometric scaling ($Q_s \equiv$ saturation scale $\equiv$ front position)

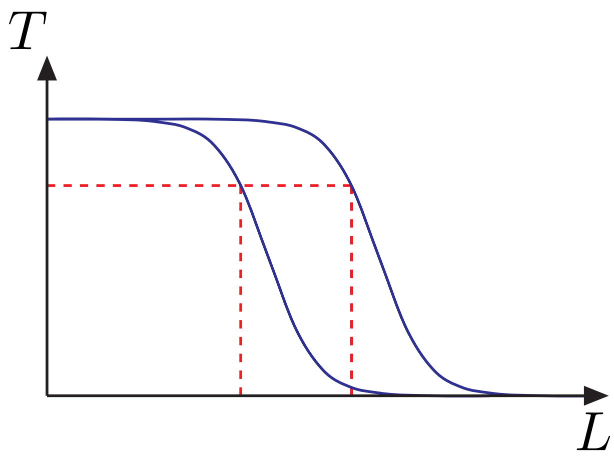
$$T(r,Y) = T\left[rQ_s(Y)\right] \quad \text{with} \quad Q_s^2(Y) = v_c\bar{\alpha}Y$$

$$\overset{rQ_s \ll 1}{=} \underbrace{\left[r^2Q_s^2(Y)\right]^{\gamma_c}}_{\text{slope } \gamma_c} \quad \underbrace{\exp\left[\frac{\log^2(r^2Q_s^2)}{2\chi''(\gamma_c)\bar{\alpha}Y}\right]}_{\text{scaling window}}$$

$$|\log(r^2Q_s^2)| \lesssim \sqrt{\chi''(\gamma_c)\bar{\alpha}Y}$$

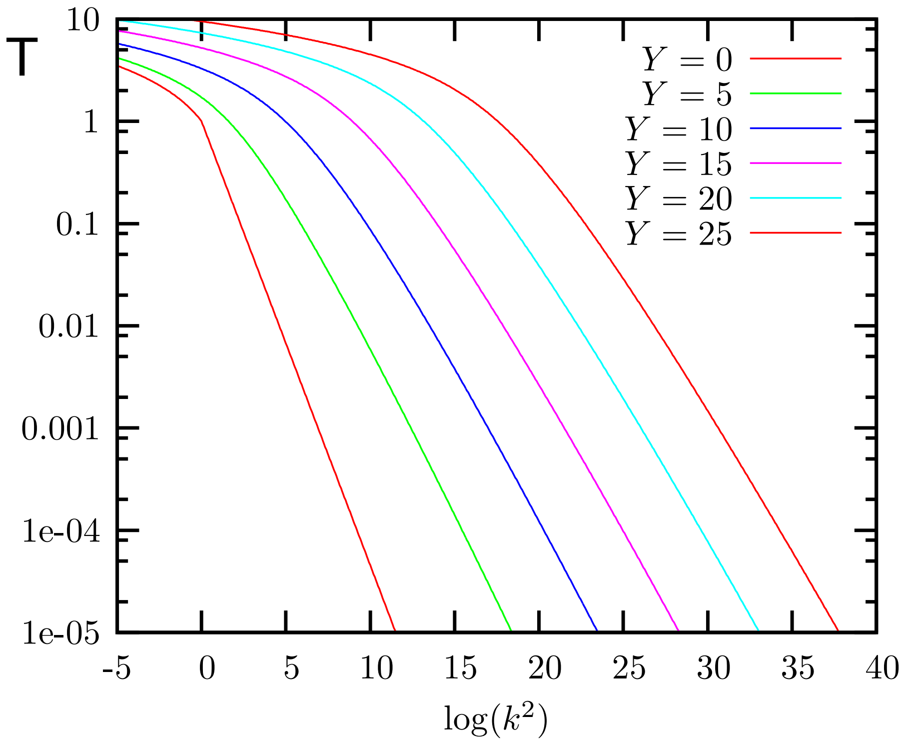
Interpretation: invariance along the saturation line





Geometric scaling

Numerical simulations:



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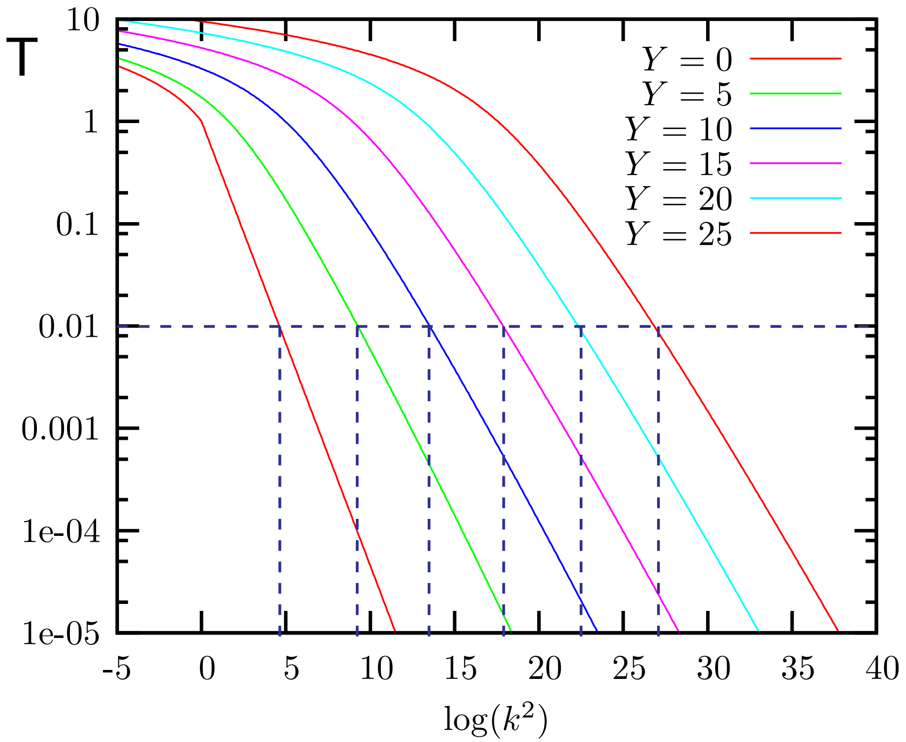
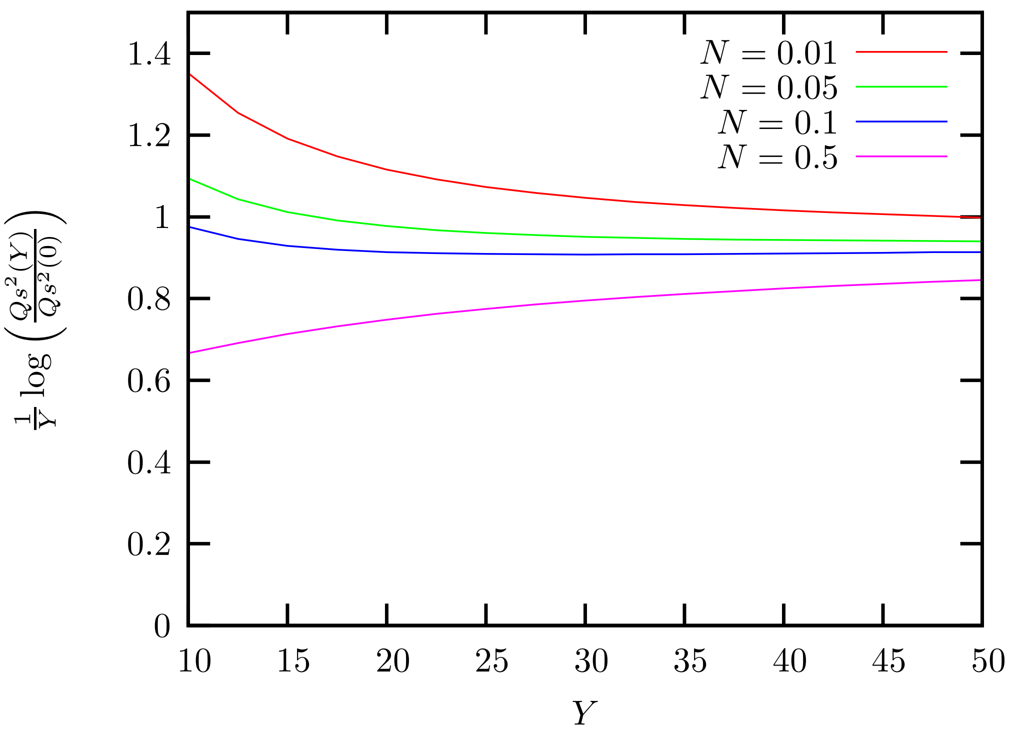




$$T(k,Y) = T\left(\frac{k^2}{Q_s^2(Y)}\right) \approx \left(\frac{k^2}{Q_s^2(Y)}\right)^{-\gamma_c}$$

$$Q_s^2(Y) \propto \exp(v_c Y)$$

The full BK equation

Case 2: including impact parameter

Go to momentum space: use momentum transfer $\mathbf{q}$

$$\tilde{T}(\mathbf{k},\mathbf{q}) = \int d^2x\, d^2y\, e^{i\mathbf{k}.\mathbf{x}} e^{i(\mathbf{q}-\mathbf{k}).\mathbf{y}} \frac{T(\mathbf{x},\mathbf{y})}{(\mathbf{x}-\mathbf{y})^2}$$

new form of the BK equation

$$\begin{aligned}
\partial_Y \tilde{T}(\mathbf{k},\mathbf{q}) \quad &= \quad \frac{\bar{\alpha}}{\pi} \int \frac{d^2k'}{(k-k')^2} \left\{ \tilde{T}(\mathbf{k}',\mathbf{q}) - \frac{1}{4}\left[\frac{k^2}{k'^2} + \frac{(q-k)^2}{(q-k')^2}\right] \tilde{T}(\mathbf{k},\mathbf{q}) \right\} \\
&- \quad \frac{\bar{\alpha}}{2\pi} \int d^2k'\, \tilde{T}(\mathbf{k},\mathbf{k}')\tilde{T}(\mathbf{k}-\mathbf{k}',\mathbf{q}-\mathbf{k}')
\end{aligned}$$

[C.Marquet, R.Peschanski, G.S., 05]

The full BK equation

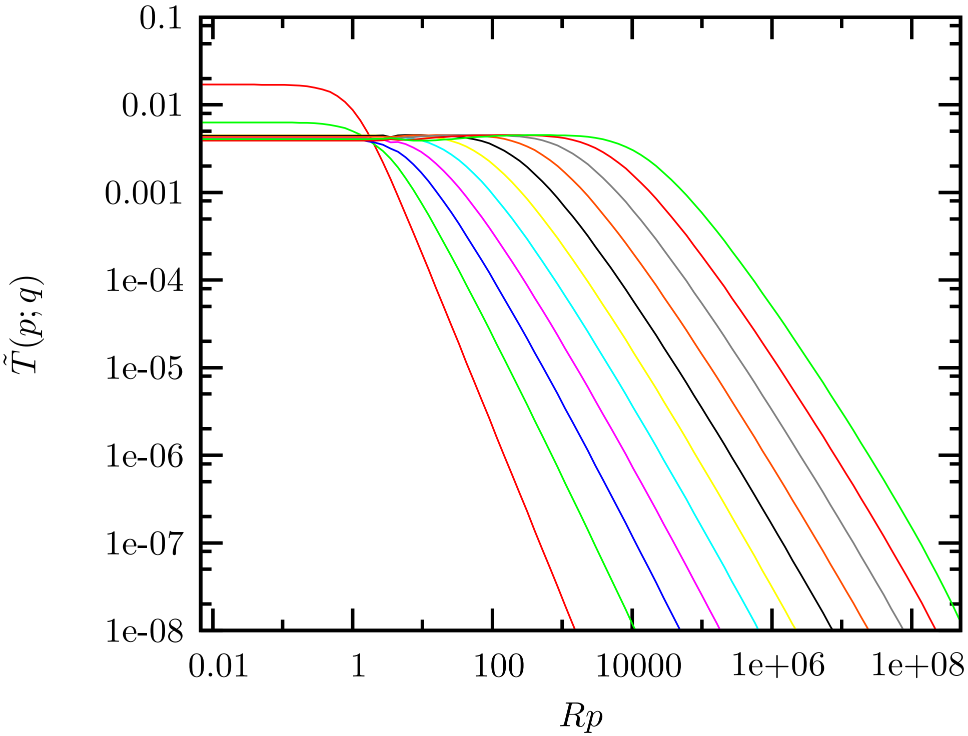
1. Study BFKL with both $\mathbf{k}$ and $\mathbf{q}$ dependences
2. Look for power decreases in the tail ($k \gg q, k_0$, $k_0 \equiv$ target scale)

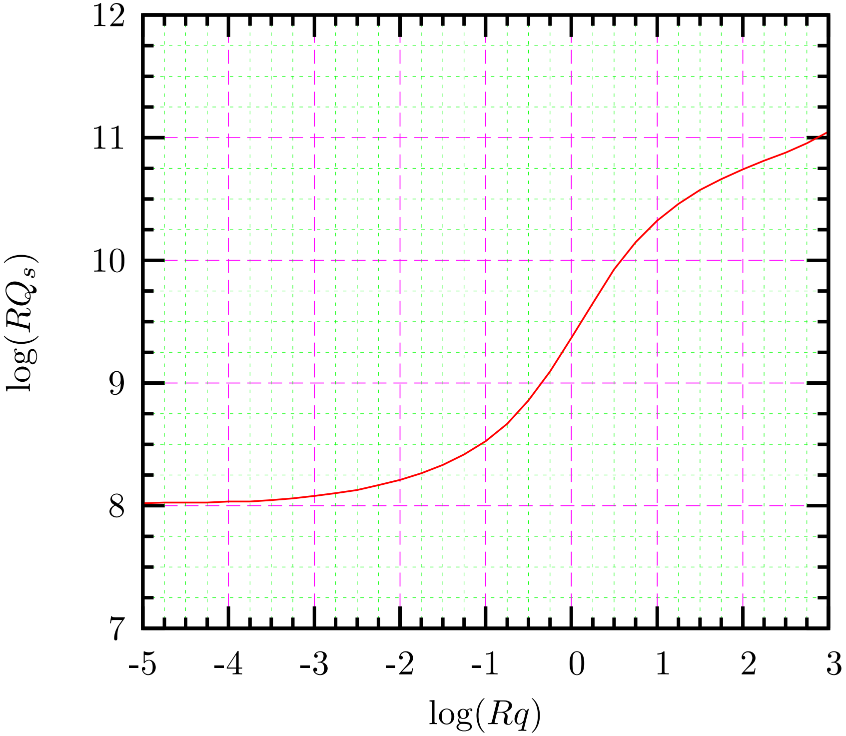
2 possible situations:

- $q \gg k_0 \Rightarrow$ tail given by $e^{\bar{\alpha}\chi(\gamma)Y}(k^2/q^2)^{-\gamma}$
- $q \ll k_0 \Rightarrow$ tail given by $e^{\bar{\alpha}\chi(\gamma)Y}(k^2/k_0^2)^{-\gamma}$

$\Rightarrow$ same selection mechanism with different reference scale:

$$T(k,q;Y) \propto \left[\frac{k^2}{Q_s^2(q,Y)}\right]^{-\gamma_c} \qquad \text{with } Q_s^2(q,Y) = \begin{cases} k_0^2\, e^{v_c Y} & \text{if } q \ll k_0 \\ q^2\, e^{v_c Y} & \text{if } q \gg k_0 \end{cases}$$





One can prove analytically that:

- traveling wave at large k: BFKL $\Rightarrow$ same γ_c, v_c
- q dependence: $Q_s^2(q,Y) = \min(k_0^2, q^2)\exp(v_c Y)$

Predicts geometric scaling for t-dependent processes

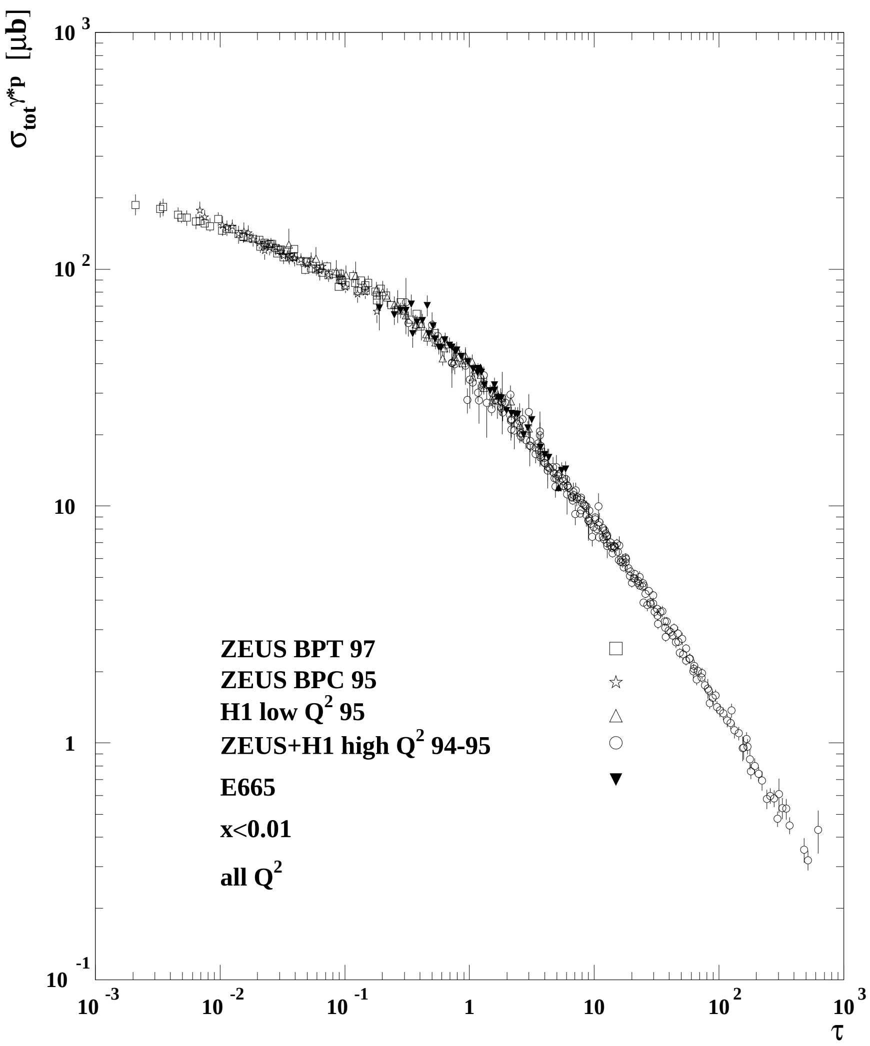
Phenomenology

1. Geometric scaling in F_2

1.1. direct observation

Observation

[A.Stasto, K.Golec-Biernat, Kwiecinski, 01]



$$\sigma^{\gamma^* p}(Q^2, x) = \sigma(\tau)$$
$$\tau = \tau(Q^2, x) = Q^2/Q_s^2(x)$$

[F.Gelis, R.Peschanski, L.Shoeffel, G.S., hep-ph/0610436]

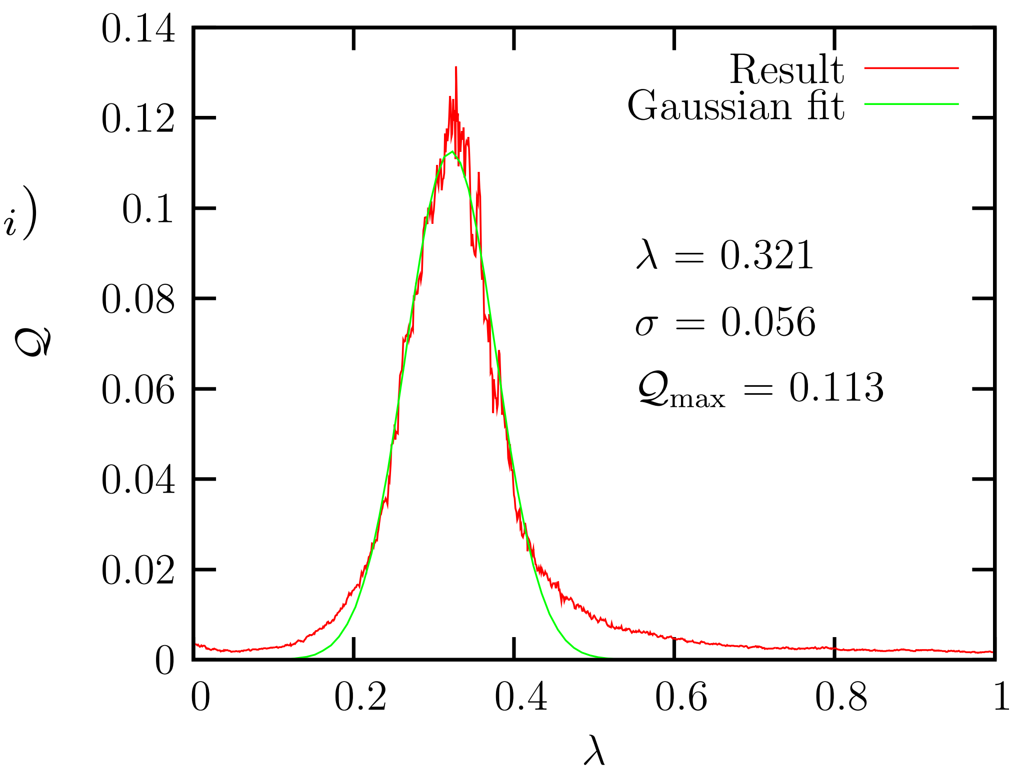
Systemtic approach to find scaling relations ?

Starting point:

Set of points: $(\tau_i = \tau(Q_i^2, Y_i), F_{2,i})$

$\longrightarrow$ Quality factor: $\mathcal{Q}$

Large when points on a curve.

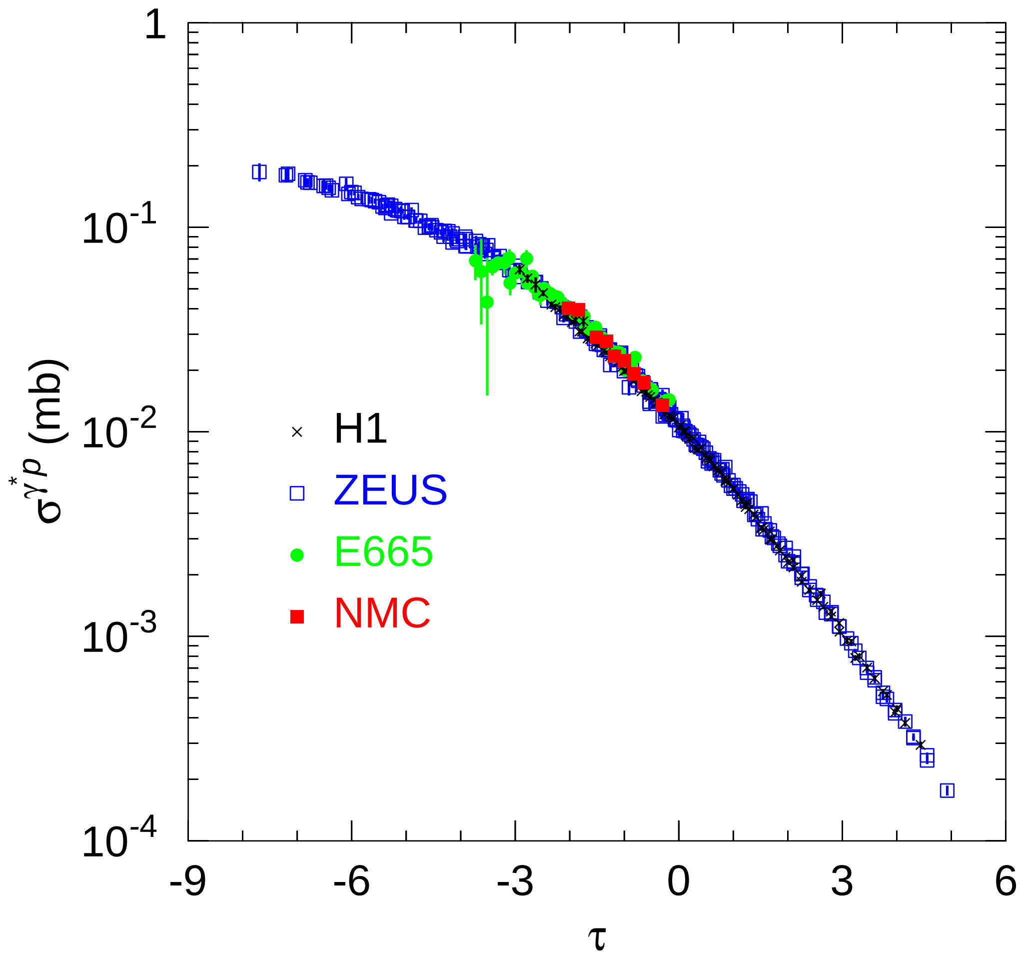


Example: $\tau = \log(Q^2) - \log(Q_s^2(Y))$ with $\log(Q_s^2(Y)) = \lambda Y$

Scan in $\lambda \Rightarrow \lambda \approx 0.321$

Observation (revisited)

[F.Gelis, R.Peschanski, L.Shoeffel, G.S., hep-ph/0610436]



$$\sigma^{\gamma^* p}(Q^2, x) = \sigma(\tau)$$
$$\tau = \log(Q^2) - \log(Q_s^2(Y))$$

- BK with fixed coupling:

$$\log(Q_s^2(Y)) = \lambda Y$$
$$\lambda \approx 0.32$$

- BK with running coupling:

$$\log(Q_s^2(Y)) = \lambda\sqrt{Y}$$
$$\lambda \approx 1.62$$

Phenomenology

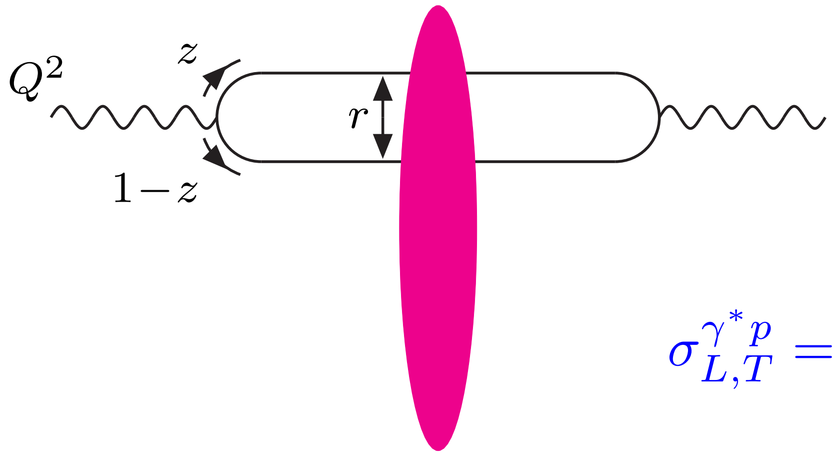
1. Geometric scaling in F_2

1.2. QCD/saturation description

Description

Factorisation formula:

[E.Iancu, K.Itakura, S.Munier, 03]



$$\sigma_{L,T}^{\gamma^* p} = \int d^2r \int_0^1 dz\, \overbrace{\left|\Psi_{L,T}(z,r;Q^2)\right|^2}^{\text{from pQED}}\, 2\pi R_p^2 T(\mathbf{r};Y)$$

param for dipole amplitude: scaling variable $\tau = \log(r^2 Q_s^2/4)$

$$T(r;Y) = \begin{cases} T_0 \exp\left(\gamma_c \tau - \frac{\tau^2}{2\bar\alpha \chi_c'' Y}\right) & \text{if } rQ_s < 2 \qquad \text{(travelling wave)} \\ 1 - \exp[-a(\tau+b)^2] & \text{if } rQ_s > 2 \qquad \text{(McLerran-Venugopalan)} \end{cases}$$

$Q_s^2(Y) = k_0^2 \exp(\lambda Y) \longrightarrow \gamma_c$, χ_c'' from LO BFKL, 3 parameters: λ, k_0, R_p

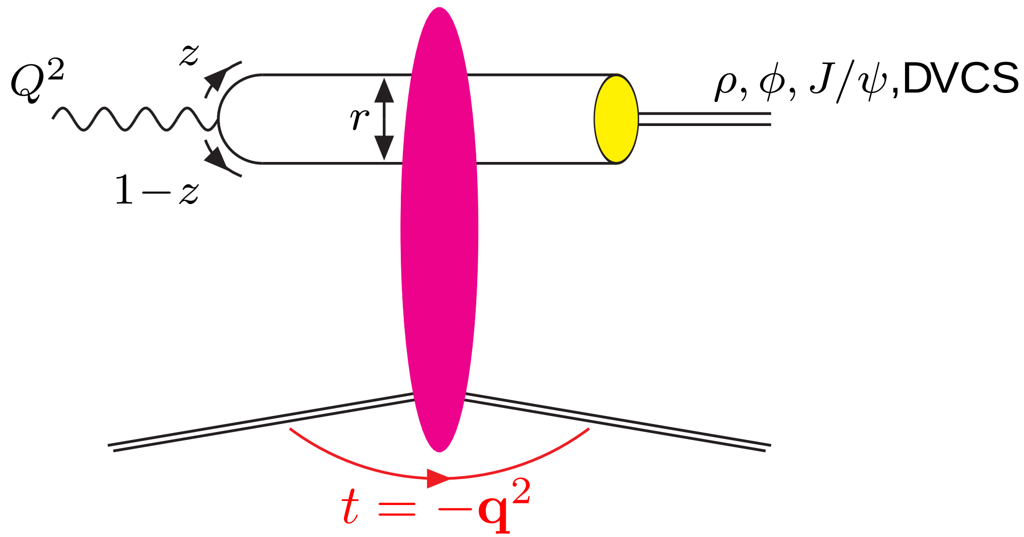
$\Rightarrow\ \lambda \approx 0.25$, in agreement with NLO BFKL predictions.

Phenomenology

2. Geometric scaling in vector-meson production

Dipole description

[C.Marquet, R.Peschanski, G.S., to appear]



Factorisation formula:

$$\mathcal{A}_{L,T}^{\gamma^* p \to V p} = i \int d^2 r \int_0^1 dz\, \Psi_{L,T}(z, r; Q^2) \Psi^*_{L,T}(z, r, q : M_V^2)\, e^{iz\mathbf{q}.\mathbf{r}}\, \sigma_{\text{dip}}(\mathbf{r}, \mathbf{q}; Y)$$

$\rightarrow \frac{d\sigma}{dt}$, σ_{el} for $\rho, \phi, J/\psi$,DVCS

QCD predictions

- photon wavefunction: from QED
  Vector-mesons wavefunction: Boosted-Gaussian model
- dipole amplitude:

$$\sigma_{\text{dip}}(r, q; Y) = 2\pi R_p^2 e^{-b|t|}\, T_{\text{IIM}}\left(r, Q_s^2(q, Y)\right)$$

  - Normalisation: only one slope b (no Q^2 dependence)
  - T-matrix: t-dependent saturation scale from theoretical predictions:

$$Q_s^2 = k_0^2\, e^{\lambda Y} \qquad \longrightarrow \qquad Q_s^2 = k_0^2\, (1 + c|t|)\, e^{\lambda Y}$$

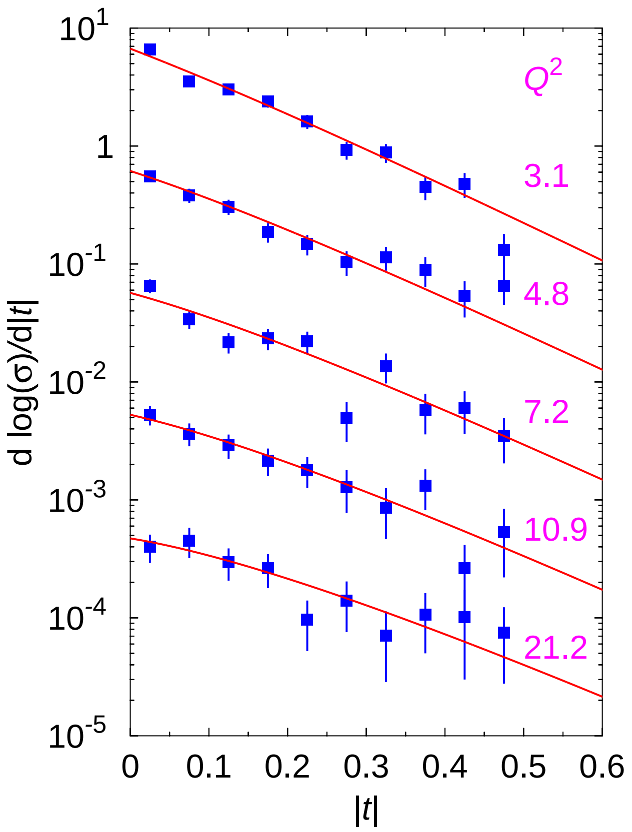
Hence:

$$b,\, c \qquad \longrightarrow \qquad \left.\frac{d\sigma}{dt},\, \sigma_{\text{el}}\right|_{\rho,\phi} \qquad \text{(201 data)}$$

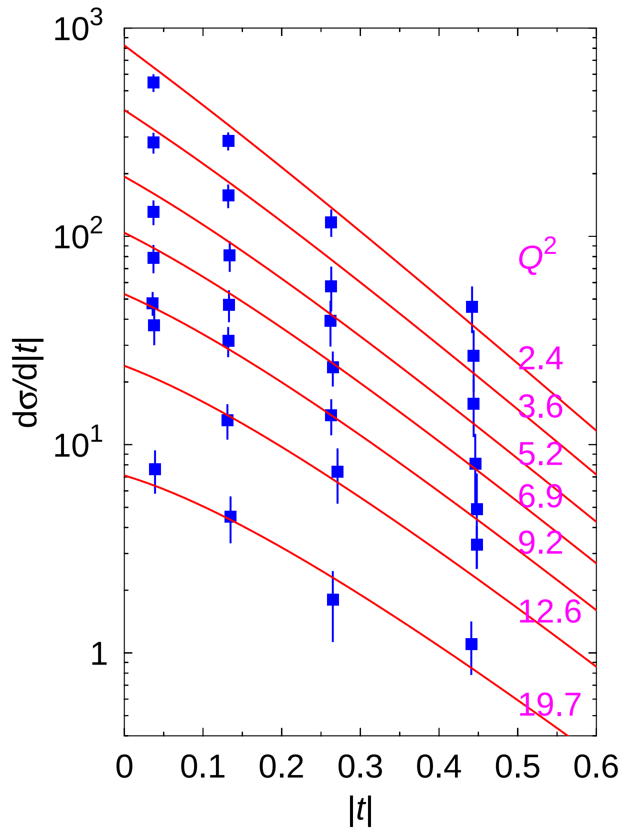
Results

Example: differential cross-section:

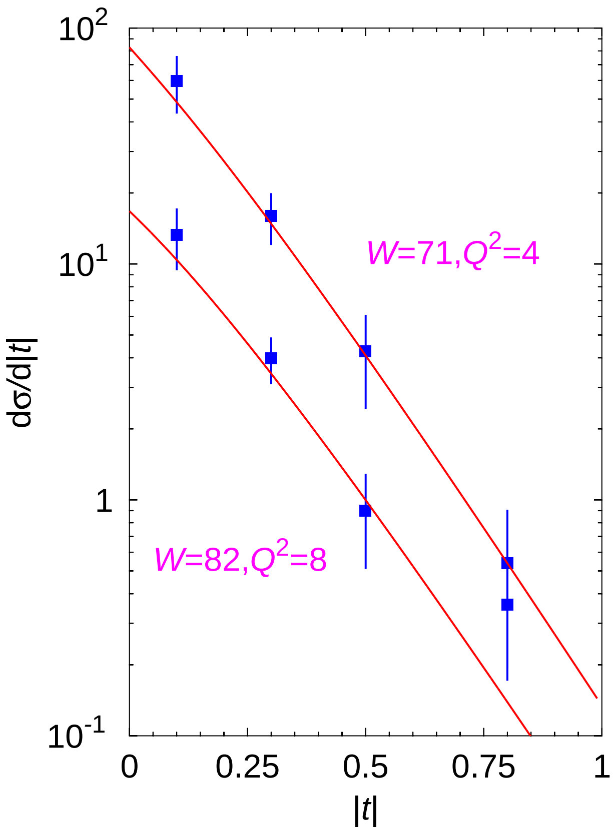
$\gamma^* p \to \rho p$



$\gamma^* p \to \phi p$



pred. for DVCS



Phenomenology

3. Massive quarks effects

Charm enters the game

[G.S. in preparation]

- No heavy quark in the IIM model
- Some other model does but (see Beatriz Gay Ducati's talk)
  - are not fully QCD-based
  - have clearly lower saturation momentum

Aim: Include the charm in the IIM model ?

Key issue: alloc γ_c to vary !

Data: ZEUS and H1 (5% renorm.) Domain: $x \leq 0.01$, $Q^2 \leq 150$ GeV^2

model	γ_c	v_c	x_0	R_p	χ^2/n
IIM	0.6275	0.253	2.67 10^{-5}	3.250	≈0.9
IIM+c	0.6275	0.195	6.42 10^{-7}	3.654	1.109
	0.7065	0.222	1.19 10^{-5}	3.299	0.963

Note: $\gamma_c \approx 0.7$ is in better agreement with NLO BFKL predictions

Conclusion

Part 1: Evolution equations towards high-energy

- The BK equation contains both BFKL exchanges and unitarity/saturation corrections
- See other talks for pomeron-loops equations

Part 2: Solutions for scattering amplitudes

- BK equation $\Rightarrow$ geometric scaling
  - At fixed impact parameter
  - At nonzero momentum transfer
- See other talks for pomeron-loops equations

Conclusion

Part 3: Phenomenological consequences

We focused on the HERA phenomenology:

- geometric scaling for F_2:
  - Direct analysis of the data ($\mathcal{Q}$uality factor)
  - Iancu-Itakura-Munier model + new extension with charm
- geometric scaling in vector-meson production and DVCS
  t-dependence from pQCD instead of b-dependence postulated

$\triangleright$ indications for saturation

$\triangleright$ can help understanding for other experiments

See other talks for phenomenology

- at RHIC/LHC
- with pomeron-loop equations