SISConet
A Seedless Infrared-Safe Cone jet algorithm

Grégory Soyez

University of Liège

In collaboration with Gavin Salam

code available at http://projects.hepforge.org/siscone

or as a FastJet plugin (http://www.lpthe.jussieu.fr/~salam/fastjet)
Outline

- Cone jet algorithms
- Infrared-Safety issues:
  - Why is this mandatory?
  - IR unsafety of the midpoint algorithm
- SIS Cone: a practical solution
- Physical consequences:
  - Algorithm speed
  - Inclusive jet spectrum
  - Jet mass spectrum in multi-jet events
- Conclusions
Given: set of $N$ particles with their 4-momentum

Quest: clustering those particles into jets

Idea: jets $\equiv$ cones around dominant energy flows

for a cone of radius $R$ in the $(y, \phi)$ plane, stable cones are such that:

- centre of the cone $\equiv$ direction of the total momentum of its particles

Algorithm: Tevatron Run II

- Step 1: find ALL stable cones of radius $R$
- Step 2: run a split-merge procedure with overlap $f$
  to deal with overlapping stable cones

This talk: Why finding all stable cones and how.

$\rightarrow$ C++ implementation: Seedless Infrared Safe Cone algorithm (SISCone)
Usual **seeded** method to search stable cones: **midpoint cone algorithm**

- **For an initial seed**
  1. sum the momenta of all particles within the cone centred on the seed
  2. use the direction of that momentum as new seed
  3. repeat 1 & 2 until stable state cone reached

- **Sets of seeds:**
  1. All particles (above a $p_t$ threshold $s$)
  2. Midpoints between stable cones found in 1.

**Problems:**

- the $p_t$ threshold $s$ is collinear unsafe
- seeded approach $\Rightarrow$ stable cones missed $\Rightarrow$ infrared unsafety
**Midpoint IR Unsafety**

Stable cones:
- Midpoint: \{1,2\} & \{3\}
- Seedless: \{1,2\} & \{3\} & \{2,3\}

Jets: \(f = 0.5\)
- Midpoint: \{1,2\} & \{3\}
- Seedless: \{1,2,3\}

\[
\text{\(\rightarrow\)} \quad \text{IR unsafety of the midpoint algorithm}
\]
**Ellipsis:** IR safety, i.e. stability upon emission of soft particles, is required for perturbative computations to make sense!

Cancellation of IR divergences between real and virtual emissions of SOFT gluons

- IF Jet clustering is different in both cases, THEN the cancellation is not done and the result is not consistent with pQCD
- Stable cones must not change upon addition of soft particles
- Divergence at NLO is parametrically of the same order as LO contribution
Naive approach: check stability of each subset of particle
Complexity is $O(N 2^N)$ i.e. definitely unrealistic ($10^{17}$ years for $N = 100$)

Idea: all enclosures are defined by a pair of points

Tricks: e.g. traversal order to avoid recomputation of the cone content

Complexity:
- SIS Cone is $O(N n \ln n)$ (with $n \sim N$ the number of points in a circle of radius $R$)
- midpoint standard implementation is $O(N^2 n)$
Hard event: 2-10 particles
Soft add-on: 1-5 particles
Run:
  “hard” only
  many “hard+soft” trials
  Search differences

<table>
<thead>
<tr>
<th>Unsafety level</th>
<th>failure rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 hard + 1 soft</td>
<td>~ 50%</td>
</tr>
<tr>
<td>3 hard + 1 soft</td>
<td>~ 15%</td>
</tr>
<tr>
<td>SISCones</td>
<td>IR safe!</td>
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</tbody>
</table>

NB: small issues in the split-merge
**Speed**

- **CDF midpoint (s=0 GeV)**
- **CDF midpoint (s=1 GeV)**
- **PxCone**
- **SISCone**
- **$k_t$ (fastjet)**

- **at least as fast as other cones**
- **IR safe**
- **collinear safe**
- **midpoint ($s > 0$) is not**

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Impact on inclusive jet spectrum

Physical impact: SISCone vs. midpoint$(s)$?

- IR unsafety of midpoint: 3 particles in the same vicinity + 1 to balance $p_t$  ⇒ starts at the $2 \to 4$ level ($\mathcal{O}(\alpha_s^4)$)

- 3 contributions at this order:
  - $2 \to 4$ at LO (tree), $2 \to 3$ at NLO (1 loop) and $2 \to 2$ at NNLO (2 loops)
  ⇒ $2 \to 4$ at LO is IR divergent
  BUT the difference between SISCone and midpoint$(s)$ in finite since it is 0 at the $2 \to 2$ and $2 \to 3$ levels
  ⇒ compute $|\text{SISCone}-\text{midpoint}(s)|$ for $2 \to 4$ diagrams

- Compare with the $2 \to 2$ (LO) spectrum to estimate effect
Impact on inclusive jet spectrum

Differences of order 1-2 %
Impact on inclusive jet spectrum

Including parton shower, hadronic corrections and/or underlying event:

Ratio midpoint/SISCone:

- Differences up to 6 %
- Less effect from underlying event in SISCone
Study of more exclusive quantity \textit{e.g.} mass spectrum in 3-jet events

1. At fixed order (NLOJet, LO, $2 \rightarrow 4$)

Differences up to 10%
2. At fixed order (PYTHIA)

- Differences of order 10 \%
- Larger effects in the tail
- Seed threshold even worse
Cone jet algorithms are widely used

seeded implementations are IR unsafe (sometimes collinear unsafe)
IR safety is a prerequisite for perturbative QCD to make sense

We propose a **new cone algorithm** (SISCone):

- IR safe (and collinear safe)
- as fast as available cone implementations
- has 10% impact on jet mass spectra
- is less affected by underlying events