Pomeron loops & running coupling effects

Gregory Soyez

A. Dumitru, E. Iancu, L. Portugal, G. S., D.N. Triantafyllopoulos,
build a simple toy-model: evolution based on:
  - based on high-energy QCD (dipole model + JIMWLK saturation)
  - easily simulated with and without running coupling

Known properties:
  - Mean-field approximation: geometric scaling
  - Fixed-coupling fluctuations: diffusive scaling

New results:
  - Pomeron loops together with Running coupling effects

Conclusions and perspectives
Dipole model:

\[ \partial_{\bar{\alpha}Y} T(r; Y) = \chi_{BFKL} \otimes T(r; Y) - T^2(r; Y) + \sqrt{\kappa \alpha_s^2 T(r; Y)} \nu(r; Y) \]

sF-KPP equation in statistical physics:

\[ \partial_{\bar{\alpha}Y} u(r; Y) = (\nabla_r^2 + 1) u(r; Y) - u^2(r; Y) + \sqrt{u(r; Y)}/N \nu(r; Y) \]

- Linear growth (BFKL)
- saturation (BK)
- discretisation effects (fluctuations, pomeron loops)
Dipole model:

\[ \partial_{\bar{\alpha}Y} T(r; Y) = \chi_{BFKL} \otimes T(r; Y) - T^2(r; Y) + \sqrt{\kappa \alpha_s^2 T(r; Y)} \nu(r; Y) \]

sF-KPP equation in statistical physics:

\[ \partial_{\bar{\alpha}Y} u(r; Y) = (\nabla^2_r + 1)u(r; Y) - u^2(r; Y) + \sqrt{u(r; Y)/N \nu(r; Y)} \]

- Linear growth (BFKL)
- saturation (BK)
- discretisation effects (fluctuations, pomeron loops)

High-energy QCD evolution \( \equiv \) reaction-diffusion problem

\[ A \rightleftharpoons A + A \quad A_i \rightarrow A_{i \pm 1} \]

Saturation \( \equiv \) recombination in the wavefunction

**BUT**: This is effective: there is no dipole recombination in QCD
The JIMWLK saturation mechanism

Colour Glass Condensate: evolution a la JIMWLK

\[ W_Y[\alpha^a] \rightarrow W_{Y+\delta Y}[\alpha^a] \equiv W_Y[\alpha^a] \]

- **Coherent emission of new gluons**
  - saturation \( \equiv \) saturation of the rate of emission

- **Classical field**: no discretisation, no fluctuation
**IDEA:** a model with fluctuations and saturation \textit{a la} JIMWLK

- stochastic with discrete particle number (fluctuations)
- only emissions with a saturating rate (JIMWLK)

Master equation \((x_i = \log(r_{2i}^2))\)

\[
\partial_Y P(\vec{n}; Y) = \sum_i f_i(\vec{n} - \vec{e}_i)P(\vec{n} - \vec{e}_i; Y) - f_i(\vec{n})P(\vec{n}; Y)
\]

- \(P(\vec{n}; Y) \equiv \text{probability to have } n_i \text{ particles at lattice site } i \text{ and rapidity } Y\)
- \(f_i(\vec{n}) \equiv \text{deposit rate} \text{ i.e. probability to emit a new gluon at site } i \text{ from the } \vec{n} \text{ gluons present}\)
A toy model for high-energy scattering (1/2)

**IDEA:** a model with fluctuations and saturation *à la* JIMWLK

- stochastic with discrete particle number (fluctuations)
- only emissions with a saturating rate (JIMWLK)

Master equation ($x_i = \log(r_i^2)$)

$$\partial_Y P(\vec{n}; Y) = \sum_i f_i(\vec{n} - \vec{e}_i) P(\vec{n} - \vec{e}_i; Y) - f_i(\vec{n}) P(\vec{n}; Y)$$

- factorized scattering between left and right movers

$$S(Y) = \sum_{\vec{m}, \vec{n}} P_L(\vec{m}; Y_0) P_R(\vec{n}; Y - Y_0) \prod_{i,j} \sigma_{i,j}^{m_i n_j}$$

$$\sigma_{i,j} = 1 - \tau_{i,j}, \text{ with } \tau_{i,j} = \mathcal{O}(\alpha_s^2) \text{ the elementary scattering}$$
A toy model for high-energy scattering (1/2)

IDEA: a model with fluctuations and saturation \textit{a la} JIMWLK

- stochastic with discrete particle number (fluctuations)
- only emissions with a saturating rate (JIMWLK)

Master equation \( (x_i = \log(r_i^2)) \)

\[
\partial_Y P(\vec{n}; Y) = \sum_i f_i(\vec{n} - \vec{\epsilon}_i) P(\vec{n} - \vec{\epsilon}_i; Y) - f_i(\vec{n}) P(\vec{n}; Y)
\]

- factorized scattering between left and right movers

\[
S(Y) = \sum_{\vec{m}, \vec{n}} P_L(\vec{m}; Y_0) P_R(\vec{n}; Y - Y_0) \prod_{i,j} \sigma_{ij}^{m_i n_j}
\]

\[
\sigma_{ij} = 1 - \tau_{ij}, \text{ with } \tau_{ij} = \mathcal{O}(\alpha_s^2) \text{ the elementary scattering}
\]

- boost invariance \( \partial_{Y_0} S(Y) = 0 \) to constrain \( f_i \)
(Modulo a couple of simple assumptions) this fixes the emission/deposit rate

\[ f_i(\vec{n}) = \frac{\alpha_s}{\tau} \prod_j (1 - \sigma_{ij}^n) \propto \begin{cases} 
\alpha_s n & \text{at small densities} \\
1/\alpha_s & \text{at saturation} \end{cases} \]
(Modulo a couple of simple assumptions) this fixes the emission/deposit rate

\[ f_i(\vec{n}) = \frac{\alpha_i}{\tau} \prod_j (1 - \sigma_{ij}^{n_j}) \propto \begin{cases} \alpha_s n & \text{at small densities} \\ 1/\alpha_s & \text{at saturation} \end{cases} \]

- **Fixed coupling:** \( \tau_{xy} = \alpha^2 \exp(|x - y|) = \alpha^2 \langle r_\leq/r_\geq \rangle \)
- **Running coupling:** \( \alpha^2 \to \alpha_x \alpha_y \)

- Same type of hierarchy as for the dipole picture e.g.

  Dipole-target amplitude:
  \[ \partial_Y \langle T_x \rangle = \alpha_x \int_z e^{|x - z|} \langle T_z (1 - T_x) \rangle \]

  - Contains “BFKL”, saturation and fluncts.; Mean-field: \( \langle T_x T_z \rangle = \langle T_x \rangle \langle T_z \rangle \)
Universality at play: everything goes as for the BK/pomeron-loops equations:

- **Linear Kernel** (by Mellin transform): \( \chi(\gamma) = \frac{1}{1-\gamma^2} \)

- **“critical” parameters** for mean-field saturation physics: \( \gamma_c, v_c, \chi''_c \)

- All (qualitative) “standard results” recovered:
  - geometric scaling for mean-field, diffusive scaling with pomeron loops

- see Guillaume Beuf’s talk for the detailed analytic results

In what follows, we compare (numerically)
  - mean field vs. pomeron loops at fixed and running coupling
Fixed coupling results:

We recover the “traditional features” of the dipole evolution
**Fixed coupling results**

Monte-Carlo simulation of the particle process
Initial condition: \( n(x, Y = 0) = N_0 \Theta(x_0 - x) \)

Universality: same properties as in QCD/sFKPP
- **Event-by-event**: geometric scaling + fluctuations in the tail
- **Average**: violations of geometric scaling
\[ \langle x_s \rangle = v \alpha_s Y \quad \text{and} \quad \sigma^2 = \langle x_s^2 \rangle - \langle x_s \rangle^2 = D \alpha_s Y \]

- **Mean-field:** \( v \to v_{\text{mean field}}, \; D = 0 \) for all \( \alpha_s \)
- **Fluct.:** \( v < v_{\text{mean field}} \cdot \alpha_s \uparrow \Rightarrow v \downarrow, \; D \uparrow \)
- **Order of magnitude:** \( \sigma^2 \gg 1, \; D \sim 1 \)
Our toy model lies in the universality class of sFKPP!

What survives with running coupling?
The “time” variable is now $\sqrt{Y/\beta}$ instead of $\alpha_s Y$

- Same effects observed

- Much less important than with fixed-coupling!!!
Again: $\sigma^2 \propto D \sqrt{Y/\beta}$ with $D$ increasing with coupling strength

Order of magnitude: $\sigma^2 \sim 1$, $D \ll 1$

(fixed coupl.: Order of magnitude: $\sigma^2 \gg 1$, $D \sim 1$)
Pomeron-loop effects much less important with running coupling

- checked up to \( Y = 400 \) !!!
- checked with different initial conditions
- checked with different IR regularisations of \( \alpha_s \)

Asymptotic results should be reached but much later than

(i) with fixed coupling,

(ii) relevant for phenomenology
Fluctuations start when geometric scaling window down to $T \approx \alpha_s^2$.

$\Rightarrow$ formation time to get a front of length $L = \log(1/\alpha_s)/\gamma_c$:

- **Fixed coupling**: $Y_{\text{form}} \approx (1/\alpha_s)L^2 \sim 5 - 10$ for $\alpha_s = 0.2$
- **Running coupling**: $Y_{\text{form}} \approx \beta L^6 \sim 400$ for $\beta = 0.72$

$\Rightarrow$ Fluctuations come out much later with running coupling
All this suggests that BK/Mean-field + running coupling is sufficient
⇒ we’re happy with BK’s geometric scaling!
All this suggests that BK/Mean-field + running coupling is sufficient
⇒ we’re happy with BK’s geometric scaling!

BUT: The reduced front \( (e^{\gamma_c (x-x_s)} T') \) shows a slowly-growing geometric-scaling window!

Haven’t we also thrown away geometric scaling?
All this suggests that BK/Mean-field + running coupling is sufficient
⇒ we’re happy with BK’s geometric scaling!

BUT: The reduced front \( e^{\gamma_c (x-x_s) T} \) shows a slowly-growing geometric-scaling window!
Hopefully, an approximate/effective scaling seems to hold

Though with a slightly larger (effective) slope
Conclusions:

- We have a toy model that mimics high-energy evolution in QCD
- Allows to study pomeron-loops effect with fixed and running $\alpha_s$
- Pomeron loop effect killed by running $\alpha_s$
- Mean-field approximation and running coupling are valid

TODO:

- Is that true for other models such as sFKPP? (under study, we do expect universality)
- Effects should subsist in dilute-dilute collisions i.e. in the approach to saturation (under study)
- More involved analysis with mean-field and running coupling to test geometric scaling
- Take into account impact parameter (under study)