Pomeron loops & running coupling effects

Gregory Soyez

Based on : E. Iancu, J.T. Amaral, G. S, D.N. Triantafyllopoulos, N. P. A786 (2007) 131, [hep-ph/0601150]

A. Dumitru, E. Iancu, L. Portugal, G. S., D.N. Triantafyllopoulos,

to appear in JHEP [arXiv:0706.2540 [hep-ph]]

Outline

- build a simple toy-model: evolution based on:
  - based on high-energy QCD QCD (dipole model + JIMWLK saturation)
  - easily simulated with and without running coupling
- Known properties:
  - Mean-field approximation: geometric scaling
  - Fixed-coupling fluctuations: diffusive scaling
- New results:
  Pomeron loops together with Running coupling effects
- Conclusions and perspectives

Reaction-diffusion in high-energy QCD

Dipole model:

$$\partial_{\bar{\alpha}Y} T(r;Y) = \chi_{BFKL} \otimes T(r;Y) - T^2(r;Y) + \sqrt{\kappa \alpha_s^2 T(r;Y)}\,\nu(r;Y)$$

sF-KPP equation in statistical physics:

$$\partial_{\bar{\alpha}Y} u(r;Y) = (\nabla_r^2 + 1) u(r;Y) - u^2(r;Y) + \sqrt{u(r;Y)/N}\,\nu(r;Y)$$

- Linear growth (BFKL)
- saturation (BK)
- discretisation effects (fluctuations, pomeron loops)

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High-energy QCD evolution $\equiv$ reaction-diffusion problem

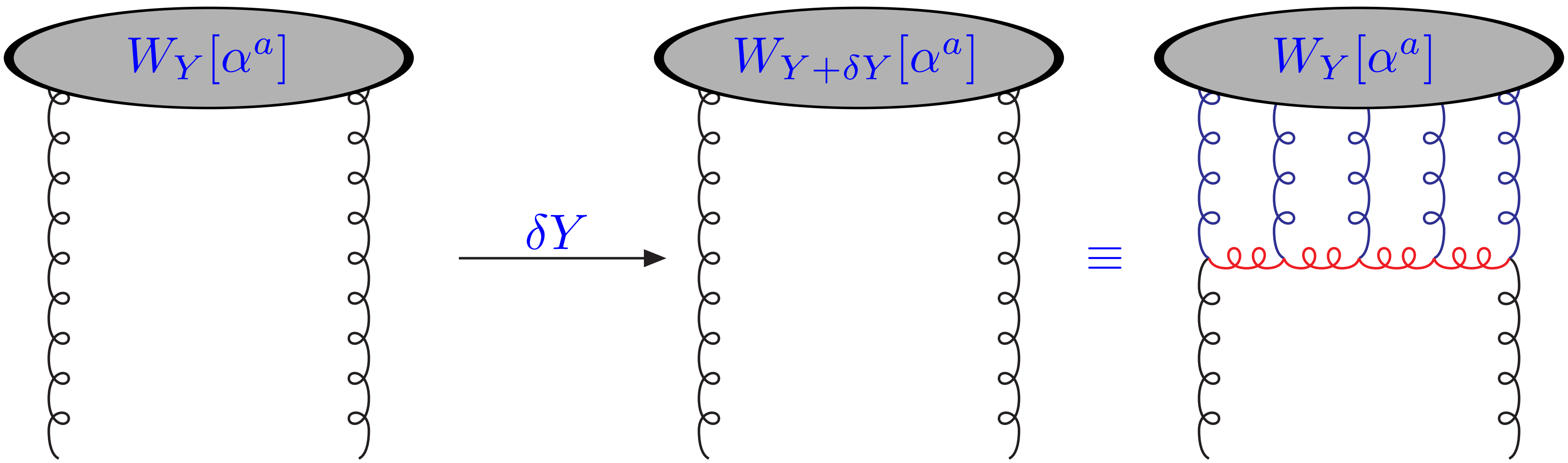
$$A \rightleftharpoons A + A \qquad A_i \to A_{i\pm 1}$$

Saturation $\equiv$ recombination in the wavefunction

BUT: This is effective: there is no dipole recombination in QCD

The JIMWLK saturation mechanism

Colour Glass Condensate: evolution a la JIMWLK



- Coherent emission of new gluons
  saturation $\equiv$ saturation of the rate of emission

- Classical field: no discretisation, no fluctuation

A toy model for high-energy scattering (1/2)

IDEA: a model with fluctuations and saturation a la JIMWLK

- stochastic with discrete particle number (fluctuations)
- only emissions with a saturating rate (JIMWLK)

Master equation ($x_i = \log(r_i^2)$)

$$\partial_Y P(\vec{n};Y) = \sum_i \underbrace{f_i(\vec{n}-\vec{e}_i)P(\vec{n}-\vec{e}_i;Y)}_{\text{gain}} - \underbrace{f_i(\vec{n})P(\vec{n};Y)}_{\text{loss}}$$

- $P(\vec{n};Y) \equiv$ probability to have n_i particles at lattice site i and rapidity Y
- $f_i(\vec{n}) \equiv$ **deposit rate** i.e. probability to emit a new gluon at site i from the $\vec{n}$ gluons present

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- factorized scattering between left and right movers

$$S(Y) = \sum_{\vec{m},\vec{n}} P_L(\vec{m};Y_0)P_R(\vec{n};Y-Y_0)\prod_{i,j}\sigma_{ij}^{m_i n_j}$$

$\sigma_{ij} = 1-\tau_{ij}$, with $\tau_{ij} = \mathcal{O}(\alpha_s^2)$ the elementary scattering

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- boost invariance $\partial_{Y_0} S(Y) = 0$ to constrain f_i

A toy model for high-energy scattering (2/2)

(Modulo a couple of simple assumptions) this fixes the emission/deposit rate

$$f_i(\vec{n}) = \frac{\alpha_s}{\tau} \prod_j \left(1 - \sigma_{ij}^{n_j}\right) \propto \begin{cases} \alpha_s n & \text{at small densities} \\ 1/\alpha_s & \text{at saturation} \end{cases}$$

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- Fixed coupling: $\tau_{xy} = \alpha^2 \exp(|x-y|) = \alpha^2 r_<^2 / r_>^2$
  Running coupling: $\alpha^2 \to \alpha_x \alpha_y$
- Same type of hierarchy as for the dipole picture e.g.

  Dipole-target amplitude: $\partial_Y \langle T_x \rangle = \alpha_x \int_z e^{|x-z|} \langle T_z (1 - T_x) \rangle$
- Contains “BFKL”, saturation and flucts.; Mean-field: $\langle T_x T_z \rangle = \langle T_x \rangle \langle T_z \rangle$

Analytic results

Universality at play: everything goes as for the BK/pomeron-loops equations:

- Linear Kernel (by Mellin transform): $\chi(\gamma) = \frac{1}{1-\gamma^2}$
- "critical" parameters for mean-field saturation physics: γ_c, v_c, χ''_c
- All (qualitative) "standard results" recovered:
  geometric scaling for mean-field, diffusive scaling with pomeron loops
- see Guillaume Beuf's talk for the detailed analytic results

In what follows, we compare (numerically)

mean field vs. pomeron loops at fixed and running coupling

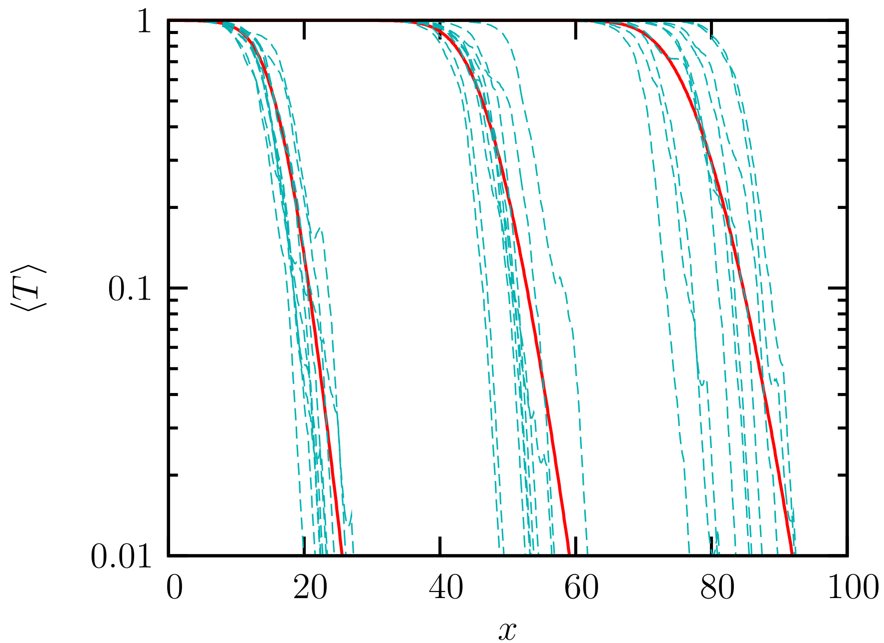
Fixed coupling results:

We recover the "traditional features" of the dipole evolution

Fixed coupling results

Monte-Carlo simulation of the particle process

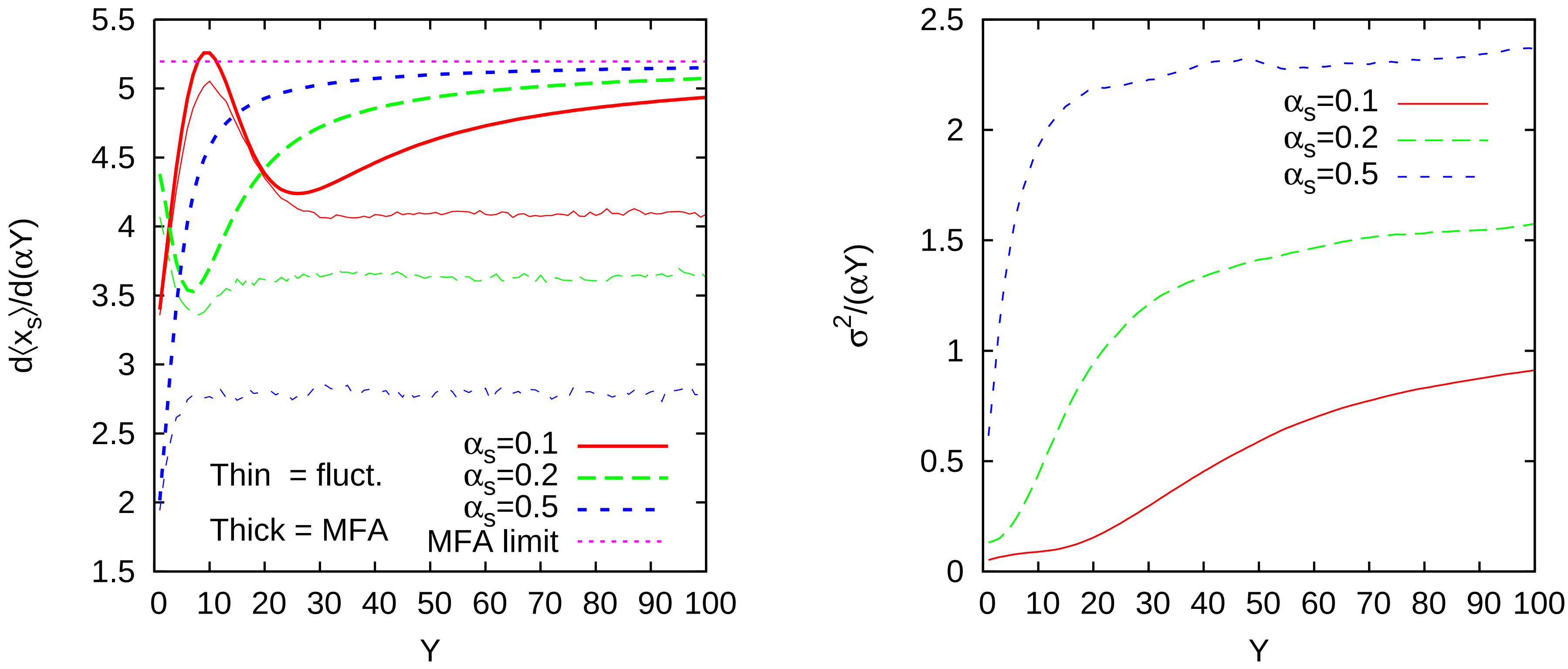
Initial condition: $n(x, Y = 0) = N_0 \Theta(x_0 - x)$



Universality: same properties as in QCD/sFKPP

- Event-by-event: geometric scaling + fluctuations in the tail
- Average: violations of geometric scaling

Fixed coupling: saturation scale statistics

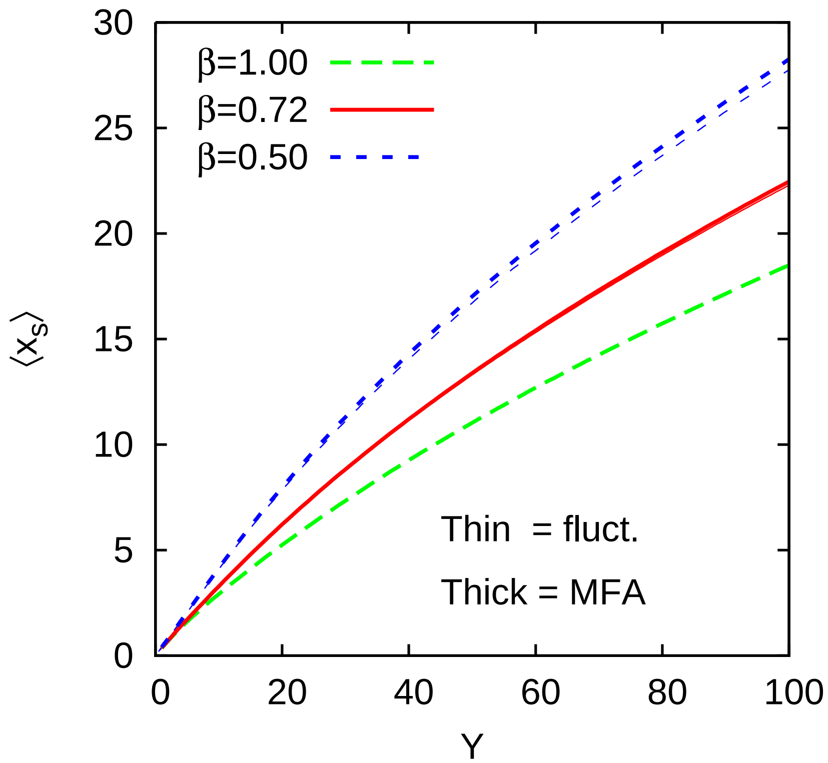


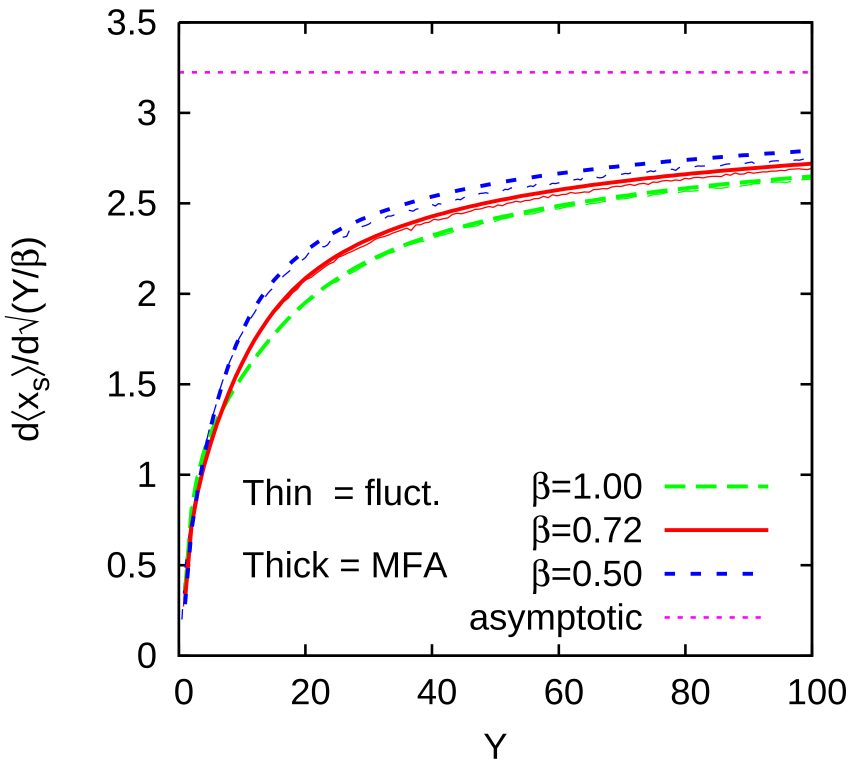
- $\langle x_s \rangle = v\alpha_s Y$ and $\sigma^2 = \langle x_s^2 \rangle - \langle x_s \rangle^2 = D\alpha_s Y$
- Mean-field: $v \to v_{\text{mean field}}$, $D = 0$ for all α_s
- Fluct.: $v < v_{\text{mean field}}$. $\alpha_s \nearrow \Rightarrow v \searrow, D \nearrow$
- Order of magnitude: $\sigma^2 \gg 1$, $D \sim 1$

Our toy model lies in the universality class of sFKPP!

What survives with running coupling?

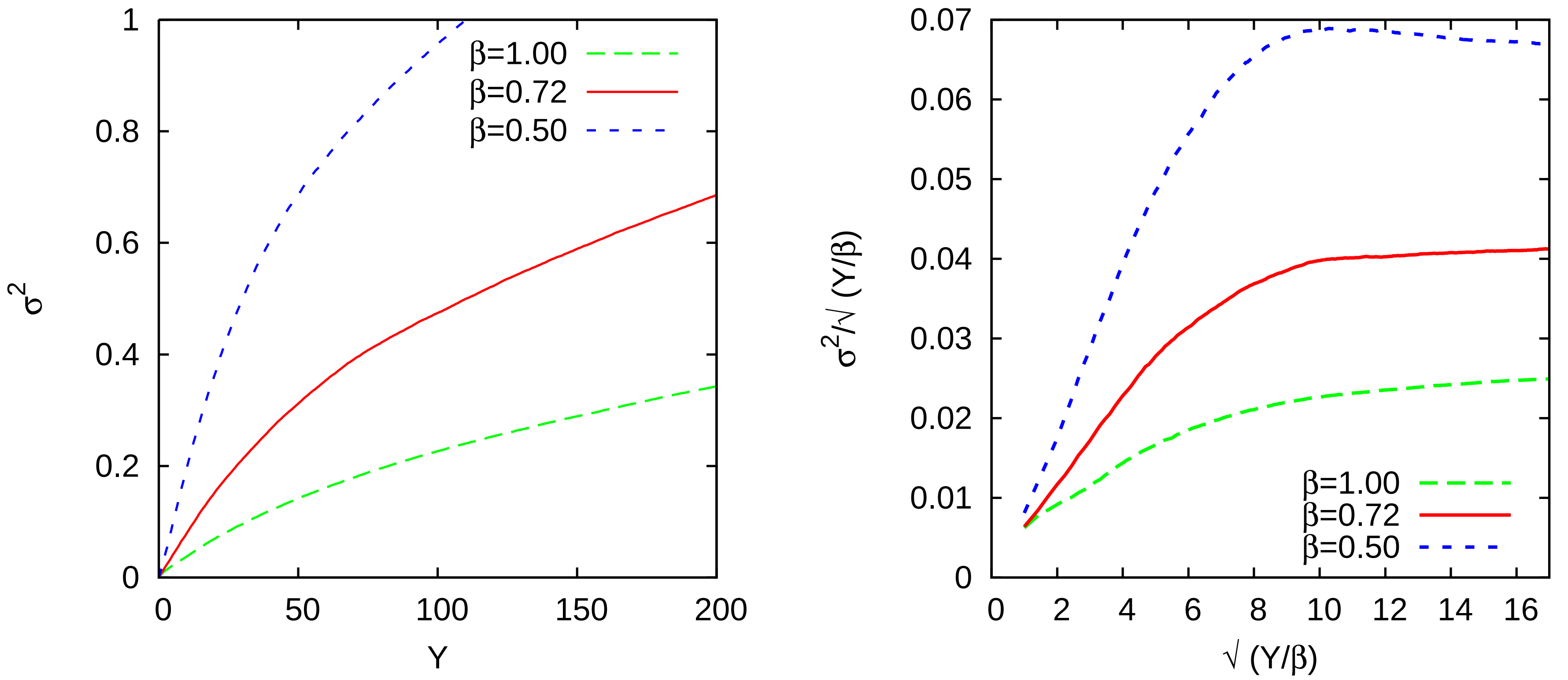
Running coupling: saturation scale





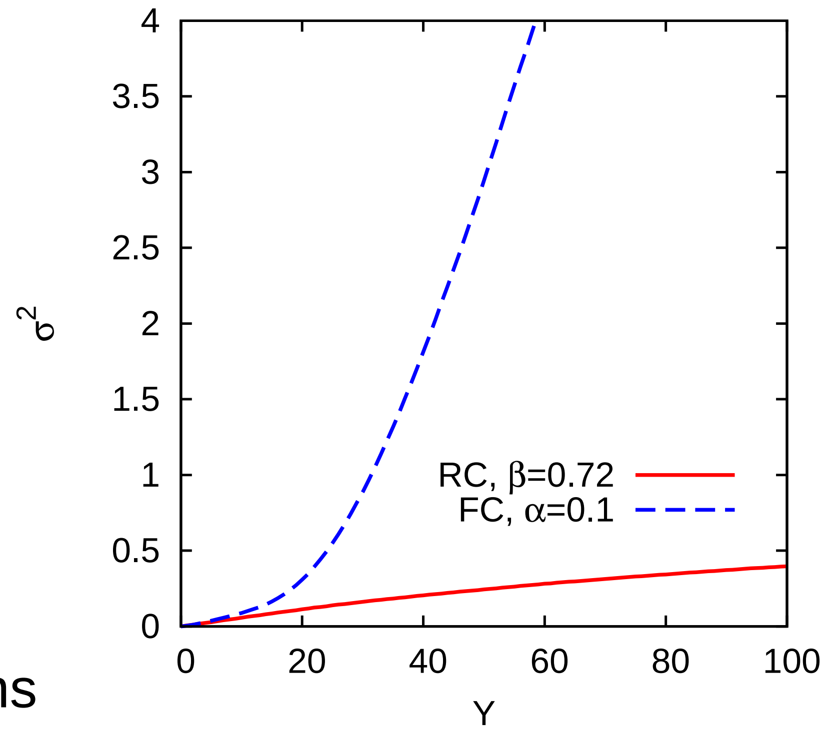
- The “time” variable is now $\sqrt{Y/\beta}$ instead of $\alpha_s Y$
- Same effects observed
- **much less important than with fixed-coupling!!!**

Running coupling: dispersion



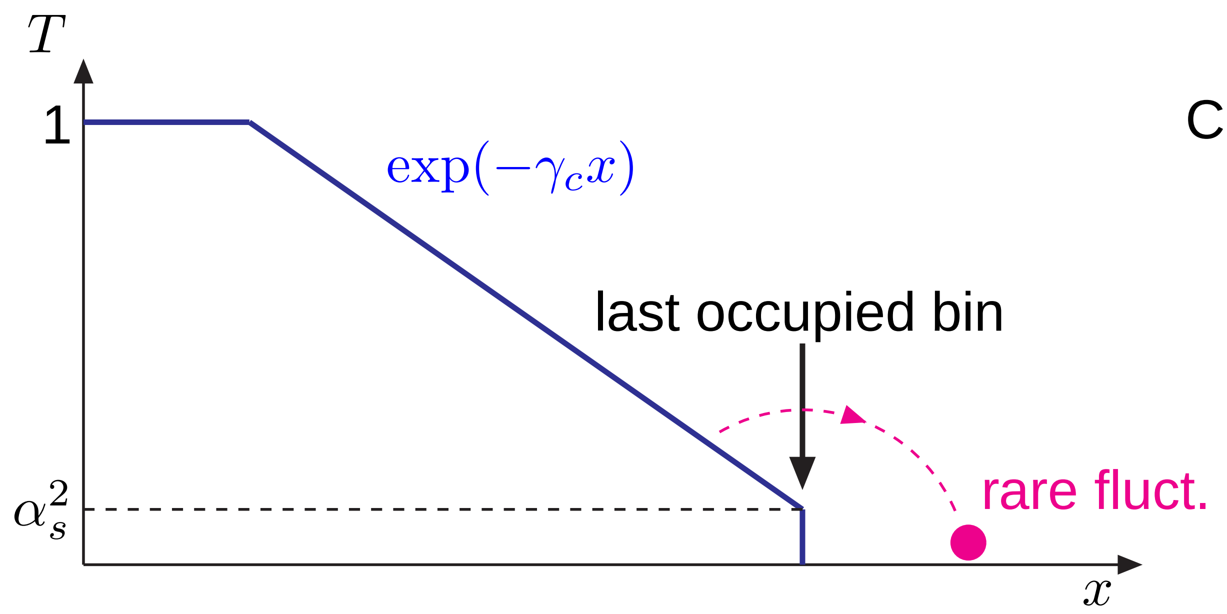
- Again: $\sigma^2 \propto D\sqrt{Y/\beta}$ with D increasing with coupling strength
- Order of magnitude: $\sigma^2 \sim 1$, $D \ll 1$
  (fixed coupl.: Order of magnitude: $\sigma^2 \gg 1$, $D \sim 1$)

Pomeron-loop effects much less important with running coupling



- checked up to $Y = 400$!!!
- checked with different initial conditions
- checked with different IR regularisations of α_s
- Asymptotic results should be reached but much later than
  (i) with fixed coupling,
  (ii) relevant for phenomenology

Argument: formation time



Competition between

1. BFKL-like evolution of the fluctuation
2. Front evolution to the right

Fluctuations start when geometric scaling window down to $T \approx \alpha_s^2$

$\Rightarrow$ formation time to get a front of length $L = \log(1/\alpha_s)/\gamma_c$:

- Fixed coupling: $Y_{\text{form}} \approx (1/\alpha_s)L^2 \sim 5 - 10$ for $\alpha_s = 0.2$
- Running coupling: $Y_{\text{form}} \approx \beta L^6 \sim 400$ for $\beta = 0.72$

$\Rightarrow$ Fluctuations come out much later with running coupling

Note on geometric scaling

All this suggests that BK/Mean-field + running coupling is sufficient

$\Rightarrow$ we're happy with BK's geometric scaling!

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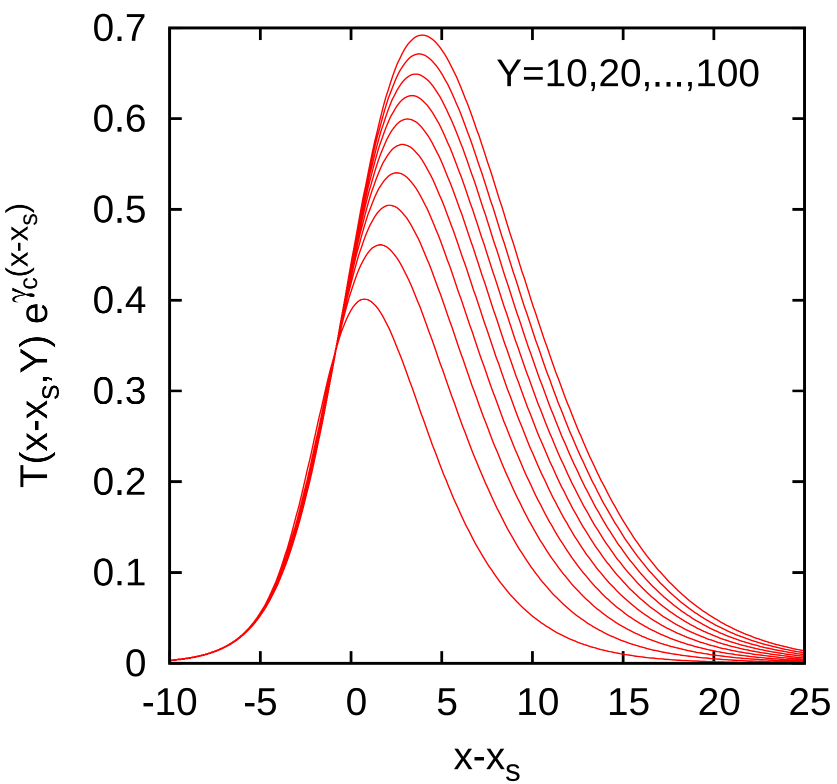
BUT: The reduced front ($e^{\gamma_c(x-x_s)}\,T$) shows a slowly-growing geometric-scaling window !



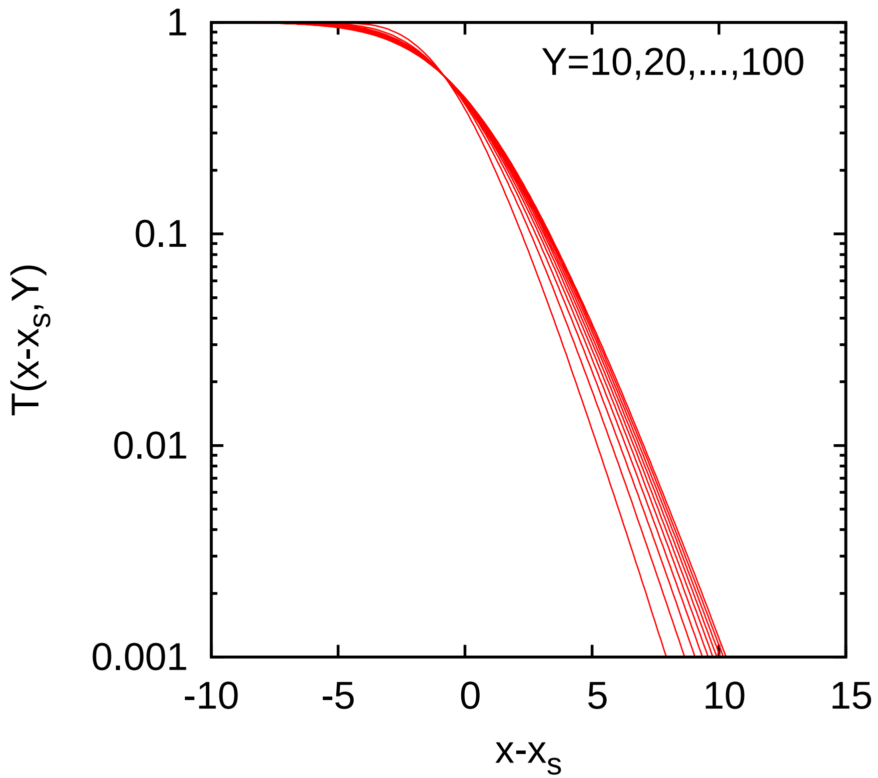
Haven't we also thrown away geometric scaling?

Note on geometric scaling

All this suggests that BK/Mean-field + running coupling is sufficient
$\Rightarrow$ we're happy with BK's geometric scaling!

BUT: The reduced front ($e^{\gamma_c(x-x_s)}\,T$) shows a slowly-growing geometric-scaling window !

Hopefully, an approximate/effective scaling seems to hold



Though with a slightly larger (effective) slope

Conclusion and perspectives

Conclusions:

- We have a toy model that mimics high-energy evolution in QCD
- Allows to study pomeron-loops effect with fixed and running α_s
- Pomeron loop effect killed by running α_s
- Mean-field approximation and running coupling are valid

TODO:

- Is that true for other models such as sFKPP?
  (under study, we do expect universality)
- Effects should subsist in dilute-dilute collisions i.e. in the approach to saturation (under study)
- More involved analysis with mean-field and running coupling to test geometric scaling
- Take into account impact parameter (under study)