New tools in jet physics:
SISCone (new cone algorithm) - jet areas (new concept)

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In collaboration with Gavin Salam


Code available at http://projects.hepforge.org/siscone
FastJet plugin: http://www.lpthe.jussieu.fr/~salam/fastjet

Paper with Matteo Cacciari and Gavin Salam in preparation
Introduction: jet algorithms in general

Infrared-Safety issues:
  - Why is this mandatory?
  - IR unsafety of the JetClu and midpoint algorithms

SISCone: a practical solution

Physical consequences:
  - Algorithm speed
  - Inclusive jet spectrum
  - Jet mass spectrum in multi-jet events

Area of a jet
  - Definition and properties
  - Applications
Why jet algorithms?

**Given**: set of $N$ particles with their 4-momentum
Why jet algorithms?

- **Given**: set of $N$ particles with their 4-momentum
- **Quest**: clustering those particles into jets

⇒ understand the original particle-level process
Two classes of algorithms

Class 1: recombination
Successive recombinations of the “closest” pair of particles

Distance:

- **$k_t$:** \[ d_{i,j} = \min(k_{t,i}^2, k_{t,j}^2)(\Delta \phi_{i,j}^2 + \Delta y_{i,j}^2) \]
- **Aachen/Cam.:** \[ d_{i,j} = \Delta \phi_{i,j}^2 + \Delta y_{i,j}^2 \]

Stop when \( d_{\text{min}} > R \)
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**Distance:**

- **Aachen/Cam.:** \[ d_{i,j} = \Delta \phi_{i,j}^2 + \Delta y_{i,j}^2 \]
- stop when \( d_{\min} > R \)
- Often used for \( e^\pm e^\pm \) or \( e^\pm p \)

**FastJet**: a fast implementation of those algorithms

Two classes of algorithms

Class 2: cone
Find directions of dominant energy flow

for a cone of radius $R$ in the $(y, \phi)$ plane, \textit{stable cones} are such that:
centre of the cone $\equiv$ direction of the total momentum of its particle contents
Two classes of algorithms

Class 2: cone
Find directions of dominant energy flow

for a cone of radius $R$ in the $(y, \phi)$ plane, stable cones are such that:
centre of the cone $\equiv$ direction of the total momentum of its particle contents

- Often used for $pp$
- Many cone algorithms: Snowmass, JetClu, PxCone, CDF Midpoint, ...
- BUT none satisfies 1990’s requirements
Snowmass Accord (FERMILAB, 1990):
any jet algorithm must satisfy

1. Can be practically used in experimental analysis
2. Can be practically used in theoretical computations
3. Can be defined at any order of the perturbation theory
4. Yields finite cross-sections at any order
5. Has a small sensitivity to hadronisation corrections
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- 1, 2 and 4 never satisfied together
- 5 is unclear (Underlying event and $R_{sep}$ issues discussed later)
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Previous cone algorithms:
- 1, 2 and 4 never satisfied together
- 5 is unclear (Underlying event and $R_{\text{sep}}$ issues discussed later)

This talk shows how to satisfy all these.
**Ellipsis:** IR safety, i.e. stability upon emission of soft particles, is required for perturbative computations to make sense!

Cancellation of IR divergences between real and virtual emissions of SOFT gluons in QCD

- IF Jet clustering is different in both cases, THEN the cancellation is not done and the result is not consistent with pQCD
  
  \[ \Rightarrow \text{Stable cones must not change upon addition of soft particles} \]

- Note: 100 GeV jet cannot change by adding a 1 GeV particle
  This would break parton/hadron correspondence
Modern cone jet algorithm (Tevatron Run II type):

- **Step 1**: find **ALL** stable cones of radius $R$
- **Step 1'**: if some of the particles are not in stable cones, rerun Step 1 with the remaining ones.
- **Step 2**: run a split-merge procedure with overlap $f$
  to deal with overlapping stable cones

$$\tilde{p}_{t,\text{shared}} > f\tilde{p}_{t,\text{min}}$$

$$\tilde{p}_{t,\text{shared}} \leq f\tilde{p}_{t,\text{min}}$$
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**Parameters:**

- **Standard parameters**: cone radius $R$, overlap parameter $f$
- **Additional controls**: number of passes $n_{\text{pass}}$, stable cone $p_{t,\text{min}}$ cut-off
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**This talk**: Why finding **all** stable cones and **how**.

→ [C++] implementation: Seedless Infrared-Safe Cone algorithm (SISCone)
Typical cone: Midpoint algorithm

Usual seeded method to search stable cones: midpoint cone algorithm

For an initial seed
1. sum the momenta of all particles within the cone centred on the seed
2. use the direction of that momentum as new seed
3. repeat 1 & 2 until stable state cone reached
Usual seeded method to search stable cones: midpoint cone algorithm

- For an initial seed
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- Sets of seeds:
  1. All particles (above a $p_t$ threshold $s$) (JetClu)
  2. Midpoints between stable cones found in 1.
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Problems:

- the \( p_t \) threshold \( s \) is collinear unsafe
- seeded approach \( \Rightarrow \) stable cones missed \( \Rightarrow \) infrared unsafety
- NB.: addition of soft particles does not modify the set of stable cone, the question is “does our algorithm find all of them”?
Stable cones:

JetClu: \{1\} & \{2\}

Hard event

Hard+soft event

\{1\} & \{2\} & \{1,2\}
JetClu IR Unsafety ($R=1$)

**Hard event**

JetClu: $\{1\}$ & $\{2\}$

Jets: ($f = 0.5$)

JetClu: $\{1\}$ & $\{2\}$

Stable cones:

JetClu: $\{1\}$ & $\{2\}$ & $\{1,2\}$

Midpoint: $\{1,2\}$ & $\{1,2\}$

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JetClu: $\{1\}$ & $\{2\}$ & $\{1,2\}$

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Stable cone missed $\rightarrow$ IR unsafety of the JetClu algorithm
JetClu IR Unsafety (R=1)

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Stable cones:

Midpoint: $\{1,2\} \& \{3\}$

Seedless: $\{1,2\} \& \{3\} \& \{2,3\}$
Midpoint IR Unsafety (R=1)

Stable cones:
- **Midpoint:** \{1,2\} & \{3\}
- **Seedless:** \{1,2\} & \{3\} & \{2,3\}

Jets: \( f = 0.5 \)
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**Stable cone missed** → IR unsafety of the midpoint algorithm
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Stable cones:
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Jets: ($f = 0.5$)
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Stable cone missed $\rightarrow$ IR unsafety of the midpoint algorithm
**is a seedless solution practical?**

- **Solution**: use a seedless approach, find **ALL** stable cones
- **Naive approach**: check stability of each subset of particle
is a seedless solution practical?

Solution: use a seedless approach, find ALL stable cones

Naive approach: check stability of each subset of particle
Complexity is $O(N2^N)$
$\Rightarrow$ definitely unrealistic: $10^{17}$ years for $N = 100$
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**Solution**: use a seedless approach, find **ALL** stable cones

**Naive approach**: check stability of each subset of particle

Complexity is $O(N 2^N)$

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**Midpoint complexity**:

- For 1 seed: build and check cone content is $O(N)$
- Initially $N$ seeds $\Rightarrow O(N)$ stable cones
  $\Rightarrow O(Nn)$ new, midpoint, seeds
  $\Rightarrow$ midpoint complexity is $O(N^2n)$
- With $n \sim N$ the number of points in a circle of radius $R$
- Note: the number of stable cones is $O(N)$
Idea: use geometric arguments

Enumerate enclosures and check if they are stable
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⇒ Enumerate all pairs of particles with 2 circle orientations and 4 possible inclusion/exclusion

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Complexity?

- Enumerate all pairs of particles: $O(Nn)$
- For each, build content and check stability
  ⇒ $O(N^2n)$
⇒ Enumerate all pairs of particles
   with 2 circle orientations and 4 possible inclusion/exclusion
   → find all enclosures

Complexity?

- Enumerate all pairs of particles: $O(Nn)$
- For each, build content and check stability
  → $O(N^2n)$

Same as midpoint... but we’ll use more tricks:

- avoid systematic recomputation of cone contents
- limit complete tests of cone stability
**Tricks:**

- For all enclosures around a particle, introduce a traversal order.

From one cone to the next one, contents only changed by “border” particles.

⇒ avoids recomputing the cone contents at each step.
**Tricks:**

- For all enclosures around a particle, introduce a **traversal order**
  ⇒ avoids recomputing the cone contents at each step

- Label the particles using a $q$-bit tag
  ⇒ checkxor to identify distinct cones

Introduces a potential “collision” problem

\[ q = 96 \quad \Rightarrow \quad P(\text{collision}) = 10^{-18} \]
**Tricks:**

- For all enclosures around a particle, introduce a **traversal order**
  ⇒ avoids recomputing the cone contents at each step

- Label the particles using a \(q\)-bit tag
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- Only test “border particles” for stability (cost \(\mathcal{O}(1)\))
  ⇒ limits the number of full stability test to \(\mathcal{O}(N)\)
  checkxor → keep trace of stability tests
How to efficiently determine all stable cones:

- For each particle $i$
  - get “partners” and associated cone centres
  - order them by angle
  - build the first candidate cone contents
  - for all those candidates
    - check stability w.r.t. border particles
      - 4 possible $\in$ or $\notin$ & keep track of tested cones
      - move to the next cone
  - Full stability test for the $O(N)$ not-yet-unstable candidate
How to efficiently determine all stable cones:

- For each particle $i$
  - get “partners” and associated cone centres $O(N)$
  - order them by angle $O(n \log(n))$
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All stable cones found in $O(Nn \log(n))$
SISCones vs. other cone algorithms

implications of a seedless cone
faster than midpoint with no seed threshold and IR safe
- faster than midpoint with no seed threshold and IR safe
- same as midpoint with 1 GeV seed and collinear safe
- faster than midpoint with no seed threshold and IR safe
- same as midpoint with 1 GeV seed and collinear safe
- slower than $k_t$/FastJet affordable for practical usage e.g. at the LHC
**Hard event**: 2-10 particles

**Soft add-on**: 1-5 particles

**Run**:
- “hard” only
- many “hard+soft” trials
- Search differences
**IR Unsafety failure rates**

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Grégory Soyez
Fermilab, USA, October 31th 2007
SISCon and jet areas – p. 19/38
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- **JetClu** 50.1%
- **SearchCone** 48.2%
- **MidPoint** 16.4%
- **Midpoint-3** 15.6%
- **PxCone** 9.3%

Fraction of hard events failing IR safety test

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NB: small issues in the split-merge
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**Graph:**
- **JetClu**: 50.1%
- **SearchCone**: 48.2%
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- **PxCones**: 9.3%
- **Seedless [SM-p]**: 1.6%
- **Seedless [SM-MIP]**: 0.17%
- **Seedless (SISCones)**: < 10^-9

Fraction of hard events failing IR safety test

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Consequences on observables

Physical impact: SIS Cone vs. midpoint(s)?

IR unsafety of midpoint: 3 particles in the same vicinity + 1 to balance $p_t$
$\Rightarrow$ starts at the $2 \rightarrow 4$ level ($O(\alpha_s^4)$)

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The IR-unsafety issue will matter at LHC
SISCone vs. midpoint(s) in inclusive jet spectrum?

- IR unsafety of midpoint: 3 particles in the same vicinity + 1 to balance $p_t$
  $\Rightarrow$ starts at the $2 \rightarrow 4$ level ($O(\alpha_s^4)$)

- 3 contributions at this order:
  2 $\rightarrow$ 4 at LO (tree), 2 $\rightarrow$ 3 at NLO (1 loop) and 2 $\rightarrow$ 2 at NNLO (2 loops)
SISCones vs. midpoint in inclusive jet spectrum?

- IR unsafety of midpoint: 3 particles in the same vicinity + 1 to balance $p_t$
  $\Rightarrow$ starts at the $2 \rightarrow 4$ level ($\mathcal{O}(\alpha_s^4)$)

- 3 contributions at this order:
  - $2 \rightarrow 4$ at LO (tree),
  - $2 \rightarrow 3$ at NLO (1 loop) and $2 \rightarrow 2$ at NNLO (2 loops)

- $2 \rightarrow 4$ at LO is IR divergent
  BUT the difference between SISCones and midpoint in finite since it is 0
  at the $2 \rightarrow 2$ and $2 \rightarrow 3$ levels

  $\Rightarrow$ compute $|\text{SISCone-midpoint}|$ for $2 \rightarrow 4$ diagrams

- Compare with the $2 \rightarrow 2$ (LO) spectrum to estimate effect
Inclusive jet spectrum: perturbative exp.

(a) Inclusive $p_T$ spectrum (all $y$)

- SISCon (Born level, $0(\alpha_s^2)$)
- $|\text{midpoint}(0) - \text{SISCon}| \; 0(\alpha_s^4)$

(b) Differences of order 1-2 %
Including parton shower, hadronic corrections and/or underlying event:

Ratio midpoint/SISCon-1:

\[ \frac{d\sigma_{\text{midpoint}}}{dp_t} / \frac{d\sigma_{\text{SISCon}}}{dp_t} - 1 \]

pp √s = 1.96 TeV

(a) hadron-level (with UE) — blue dashed
hadron-level (no UE) — green dashdot
parton-level — red solid

Pythia 6.4
R=0.7, f=0.5, |y|<0.7
Including parton shower, hadronic corrections and/or underlying event:

**Ratio midpoint/SISCone-1:**

- **pp \( \sqrt{s} = 1.96 \) TeV**
  - Hadron-level (with UE)
  - Hadron-level (no UE)
  - Parton-level

- **pp \( \sqrt{s} = 14 \) TeV**
  - Hadron-level (with UE)
  - Hadron-level (no UE)
  - Parton-level

- Differences up to 5% (with a change of sign)
- Raise up to 10% at LHC energy!
Inclusive jet spectrum: hadron level

Including parton shower, hadronic corrections and/or underlying event:

Ratio midpoint/SISCone-1:

- Differences up to 5% (with a change of sign)
- Raise up to 10% at LHC energy!
- Less effect from underlying event in SISCone (i.e. better agreement with parton level)
Inclusive jet spectrum

→ effect at NNLO i.e. $\mathcal{O}(\alpha_s^2)$ w.r.t. LO
⇒ want to look at more exclusive processes

Example: mass spectrum in 3-jet events (or W/Z/H+2j)

\[ \begin{align*}
2 \rightarrow 2 & \text{ has only 2 jets} \\
2 \rightarrow 3 & \text{ has zero masses}
\end{align*} \]

⇒ first contribution from $2 \rightarrow 4$

⇒ Expect modifications at LO!

Ratio \( \frac{\text{midpoint} - \text{SISCone}}{\text{SISCone}} \) for masses spectra in 3-jet events

cuts: \( p_{t,1} \geq 120 \text{ GeV}, p_{t,2} \geq 80 \text{ GeV}, p_{t,3} \geq 40 \text{ GeV} \)
1. Fixed order computation (NLOJet, LO, $2 \rightarrow 4$)

Differences up to 10 %
1. Fixed order computation (NLOJet, LO, \(2 \rightarrow 4\))

\[
\begin{array}{c}
\text{rel. diff. for } \frac{d\sigma}{dM^2} \\
0 & 0.05 & 0.1 & 0.15
\end{array}
\]

\[
\begin{array}{c}
\text{SISCone} \\
\text{NLOJet} \\
R=0.7, f=0.5
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\]

Mass spectrum of jet 2 midpoint(0) – SISCone

Differences up to 10 %

2. Also require jets 2 and 3 within distance \(\leq 2R\)

\[
\begin{array}{c}
\text{rel. diff. for } \frac{d\sigma}{dM^2} \\
0 & 0.1 & 0.2 & 0.3 & 0.4 & 0.5
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\[
\begin{array}{c}
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\text{NLOJet} \\
R=0.7, f=0.5 \\
\Delta R_{23} < 1.4
\end{array}
\]

Mass spectrum of jet 2 midpoint(0) – SISCone

Differences up to 40 %
3. At hadron level (PYTHIA)

- Differences of order 10 %
- Larger effects in the tail
- Seed threshold even worse
Jets are present everywhere: $k_t$ and cone are widely used
seeded implementations are **IR unsafe** (sometimes **collinear unsafe**)
IR safety is a prerequisite for perturbative QCD to make sense

We propose a **new cone algorithm (SISCon)**:
- **IR safe** (and **collinear safe**)
- as **fast** as available cone implementations
- has **10% impact on jet mass spectra** (can be up to 40%)
- is **less affected by underlying events**
Jet area

Everyone has an idea of what a jet area is

but can we define that properly?

[M. Cacciari, G. Salam, G.S., in preparation]
[M. Cacciari, G. Salam, arXiv:0707.1378]
Idea: add soft particle (ghosts)

- with IR-safe algorithms such as $k_t$, Aachen/Cambridge and SIS Cone, clustering is unchanged
- look in which jets added particles are caught
**Idea**: add soft particle (ghosts)

- with IR-safe algorithms such as $k_t$, Aachen/Cambridge and SISCones, clustering is unchanged
- look in which jets added particles are caught

**Passive area**

add one ghost and look where it ends. repeat to cover the $(y, \phi)$ plane
**Area definition**

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  - with IR-safe algorithms such as $k_t$, Aachen/Cambridge and SISCone, clustering is unchanged
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- **Active area**
  - add a large amount of ghosts and cluster everything
  - also gives purely ghosted jets
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add a large amount of ghosts and cluster everything
also gives purely ghosted jets

**Voronoi area**
~ Area of the Voronoi cells
Small $N$: active area is usually smaller than passive area (especially for the cone)
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For more dense events (e.g. Pythia with underlying event) they tend to be the same
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## Examples: 1-particle cases

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<th>$k_t$</th>
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<tbody>
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<td>Passive</td>
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<tr>
<td>Passive</td>
<td><img src="chart.png" alt="circle" /> $\pi R^2$</td>
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Grégory Soyez

Fermilab, USA, October 31th 2007

SISCon and jet areas – p. 31/38
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<td>![Circle]</td>
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<td><img src="image" alt="Graph" /></td>
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- $A_{\text{hard}} / \pi R^2 \approx 0.812 \pm 0.277$
- $A_{\text{ghost}} / \pi R^2 \approx 0.554 \pm 0.174$
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<tr>
<td>Active</td>
<td><img src="image" alt="kt algorithm" /></td>
<td><img src="image" alt="Cam/Aachen algorithm" /></td>
<td><img src="image" alt="possible monster jets" /></td>
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For the passive case:
- $A_{hard}/\pi R^2 \approx 0.812 \pm 0.277$
- $A_{ghost}/\pi R^2 \approx 0.554 \pm 0.174$

For the active case:
- $A_{hard}/\pi R^2 \approx 0.814 \pm 0.261$
- $A_{ghost}/\pi R^2 \approx 0.551 \pm 0.176$

$A_{hard}/\pi R^2 = 0.25$

$A_{ghost}$ depends on $f$
2-particle cases

**Passive area**: 1 hard particle + 1 soft

- **a)** \( k_t \)
- **b)**
- **c)**
- **d)** Cam/Aachen
- **e)**
- **f)**
- **g)** cone
- **h)** \( 0 < \Delta_{12} < R/2 \)
- **i)** \( R < \Delta_{12} < 2R \)
Active area: 1 hard particle + 1 soft: analytic result for cone only

d < R

R < d < \sqrt{2} R

\sqrt{2} R < d < 2R
**Active area:** 1 hard particle + 1 soft: analytic result for cone only

Alltogether, we have:

- **Area** \( \neq \text{cst. } \pi R^2 \)
- **\( \Delta_{12} \)** dependence under control
QCD probability of emitting a small-angle soft gluon:

$$\frac{dP}{d\Delta_{12} dp_{t,2}} = C_{F,A} \frac{2\alpha_s}{\pi} \frac{1}{\Delta_{12}} \frac{1}{p_{t,2}}$$

Hence the average area is

$$\langle A(p_{t,1}, R) \rangle = A_{\text{hard}}(R) + \int d\Delta d\Delta_{12} dp_{t,2} \frac{dP}{d\Delta_{12} dp_{t,2}} \left[ A_{\text{hard+1 soft}}(\Delta, R) - \pi R^2 \right]$$
QCD probability of emitting a small-angle soft gluon:

\[
\frac{dP}{d\Delta_{12} dp_{t,2}} = C_{F,A} \frac{2\alpha_s}{\pi} \frac{1}{\Delta_{12}} \frac{1}{p_{t,2}}
\]

Hence the average area is

\[
\langle A(p_{t,1}, R) \rangle = A_{1\text{hard}}(R) + \int d\Delta \ dp_{t,2} \ \frac{dP}{d\Delta_{12} dp_{t,2}} \left[ A_{\text{hard+1 soft}}(\Delta, R) - \pi R^2 \right]
\]

\[
= C_{F,A} \frac{1}{\pi b_0} \log \left( \frac{\alpha_s(\Lambda)}{\alpha_s(R p_t)} \right) \pi R^2 \ d
\]

Scaling violation
QCD probability of emitting a small-angle soft gluon:

$$\frac{dP}{d\Delta_1} = C_{F,A} \frac{2\alpha_s}{\pi} \frac{1}{\Delta_1} \frac{1}{p_{t,2}}$$

Hence the average area is

$$\langle A_{(p_t,1,R)} \rangle = A_{\text{hard}}(R) + \int d\Delta dp_{t,2} \frac{dP}{d\Delta_1 dp_{t,2}} [A_{\text{hard}+1 \text{ soft}}(\Delta, R) - \pi R^2]$$

$$= \frac{C_{F,A}}{\pi b_0} \log \left( \frac{\alpha_s(\Lambda)}{\alpha_s(Rp_t)} \right) \pi R^2 d$$

- Scaling violation
- gluon > quark
QCD probability of emitting a small-angle soft gluon:

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\frac{dP}{d\Delta_{12}dp_{t,2}} = C_{F,A} \frac{2\alpha_s}{\pi} \frac{1}{\Delta_{12}} \frac{1}{p_{t,2}}
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<tbody>
<tr>
<td>(k_t)</td>
<td>0.5638</td>
<td>0.519</td>
</tr>
<tr>
<td>Cam</td>
<td>0.07918</td>
<td>0.0865</td>
</tr>
<tr>
<td>Cone</td>
<td>-0.06378</td>
<td>0.1246</td>
</tr>
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Herwig simulations of $qq$ or $gg$ processes at hadron level with underlying event: area vs. $p_t$ of the jet
Herwig simulations of $qq$ or $gg$ processes at hadron level with underlying event: area vs. $p_t$ of the jet

![Graph showing area vs. log($p_t$) for different jet cone definitions.](image)
"Real-life" anomalous dimension

Herwig simulations of $qq$ or $gg$ processes at hadron level with underlying event: area vs. $p_t$ of the jet

- good agreement with LO predictions
- $k_t$ bigger $\Rightarrow$ NLO?
Dense event with pile-up:

$k_t$ algorithm, $R=0.5$
Area $\propto p_t$ of the jet

$p_t/\text{area}$ is constant $\rightarrow \rho = \text{median } p_t/\text{area}$
What can area be used for?

Dense event with pile-up:

- Area $\propto p_t$ of the jet
- $p_t/\text{area}$ is constant $\rightarrow \rho = \text{median } p_t/\text{area}$

Area can be used to remove pileup pollution

\[ \text{e.g. by removing } \rho\cdot\text{area} \]
**Subtraction in action**

\[ t\bar{t} + W \quad (t\bar{t} \rightarrow \ell^+ \nu_\ell b + q\bar{q}b) \]

\[ (W \rightarrow q\bar{q}) \]

**Graphs:**

- **kt, R=0.4, no pileup**
  - \(1/N \cdot dN/dm \) [GeV\(^{-1}\)]
  - Reconstructed \( W \) / top mass [GeV]

- **Cam/Aachen, R=0.4**
  - \(1/N \cdot dN/dm \) [GeV\(^{-1}\)]
  - Reconstructed \( W \) / top mass [GeV]

- **SISCon, R=0.4, f=0.5**
  - \(1/N \cdot dN/dm \) [GeV\(^{-1}\)]
  - Reconstructed \( W \) / top mass [GeV]

**Legend:**

- \( W \)
- \( \text{top} \)

**Observations:**

- LHC at high lumi
  - no pileup ➞ good result
Subtraction in action

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LHC at high lumi

- no pileup \( \Rightarrow \) good result
- no subtraction effect
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  - subtraction works

Grégory Soyez, Fermilab, USA, October 31th 2007
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Background suppression in heavy ions!
SIS Cone: a new cone jet algorithm
- first to satisfy requirements of the 90’s!
- mandatory for LHC
Conclusions

**SISCon**: a new cone jet algorithm
- first to satisfy requirements of the 90’s!
- mandatory for LHC
- Get it at [http://projects.hepforge.org/siscone](http://projects.hepforge.org/siscone)
  or [http://www.lpthe.jussieu.fr/~salam/fastjet](http://www.lpthe.jussieu.fr/~salam/fastjet)

**TODO**: in-depth study of $k_t$/Cam vs. cone.
Under study with the “Les Houches jet benchmarks”
**Conclusions and perspectives**

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  - mandatory for LHC

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- New concept: the area of a jet
  - active, passive and Voronoi: definition and basic properties
  - pileup effects subtraction, background subtraction in heavy ions
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TODO:
- anomalous dimension resummation
- only the beginning...