Progress in defining jets

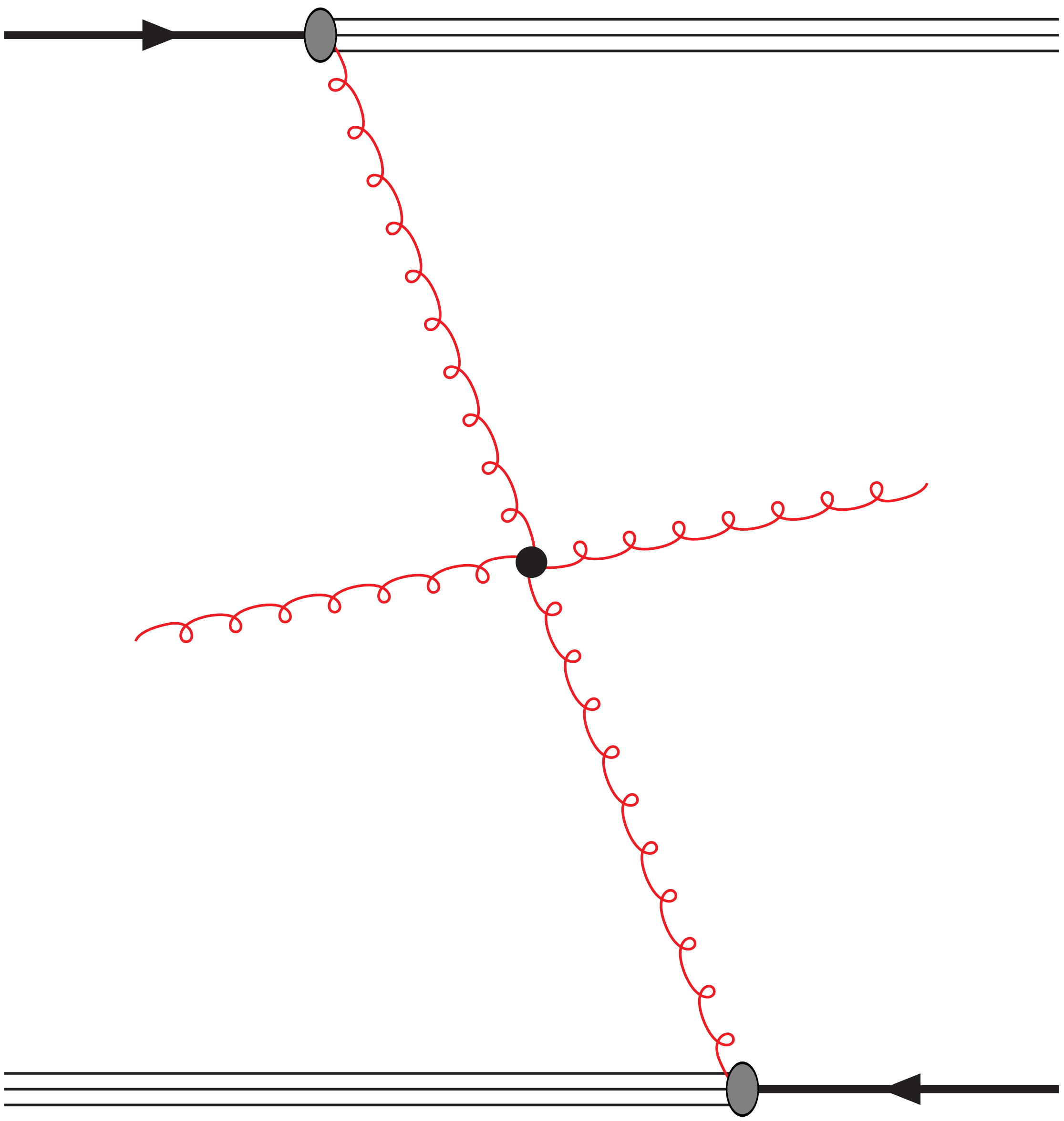
Grégory Soyez

Brookhaven National Laboratory

in collaboration with G. Salam, M. Cacciari

arXiv:0704:0292, arXiv:0802:1188, arXiv:0802.1189

General (over)simplified picture

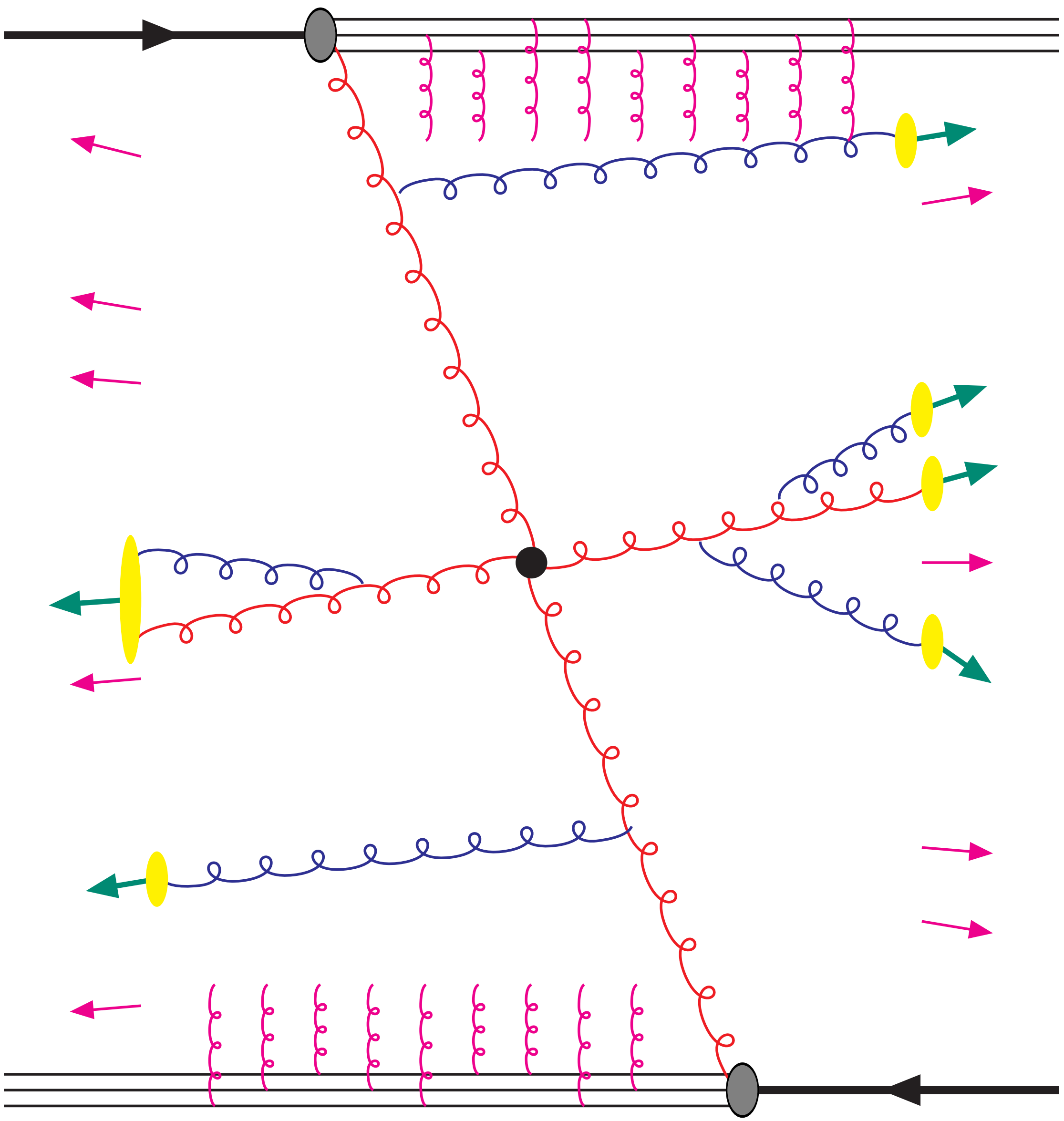


Hard scattering ($2 \to n$)

computed exactly at $\mathcal{O}(\alpha_s^p)$

$gg \to gg$, $gg \to ggg$,
$gg \to gggg$,
$gg \to H \to b\bar{b}$,
$gg \to t\bar{t} \to \mu\nu_\mu b\bar{b}q\bar{q}$,
$gg \to Z' \to q\bar{q}$, ...

General (over)simplified picture



Hard scattering ($2 \to n$)

Parton level

$\approx$ resummed collinear div.

$\sum_i \alpha_s^i \log^i(p_t^2/\mu^2)$

Hadron level: hadronisation

Underlying event

beam remnants interactions

$\Rightarrow$ soft background

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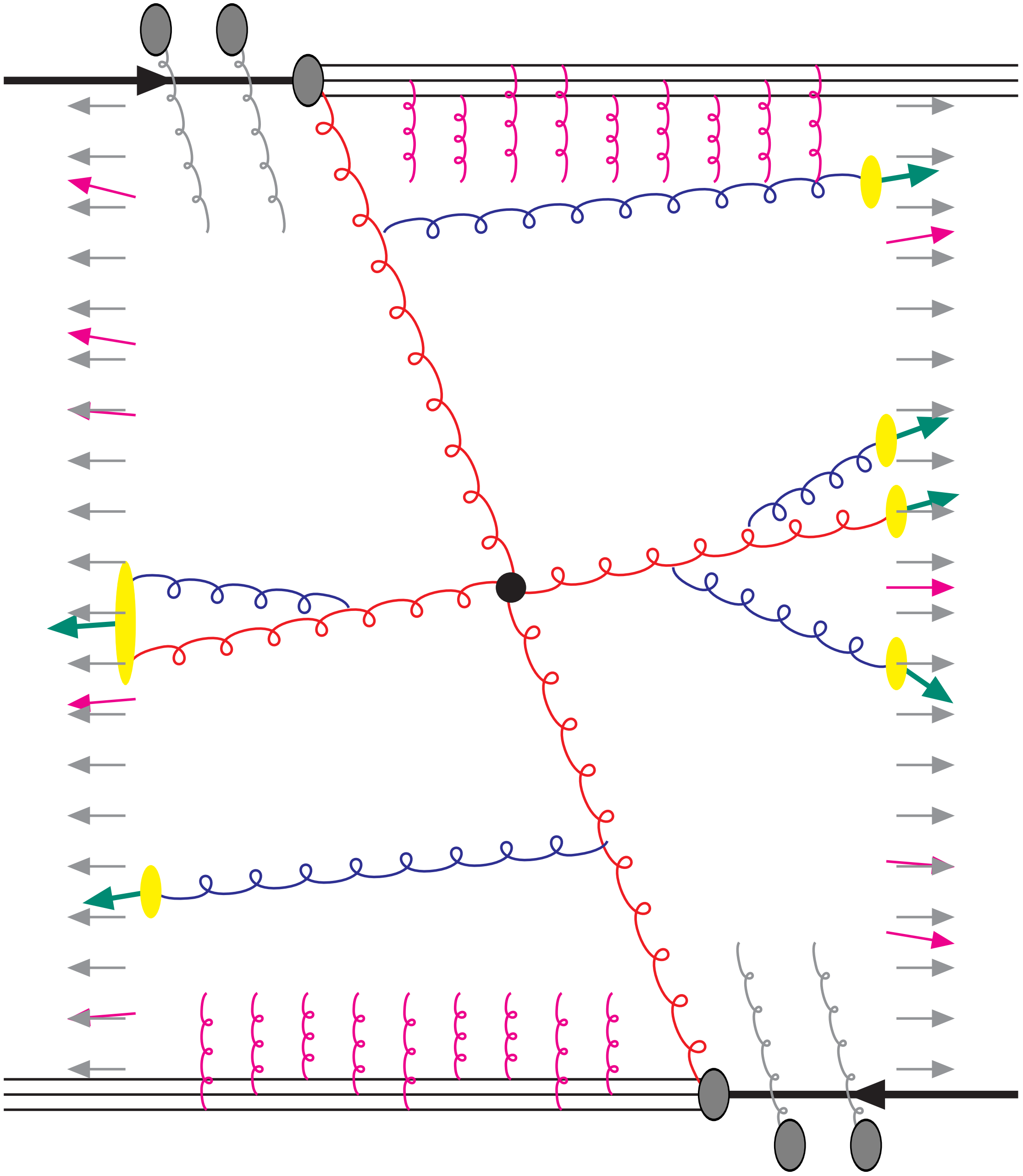

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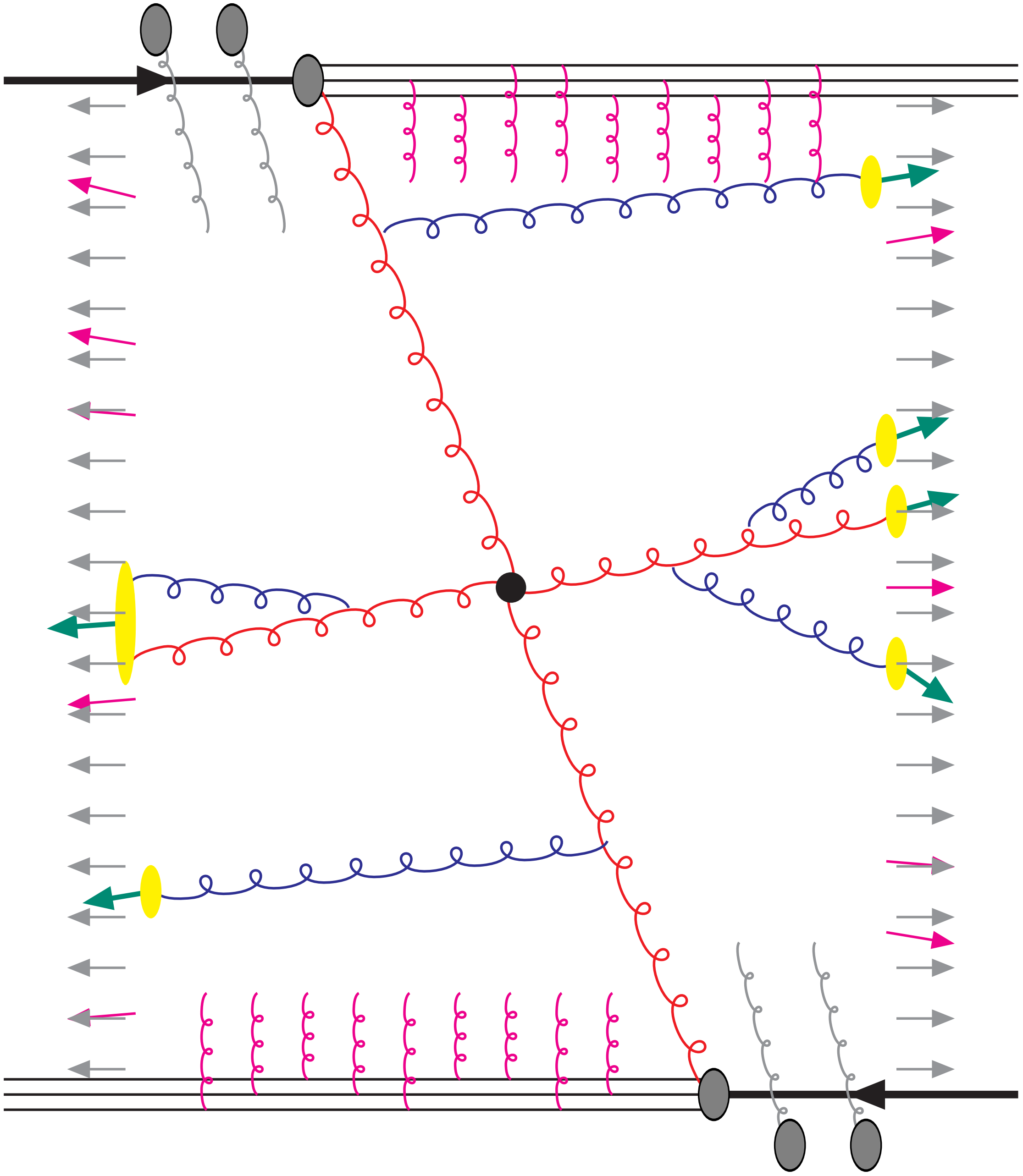

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How to access the hard scattering?

Plan

Define jets:

- Idea: a bunch of collimated partons/particles
- But: partons are ambiguous

  $\Rightarrow$ different *jet definitions*

  $\Rightarrow$ jets are really what they're defined to be

Plan

Define jets:

- Idea: a bunch of collimated partons/particles
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  $\Rightarrow$ different *jet definitions*
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Outline:

- Part 1: Failures to meet the fundamental requirements
  $\Rightarrow$ introduce *new algorithms*
- Part 2: Deal with the soft background contamination (pileup or HI background) $\Rightarrow$ introduce *jet areas* (how to control and use them)

Note: Jet definition = algorithm + parameters
Also valid for jets in heavy-ion collisions

The main recipes (1/2)

Algorithm Class 1: *successive recombinations*

- define a distance d_{ij} between any pair of particles
- repeatedly recombine the 2 closest "particles" into one
- stop when they are all more than R apart

Common distance choices:

$$k_t: \quad d_{i,j} = \min(k_{t,i}^2, k_{t,j}^2)(\Delta\phi_{i,j}^2 + \Delta y_{i,j}^2)$$

$$\text{Aachen/Cam.:} \quad d_{i,j} = \Delta\phi_{i,j}^2 + \Delta y_{i,j}^2$$

The main recipes (2/2)

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Algorithm Class 2: *cone*

> **stable cones** (radius R) such that:
> the total momentum of its contents points in the direction of its centre

Seeded (iterative) approaches: iterate from an initial position until stable

- seed = initial particle
- seed = midpoint between stable cones found at first step

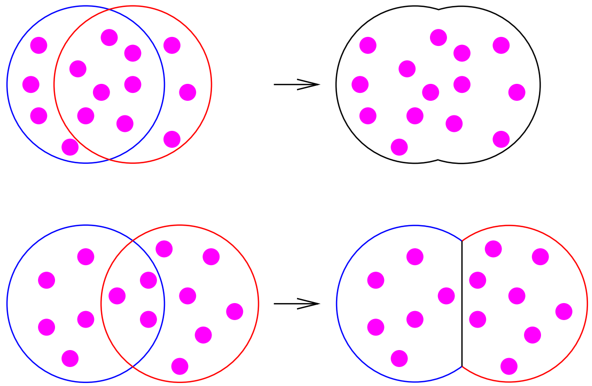
Algorithm Class 2: *cone*

stable cones (radius R) such that:
the total momentum of its contents points in the direction of its centre

Class 2(a): cone with split-merge (ex.: JetClu, Atlas, MidPoint):

$\tilde{p}_{t,\text{shared}} > f\tilde{p}_{t,\text{min}}$

$\tilde{p}_{t,\text{shared}} \le f\tilde{p}_{t,\text{min}}$



Class 2(b): cone with progressive removal (ex.: Iterative Cone)

- iterate from the hardest seed
- remove the stable cone as a jet and start again $\Rightarrow$ "regular/circular" jets

20th century jet finders

Recombination:

- k_t algorithm
- Cambridge/Aachen alg.

Cone:

- CDF JetClu
- CDF MidPoint
- D0 (run II) Cone
- PxCone
- ATLAS Cone

- CMS Iterative Cone
- PyCell/CellJet
- GetJet

Part 1
21st century: towards a solid toolkit

1990: fixing the rules

SNOWMASS accords, Tevatron 1990 (i.e. old!):

Several important properties that should be met by a jet definition are [3]:

1. Simple to implement in an experimental analysis;
2. Simple to implement in the theoretical calculation;
3. Defined at any order of perturbation theory;
4. Yields finite cross section at any order of perturbation theory;
5. Yields a cross section that is relatively insensitive to hadronization.

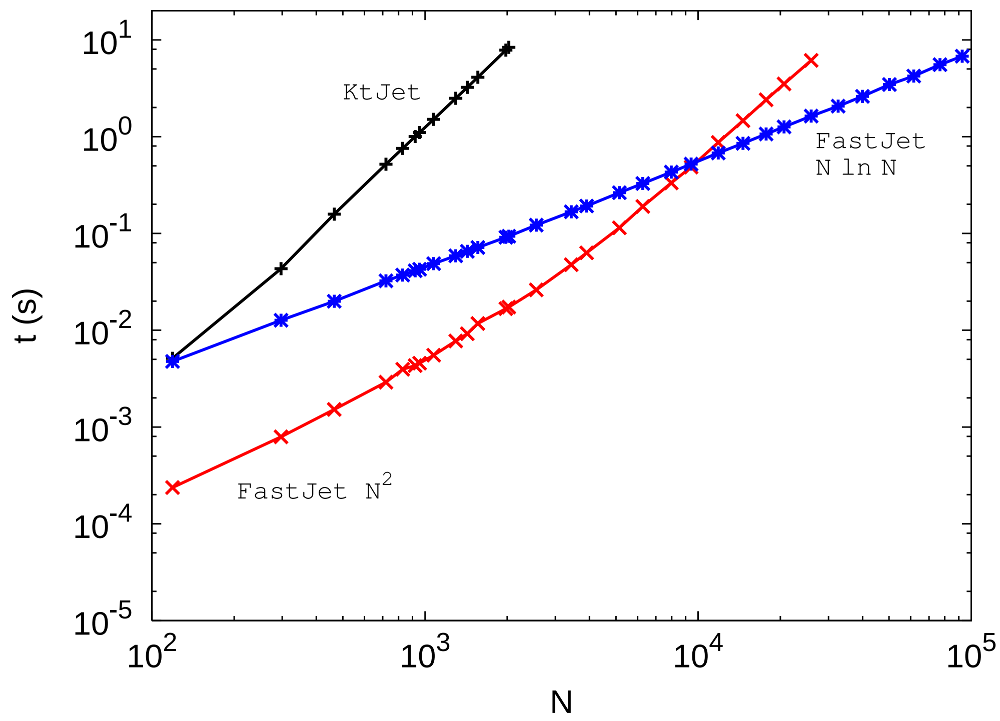
i.e. usable by theoreticians (*e.g.* finite perturbative results)

and experimentalists (*e.g.* fast enough, not much UE sensitivity)

Speed improvement

[M. Cacciari, G. Salam, 06]

- Speeding up the k_t and Cam/Aachen algorithms
  using computational-geometry techniques: $\mathcal{O}\left(N^3\right) \to \mathcal{O}\left(N \log N\right)$



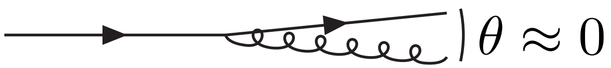
- C++ implementation in `FastJet`
  `http://www.fastjet.fr` (M. Cacciari, G. Salam, G.S.)

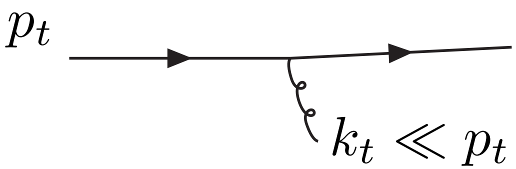
QCD divergences

QCD probability for gluon bremsstrahlung at angle θ and $\perp$-mom. k_t:

$$dP \propto \alpha_s \frac{d\theta}{\theta} \frac{dk_t}{k_t}$$

Two divergences:





Collinear

Soft

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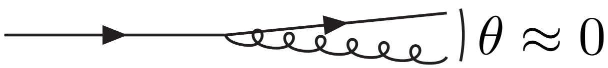
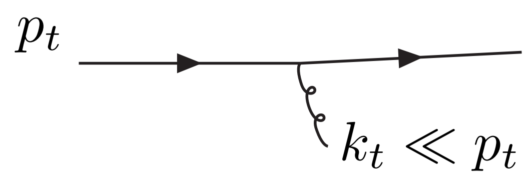


Collinear

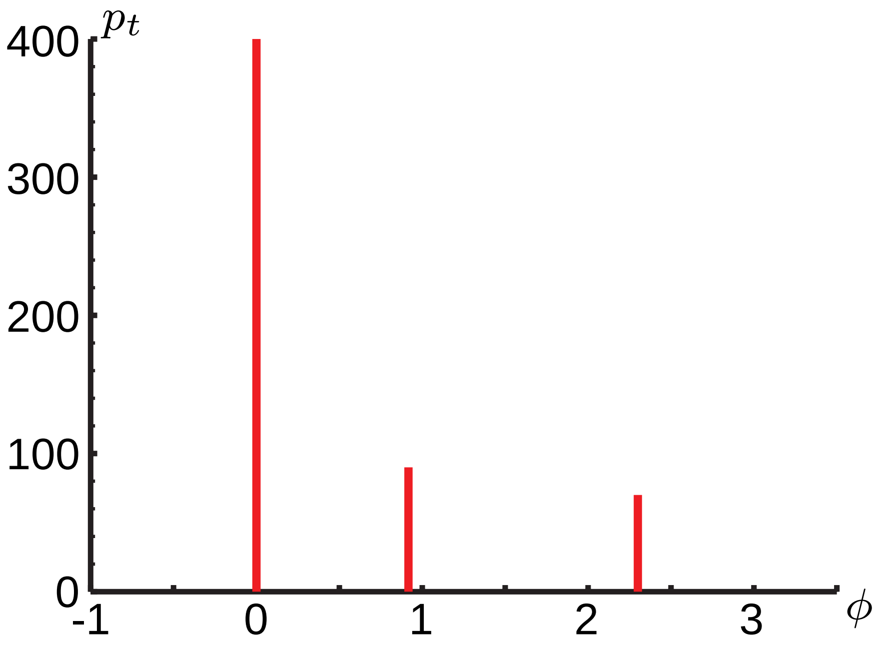


Soft

For pQCD to make sense, the (hard) jets should not change when

- one has a collinear splitting
  i.e. replaces one parton by two at the same place (η, ϕ)
- one has a soft emission *i.e.* adds a very soft gluon

IR unsafety of the Midpoint alg



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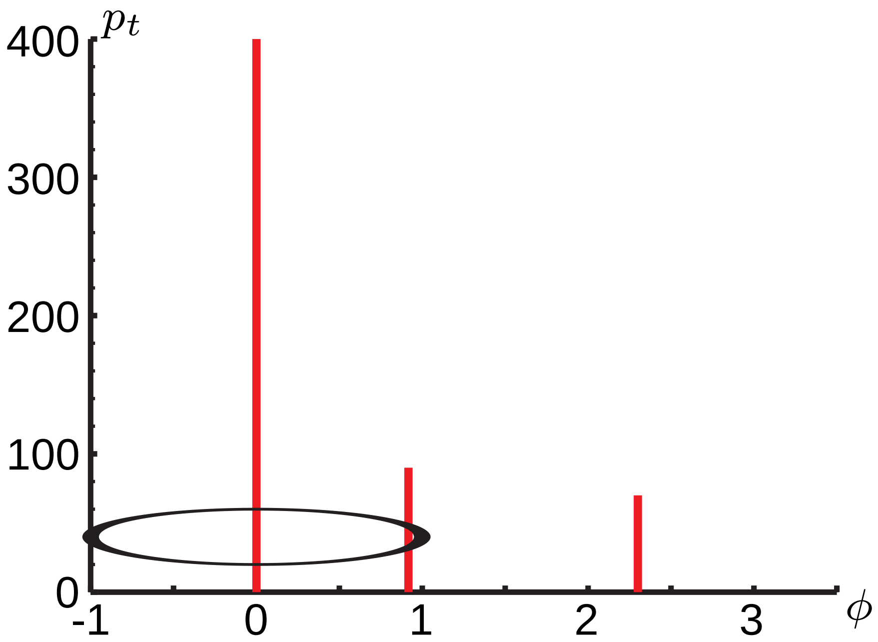

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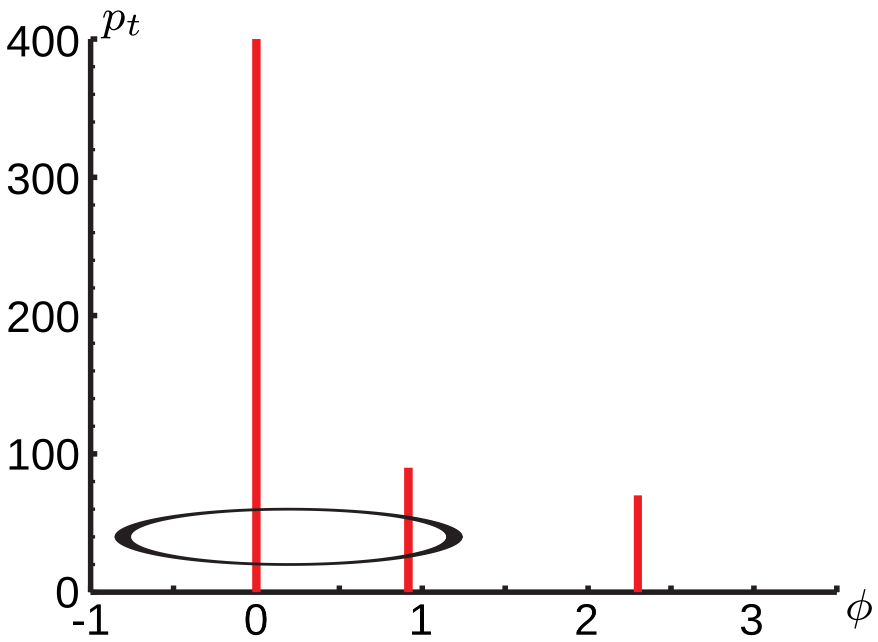

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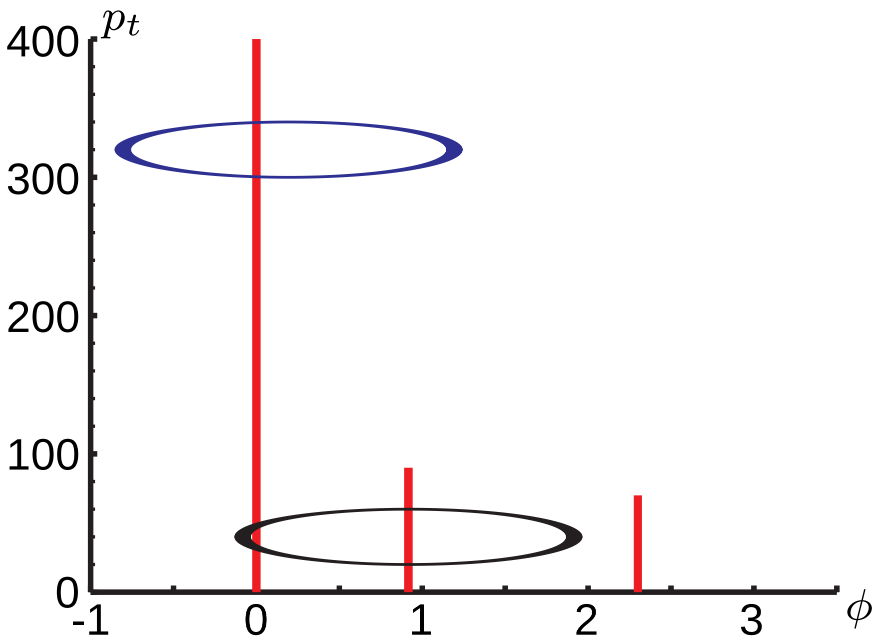

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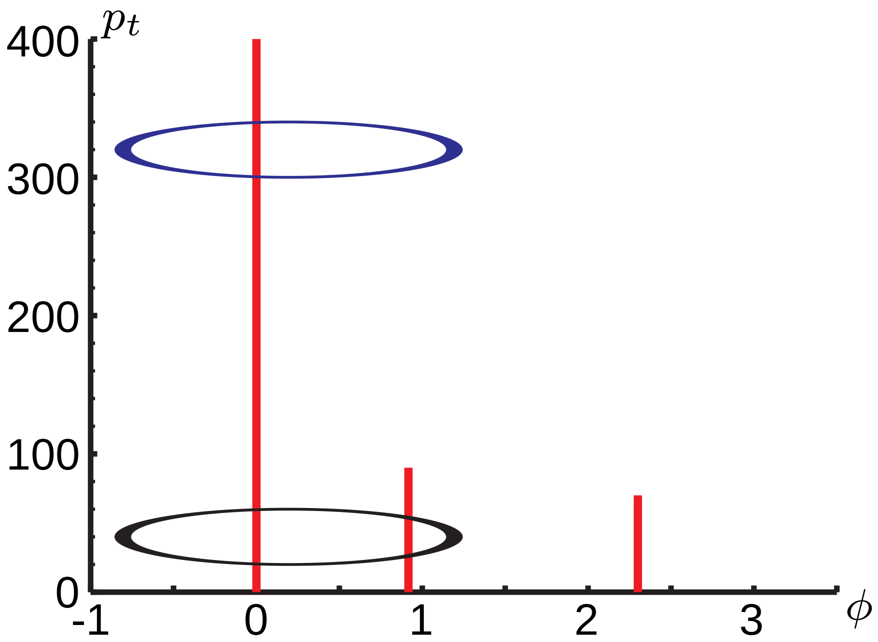

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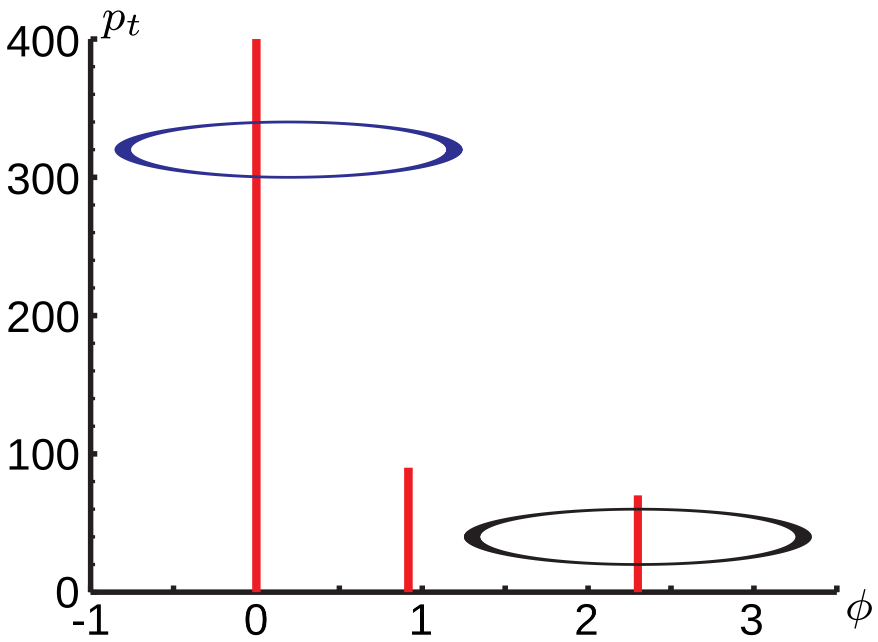

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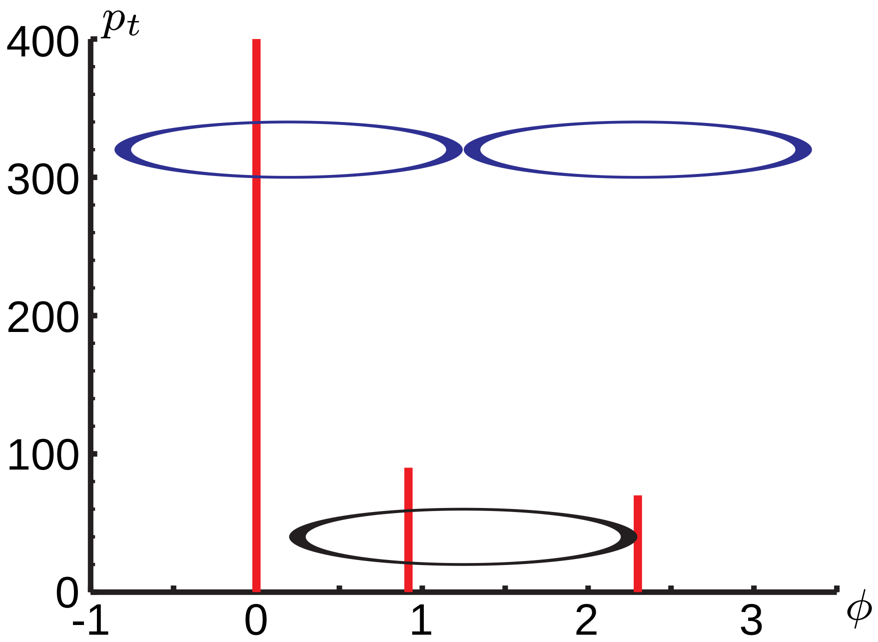

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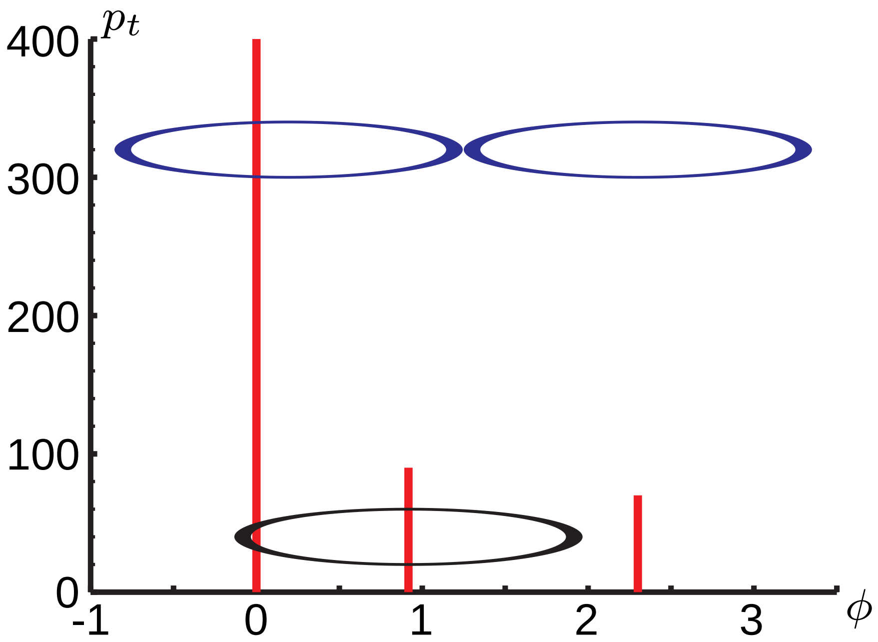

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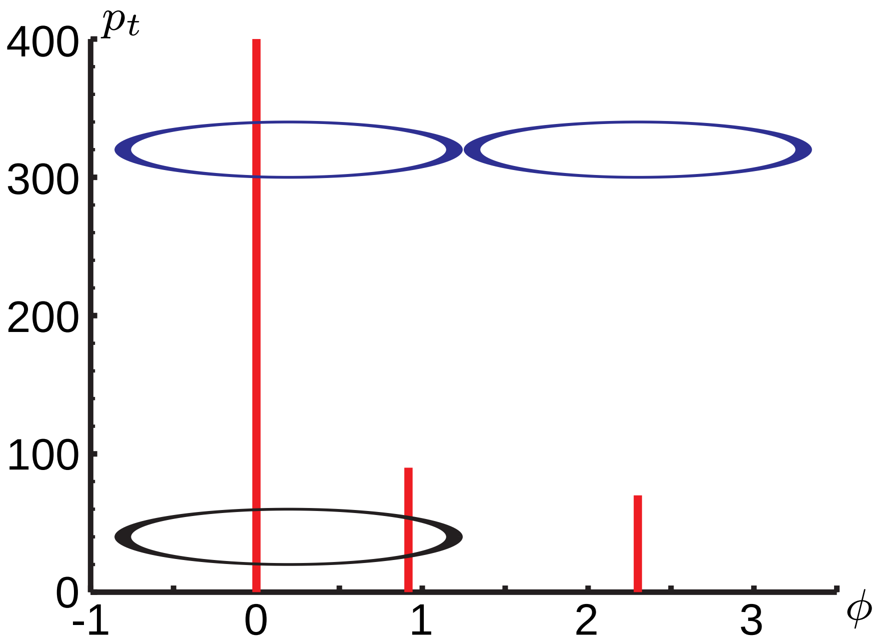

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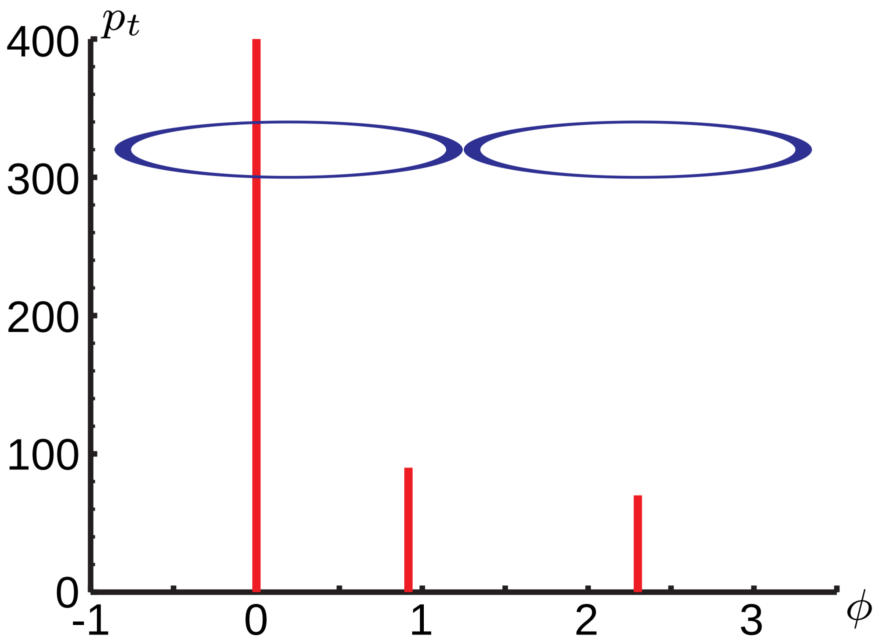

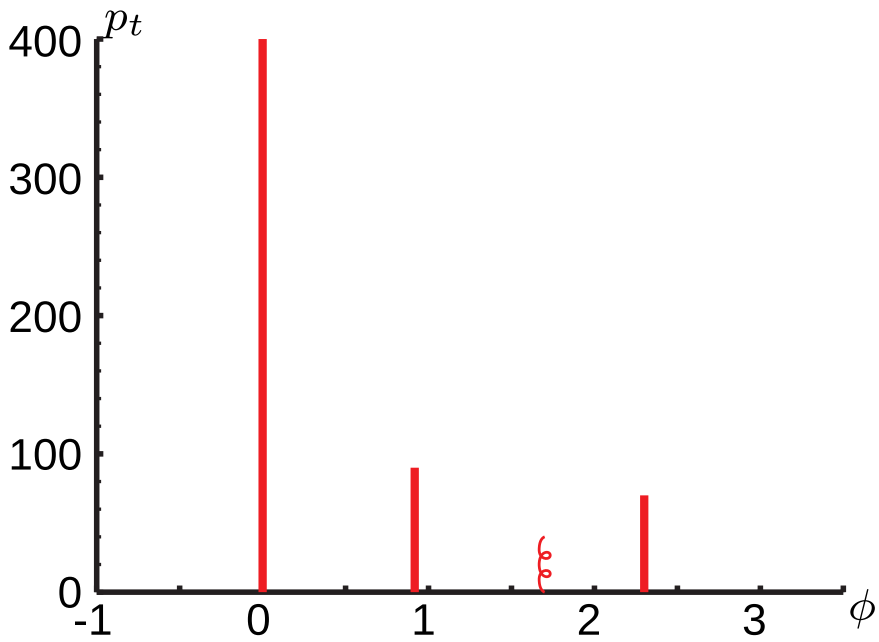

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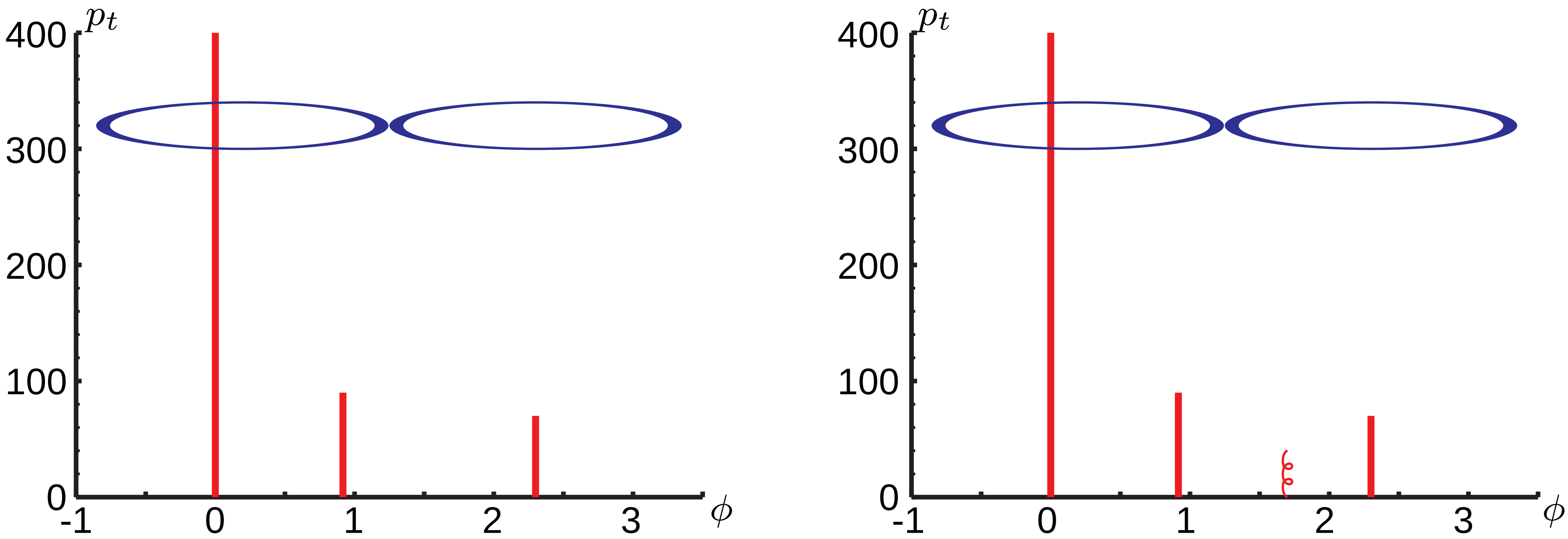

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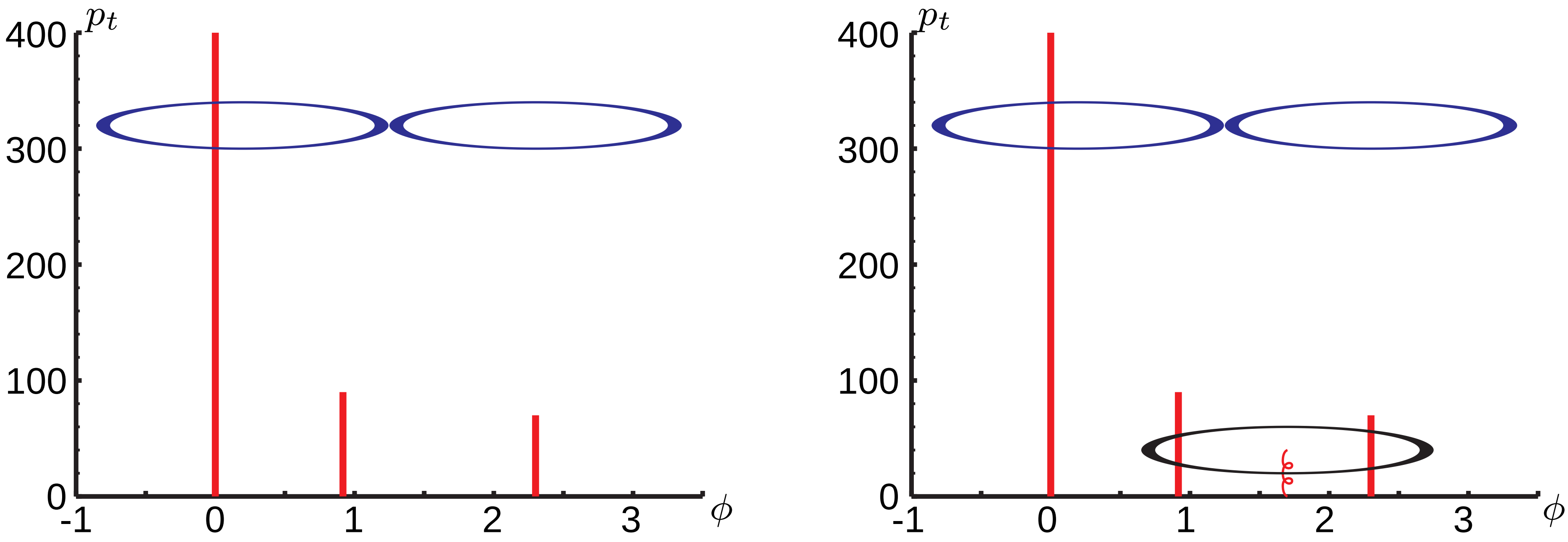

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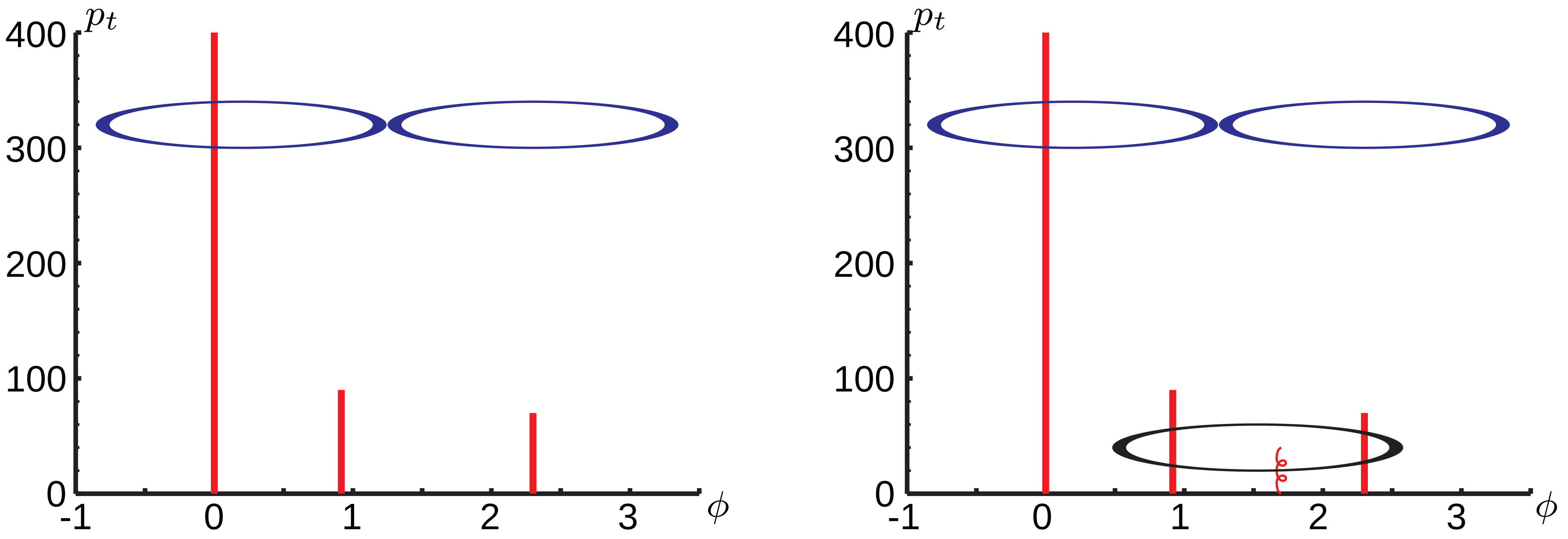

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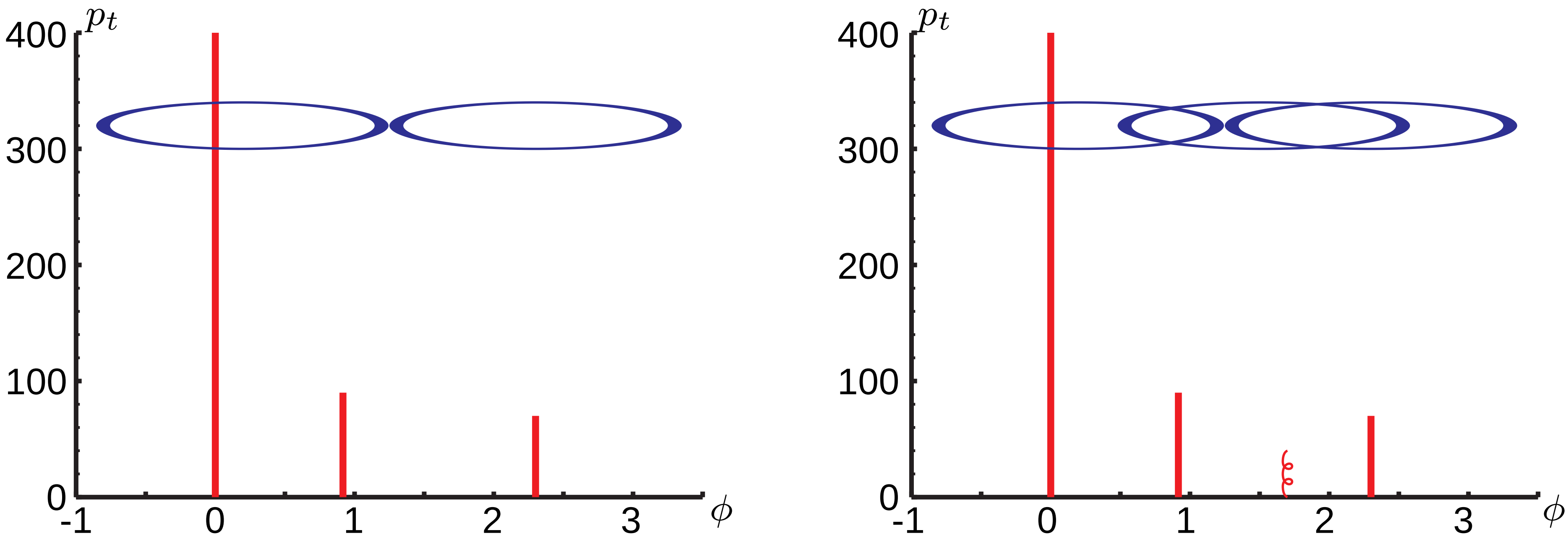
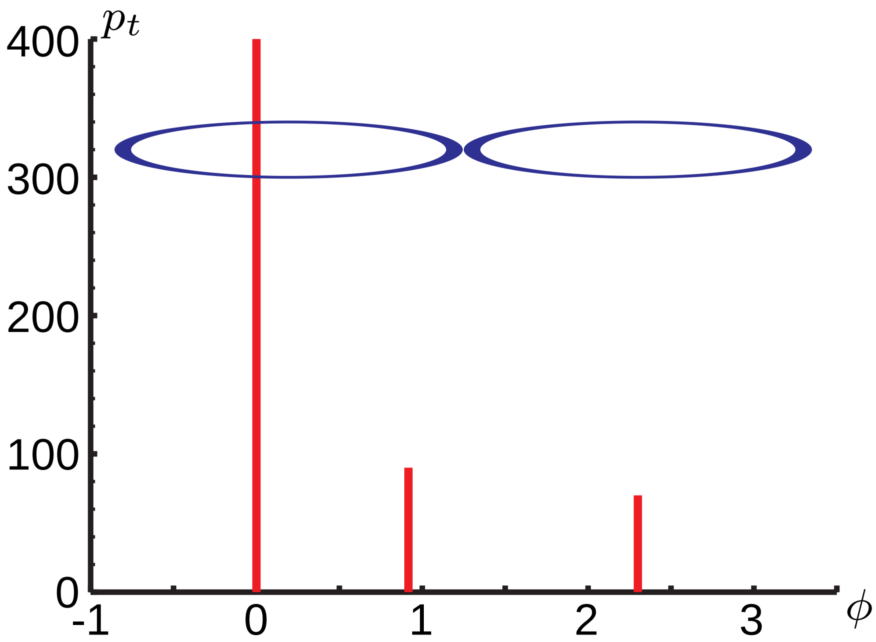
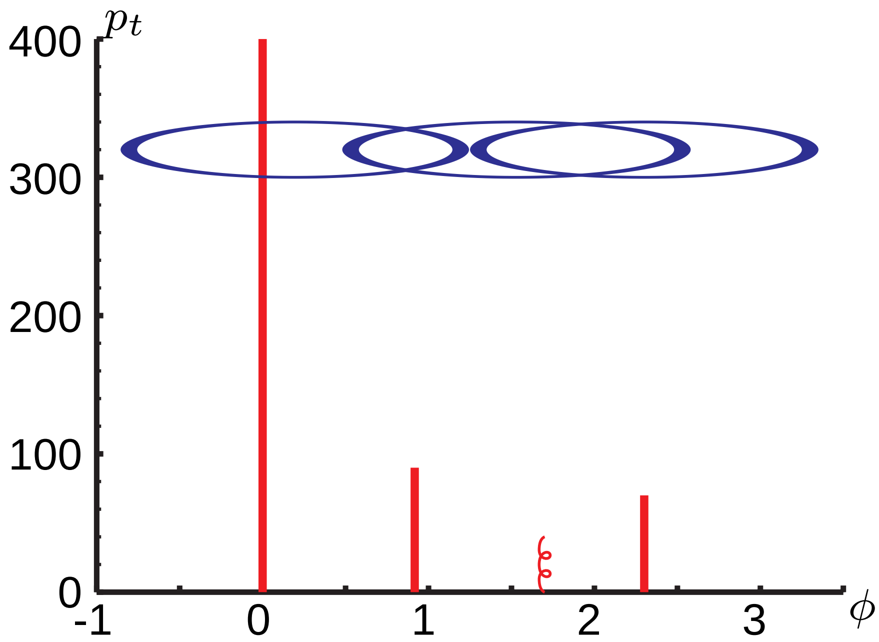


Stable cones:

Midpoint: {1,2} & {3} | {1,2} & {3} & {2,3}

IR unsafety of the Midpoint alg





Stable cones:

Midpoint:	{1,2} & {3}	{1,2} & {3} & {2,3}

Jets: ($f = 0.5$)

Midpoint:	{1,2} & {3}	{1,2,3}

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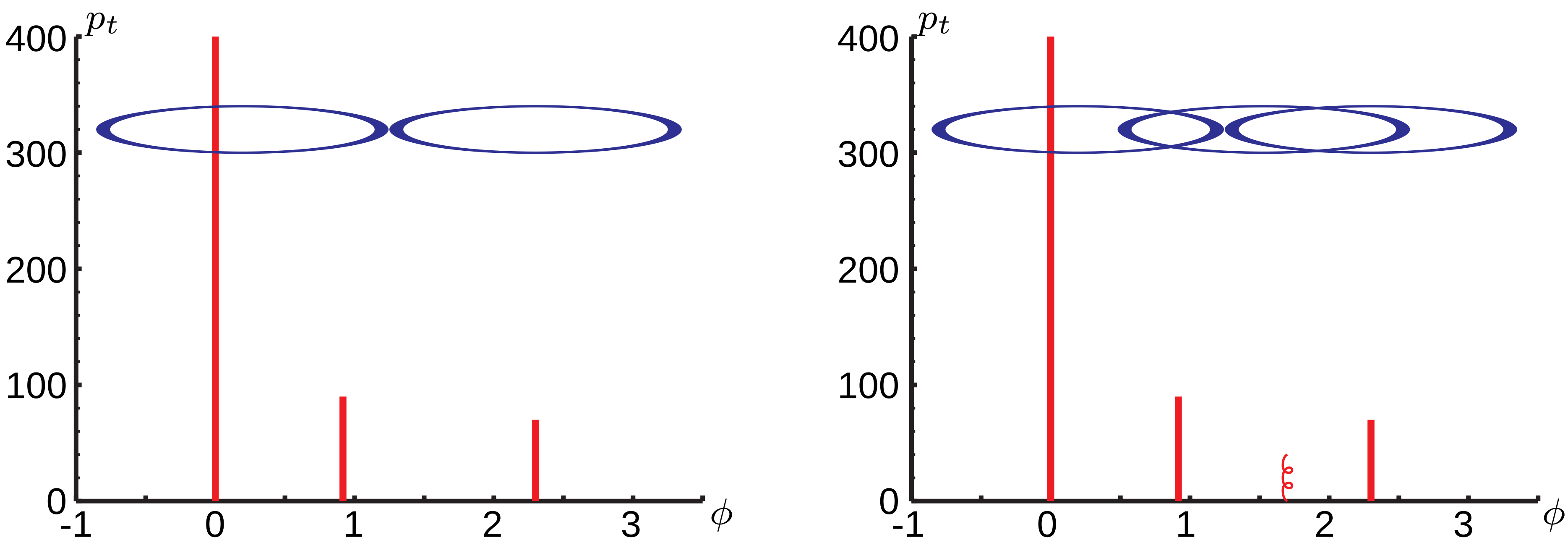

Stable cones:

Midpoint:	{1,2} & {3}	{1,2} & {3} & {2,3}
Seedless:	{1,2} & {3} & {2,3}	{1,2} & {3} & {2,3}

Jets: ($f = 0.5$)

Midpoint:	{1,2} & {3}	{1,2,3}
Seedless:	{1,2,3}	{1,2,3}

Stable cone missed $\longrightarrow$ IR unsafety of the midpoint algorithm

Solution: SISCone

Solution: use a seedless approach, find ALL stable cones

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Issue: watch your speed!

- check each subset of particle: $\mathcal{O}\left(N2^N\right)$

  irrealistically slow: $\sim 10^{17}$ years for $N = 100$
- midpoint: $\mathcal{O}\left(N^3\right)$
- New solution: use geometry again $\Rightarrow \mathcal{O}\left(N^2 \log(N)\right)$

  Idea: enumerate enclosures by enumerating pairs of particles

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$\longrightarrow$ SISCone: Seedless Infrared-Safe Cone algorithm

http://projects.hepforge.org/siscone [G.Salam, G.S., 07]

Notes:

- first cone to be infrared-and-collinear safe
- also available from FastJet

Physical impact

Midpoint IR unsafe at $\mathcal{O}(\alpha_s^4)$ (or $\mathcal{O}(\alpha_{\rm ew}\alpha_s^3)$)

Observable	1st miss cones at	Last meaningful order
Inclusive jet cross section	NNLO	NLO
3 jet cross section	NLO	LO (NLO in NLOJet)
$W/Z/H$ + 2 jet cross sect.	NLO	LO (NLO in MCFM)
jet masses in 3 jets	LO	none (LO in NLOJet)

Huge theoretical effort to compute multileg/NLO processes

That can be wasted by using unappropriate tools.

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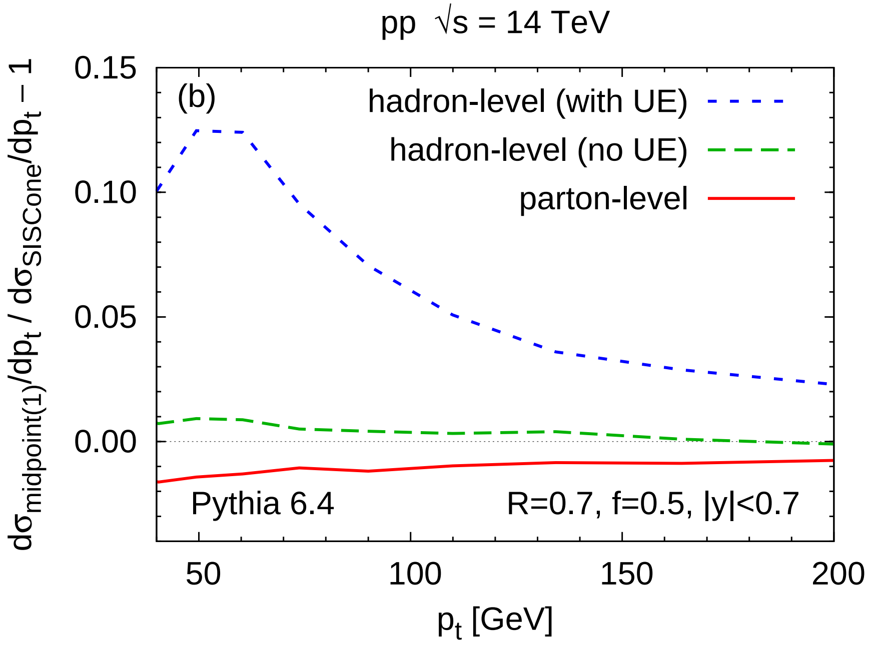
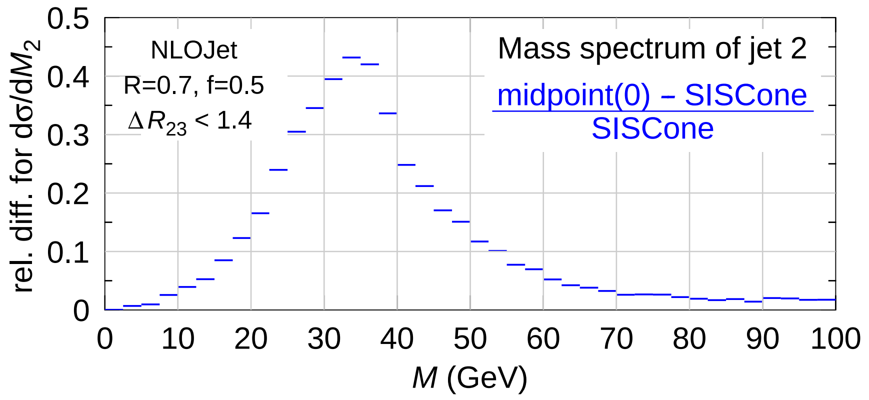
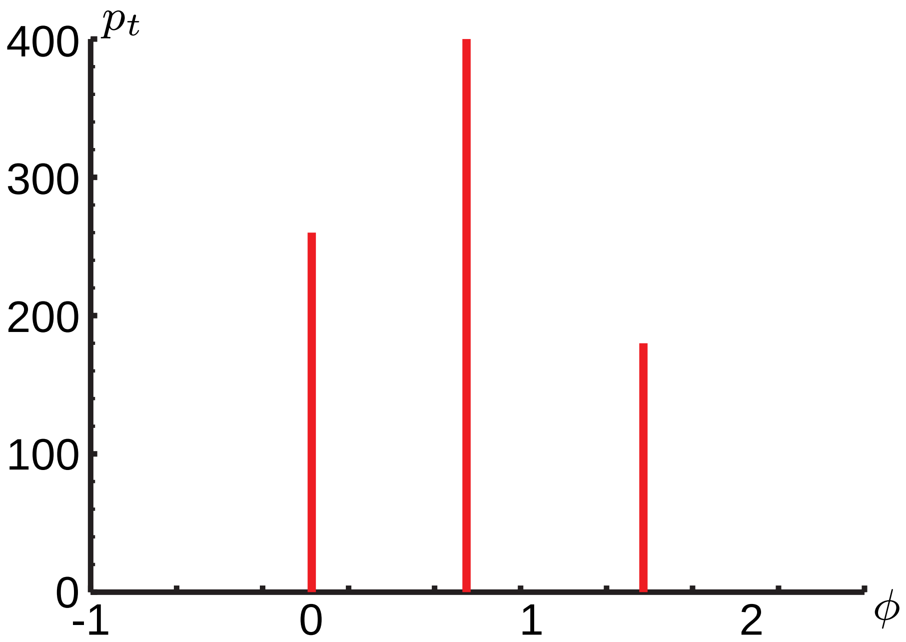
Example: (Midpoint-SISCone)/SISCone



- Incl. cross-section: a few %
- Masses in 3-jet events: $\sim 45\%$



Coll. unsafety of the iterative cone



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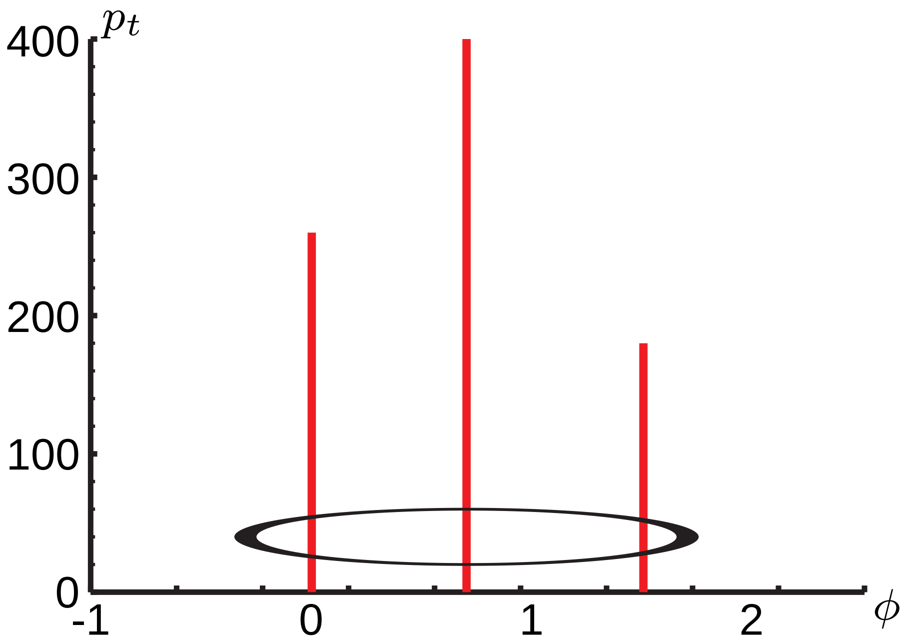

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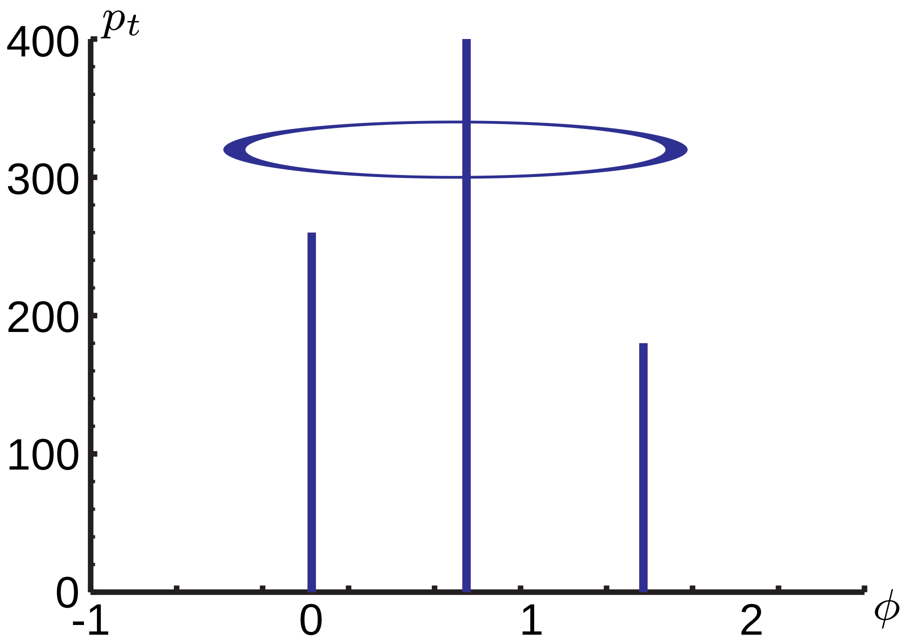

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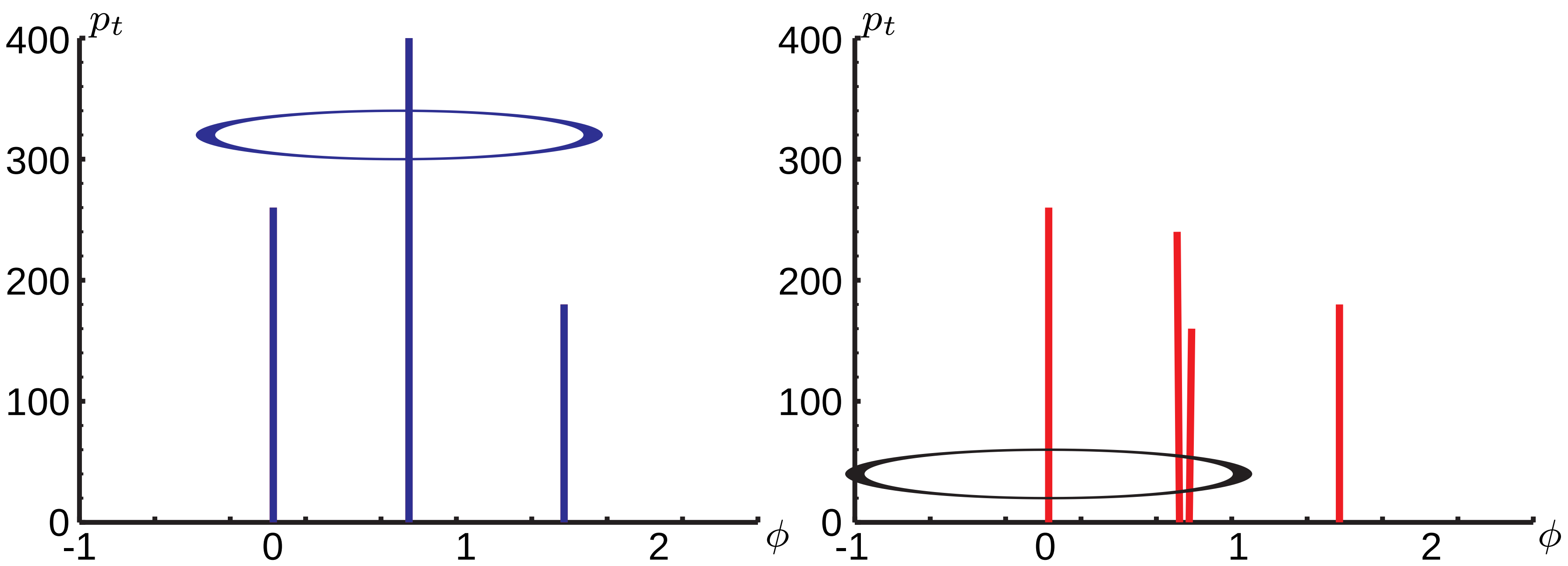

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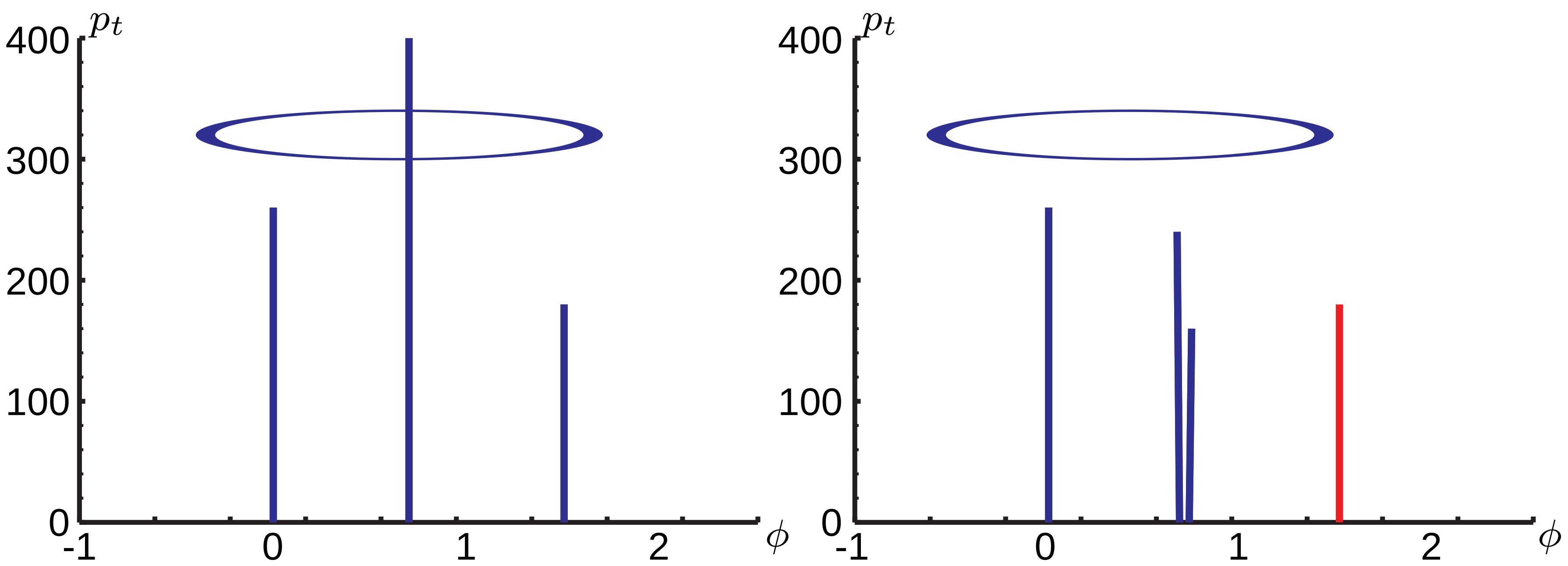

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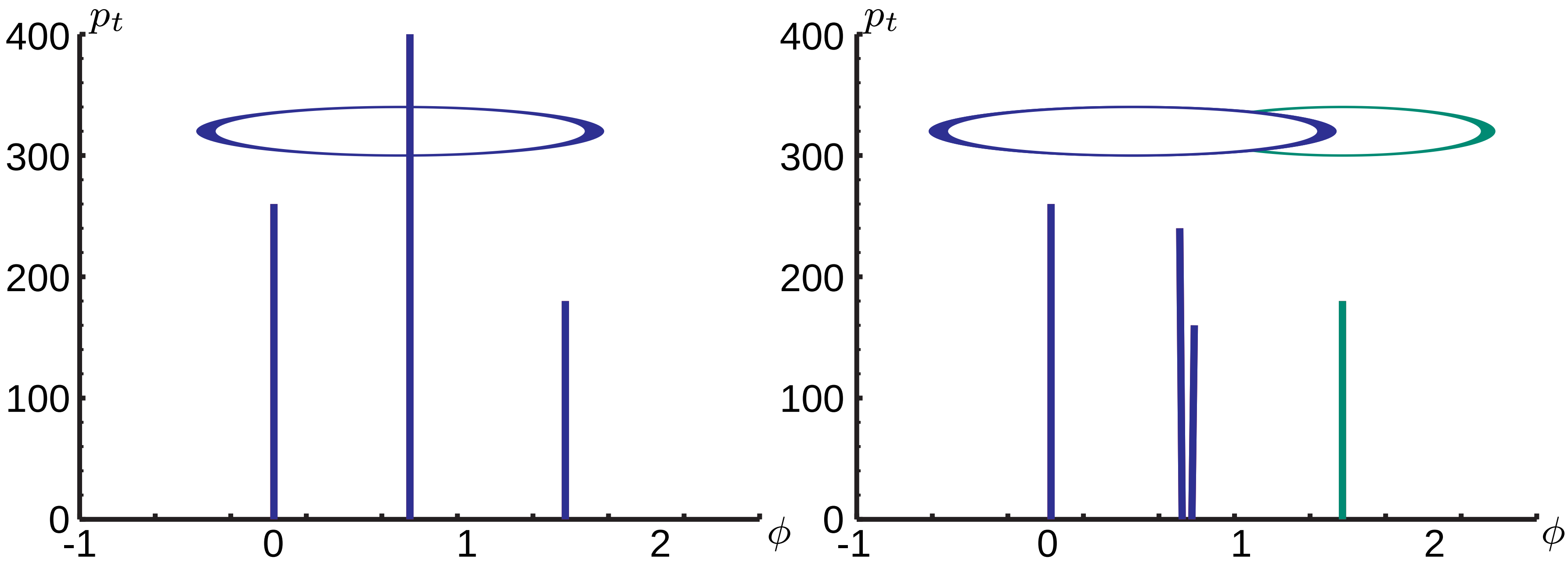

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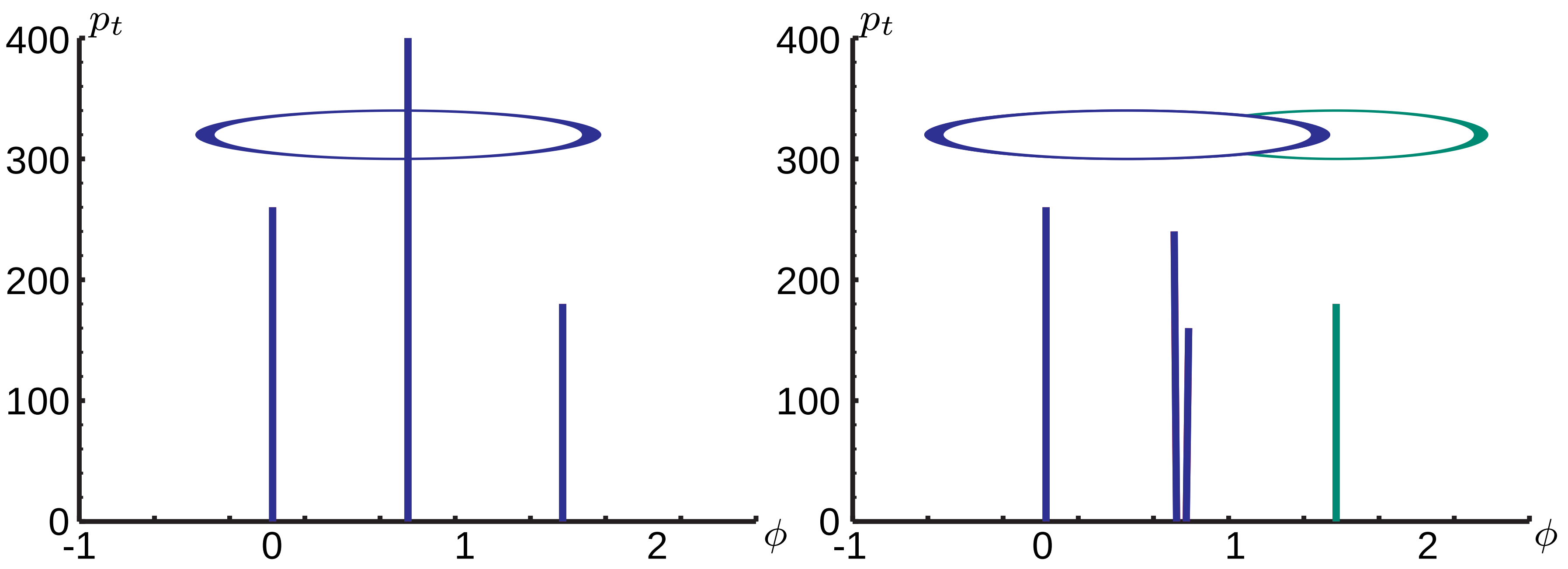

- Before collinear spliting: 1 jet
- After collinear spliting: 2 jets

$\longrightarrow$ **collinear unsafety of the iterative cone algorithm**

Come back to recombination-type algorithms:

$$d_{ij} = \min(k_{t,i}^{2p}, k_{t,j}^{2p})\ (\Delta\phi_{ij}^2 + \Delta\eta_{ij}^2)$$

- $p = 1$: k_t algorithm
- $p = 0$: Aachen/Cambridge algorithm

Anti-k_t

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- $p = 1$: k_t algorithm
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- $p = -1$: anti-k_t algorithm [M.Cacciari, G.Salam, G.S.,JHEP 04 (08) 063]

Anti-k_t

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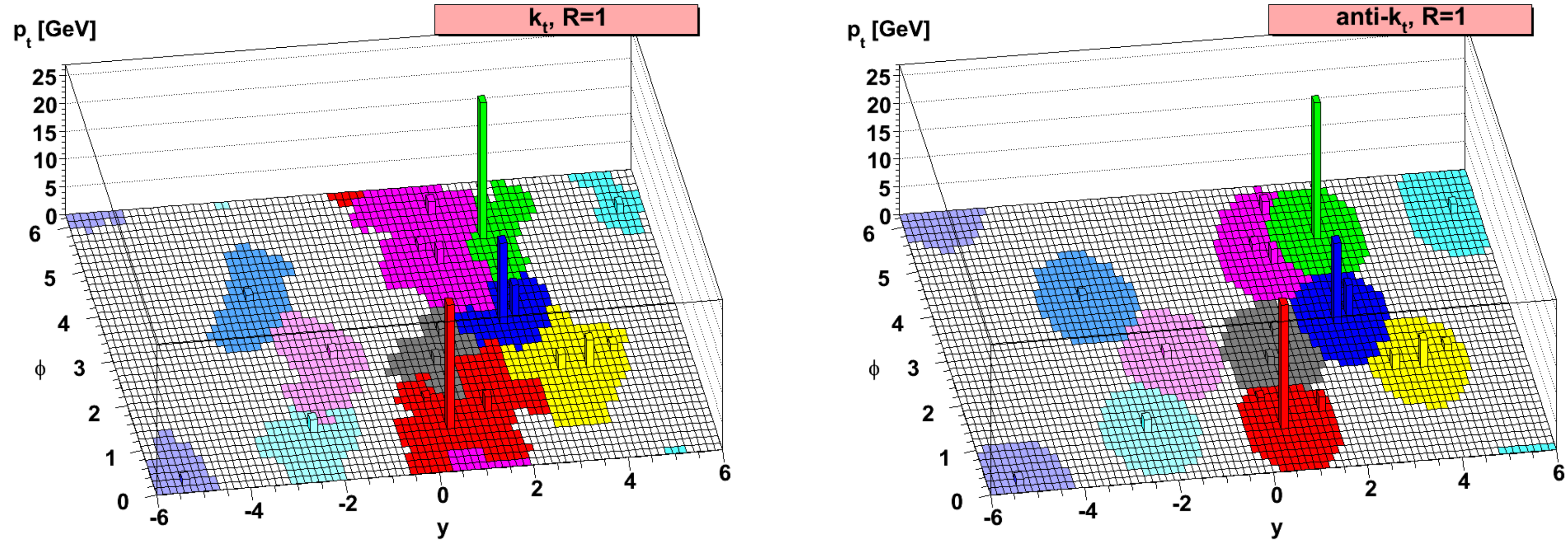
Why should that be related to the iterative cone ?!?

- "large $k_t \Rightarrow$ small distance"
  i.e. hard partons "eat" everything up to a distance R
  i.e. circular/regular jets, jet borders unmodified by soft radiation
- infrared and collinear safe

Hard event + homogeneous soft background



- anti-k_t is soft-resilient
- fast implementation as for k_t and Cambrigde/Aachen
- more later in this talk...

21st century jet finders

Recombination:

- k_t algorithm
- Cambridge/Aachen alg.
- anti-k_t algorithm

Cone:

- CDF JetClu
- CDF MidPoint
- D0 (run II) Cone
- PxCone
- ATLAS Cone
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- PyCell/CellJet
- GetJet
- SISCone

4 available
safe algorithms

All accessible from FastJet

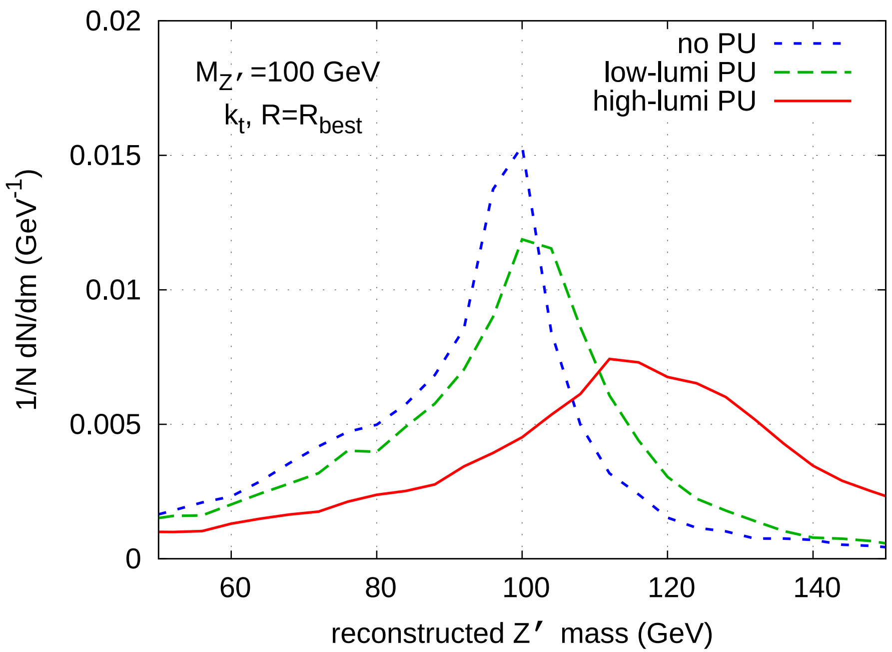
Part 2
Jets areas
Dealing with the background

[M.Cacciari,G.Salam,G.S., 07-08]

Need for subtraction

Pileup ≈ uniform soft background that shifts jets to higher p_t

Example: reconstructions of a 100 GeV Z' through $Z' \to q\bar{q} \to$ 2 jets



low-lumi = $2\ 10^{33}$ cm^{-1} s^{-1}
high-lumi = 10^{34} cm^{-1} s^{-1}

- peak shifted (because of pileup particles adding into the jets
- peak smeared (due to pileup fluctuations)

Area definition

- Idea: add soft particles (ghosts) and look in which jets they are caught

jet area = region where it catches ghosts

Area definition

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jet area = region where it catches ghosts

- 2 definitions
  - Passive area
    add one ghost and look where it ends. repeat to cover the (y, ϕ) plane
    mimics pointlike background
  - Active area
    add a large amount of ghosts and cluster everything
    mimics uniform background
    also gives purely ghosted jets
- Both practical and tractable analytically

Examples

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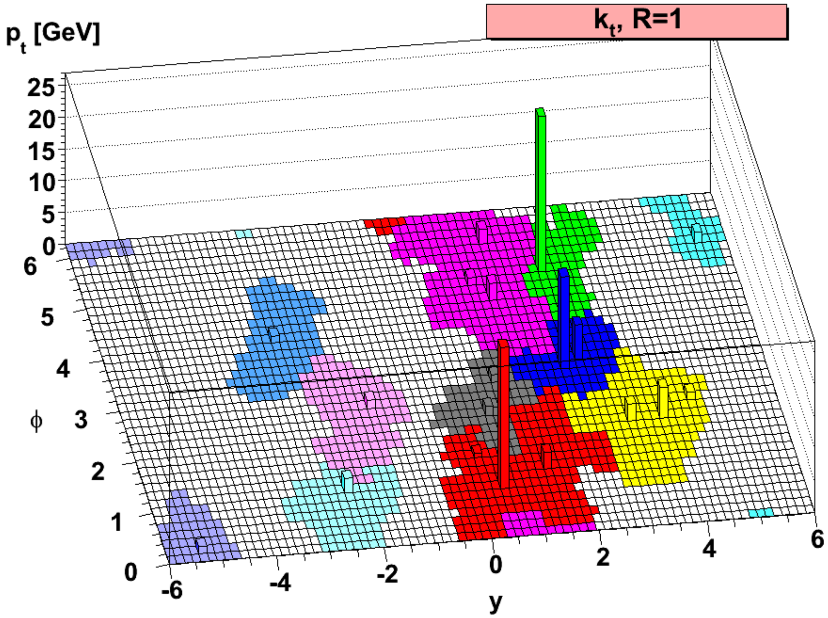
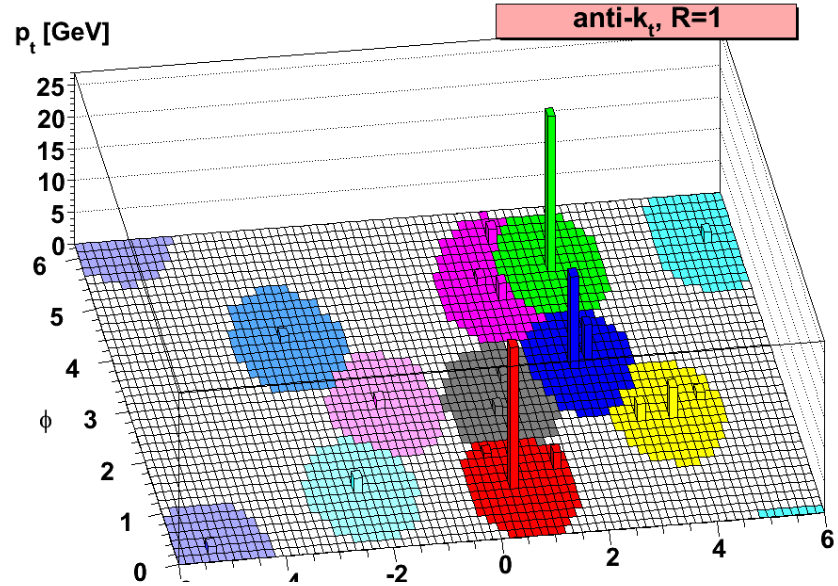


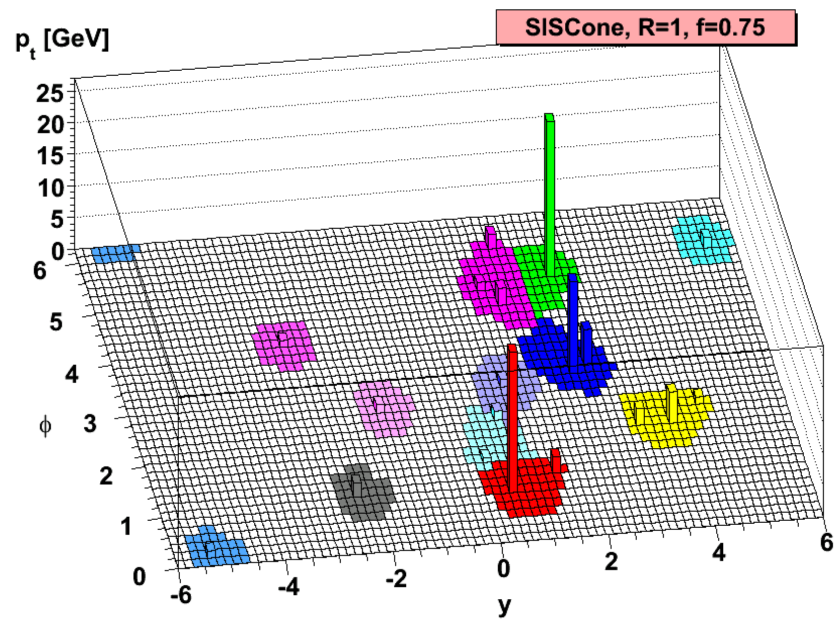


Notes: (active area)

- k_t: irregular “boundaries”
- anti-k_t: regular “boundaries”
- SISCone: smaller area

  risks of *monster jets* with $f = 0.5$

  $\Rightarrow$ use $f = 0.75$



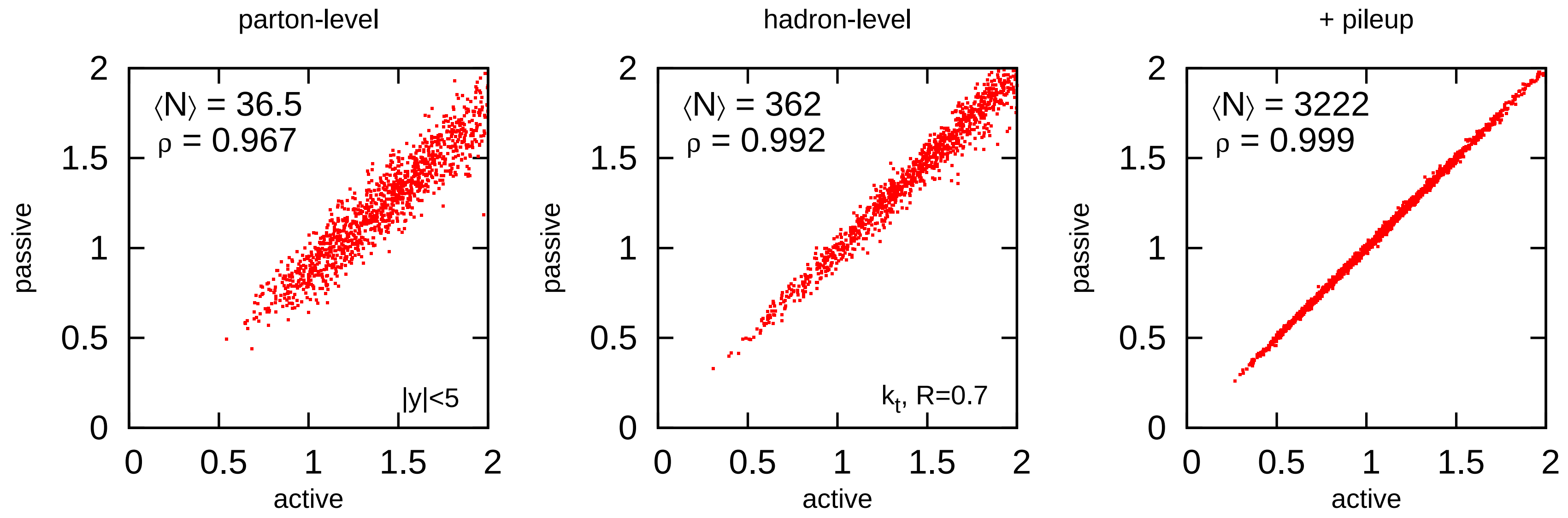
Density dependence

Dilute event:

active $\neq$ passive

Dense event

active $\simeq$ passive



Area scaling violations

Tractable analytically

Example: Average jet area (0^{th} order: 1 particle, 1^{st} order: 2 particles) :

$$\begin{aligned}\langle \mathcal{A}(p_{t,1}, R)\rangle &= \mathcal{A}_{1\text{hard}}(R) + \int d\Delta\, dp_{t,2} \frac{dP}{d\Delta_{12} dp_{t,2}} \left[\mathcal{A}_{\text{hard+1 soft}}(\Delta, R) - \pi R^2\right] \\ &= \mathcal{A}_{1\text{hard}}(R) + \frac{C_{F,A}}{\pi b_0} \log\left(\frac{\alpha_s(Q_0)}{\alpha_s(R p_t)}\right) \pi R^2\, d\end{aligned}$$

- Scaling violation (area $\neq \pi R^2$)
- gluon > quark
- know LO anomalous dimension

	passive		active	
	$\frac{\mathcal{A}_1}{\pi R^2}$	d	$\frac{\mathcal{A}_1}{\pi R^2}$	d
k_t	1	0.5638	0.81	0.519
Cam	1	0.07918	0.81	0.0865
SISCone	1	-0.06378	1/4	0.1246
anti-k_t	1	0	1	0

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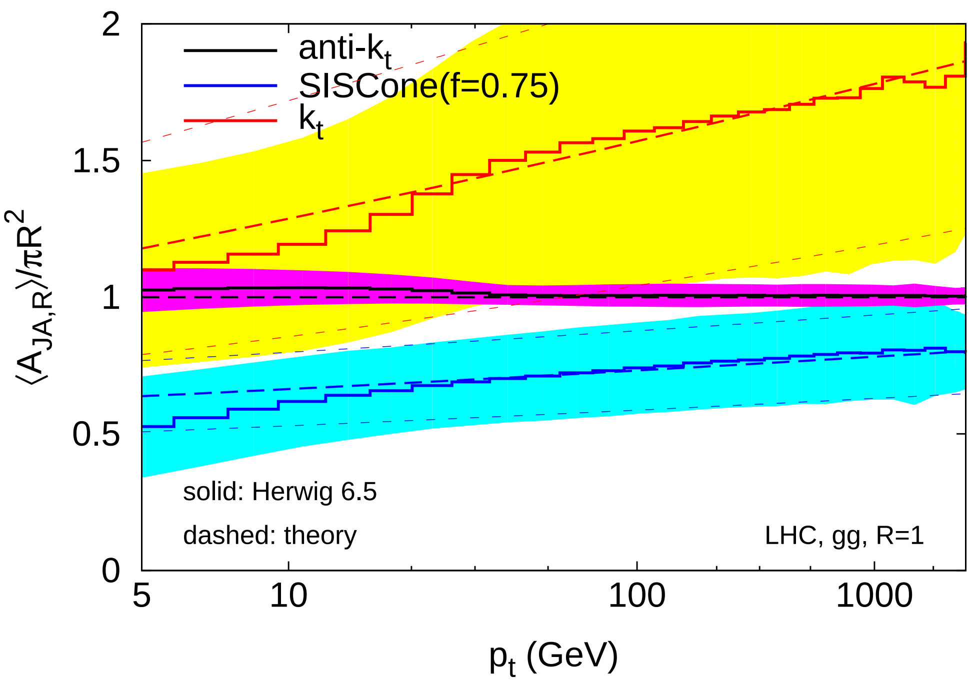
Note the Q_0 in the scaling violations:

- $Q_0 \equiv$ IR cut-off
- vacuum $Q_0 \propto \Lambda_{QCD}$,
  with background $Q_0 \propto \rho_{\rm bkg}$
- good agreement with MC



Pileup subtraction

Basic idea: [M.Cacciari, G.Salam, 08]

$$p_{t,\text{subtracted}} = p_{t,\text{jet}} - \rho_{\text{pileup}} \times \text{Area}_{\text{jet}}$$

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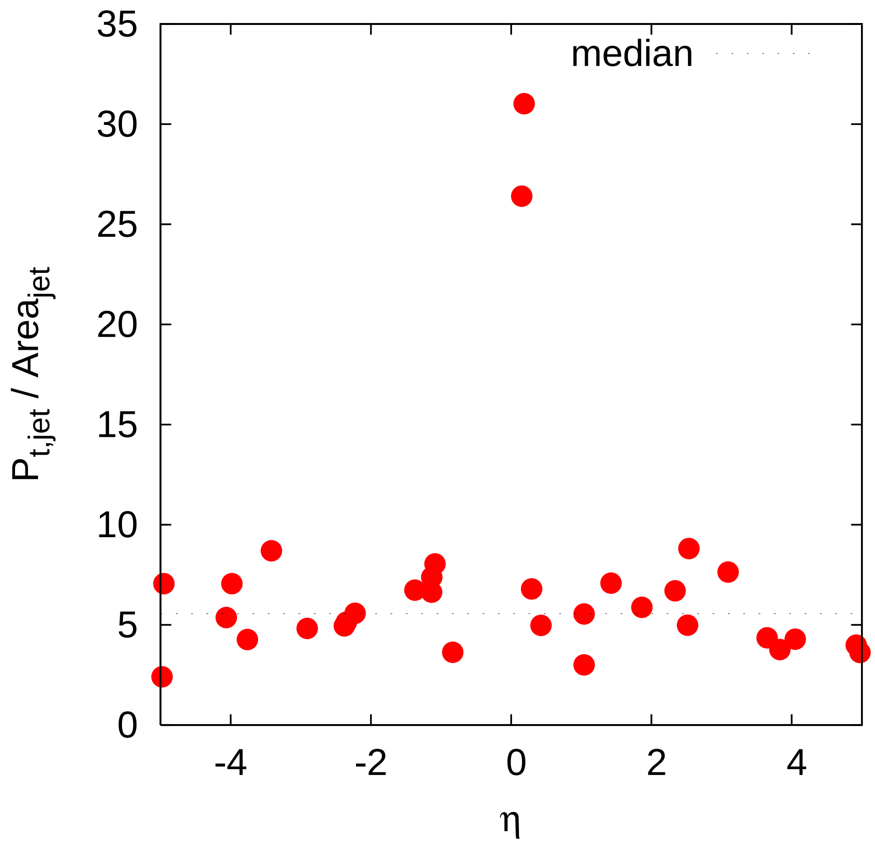
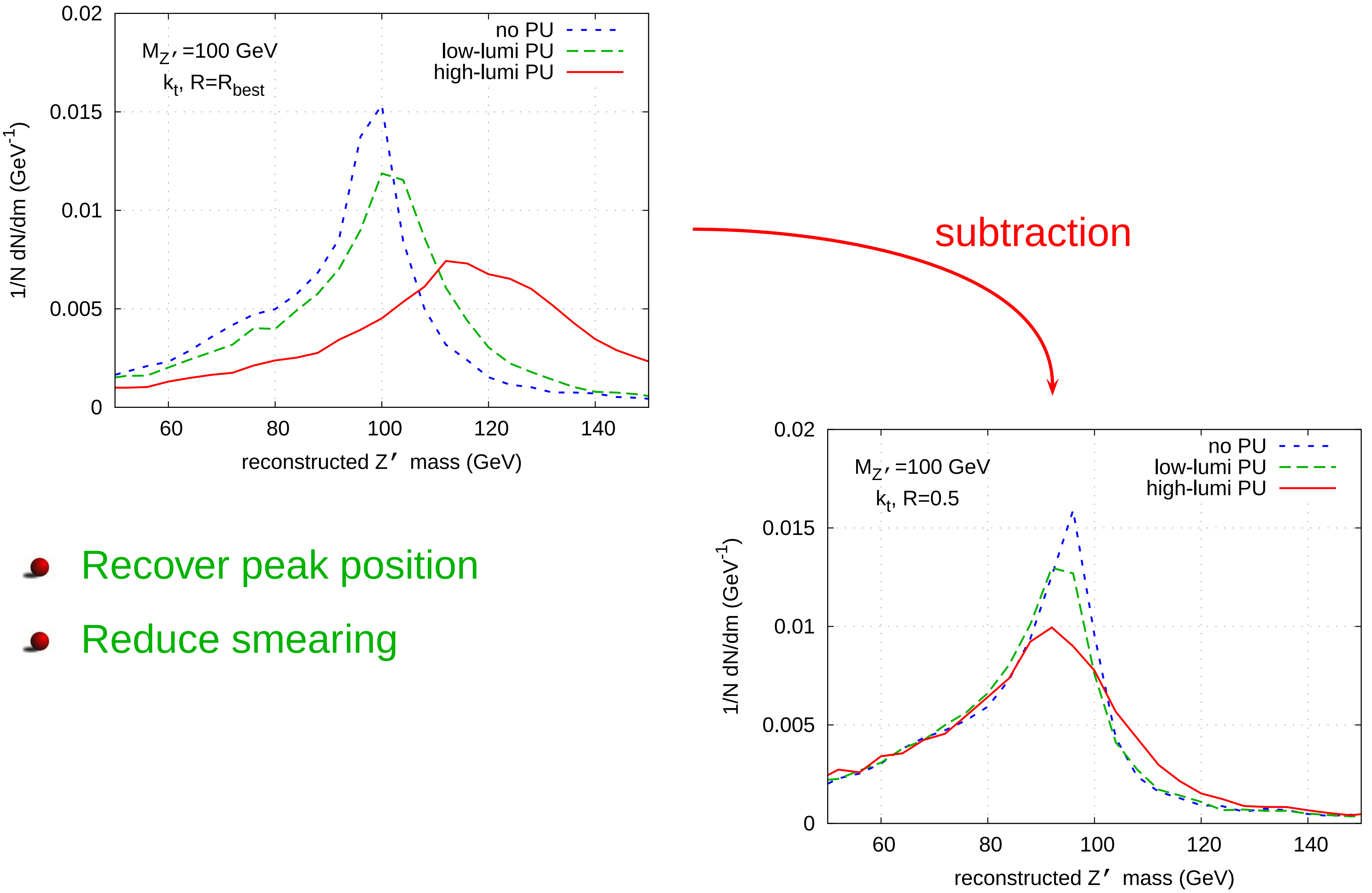


- Pileup density per unit area: ρ_{pileup}

  e.g. estimated from the median of $p_{t,\text{jet}}/\text{Area}_{\text{jet}}$

implemented in FastJet
on an event-by-event basis

Subtraction at work

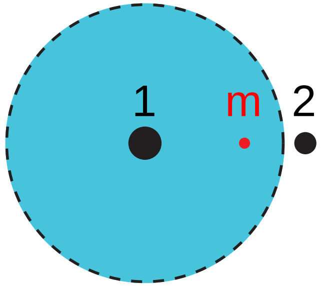


- Recover peak position
- Reduce smearing

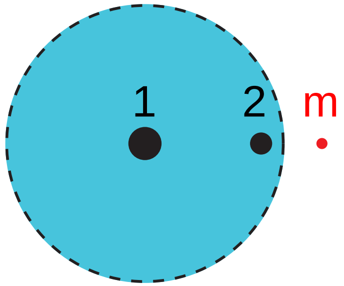
Back-reaction

Additional soft background has 2 effects:

- Throw soft particles in the hard jet: dealt with by subtraction
- Modify the hard scattering (back-reaction)
  - can be pointlike or diffuse
  - gain: p_2 gained when adding p_m



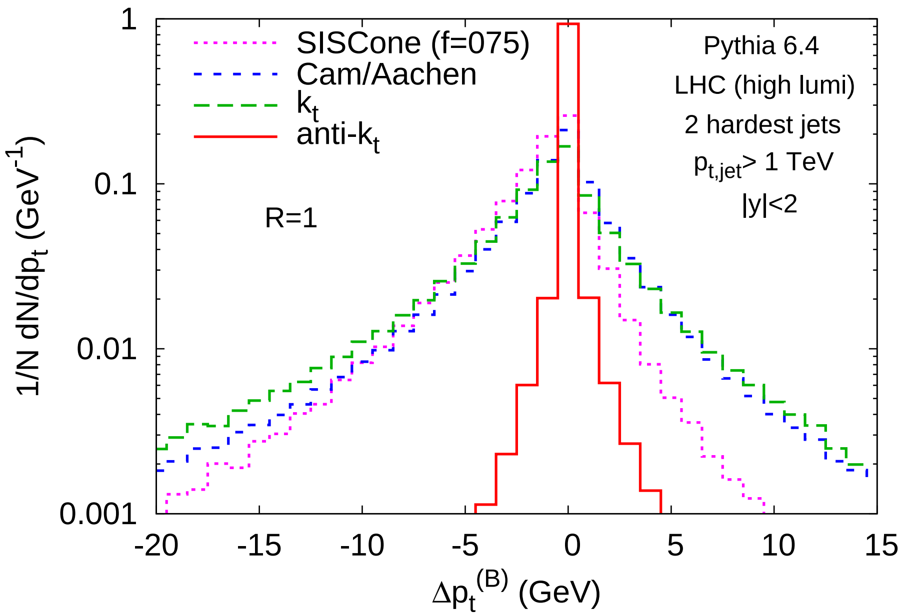
  - loss: p_2 lost when adding p_m



Back-reaction

Additional soft background has 2 effects:

- Throw soft particles in the hard jet: dealt with by subtraction
- Modify the hard scattering (back-reaction)
  - can be pointlike or diffuse
  - tractable analytically (similar to areas)
  - $k_t \gtrsim$ Cambridge > SISCone $\gg$ anti-k_t



Conclusions

- Use acceptable jet algorithms
  - 4 (fast-enough) IRC-safe algorithms:
    k_t, Cambridge/Aachen, anti-k_t and SISCone
  - IRC safety will matter at the LHC

- Definition of “the *area* of a jet”
  - defined as the region where it catched soft background
  - tractable analytically
  - can be use to subtract soft-background contamination
  - same technique holds for heavy-ion background
    some refinements are under study