Saturation in High-Energy QCD

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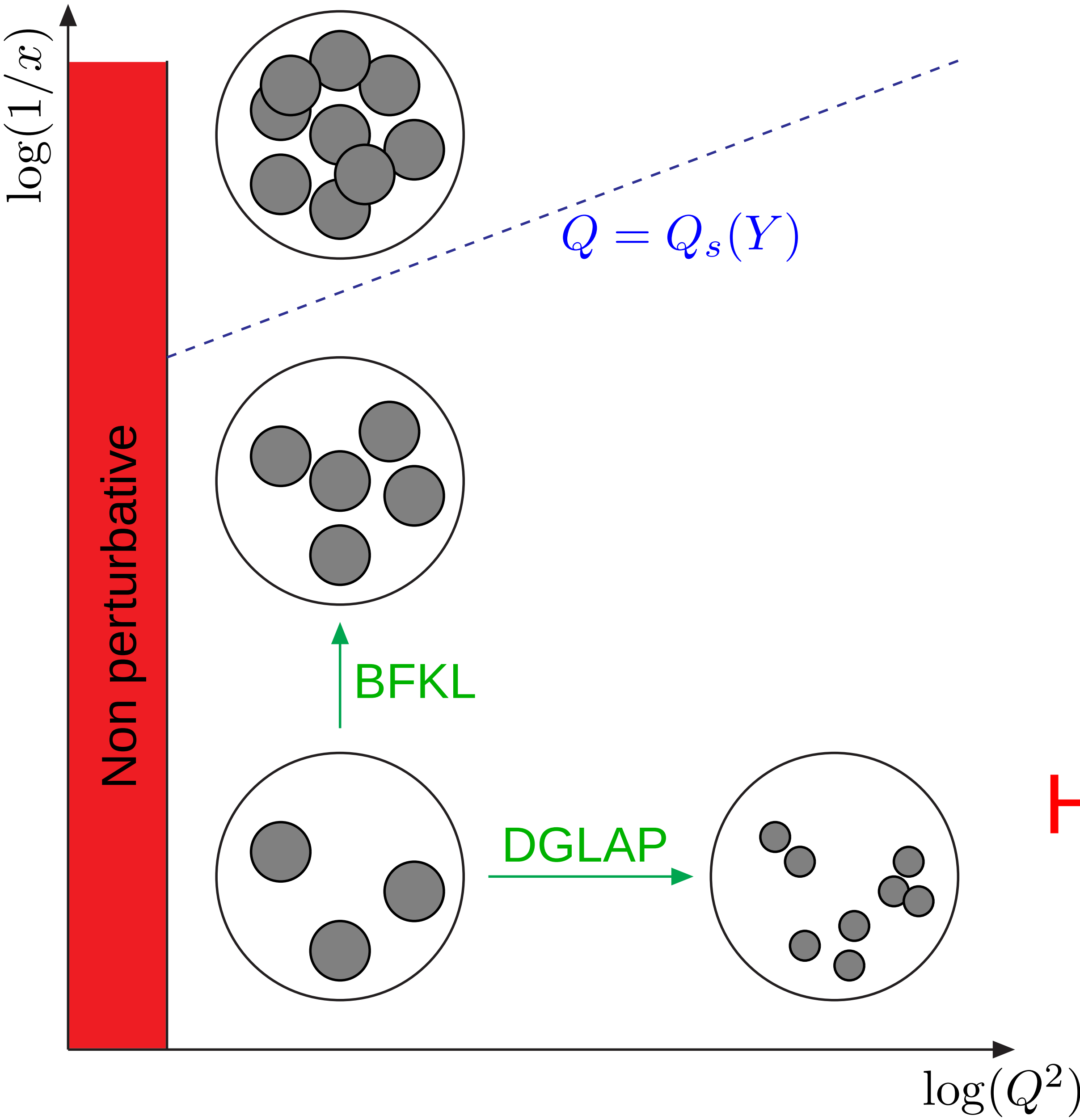
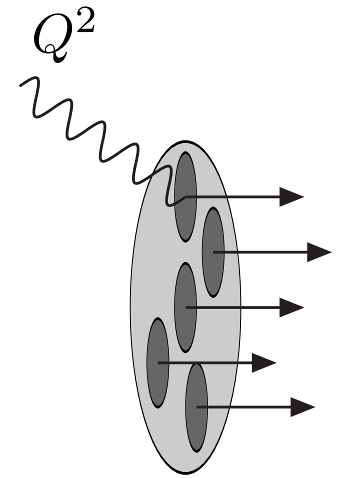
in collaboration with R. Peschanski, E. Iancu, C. Marquet, R. Venugopalan,...

Outline

- General framework
- Perturbative evolution in high-energy QCD:
  - Leading log approx.: BFKL equation
  - Unitarity/Saturation effects: Balitsky/JIMWLK and BK equation
  - Beyond saturation: fluctuations; Pomeron loops
- Asymptotic solutions:
  - saturation $\Rightarrow$ geometric scaling
  - pomeron loops $\Leftrightarrow$ reaction-diffusion $\Rightarrow$ diffusive scaling
- Phenomenological consequences
  - The dipole model
  - Description of inclusive/diffractive/exclusive DIS data
  - Forward particle production at RHIC

Motivation: why saturation ?





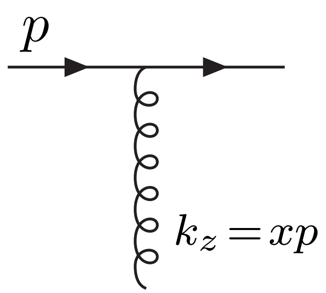
Size $\sim 1/Q$

Energy $\sim Q^2/x$

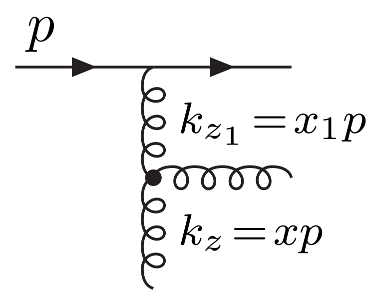
How to describe this in QCD ?

Motivation: why resummation ?

Bremsstrahlung:



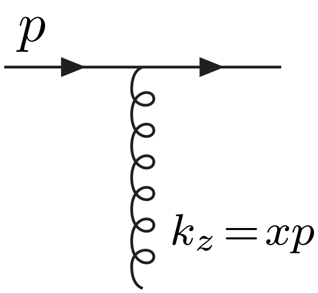
$x \ll 1$



$x \ll x_1 \ll 1$

Motivation: why resummation ?

Bremsstrahlung:



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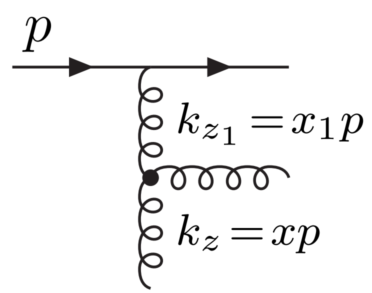

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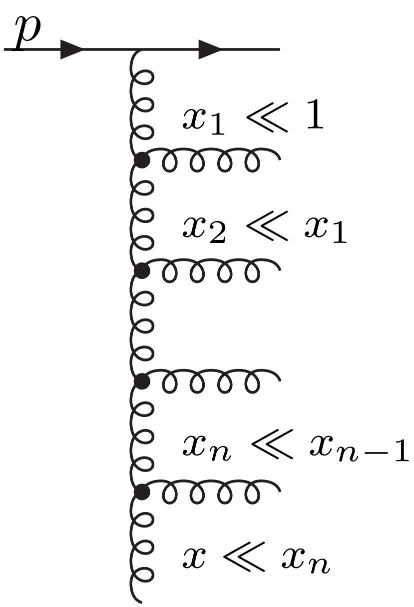


Probability of emission

$$dP \sim \alpha_s \frac{dk_\perp^2}{k_\perp^2} \frac{dx}{x}$$

In the small-x limit

$$\int_x^1 \frac{dx_n}{x_n} \cdots \int_{x_2}^1 \frac{dx_1}{x_1} \sim \frac{1}{n!} \alpha_s^n \log^n(1/x)$$

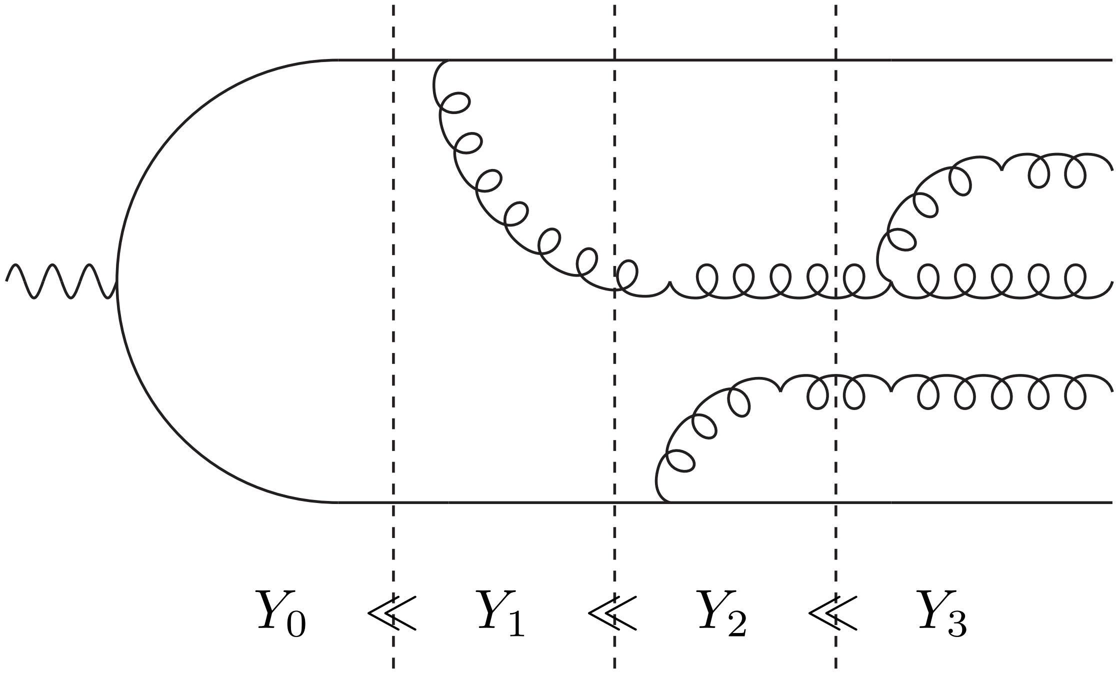
Same order when $\alpha_s \log(1/x) \sim 1 \Rightarrow$ need to be resummed

Perturbative evolution in high-energy QCD

Dipole picture

Consider a fast-moving $q\bar{q}$ dipole (Rapidity: $Y = \log(s)$)

[Mueller,93]



- Probability $\bar{\alpha}K$ of emission
- degree of freedom: transverse position of the gluon

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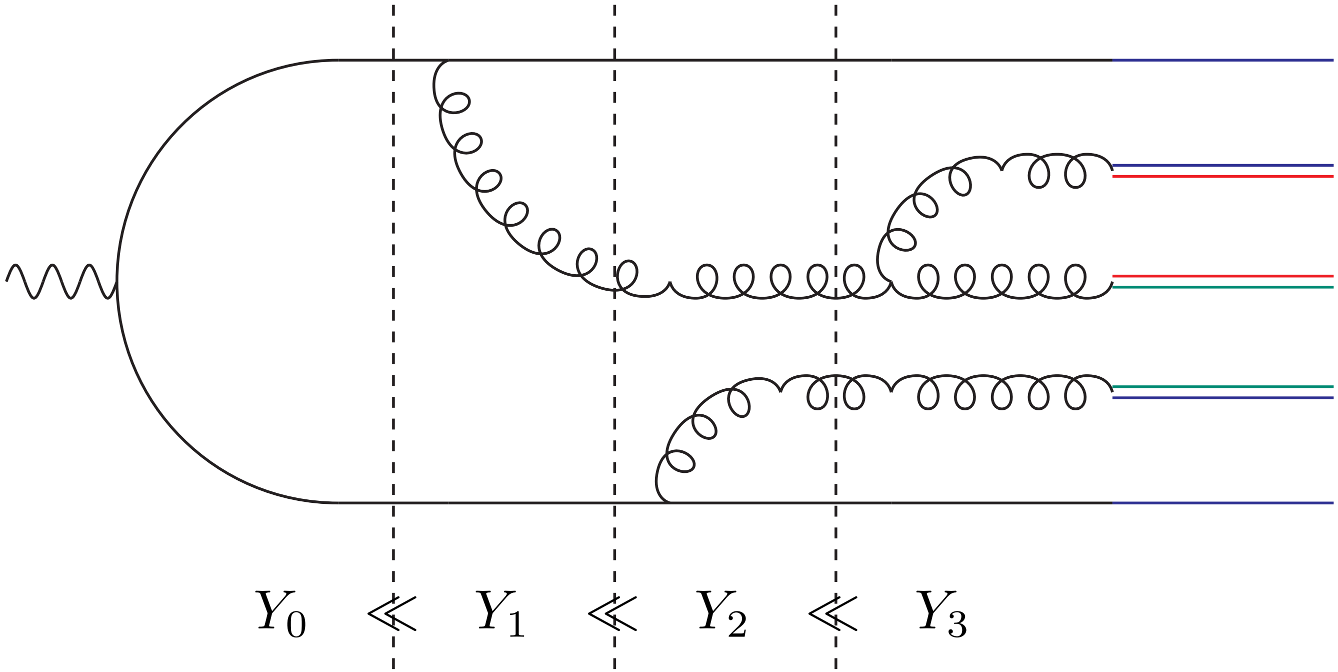
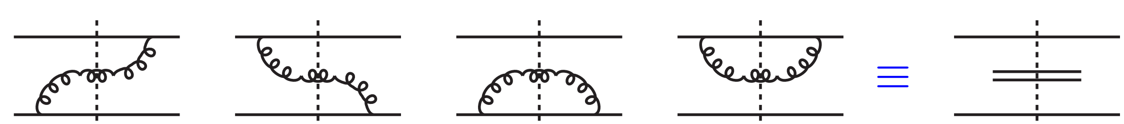


$n(r, Y)$ dipoles

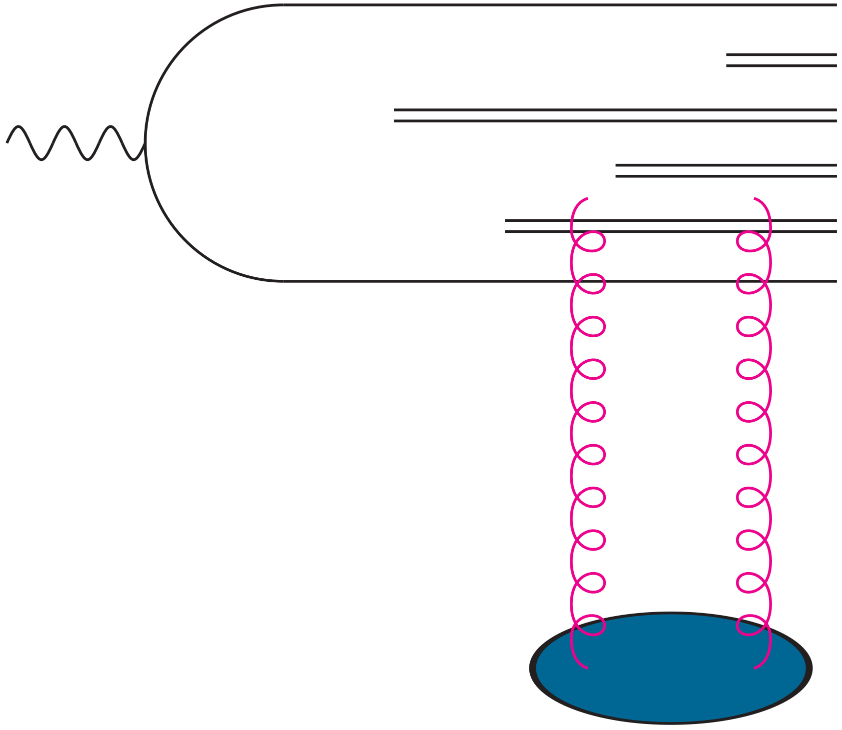
of size r

- Probability $\bar{\alpha}K$ of emission $\equiv$ dipole splitting
- degree of freedom: transverse position of the gluon
- Large-N_c approximation



BFKL evolution (1/3)

Projectile-target interaction:

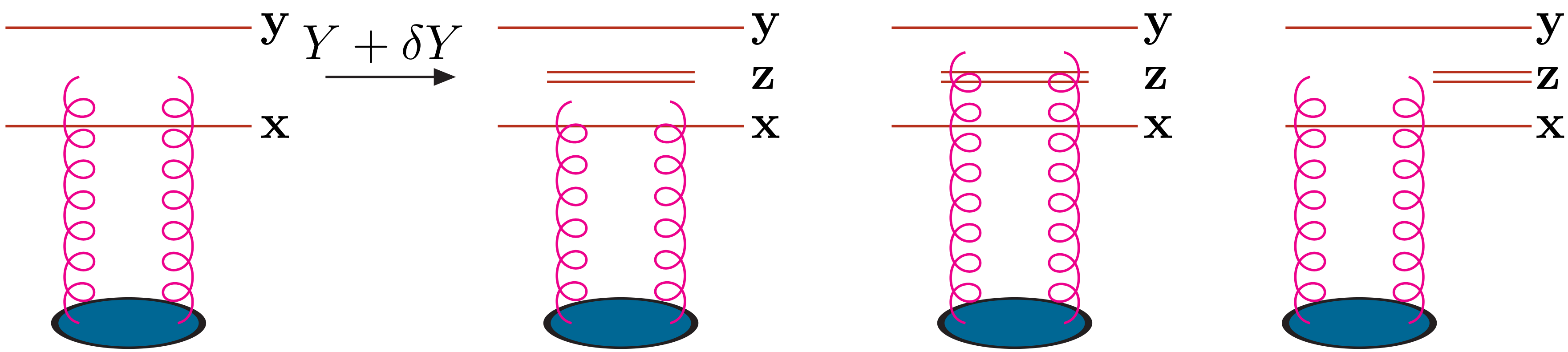


$$T(r, Y) \approx \alpha_s^2 n(r, Y)$$

Count the number of dipoles of a given size

BFKL evolution (2/3)

Consider a small increase in rapidity $\Rightarrow$ splitting



$$\partial_Y T(\mathbf{x},\mathbf{y};Y)$$

$$= \underbrace{\bar{\alpha}\int d^2z\,\frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{z}-\mathbf{y})^2}}_{\text{Emission proba from pQCD}}\underbrace{\left[T(\mathbf{x},\mathbf{z};Y)+T(\mathbf{z},\mathbf{y};Y)-T(\mathbf{x},\mathbf{y};Y)\right]}_{\text{all possible interactions}}$$

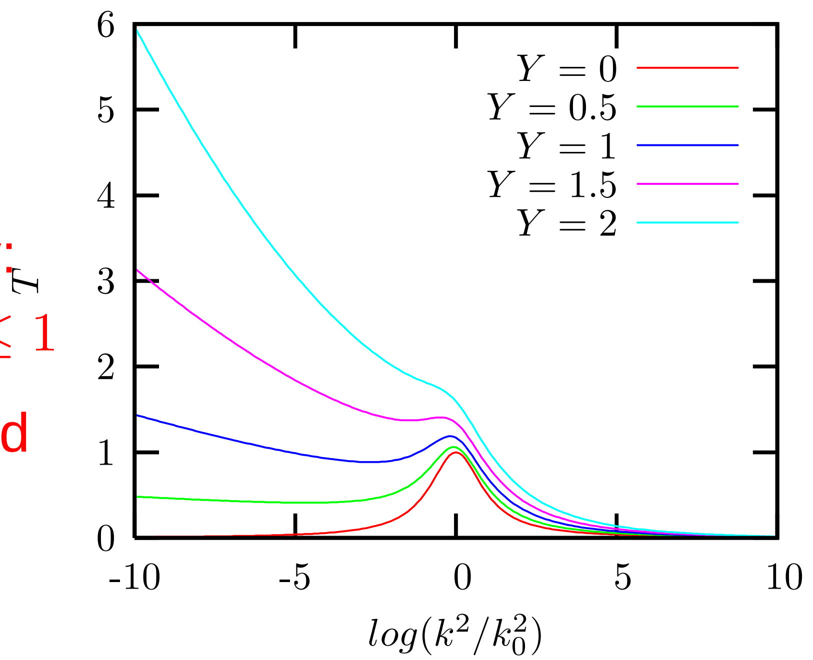
[Balitsky,Fadin,Kuraev,Lipatov,78]

BFKL evolution (3/3)

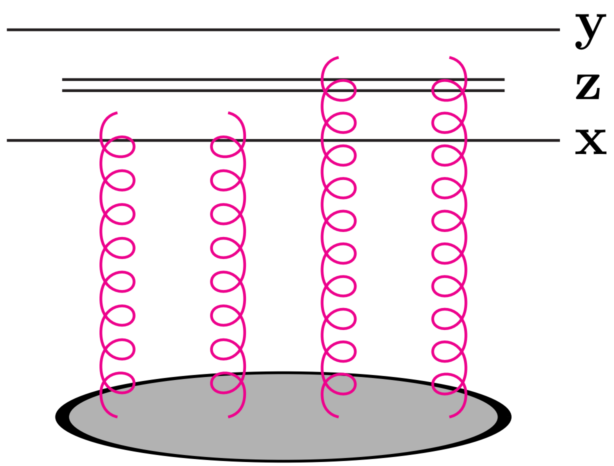
The solution goes like

$$T(Y) \sim e^{\omega Y} \qquad \text{with} \quad \omega = 4\bar{\alpha}\log(2) \approx 0.5$$

- Fast growth of the amplitude
- Intercept value too large
- Violation of the Froissart unitarity:
  $T(Y) \leq C\log^2(s) \qquad T(r,b) \leq 1$
- problem of diffusion in the infrared



Saturation effects



Multiple scattering

- ⋆ Proportional to T^2
- ⋆ important when $T \approx 1$

$\langle \cdot \rangle \equiv$ average over target field

$$\partial_Y \langle T(\mathbf{x},\mathbf{y};Y)\rangle$$
$$= \bar{\alpha}\int d^2z\,\frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{z}-\mathbf{y})^2}\left[\langle T(\mathbf{x},\mathbf{z};Y)\rangle + \langle T(\mathbf{z},\mathbf{y};Y)\rangle - \langle T(\mathbf{x},\mathbf{y};Y)\rangle - \langle T(\mathbf{x},\mathbf{z};Y)T(\mathbf{z},\mathbf{y};Y)\rangle\right]$$

But

$\partial_Y \langle T(\mathbf{x},\mathbf{y};Y)\rangle$ contains a new object: $\langle T(\mathbf{x},\mathbf{z};Y)T(\mathbf{z},\mathbf{y};Y)\rangle$

Balitsky, Kovchegov and JIMWLK

In general: complete hierarchy [Balitsky, 96]

$$\partial_Y \langle T^k \rangle \longrightarrow \underbrace{\langle T^k \rangle}_{\text{BFKL}}, \underbrace{\langle T^{k+1} \rangle}_{\text{saturation}}$$

- Beyond large-N_c: involves quadrupoles, sextupoles, ...
- Balitsky hierarchy $\equiv$ JIMWKL eq. (Colour Glass Condensate formalism)

Mean field approx.: $\langle T_{\mathbf{xz}} T_{\mathbf{zy}} \rangle = \langle T_{\mathbf{xz}} \rangle \langle T_{\mathbf{zy}} \rangle$

$$\partial_Y \langle T_{\mathbf{xy}} \rangle = \frac{\bar{\alpha}}{2\pi} \int d^2 z \, \frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{z}-\mathbf{y})^2} \left[\langle T_{\mathbf{xz}} \rangle + \langle T_{\mathbf{zy}} \rangle - \langle T_{\mathbf{xy}} \rangle - \langle T_{\mathbf{xz}} \rangle \langle T_{\mathbf{zy}} \rangle \right]$$

[Balitsky 96,Kovchegov 99]

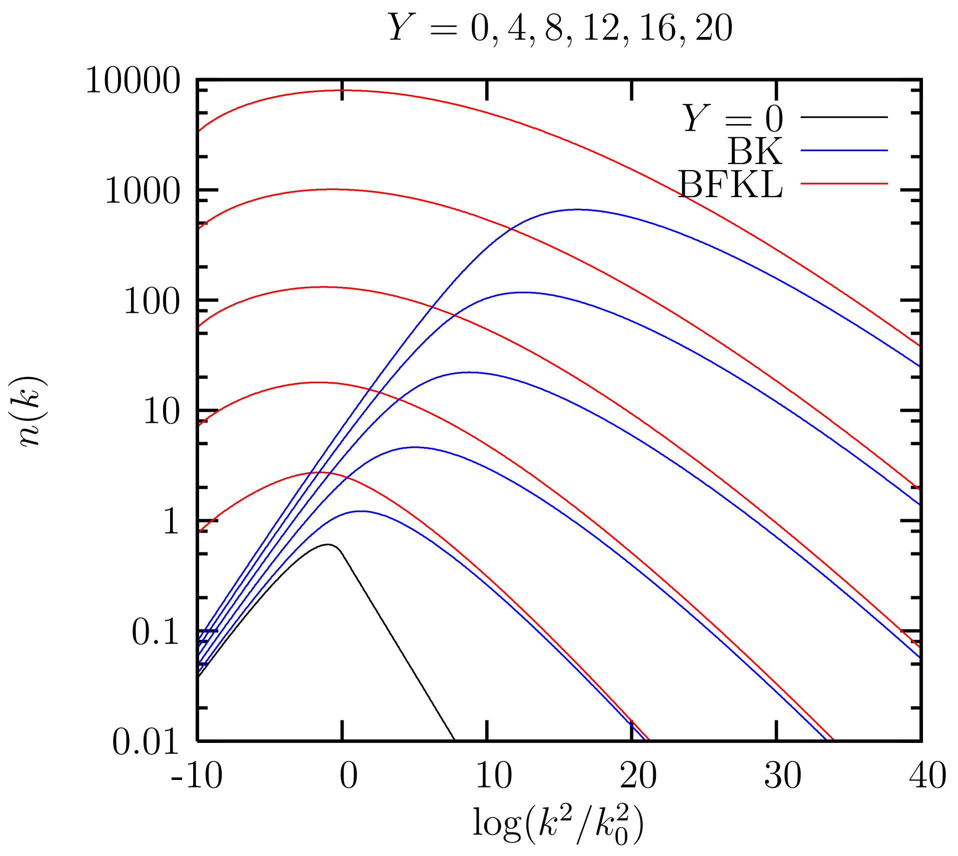
Simplest perturbative evolution equation satisfying unitarity constraint

Saturation effects

Improvements due to this new term:

- $0 \leq T(\mathbf{x}, \mathbf{y}) \leq 1 \Rightarrow$ unitarity preserved
- Cut the diffusion to the infrared

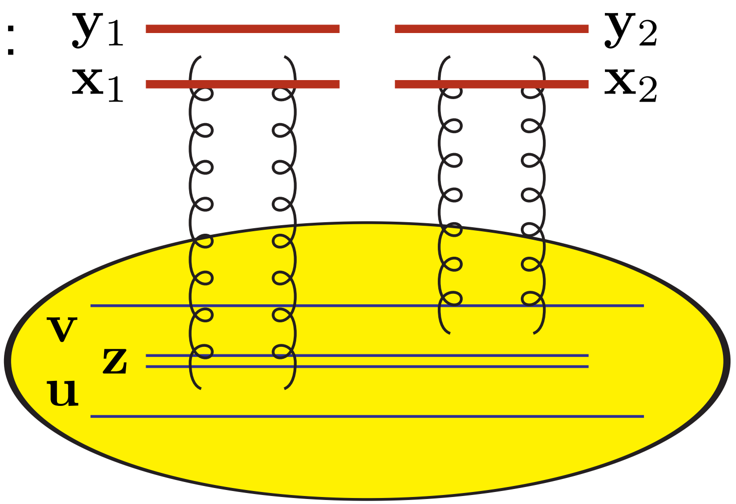
unintegrated
gluon distribution



Beyond saturation: fluctuations

[Iancu,Triantafyllopoulos,Mueller,Shoshi,Kovner,Lublinksi,...,05]

Asymptotically, an extra piece is needed:



$$\underbrace{\int_{\mathbf{uvz}} \frac{\bar{\alpha}}{2\pi} \frac{(\mathbf{u}-\mathbf{v})^2}{(\mathbf{u}-\mathbf{z})^2(\mathbf{z}-\mathbf{v})^2}}_{\text{BFKL splitting}} \underbrace{\alpha_s^2 \mathcal{A}_0(\mathbf{x}_1\mathbf{y}_1|\mathbf{uz})\, \alpha_s^2 \mathcal{A}_0(\mathbf{x}_2\mathbf{y}_2|\mathbf{zv})}_{\text{interaction (2GE)}} \underbrace{\frac{1}{\alpha_s^2} \nabla_{\mathbf{u}}^2 \nabla_{\mathbf{v}}^2 \langle T_{\mathbf{uv}} \rangle}_{\text{dipole density}}$$

- “reverse” of saturation (*i.e.* related to boost invariance)
- generates correlations throughout the evolution
- combined with saturation: generates Pomeron loops

Evolution hierarchy

$\Rightarrow$ complicated hierarchy

$$\partial_Y \langle T^{(2)}(\mathbf{x}_1,\mathbf{y}_1;\mathbf{x}_2,\mathbf{y}_2)\rangle$$
$$= \frac{\bar{\alpha}}{2\pi}\int d^2z\,\frac{(\mathbf{x}_2-\mathbf{y}_2)^2}{(\mathbf{x}_2-\mathbf{z})^2(\mathbf{z}-\mathbf{y}_2)^2}\Big[\langle T^{(2)}(\mathbf{x}_1,\mathbf{y}_1;\mathbf{x}_2,\mathbf{z})\rangle + \langle T^{(2)}(\mathbf{x}_1,\mathbf{y}_1;\mathbf{z},\mathbf{y}_2)\rangle$$
$$-\langle T^{(2)}(\mathbf{x}_1,\mathbf{y}_1;\mathbf{x}_2,\mathbf{y}_2)\rangle - \langle T^{(3)}(\mathbf{x}_1,\mathbf{y}_1;\mathbf{x}_2,\mathbf{z};\mathbf{z},\mathbf{y}_2)\rangle + (1\leftrightarrow 2)\Big]$$
$$+ \frac{\bar{\alpha}}{2\pi}\left(\frac{\alpha_s}{2\pi}\right)^2\int_{\mathbf{uvz}} \mathcal{M}_{\mathbf{uvz}}\mathcal{A}_0(\mathbf{x}_1\mathbf{y}_1|\mathbf{uz})\mathcal{A}_0(\mathbf{x}_2\mathbf{y}_2|\mathbf{zv})\,\nabla^2_{\mathbf{u}}\nabla^2_{\mathbf{v}}\langle T^{(1)}(\mathbf{u},\mathbf{v})\rangle$$

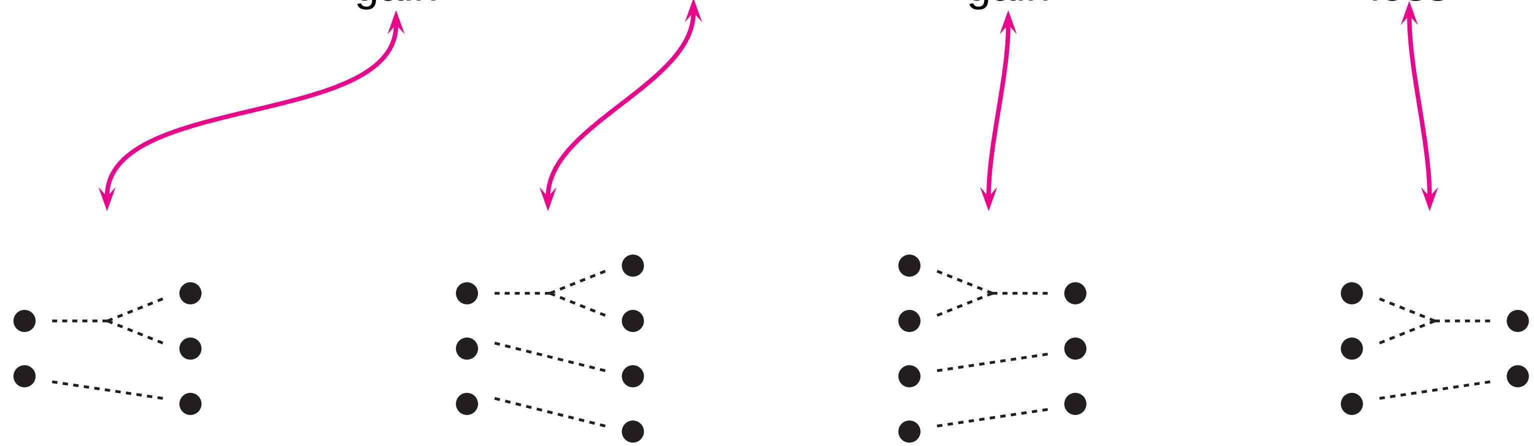
- Saturation: important when $T^{(2)}\sim T^{(1)}\sim 1$ i.e. near unitarity
- Fluctuations: important when $T^{(2)}\sim \alpha_s^2 T^{(1)}$ or $T\sim\alpha_s^2$ i.e. dilute regime

Reaction-diffusion

<u>Reaction-diffusion process</u> $A \underset{\sigma}{\overset{\gamma}{\rightleftharpoons}} A + A$

Master equation: $P_n \equiv$ proba to have n particles

$$\partial_t P_n = \underbrace{\gamma\,(n-1)P_{n-1}}_{\text{gain}} - \underbrace{\gamma\,nP_n}_{\text{loss}} + \underbrace{\sigma\,n(n+1)P_{n+1}}_{\text{gain}} - \underbrace{\sigma\,n(n-1)P_n}_{\text{loss}}$$



Particle densities: we observe a subset of k particles

$$\langle n^k \rangle \equiv \sum_{N=k}^{\infty} \frac{N!}{(N-k)!} P_N$$

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Evolution equation: $\langle n^k \rangle \equiv$ particle density/correlators

$$\partial_t \langle n^k \rangle = \gamma\,k\langle n^k \rangle + \gamma\,k(k-1)\langle n^{k-1} \rangle - \sigma\,k(k+1)\langle n^{k+1} \rangle$$

Scattering amplitude for this system off a target

$$\mathcal{A}(t) = \sum_{k=0}^{\infty} (-)^k \,\langle n^k \rangle_{t_0}\, \langle T^k \rangle_{t-t_0}$$

t_0-independent $\Rightarrow$

$$\partial_t \langle T^k \rangle = \underbrace{\gamma \langle T^k \rangle}_{\text{BFKL}} - \underbrace{\gamma \langle T^{k+1} \rangle}_{\text{sat.}} + \underbrace{\sigma \langle T^{k-1} \rangle}_{\text{fluct.}}$$

Reaction-diffusion & QCD

[E. Iancu,GS,D. Triantafyllopoulos,06]

For QCD particle = (effective) dipoles

Dipole plitting $\equiv$ BFKL kernel

$$\gamma \sim \bar{\alpha} \frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{z}-\mathbf{y})^2}$$

Effective dipole merging

$$\sigma(\mathbf{x}_1\mathbf{y}_1, \mathbf{x}_2\mathbf{y}_2 \to \mathbf{uv})$$

$$\sim \bar{\alpha}\alpha_s^2 \nabla_{\mathbf{u}}^2 \nabla_{\mathbf{v}}^2 \left\{ \mathcal{M}_{\mathbf{uvz}} \log^2 \left[\frac{(\mathbf{x}_1-\mathbf{u})^2(\mathbf{y}_1-\mathbf{z})^2}{(\mathbf{x}_1-\mathbf{z})^2(\mathbf{y}_1-\mathbf{u})^2} \right] \log^2 \left[\frac{(\mathbf{x}_2-\mathbf{v})^2(\mathbf{y}_2-\mathbf{z})^2}{(\mathbf{x}_2-\mathbf{z})^2(\mathbf{y}_2-\mathbf{v})^2} \right] \right\}$$

Remarks:

- merging not always positive
- fluctuations = gluon-number fluctuations
- Can be obtained from projectile or target point of view
- Known at large N_c in the 2-gluon-exchange limit.

Solutions

The BK equation

b-independent case

Case 1: no impact parameter dependence

$$T_{\mathbf{xy}} \to T\left(\mathbf{r} = \mathbf{x} - \mathbf{y}, \mathbf{b} = \frac{\mathbf{x}+\mathbf{y}}{2}\right) \to T(r)$$

Note:

- all arguments work for $T(r)$ or its Fourier transform $\tilde{T}(k)$
- for $\tilde{T}$, the non-linear term is simply $-\tilde{T}^2(k)$

$$L = \log\left(k^2/k_0^2\right) \quad \text{or} \quad \log\left(r_0^2/r^2\right)$$

Evolution mechanism

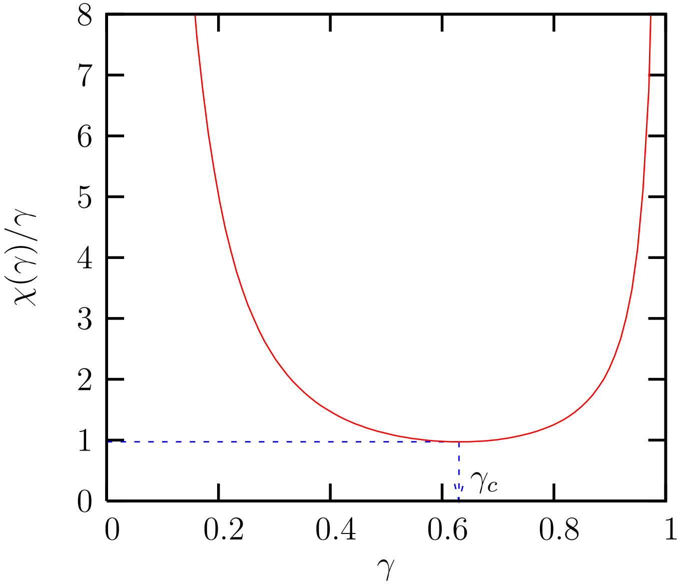
[S.Munier,R.Peschanski,03]

BK equation: $\partial_Y T = \underbrace{\chi(-\partial_L)T}_{\text{BFKL}} - T^2$

When $T \ll 1$ BFKL works:

$$\partial_Y T = \chi(-\partial_L)T$$

Solution known:



$$T(k) = \int \frac{d\gamma}{2i\pi} T_0(\gamma) \exp\left[\chi(\gamma)\bar{\alpha}Y - \gamma L\right]$$

$$= \int \frac{d\gamma}{2i\pi} T_0(\gamma) \exp\left[-\gamma\left(L - \frac{\chi(\gamma)}{\gamma}\bar{\alpha}Y\right)\right]$$

$\frac{\chi(\gamma)}{\gamma}$ min. when $\gamma = \gamma_c$

$\Rightarrow$ Wave of slope γ travels at speed

$$v = \chi(\gamma)/\gamma$$

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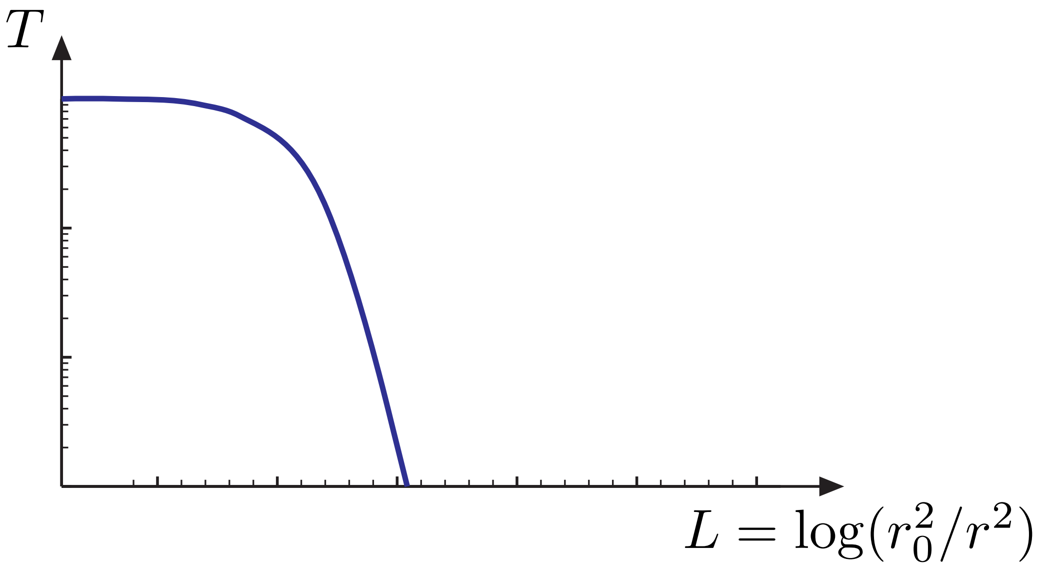

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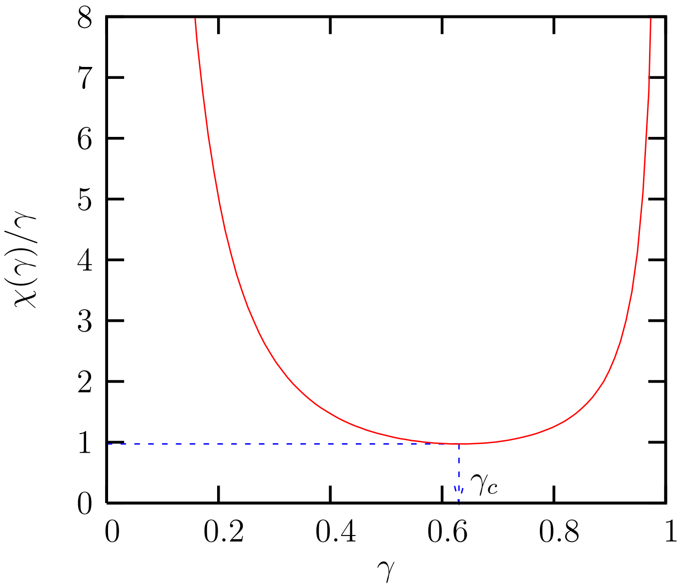

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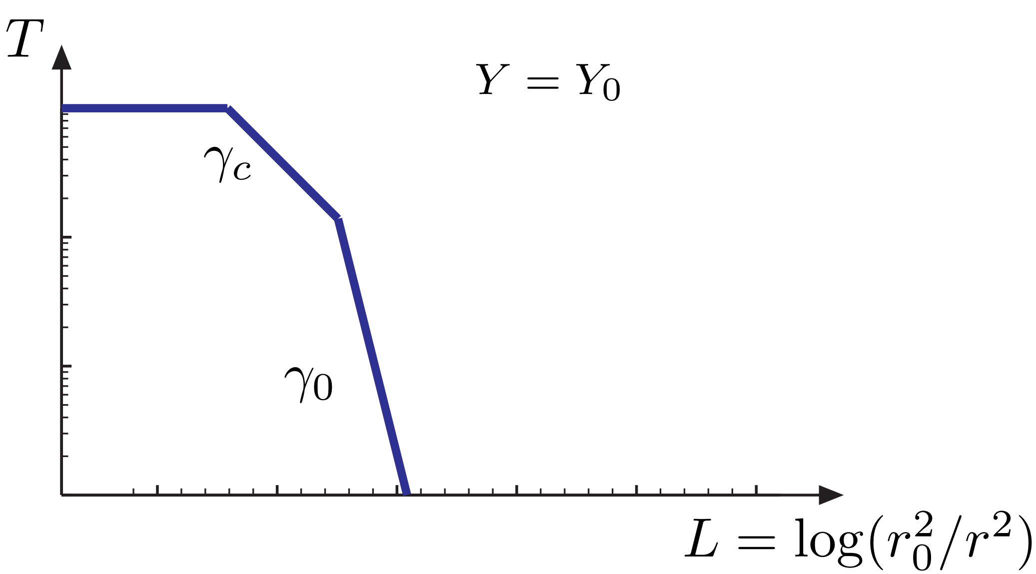

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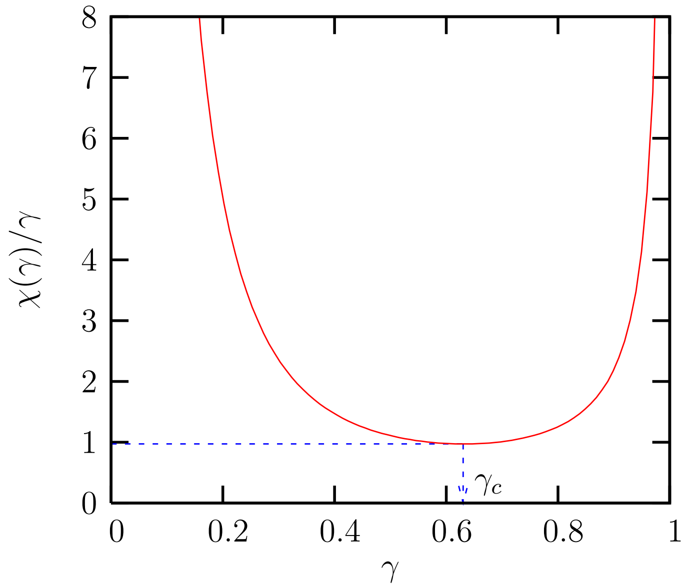

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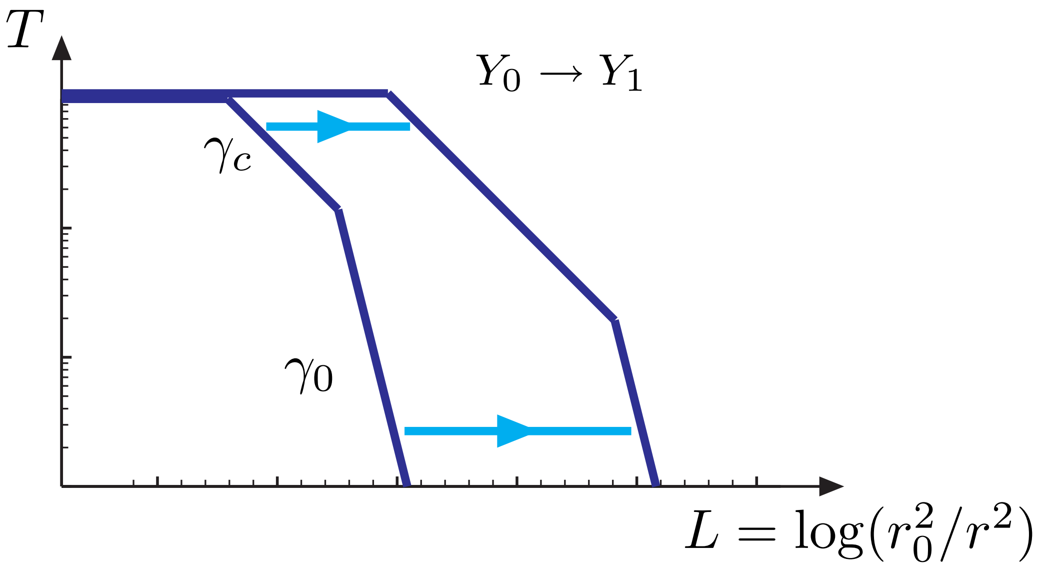

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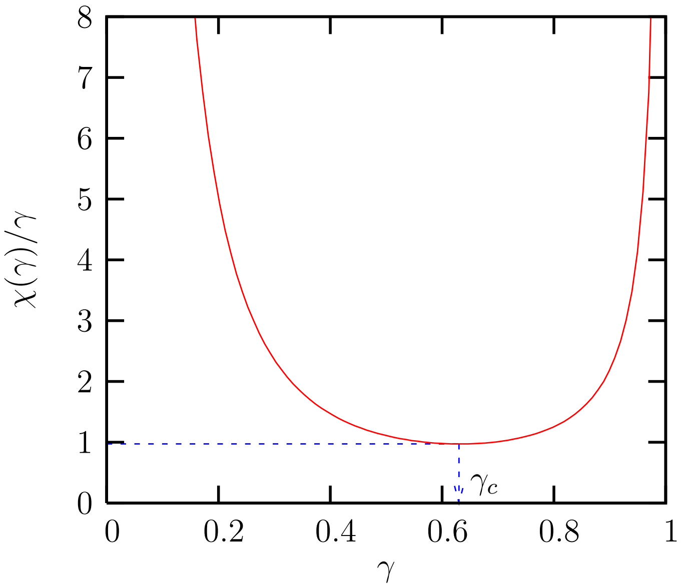

$\frac{\chi(\gamma)}{\gamma}$ min. when $\gamma = \gamma_c$

The minimal speed is selected during evoution

Geometric scaling

Consequence: geometric scaling ($Q_s \equiv$ sat. scale $\equiv$ front position)

$$T(r,Y) = T\left[rQ_s(Y)\right] \qquad \text{with} \qquad Q_s^2(Y) = v_c\bar{\alpha}Y$$

$$\stackrel{rQ_s\ll 1}{=} \underbrace{\left[r^2Q_s^2(Y)\right]^{\gamma_c}}_{\text{slope } \gamma_c} \underbrace{\exp\left[\frac{\log^2(r^2Q_s^2)}{2\chi''(\gamma_c)\bar{\alpha}Y}\right]}_{\text{scaling window}}$$

$$|\log(r^2Q_s^2)| \lesssim \sqrt{\chi''(\gamma_c)\bar{\alpha}Y}$$

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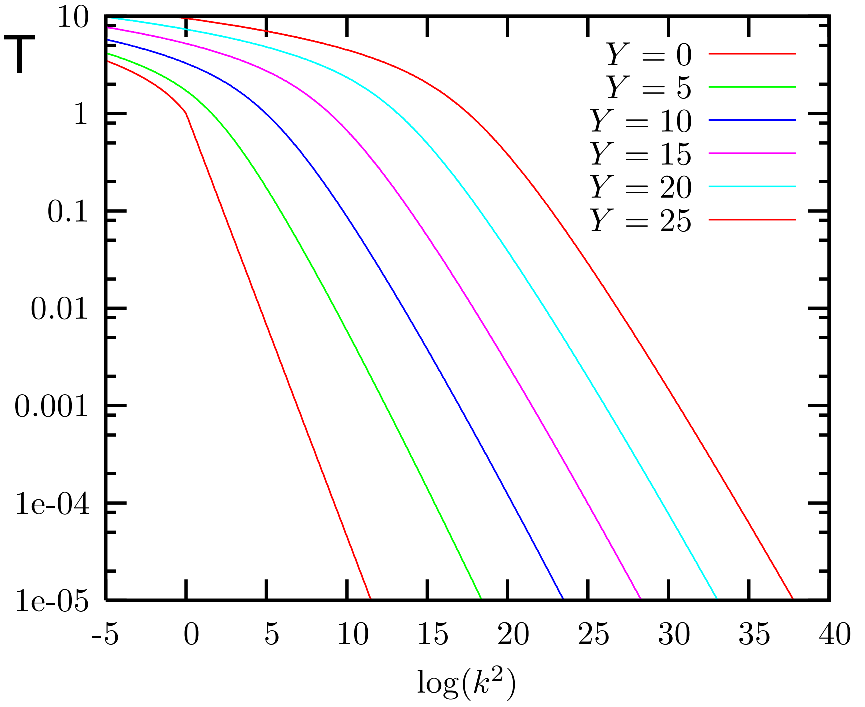
$$|\log(r^2Q_s^2)| \lesssim \sqrt{\chi''(\gamma_c)\bar{\alpha}Y}$$

- Generic arguments: exponential rise + saturation $\Rightarrow$ select γ_c
- Parameters fixed by linear kernel only
- Saturation effects even though $T \ll 1$
- initially derived for the F-KPP equation

[Fisher,Kolmogorov,Petrovsky,Piscunov,37]

Geometric scaling

Numerical simulations:



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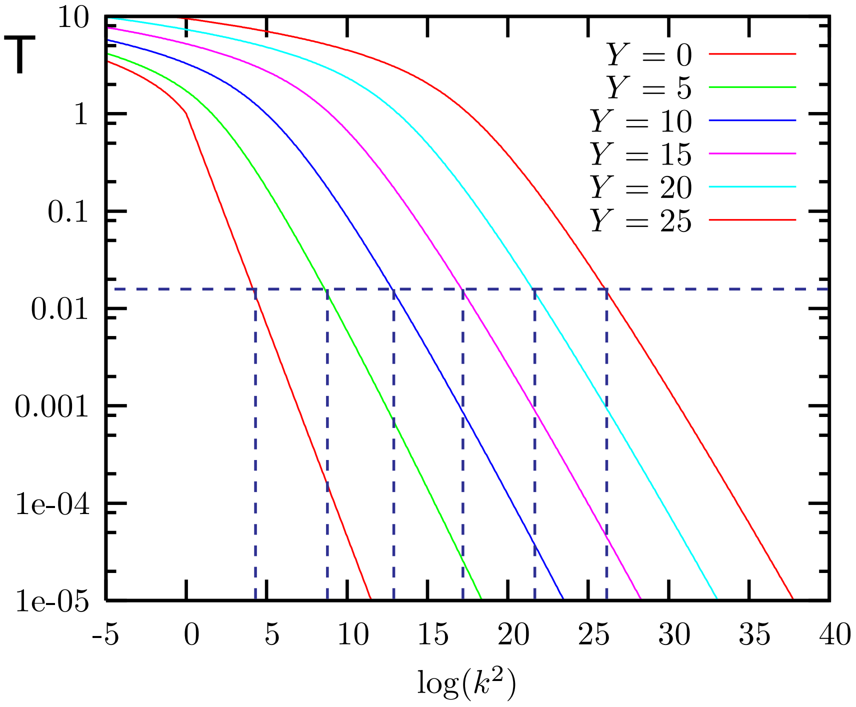
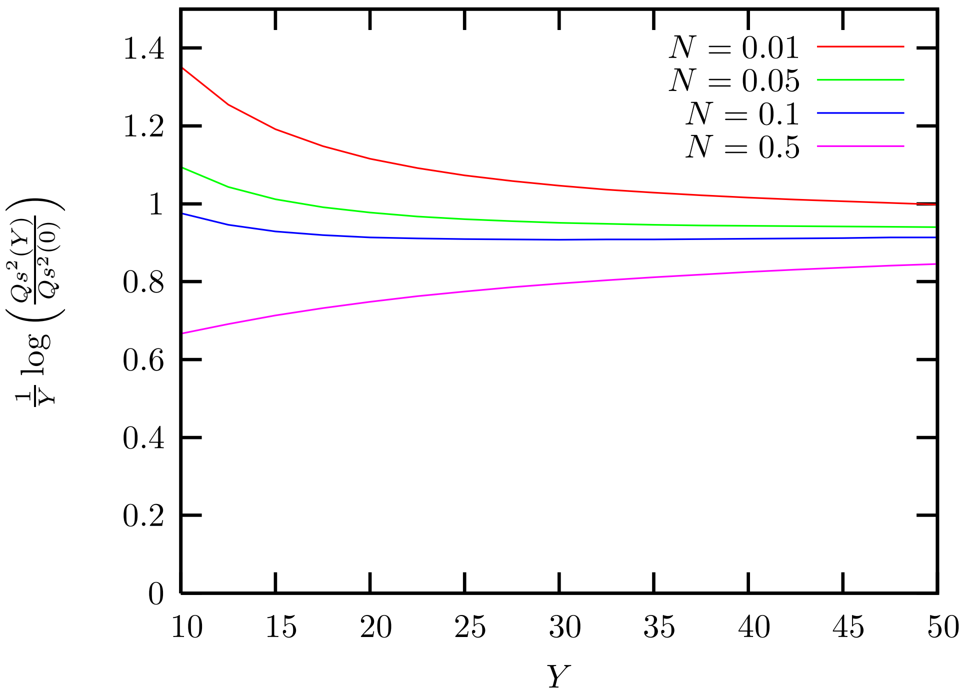




$$T(k,Y) = T\left(\frac{k^2}{Q_s^2(Y)}\right) \approx \left(\frac{k^2}{Q_s^2(Y)}\right)^{-\gamma_c}$$

$$Q_s^2(Y) \propto \exp(v_c Y)$$

NB.: Independent of the initial condition

The full BK equation

[C.Marquet, R.Peschanski, G.S., 05]

Case 2: including impact parameter

Go to momentum space: use momentum transfer $\mathbf{q}$

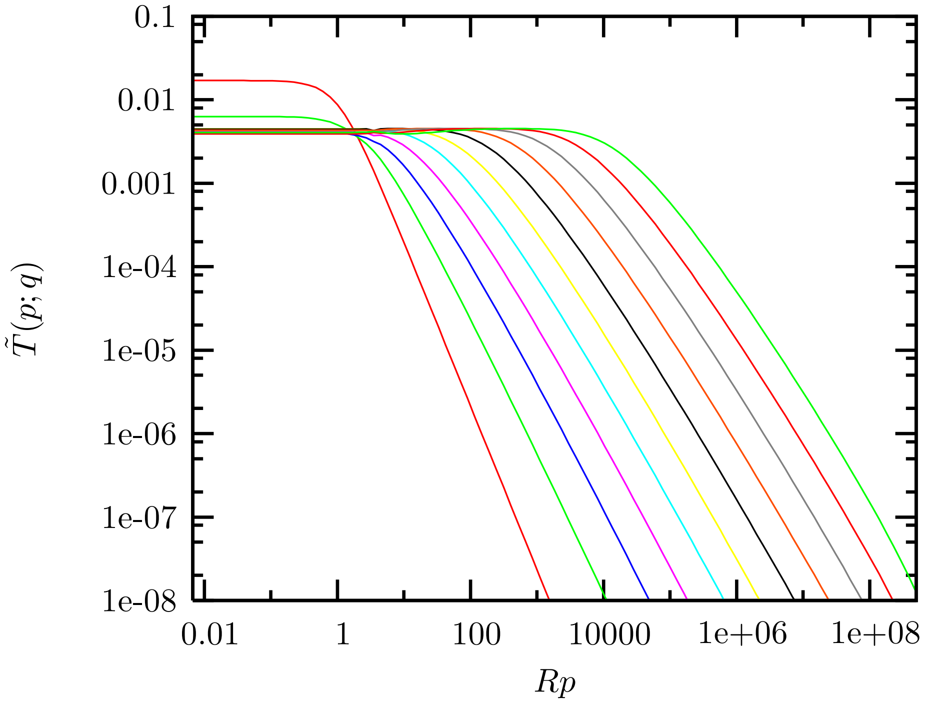
$$\tilde{T}(\mathbf{k},\mathbf{q}) = \int d^2x\, d^2y\, e^{i\mathbf{k}.\mathbf{x}} e^{i(\mathbf{q}-\mathbf{k}).\mathbf{y}} \frac{T(\mathbf{x},\mathbf{y})}{(\mathbf{x}-\mathbf{y})^2}$$

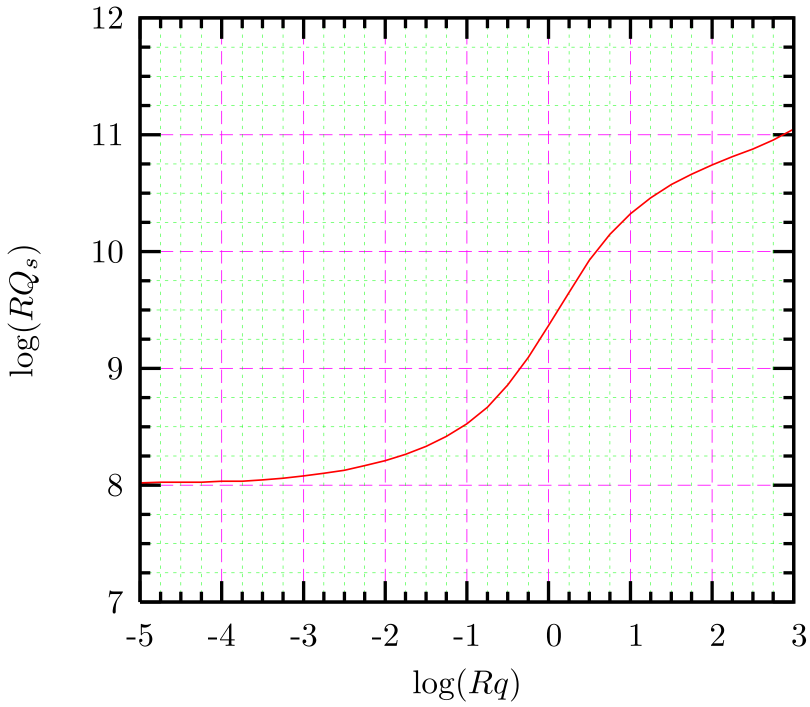
new form of the BK equation

$$\partial_Y \tilde{T}(\mathbf{k},\mathbf{q}) = \frac{\bar{\alpha}}{\pi} \int \frac{d^2k'}{(k-k')^2} \left\{ \tilde{T}(\mathbf{k}',\mathbf{q}) - \frac{1}{4}\left[\frac{k^2}{k'^2} + \frac{(q-k)^2}{(q-k')^2}\right] \tilde{T}(\mathbf{k},\mathbf{q}) \right\} - \frac{\bar{\alpha}}{2\pi} \int d^2k'\, \tilde{T}(\mathbf{k},\mathbf{k}')\tilde{T}(\mathbf{k}-\mathbf{k}',\mathbf{q}-\mathbf{k}')$$

Numerical simulations

[C.Marquet, R.Peschanski, G.S., 05]





One can prove analytically that:

- traveling wave at large k: BFKL $\Rightarrow$ same γ_c, v_c
- q dependence: $Q_s^2(q,Y) = \min(k_0^2, q^2)\exp(v_c Y)$

Predicts geometric scaling for t-dependent processes

Solutions

Including Pomeron loops

Event evolution

[E. Iancu, A. Mueller, S. Munier, 04]

no b-dependence + coarse-graining $\longrightarrow$ Langevin equation

$$\partial_Y T(k,Y) = \bar{\alpha} K_{\text{BFKL}} \otimes T(k,Y) - \bar{\alpha} T^2(k,Y) + \bar{\alpha}\sqrt{\kappa \alpha_s^2 T(k,Y)}\,\nu(k,Y)$$

with $\langle \nu(k,Y) \rangle = 0$

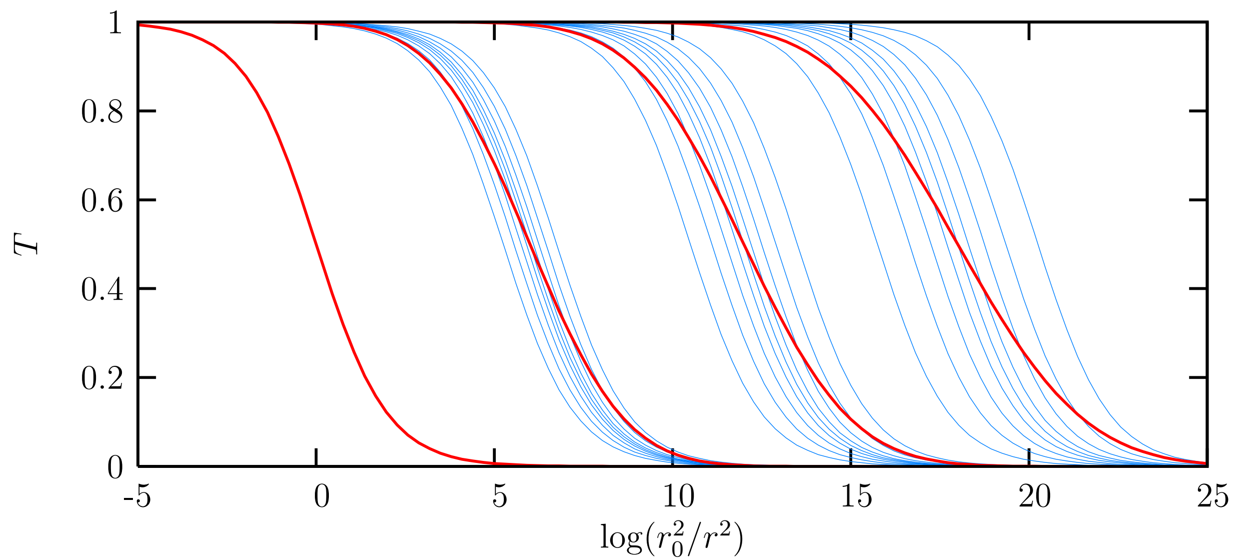
$$\langle \nu(k,Y)\nu(k',Y') \rangle = \frac{1}{\bar{\alpha}}\delta(Y-Y')\,k\delta(k-k')$$

Diffusive approximation

$$\partial_t u(x,t) = \partial_x^2 u + u - u^2 + \sqrt{2\kappa u}\,\nu(x,t)$$

stochastic F-KPP

Typical behaviour



Generic event-by-event behaviour: travelling wave

- speed $v^* \sim \lambda Y$, dispersion $\sigma^2 \sim DY$ ($\lambda < \lambda_{\rm BK}$)
- Q_s now varies event-by-event with a Gaussian distribution

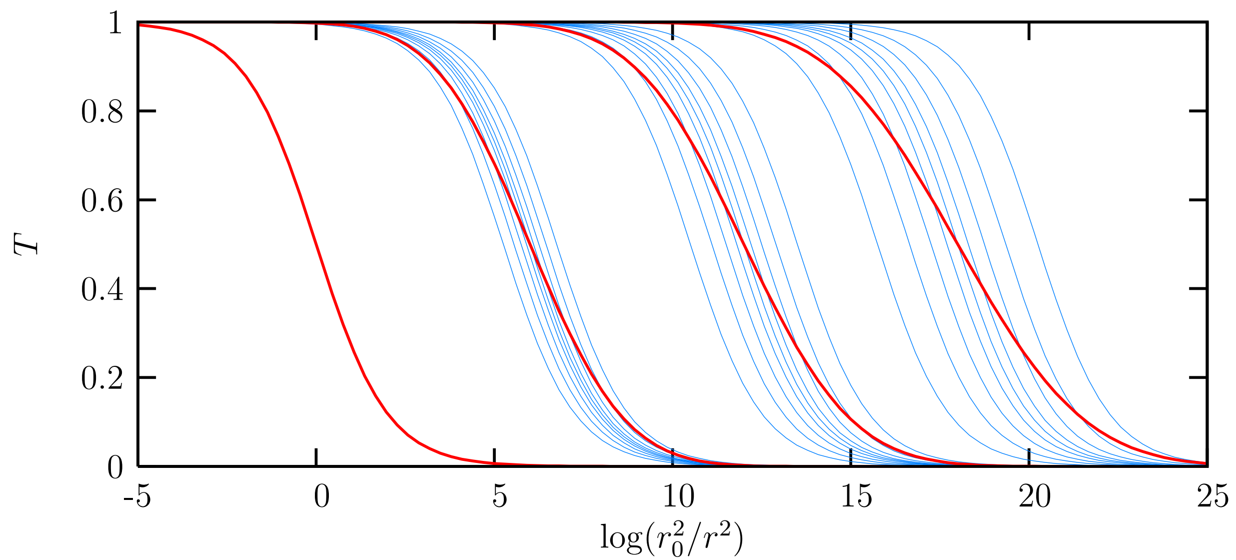
[Marquet,GS,Xiao,07]

- λ and D computable in the small and strong coupling limits

[Brunet,Derrida,Mueller,Munier,07]

[Marquet,Peschanski,GS,07]

Typical behaviour



Average amplitude:

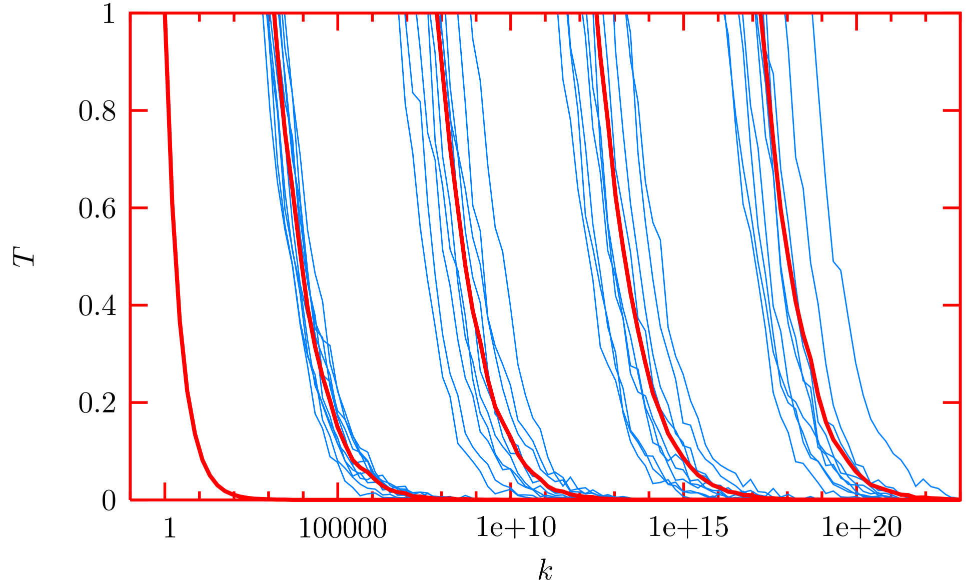
- Violations of geometric scaling
- New scaling: diffusive scaling (when $\sigma^2 \gg 1$, $T = 0$ or 1)

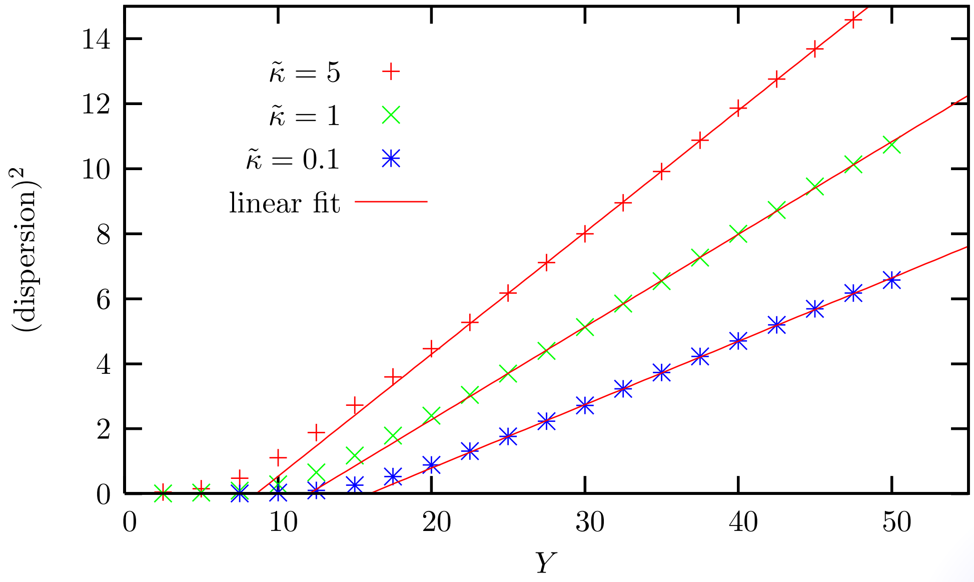
[Hatta,Iancu,Marquet,GS,Triantafyllopoulos,07]

- Generic properties of (many) reaction-diffusion systems

QCD with Pomeron loops

Numerical simulation for the QCD Pomeron loops equations [GS,05]

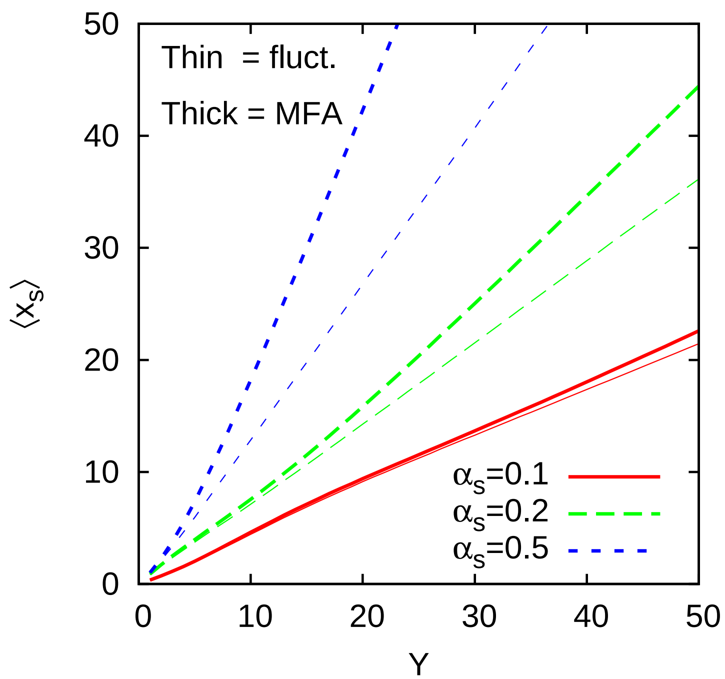


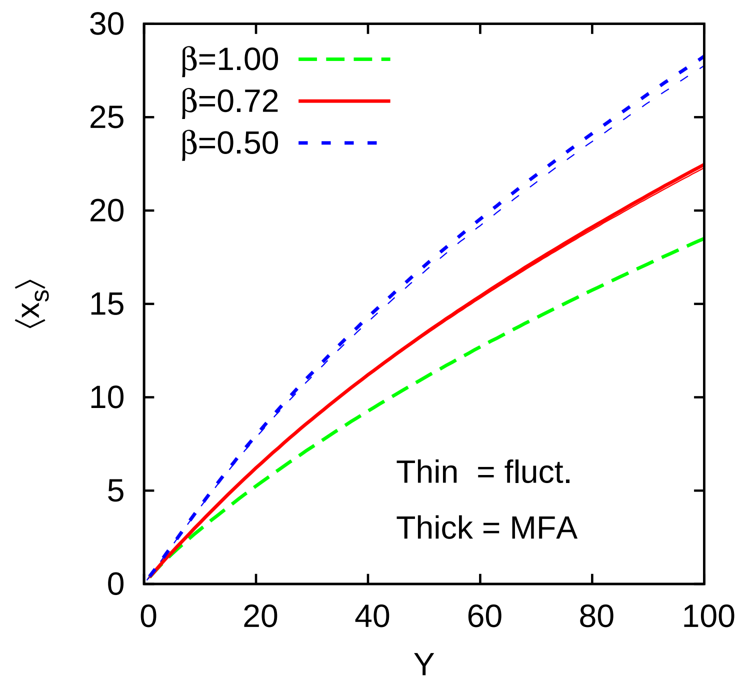


Running coupling problem

[Dumitru,Iancu,Portugal,GS,Triantafyllopoulos,07]

Toy model to study running-coupling effects





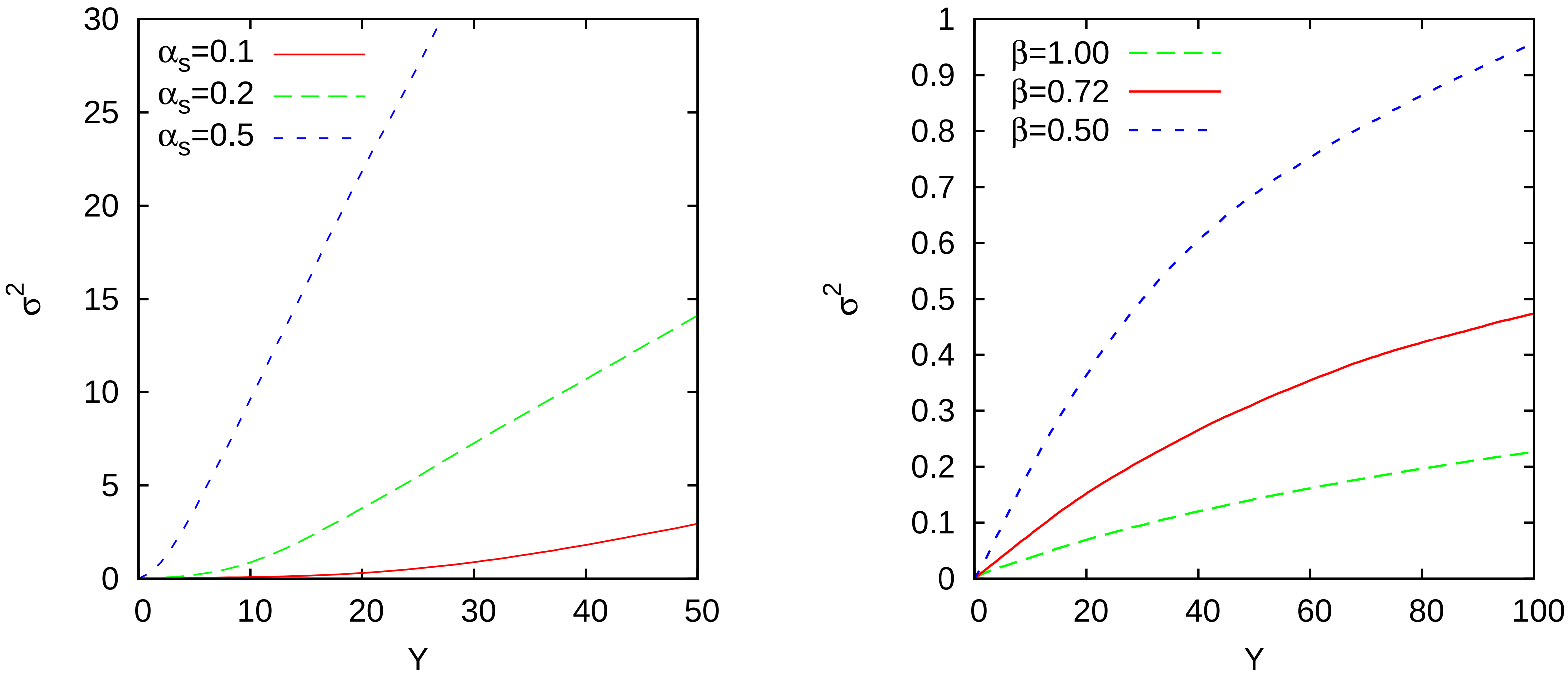
no reduction of the speed

NB: fixed $\rightarrow$ running coupling: $Y \rightarrow \sqrt{Y}$

Running coupling problem

[Dumitru,Iancu,Portugal,GS,Triantafyllopoulos,07]

Toy model to study running-coupling effects



dispersion suppressed ⇒ geometric scaling OK

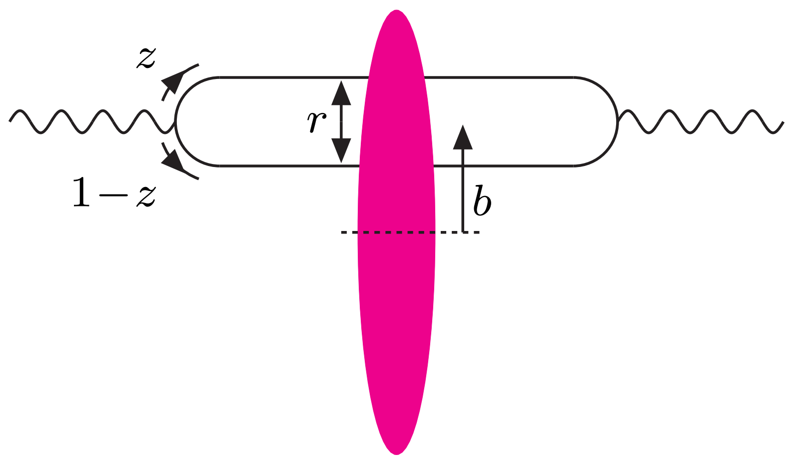
NB: fixed $\rightarrow$ running coupling: $Y \rightarrow \sqrt{Y}$

Phenomenology

F_2 *in the dipole picture*

Factorisation formula at small x:

$$\frac{\sigma_{L,T}^{\gamma^* p}}{d^2 b} = \int d^2 r \int_0^1 dz \left|\Psi_{L,T}(z, r; Q^2)\right|^2 T(\mathbf{r}, \mathbf{b}; Y)$$



- $\Psi \equiv$ photon wavefunction $\gamma^* \to q\bar{q}$: QED process
- $T \equiv$ scattering amplitude from high-energy QCD.

$$\int d^2 b\, T(r, b, Y) = 2\pi R_p^2\, T(r; Y) \qquad\qquad F_2 = \frac{Q^2}{4\pi\alpha_e}\left[\sigma_L^{\gamma^* p} + \sigma_T^{\gamma^* p}\right]$$

A guideline: geometric scaling

[A. Stasto, K. Golec-Biernat, J. Kwiecinski, 01]
[F. Gelis, R. Peschanski, L. Schoeffel, GS, 06]

Parametrisation of T motivated by one observation:

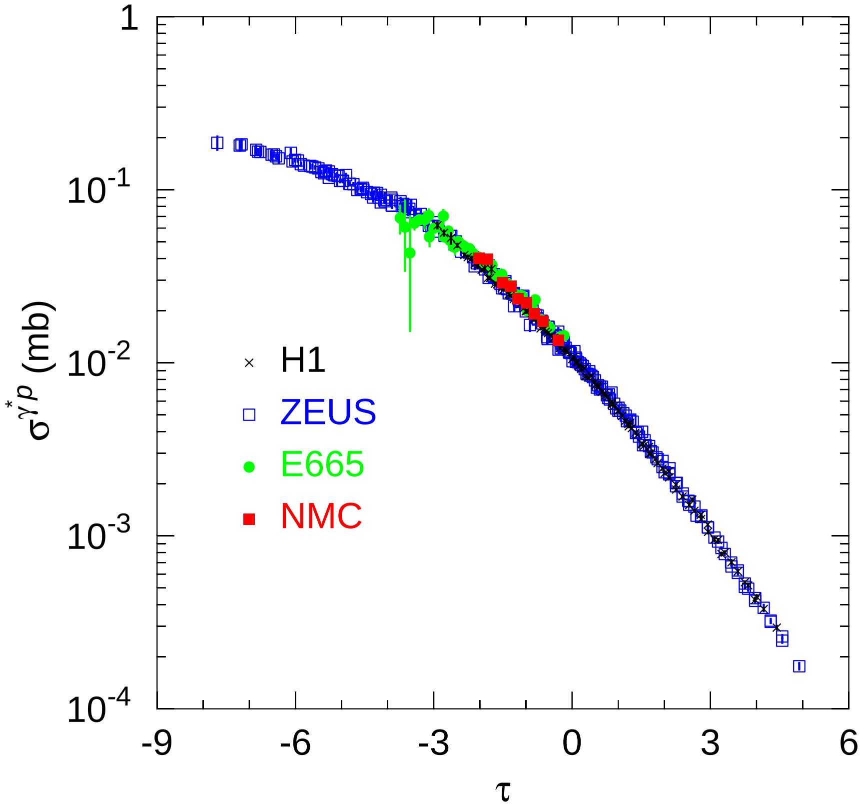
$$\sigma^{\gamma^* p}(x, Q^2) = \sigma^{\gamma^* p}(\tau)$$

with $\tau = \log(Q^2) - \lambda \log(1/x)$
$= \log(Q^2/Q_s^2)$

Geometric scaling

Saturation scale:

$$Q_s^2(x) = Q_0^2\, x^{-\lambda}$$



Since $Q \sim 1/r$ this suggests

$$T(r, x) = T(rQ_s)$$

HERA: $Q_s \sim 1$ GeV

BK equation and geometric scaling

Idea: use QCD predictions from BK: $\rho = \log\left(\frac{4}{r^2 Q_s^2}\right)$; $Q_s = Q_0 x^{-\lambda}$

$$T(r;x) \overset{rQ_s \ll 1}{\propto} \underbrace{\exp\left(-\gamma_c \rho\right)}_{\text{geometric scaling}} \quad \underbrace{\exp\left(-\frac{\rho^2}{2\bar{\alpha}\chi"_c Y}\right)}_{\text{scaling violations; window width}}$$

- High-energy QCD predicts

  geometric scaling as a consequence of saturation

- Validity in the scaling window

$$\log(1/r^2) \lesssim \log(Q_s^2) + \sqrt{2\bar{\alpha}\chi"_c Y}$$

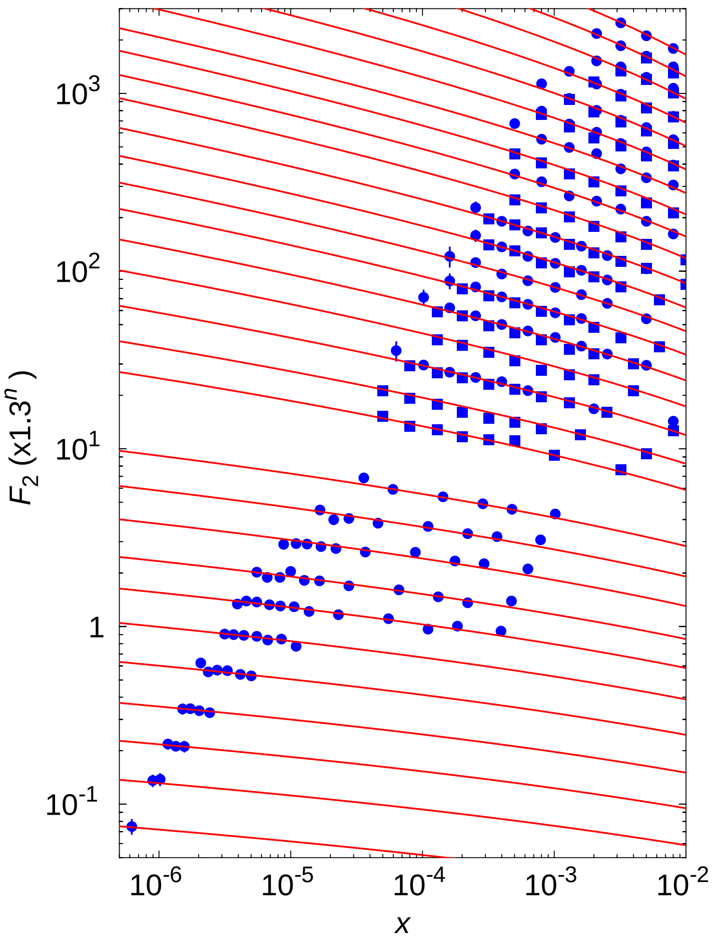
  i.e. beyond saturation scale

Note:

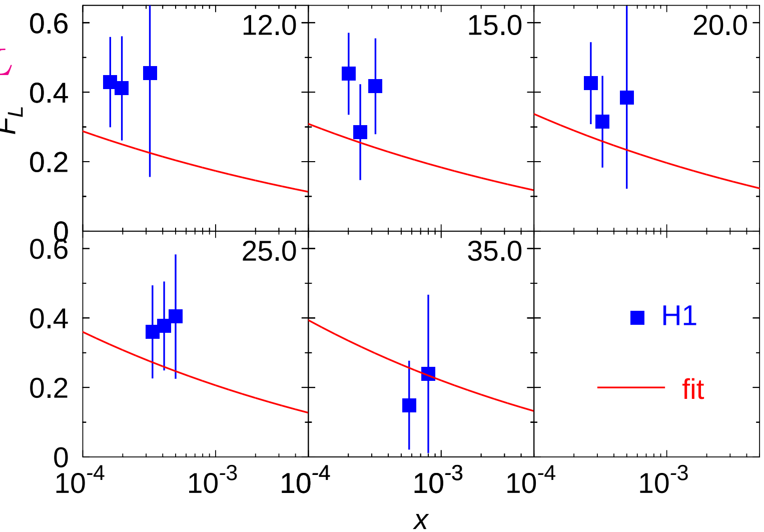
- Light quarks only [Iancu, Itakura, Munier, 03]
- With heavy quarks (longstanding issue) [GS, 07]

Latest results

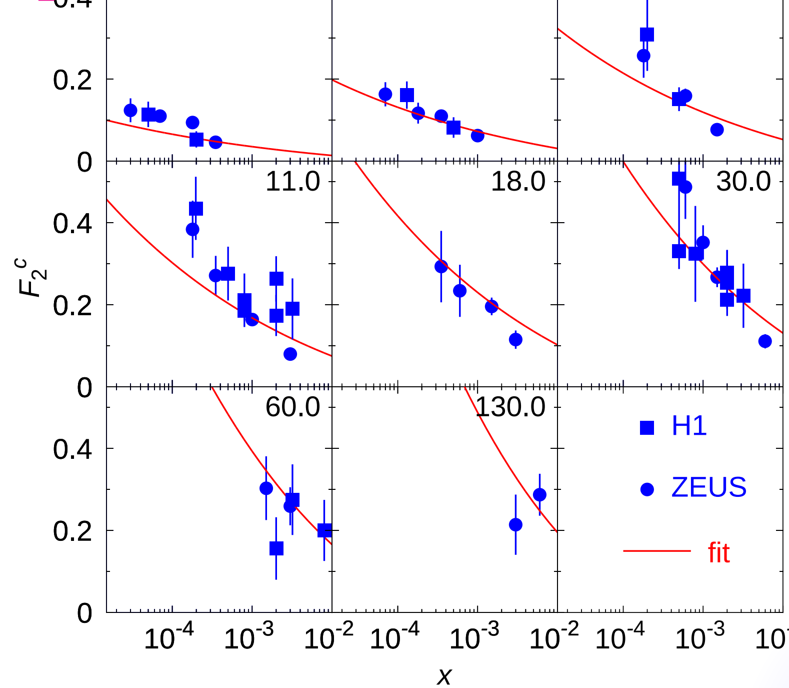
F_2^p



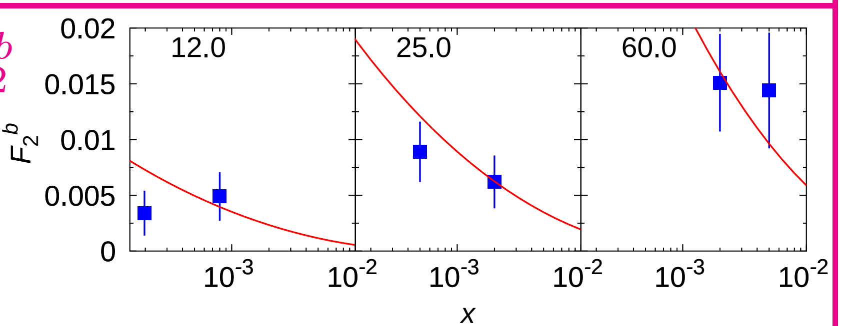
F_L



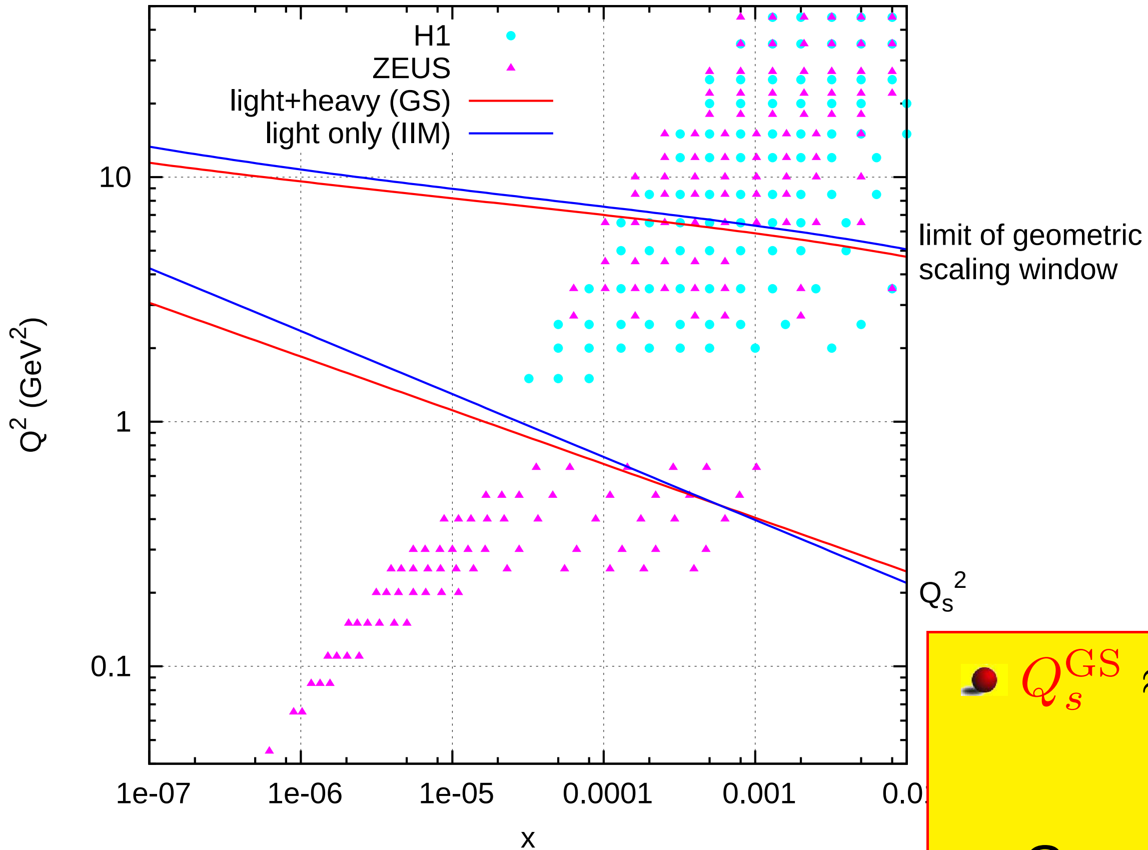
F_2^c



F_2^b



New fit results



- $Q_s^{\mathrm{GS}} \approx Q_s^{\mathrm{IIM}}$: NO drop down

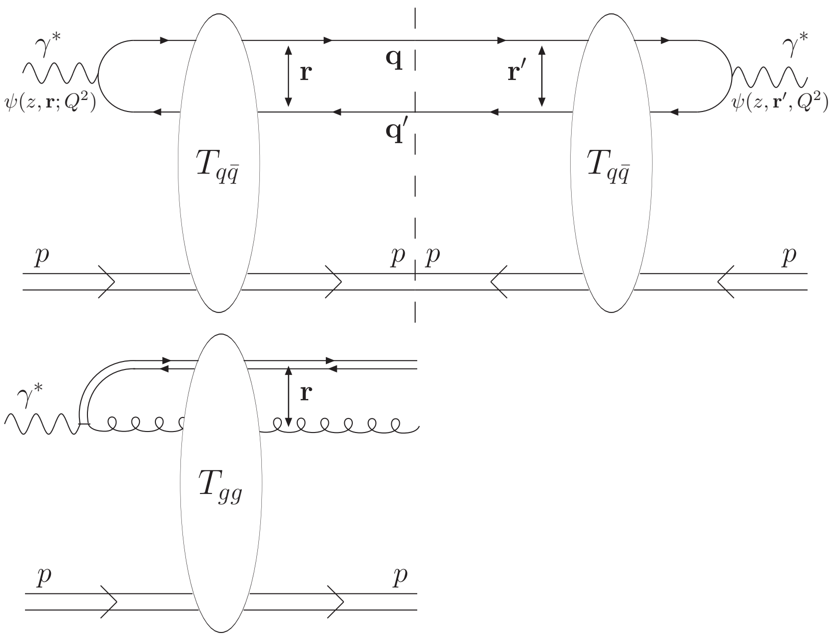
- Geometric scaling window
  $\equiv$ we “feel” saturation
  $\longrightarrow$ up to $Q^2 \sim 5 - 7$ GeV^2

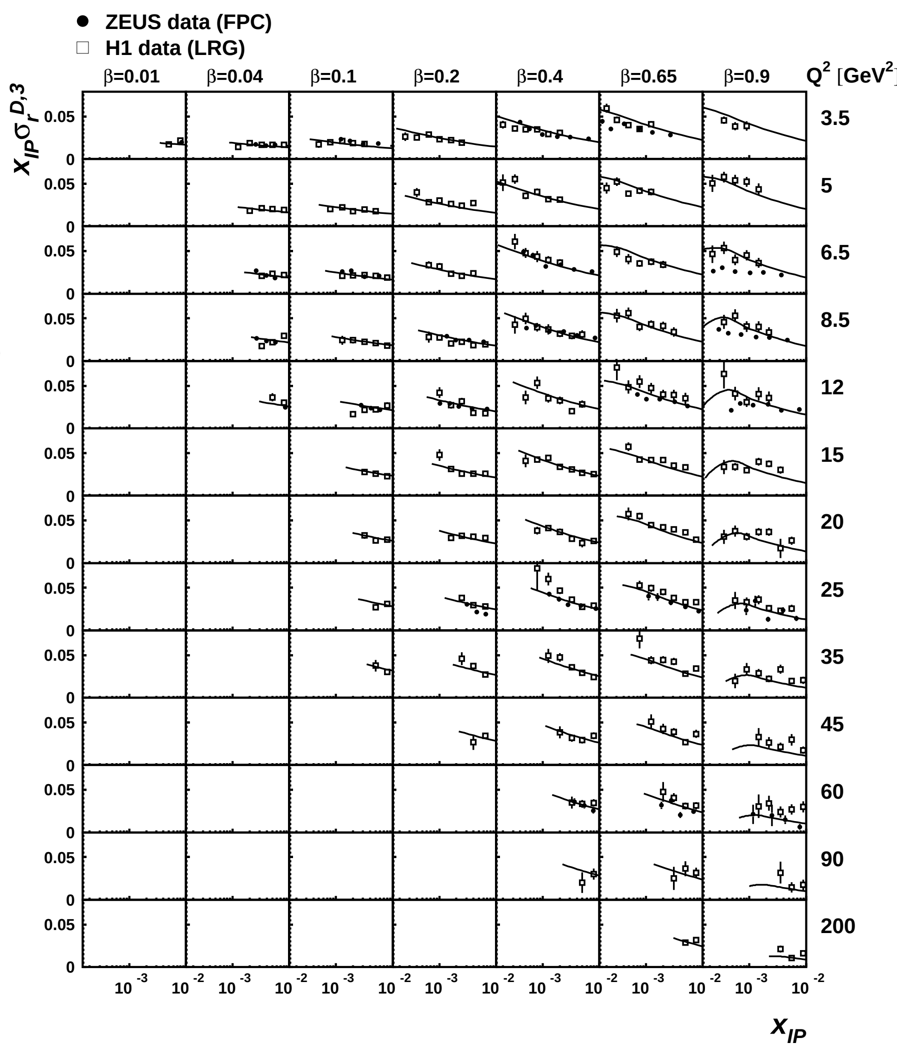
F_2^D

Same kind of factorisation

but more contribs:

$$F_2^D = F_2^{D(q\bar{q})} + F_2^{D(q\bar{q}g)} + \dots$$





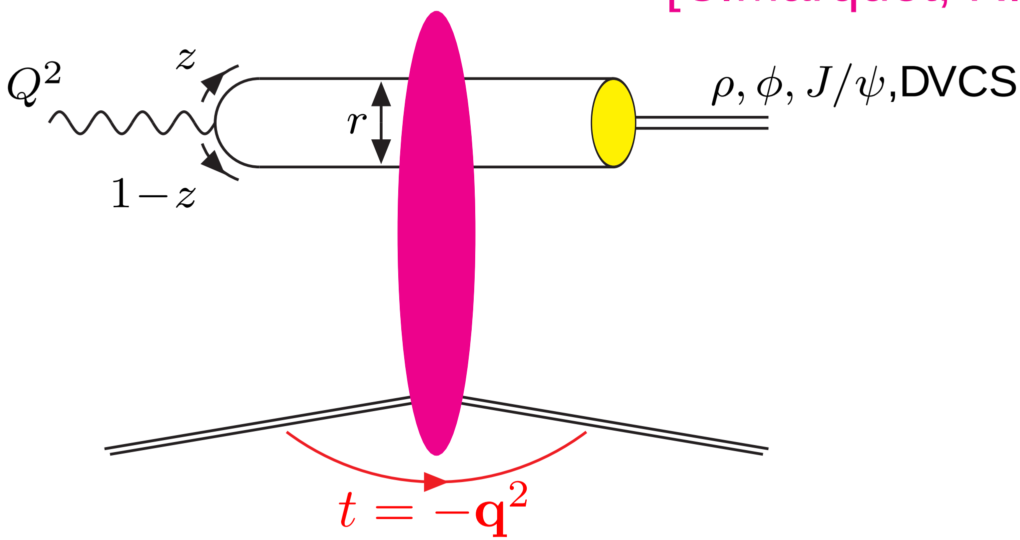
Basically: $F_2^D \propto T^2$

with the same T as for F_2

[Marquet, 07]

Vector mesons and DVCS

[C.Marquet, R.Peschanski, GS, 07]
[GS, in prep]



- Factorisation formula for

$$\mathcal{A}_{L,T}^{\gamma^* p \to V p} = \Psi^{\gamma^*} \otimes \sigma_{\text{dip}}(\mathbf{r}, \mathbf{q}; Y) \otimes \Psi^{\text{VM}}$$

- t dependence from BK (NB: BK predicts t-dep, not b-dep)

$$\sigma_{\text{dip}}(r, q; Y) = 2\pi R_p^2 e^{-b|t|}\, T_{\text{GS}}\left(r, Q_s^2(q, Y)\right)$$

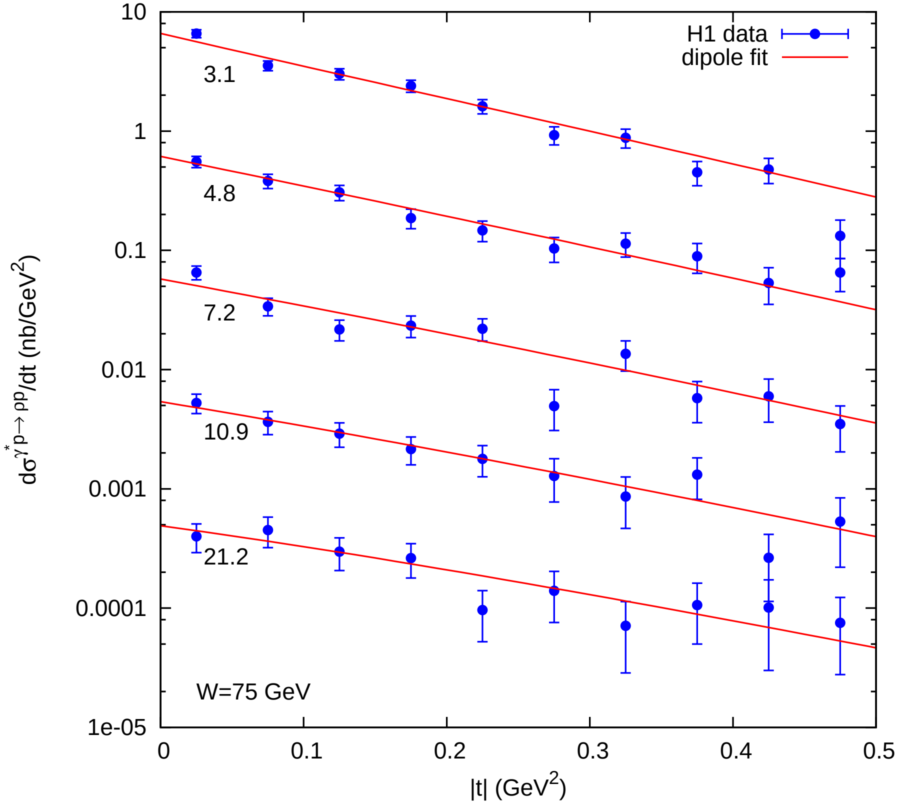
$$Q_s^2 = Q_0^2\, e^{\lambda Y} \qquad \to \qquad Q_s^2 = Q_0^2\,(1 + c|t|)\, e^{\lambda Y}$$

- b, c $\quad\to\quad$ $\frac{d\sigma}{dt}$, σ_{el} (including $\mathcal{R}e\,\mathcal{A}$ and skewedness)

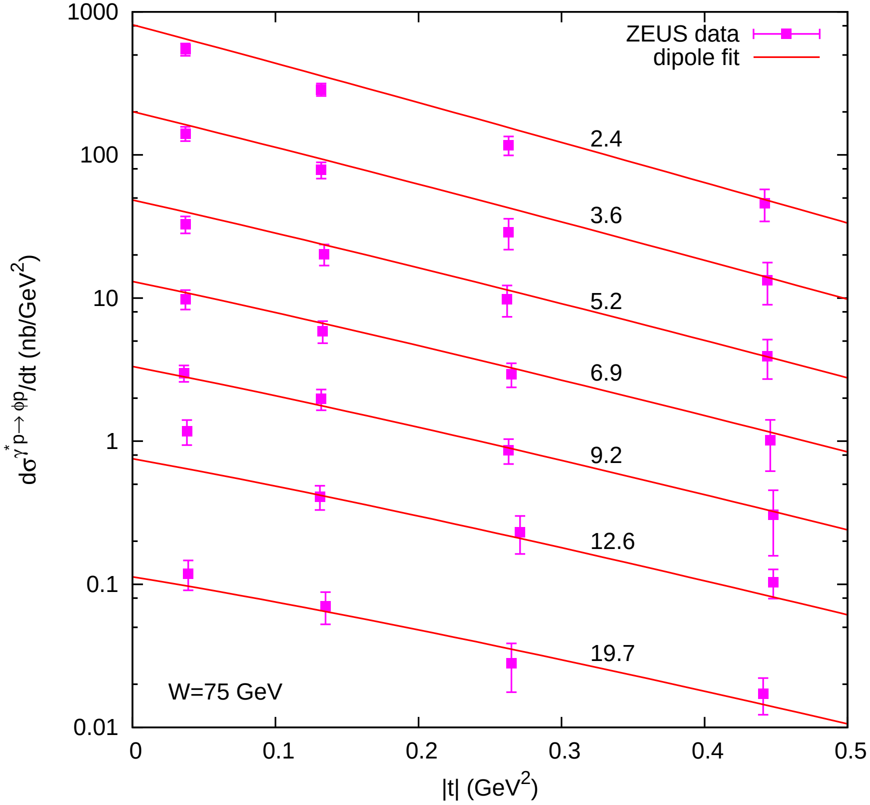
Vector mesons and DVCS

Example: differential cross-section:

$\gamma^* p \rightarrow \rho p$



$\gamma^* p \rightarrow \phi p$

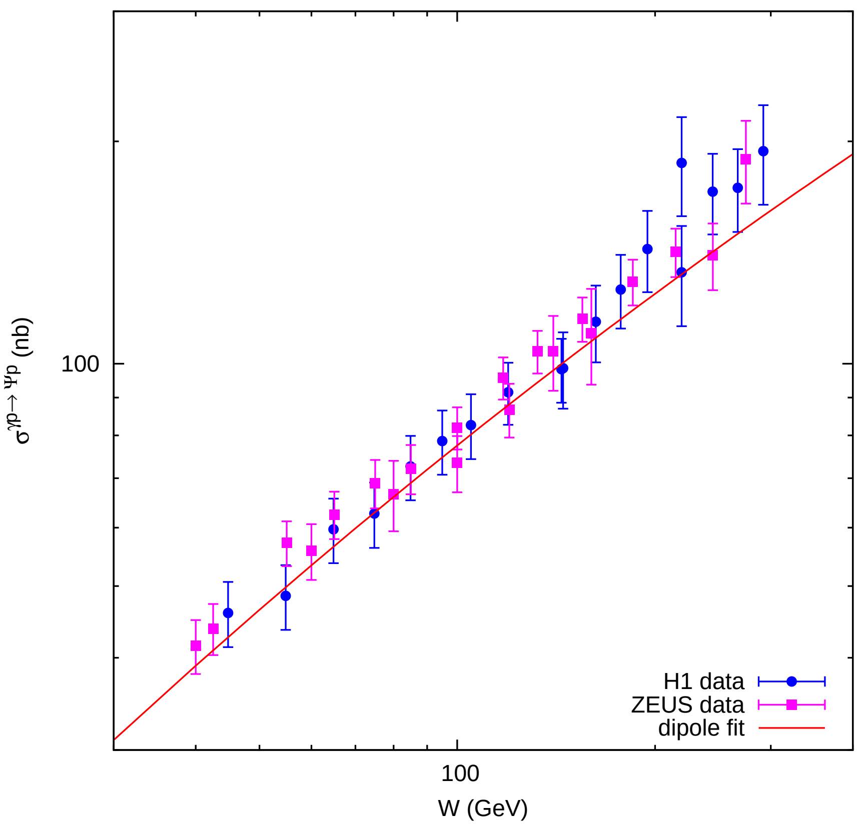


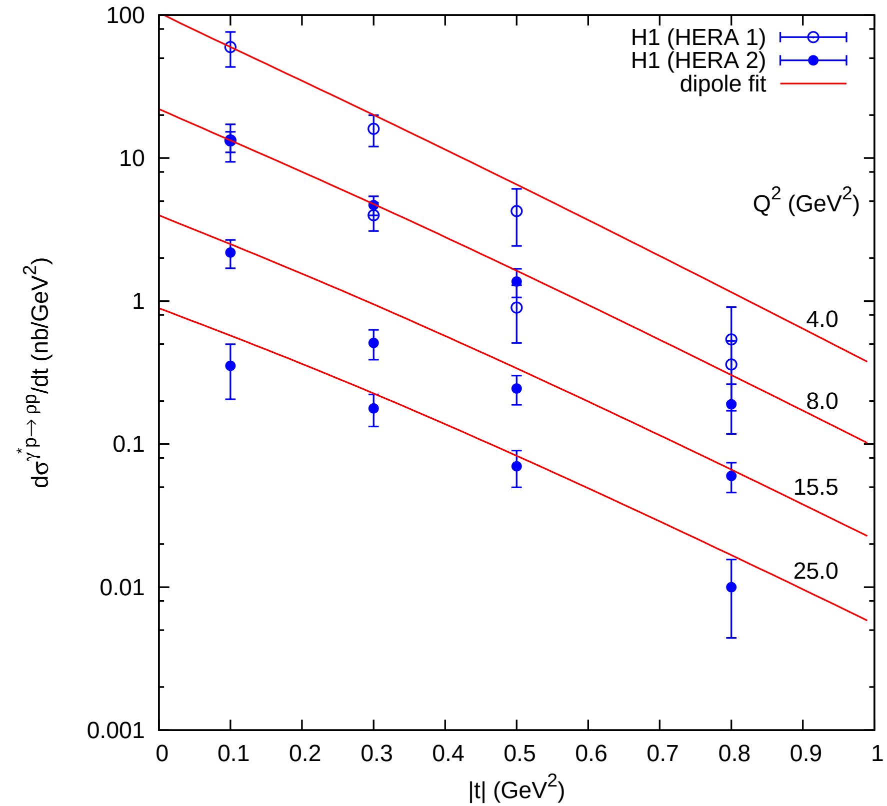
Vector mesons and DVCS

Example: continued

$$\gamma p \to J/\Psi p$$

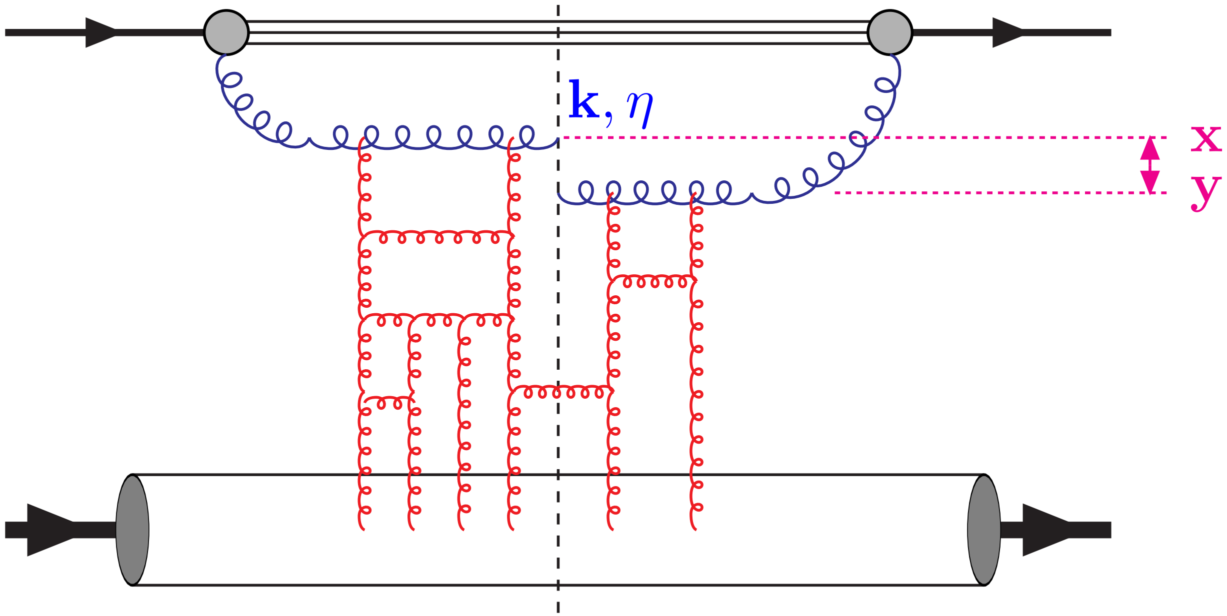
Pred. for DVCS





Particle production at RHIC

Production of a (forward) gluon, momentum $\mathbf{k}$, rapidity η



Including fragmentation to produce hadrons, this gives:

$$\frac{d^3N}{d\eta\, d^2k_t} = \frac{K}{(2\pi)^2}\int_{x_F}^{1} d\xi\, \frac{\xi}{x_F} q(\xi, k_t^2) D_{h/q}\left(\frac{x_F}{\xi}, k_t^2\right) \tilde{T}\left(\frac{\xi k_t}{x_F}, 2\eta + \log\left(\frac{1}{\xi}\right)\right) + \text{gluon}$$

with $x_F = \frac{k_t}{\sqrt{s}} \exp(-\eta)$ and $\tilde{T}(k_t, y) = \int d^2r\, e^{i\mathbf{k_t}\cdot\mathbf{r}}\, T(r, y)$

Particle production at RHIC

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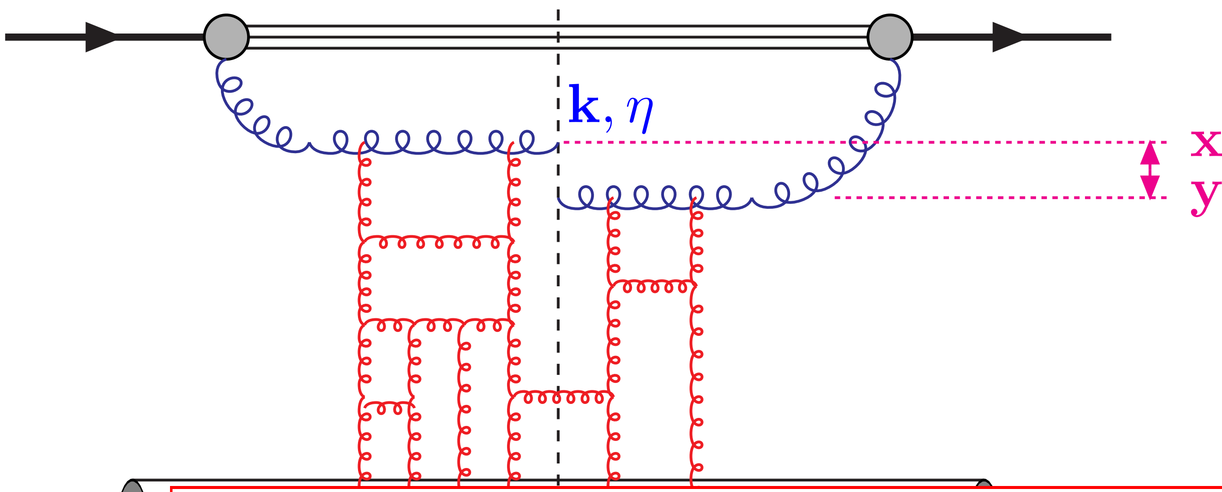

- Kharzeev, Kovchegov, Tuchin (04)
  Dumitru, Hayashigaki and Jalilian-Marian (05)
  Boer, Uterman, Wessels (07)
- no (convincing) descriptions of HERA and RHIC at the same time

Including

$$\frac{d^3N}{d\eta\, d^2k_t} \quad \ldots \text{og}\left(\frac{1}{\xi}\right)\Big) + \text{gluon}$$

with $x_F = \frac{k_t}{\sqrt{s}} \exp(-\eta)$ and $\tilde{T}(k_t, y) = \int d^2r\, e^{i\mathbf{k_t}\cdot\mathbf{r}}\, T(r, y)$

Conclusions

- Part 1: QCD evolution
  - BFKL resums $\alpha_s^n \log^n(1/x)$ to all orders
  - Saturation introduces non-linearities (BK, restores unitarity)
  - fluctuations $\leftrightarrow$ pomeron loops: reaction-diffusion system
- Part 2: solutions for the amplitude
  - Geometric scaling : predicted from the BK equation
    valid for inclusive and t-dependent processes
  - Larger rapidities: Pomeron loops $\Rightarrow$ diffusive scaling
  - Deep links with statistical phyics ((s)F-KPP equation)
- Part 3: Phenomenology
  - common descriptions of inculsive and exclusive DIS data
  - Models for particle production at RHIC (probably incomplete)

TODO

Generic/interesting open-problems

- Part 1: QCD evolution
  - NLO BK
  - pomeron loops: relaxing assumptions
  - links with confinement
- Part 2: solutions for the amplitude
  - cornerstone for understanding
  - cornerstone between “formal” equations and phenomenology
  - better understanding of the links with stat.phys. + proper QCD props.
- Part 3: Phenomenology
  - combined RHIC/HERA description $\longrightarrow$ LHC forward physics
  - links saturation $\longleftrightarrow$ underlying event