

A History of defining jets

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In collaboration with Gavin Salam, Matteo Cacciari and Juan Rojo

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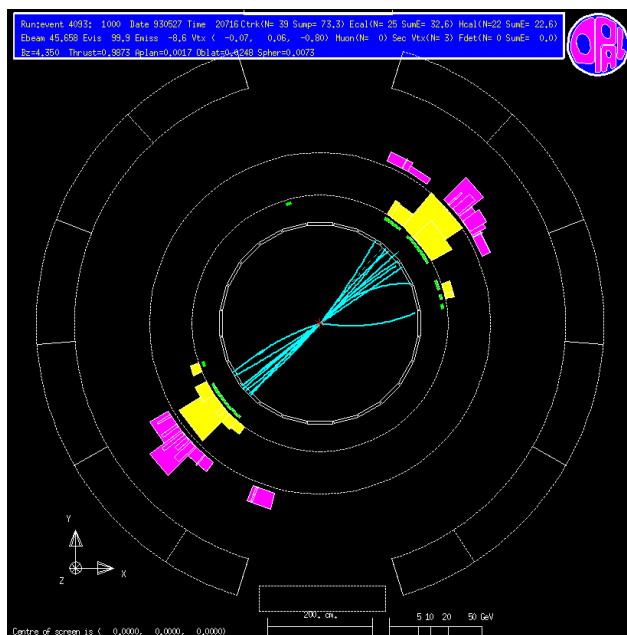
- **The past/present:** Jet algorithms and jet definitions
 - **basic ideas:** why jets? recombinations and cones
 - **failures** of the 20th-century cone algorithms
 - **new algorithms** in the 21st-century without the failures

- **The present/future:** how to better use the tools we have?
 - UE sensitivity
keywords: *jet areas, subtraction, filtering*
 - boosted taggers (Higgs, top)
keywords: *same as above + subjets*

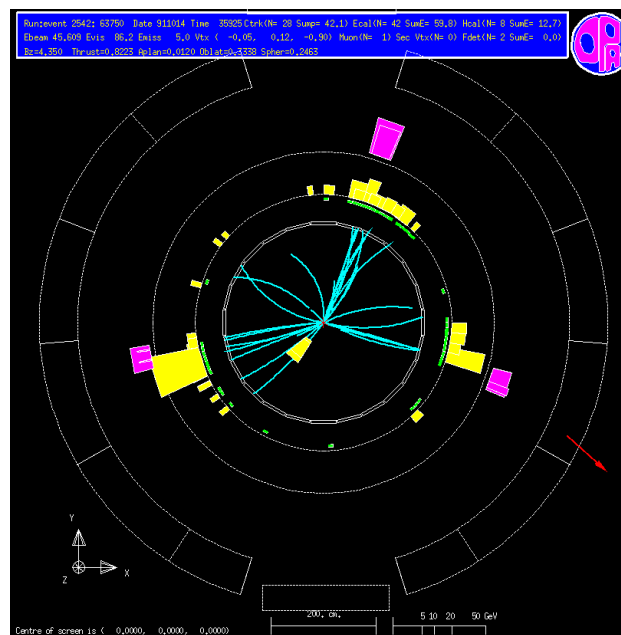
Motivation: why jets

Collinear divergence \Rightarrow QCD produces “jetty” showers

Example: LEP (OPAL) events



2 jets



3 jets

“Jets” \equiv bunch of collimated particles \cong hard partons

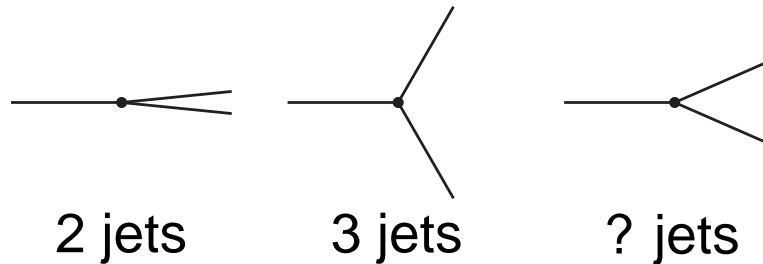
Motivation: why jets

Collinear divergence \Rightarrow QCD produces “jetty” showers

“Jets” \equiv bunch of collimated particles \cong hard partons

BUT

- a “parton” is an ambiguous concept (NLO)
- “collinear” has some arbitrariness



Motivation: why jets

Collinear divergence \Rightarrow QCD produces “jetty” showers

“Jets” \equiv bunch of collimated particles \cong hard partons

BUT

- a
- “c

In practice: use of a jet definition

particles $\{p_i\}$ $\xrightarrow[\text{definition}]{\text{jet}}$ jets $\{j_k\}$

Jet algorithm: the recipe (insufficient!)

Jet definition: algorithm + the parameters

20th century jet algorithms

Recombination:

- k_t algorithm
- Cambridge/Aachen alg.

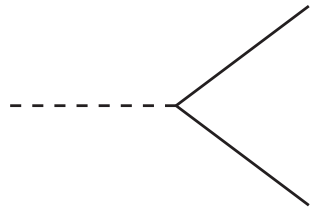
Cone:

- CDF JetClu
- CDF MidPoint
- D0 (run II) Cone
- PxCone
- ATLAS Cone
- CMS Iterative Cone
- PyCell/CellJet
- GetJet

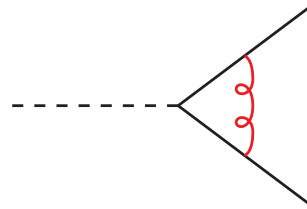
How has that list changed recently?

IR-Coll safety for jets

Ingredient: QCD soft and collinear divergencies

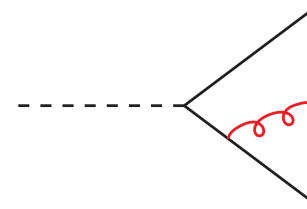


LO



NLO(virt)

∞



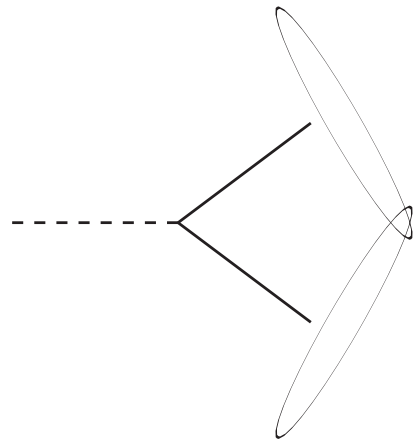
NLO(real)

∞

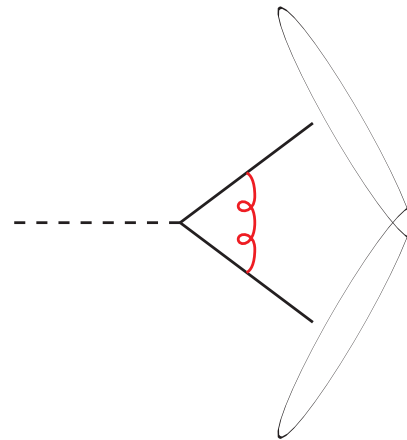
• ∞ (from soft gluons) cancel (inclusive x-section)

IR-Coll safety for jets

Ingredient: QCD soft and collinear divergencies

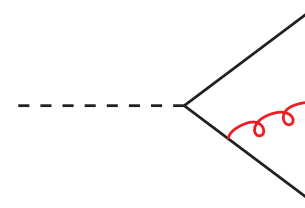


LO



NLO(virt)

∞



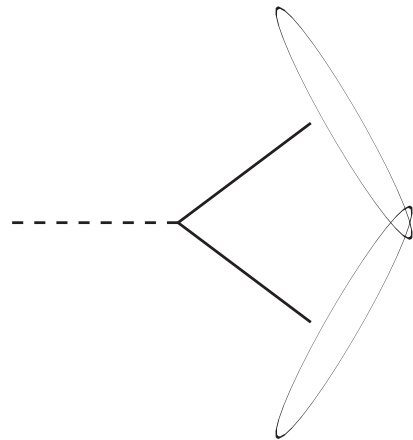
NLO(real)

∞

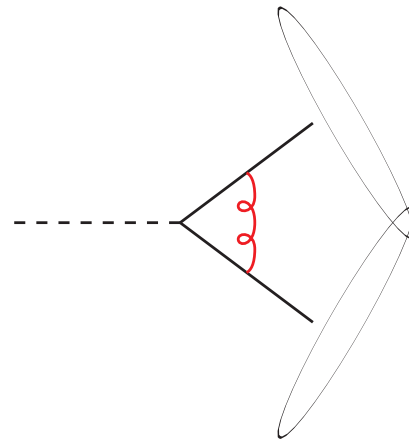
- Consider an extra (NLO) **soft** gluon
- Assume LO gives 2 jets \Rightarrow NLO(virt) gives 2 jets

IR-Coll safety for jets

Ingredient: QCD soft and collinear divergencies

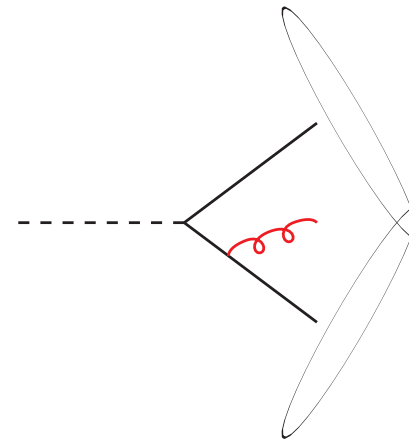


LO



NLO(virt)

∞



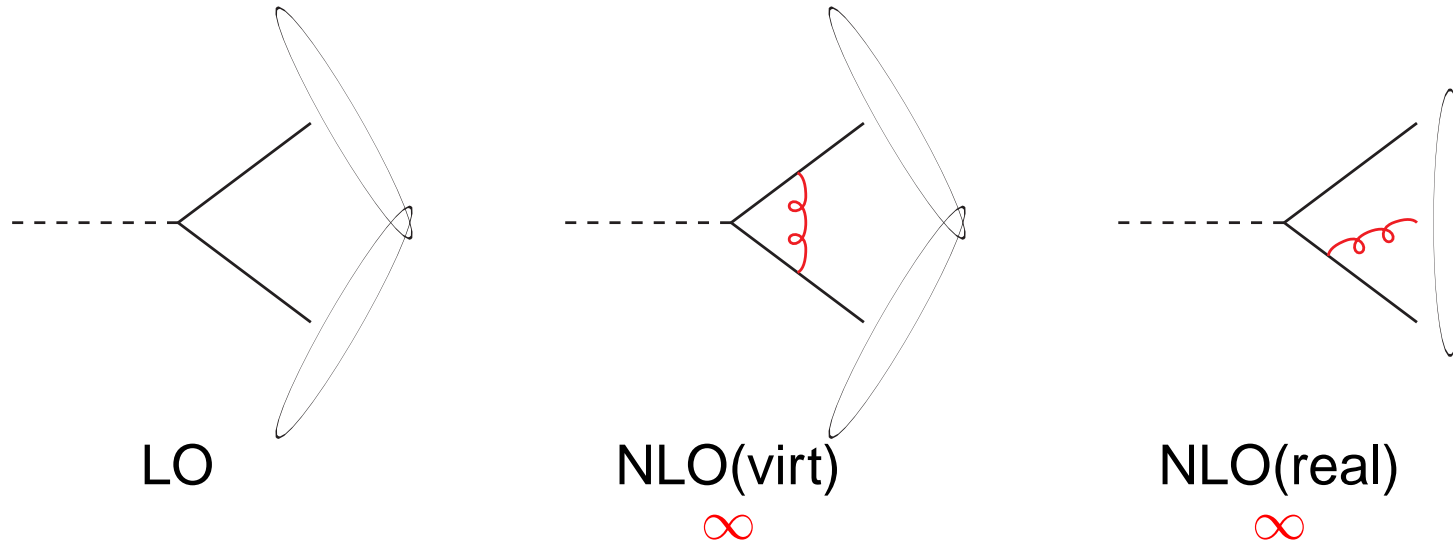
NLO(real)

∞

- Consider an extra (NLO) **soft** gluon
- Assume LO gives 2 jets \Rightarrow NLO(virt) gives 2 jets
- NLO(real) gives 2 jets $\Rightarrow \infty$ cancel \Rightarrow finite jet cross-section

IR-Coll safety for jets

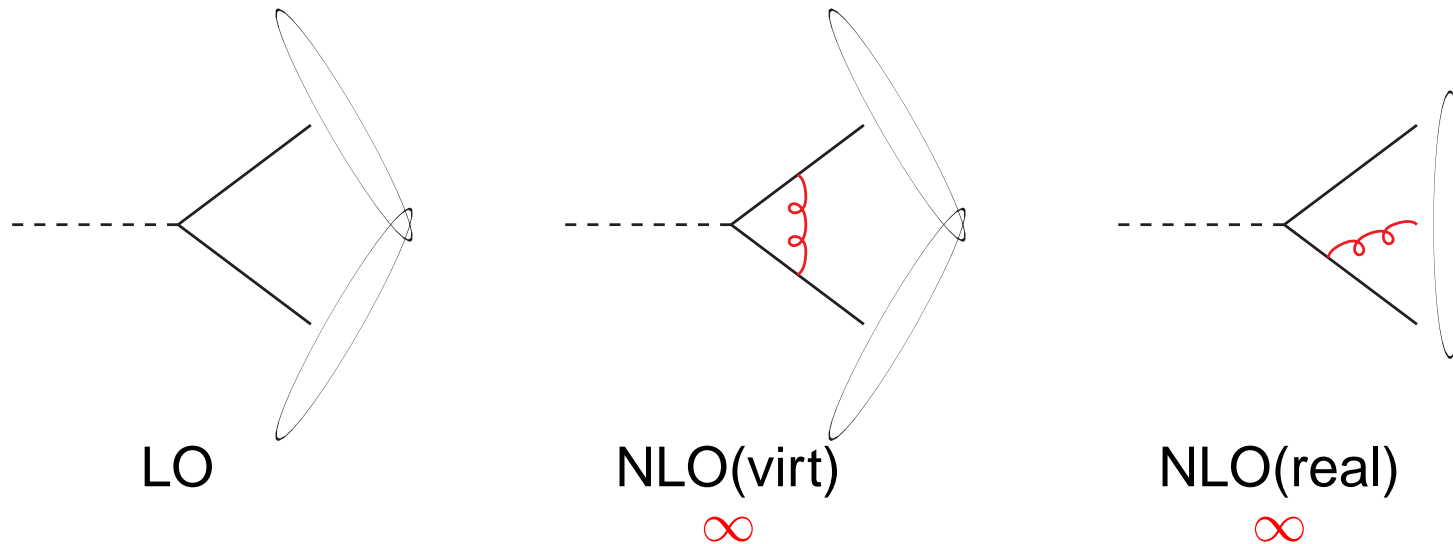
Ingredient: QCD soft and collinear divergencies



- Consider an extra (NLO) **soft** gluon
- Assume LO gives 2 jets \Rightarrow NLO(virt) gives 2 jets
- NLO(real) gives 2 jets $\Rightarrow \infty$ cancel \Rightarrow finite jet cross-section
NLO(real) gives 1 jets $\Rightarrow \infty$ do not cancel \Rightarrow infinite jet x-section

IR-Coll safety for jets

Ingredient: QCD soft and collinear divergencies



For pQCD to make sense, the (hard) jets should not change when

- one has a soft emission *i.e.* adds a very soft gluon
- one has a collinear splitting
i.e. replaces one parton by two at the same place (η, ϕ)

[SNOWMASS Accords, Fermilab, 1990]

21st century jet algorithms

Recombination:

- k_t algorithm
- Cambridge/Aachen alg.
- **anti- k_t algorithm**

4 available
safe algorithms

anti- k_t adopted as default
by ATLAS and CMS

Cone:

- CDF JetClu
- CDF MidPoint
- D0 (run II) Cone
- PxCone
- ATLAS Cone
- CMS Iterative Cone
- PyCell/CellJet
- GetJet
- **SISCone**

[G.Salam, GS, 07]

[M.Cacciari, G.Salam, GS, 08]

Inside the black box

- **Recombination**: recombine closest pair until $d > R$

$$d_{ij} = \min \left(k_{t,i}^{2p}, k_{t,j}^{2p} \right) \left(\Delta y_{ij}^2 + \Delta \phi_{ij}^2 \right)$$

- k_t ($p = 1$): **the closest to QCD**
matches the soft and collinear divergences of QCD
 - Cambridge/Achem ($p = 0$): **the simplest**
close to k_t in many respects
 - anti- k_t ($p = -1$): **gives circular/rigid hard jets**
calibration advantages, a safe CMS iterative cone
- **Cone**: find a direction of energy flow stable cone \equiv the total momentum points towards the centre of the cone + split-merge for the overlap
 - SISCone: **reduced sensitivity to the UE**
a safe CDF JetClu/Midpoint, D0 MidPoint, ATLAS Cone

Inside the black box

- **Recombination**: recombine closest pair until $d > R$

$$d_{ij} = \min \left(k_{t,i}^{2p}, k_{t,j}^{2p} \right) \left(\Delta y_{ij}^2 + \Delta \phi_{ij}^2 \right)$$

- k_t ($p = 1$): the closest to QCD

matches the soft and collinear divergences of QCD

```
#-----  
#                               FastJet release 2.4  
#       Written by M. Cacciari, G.P. Salam and G. Soyez  
#                               http://www.fastjet.fr  
#-----
```

All those algorithms (and much more)
implemented (efficiently) in FastJet

points towards the centre of the cone + split-merge for the overlap

- SISCone: reduced sensitivity to the UE

a safe CDF JetClu/Midpoint, D0 MidPoint, ATLAS Cone

We (finally) have a good set of tools

Can we do better?

A growing list

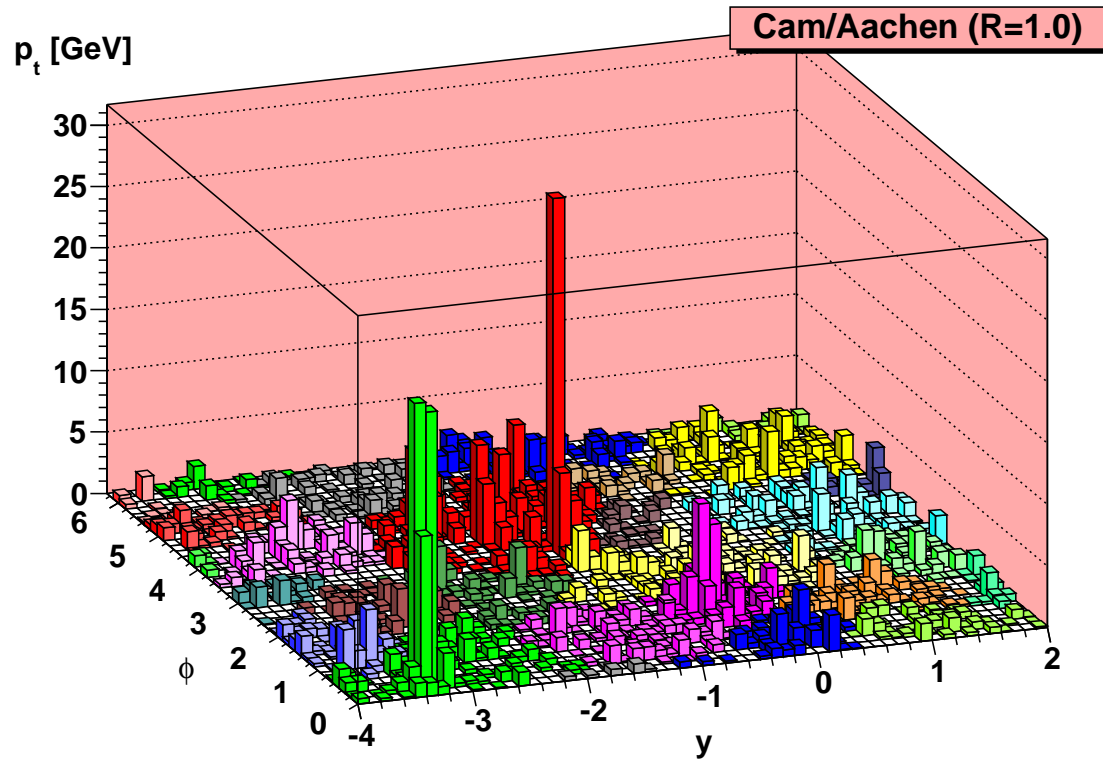
Many ideas and applications:

- Handle soft backgrounds
 - ✓ tune the algorithm, filter
 - ex.: kinematic dijet reconstruction
 - ✓ jet areas and background subtraction
 - UE, pileup, heavy-ion background subtraction
- Tag boosted objects (mostly for discovery purposes)
 - ✓ subjet analysis
 - $H \rightarrow b\bar{b}$, $t \rightarrow bq\bar{q}$, $\tilde{\chi}_0^1 \rightarrow qqq$, ...

I will only give a brief overview of what can be done

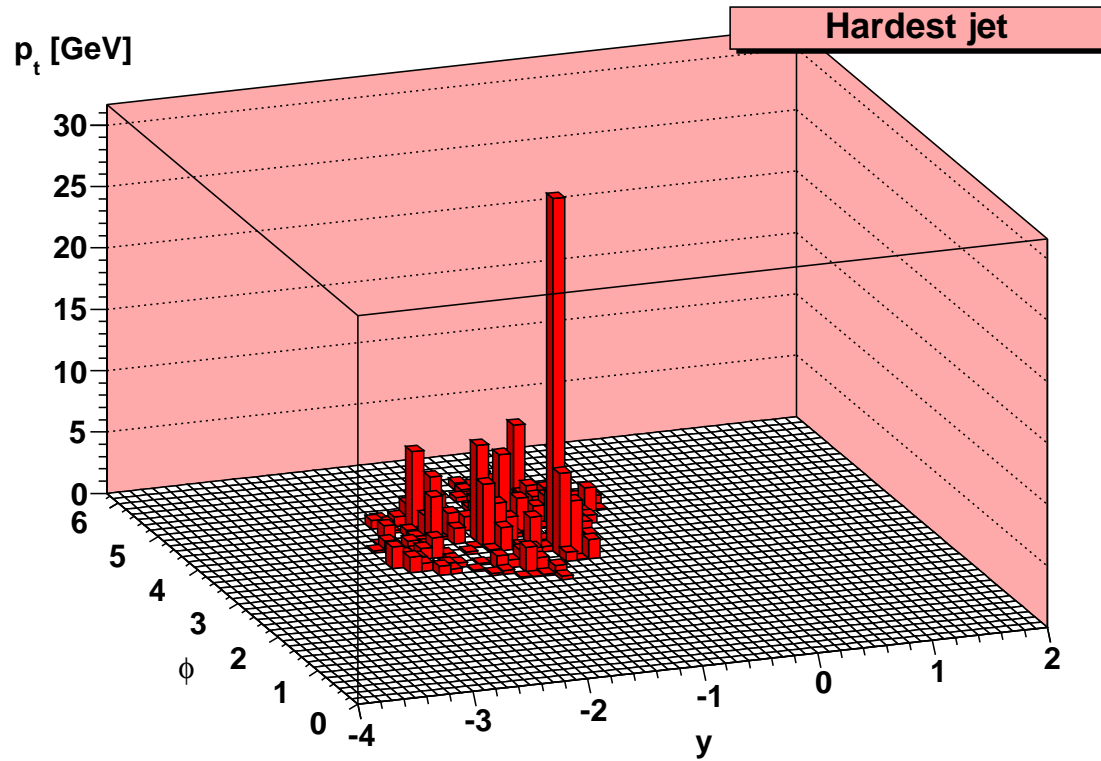
New idea #1: filtering

Filtering



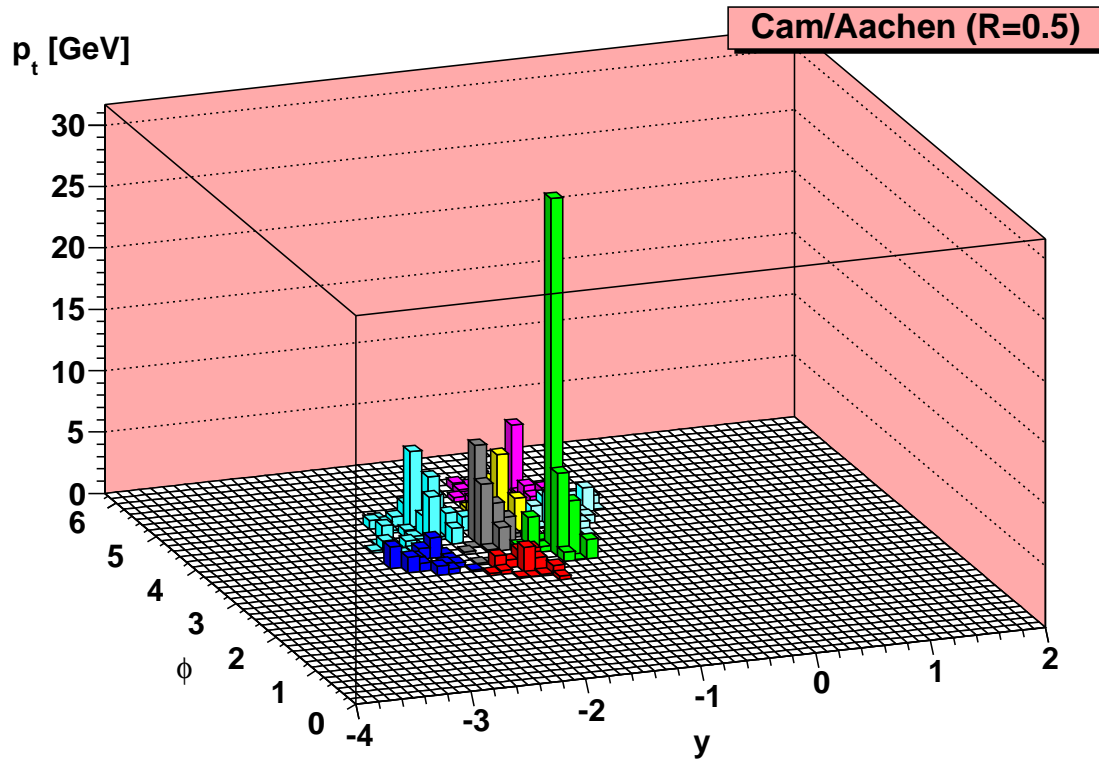
- cluster with Cambridge/Aachen(R)

Filtering



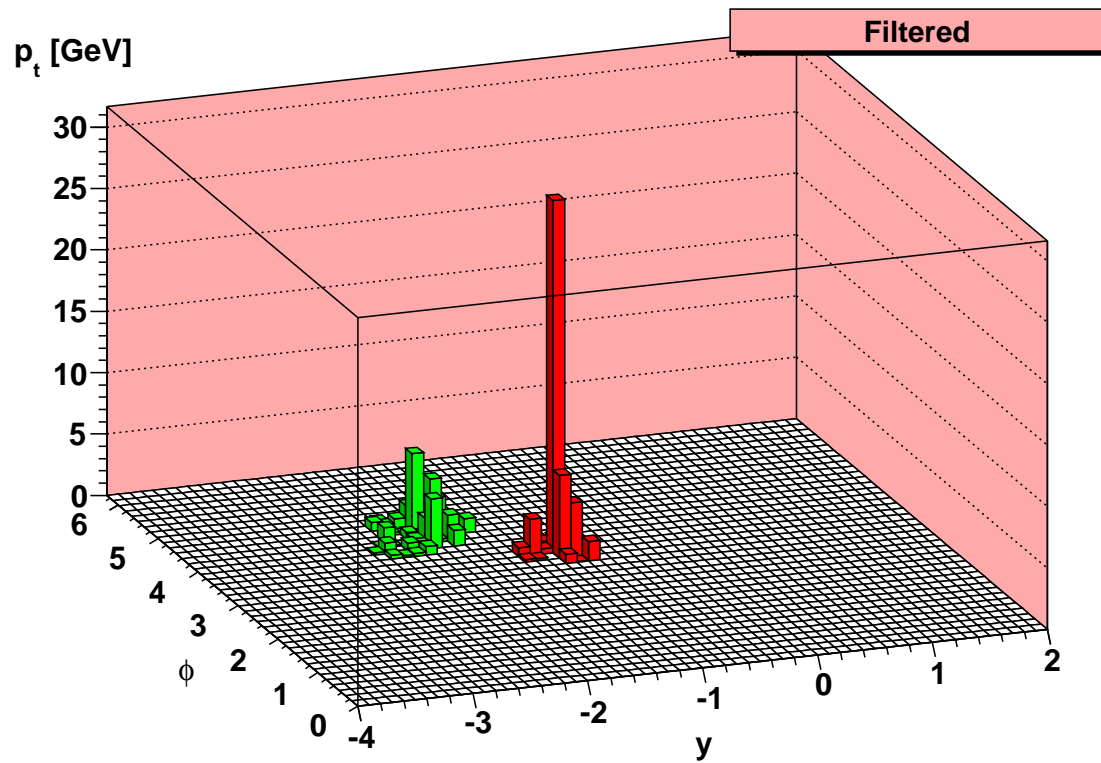
- cluster with Cambridge/Aachen(R)
- for each jet

Filtering



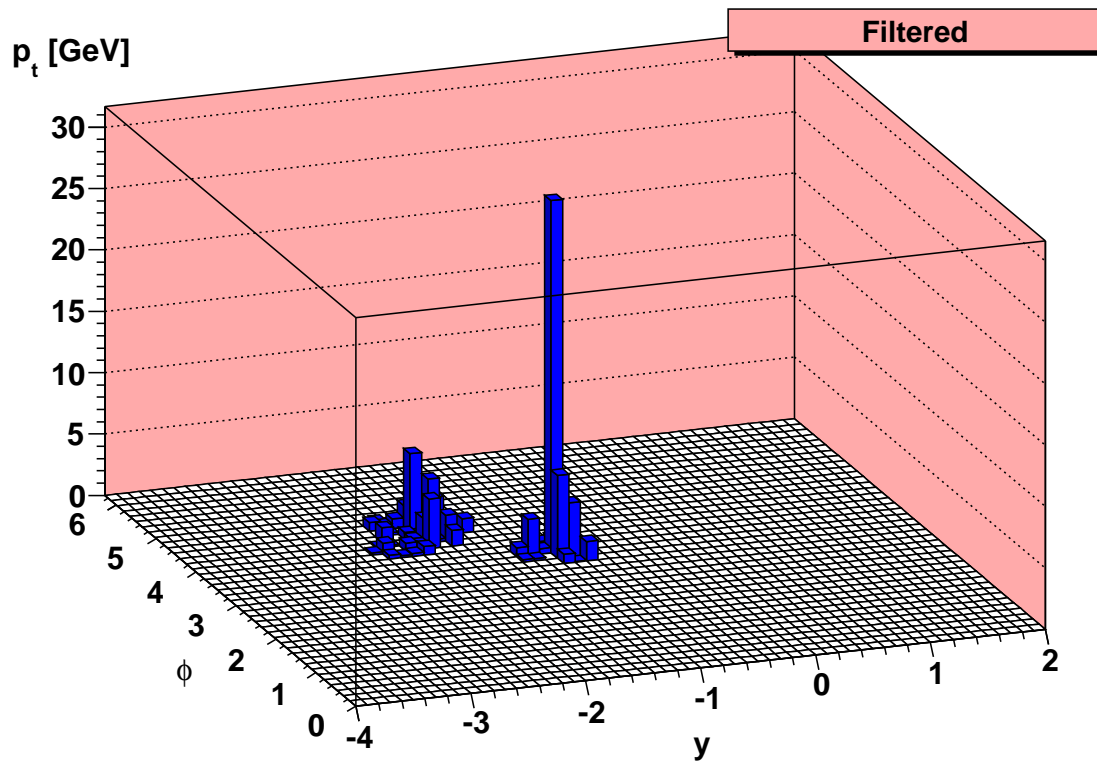
- cluster with Cambridge/Aachen(R)
- for each jet
 - recluster with Cambridge/Aachen(R/2)

Filtering



- cluster with Cambridge/Aachen(R)
- for each jet
 - recluster with Cambridge/Aachen(R/2)
 - keep the 2 hardest subjects

Filtering



- cluster with Cambridge/Aachen(R)
- for each jet
 - recluster with Cambridge/Aachen(R/2)
 - keep the 2 hardest subjects

Idea:

- ✓ keep perturb. radiation
- ✓ remove UE

- Proven useful for boosted jet $H \rightarrow b\bar{b}$ tagging

[J.Butterworth, A.Davison, M.Rubin, G.Salam, 08]

- Proven useful for kinematic reconstructions

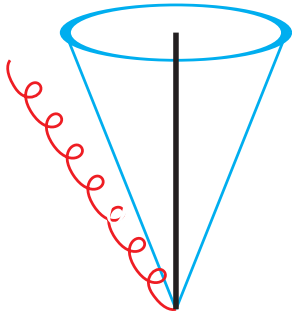
[M.Cacciari, J.Rojo, G.Salam, GS, 08]

***Application:
optimise dijet kinematic reconstruction***

The pQCD-vs-UE paradox

Competition between

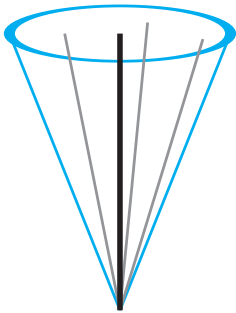
- catching perturbative radiation



Out-of-cone radiation:

$$\langle \delta p_t \rangle \propto - \int_R \frac{d\theta}{\theta} \sim -\log(1/R)$$

- not catching soft background radiation (underlying event)



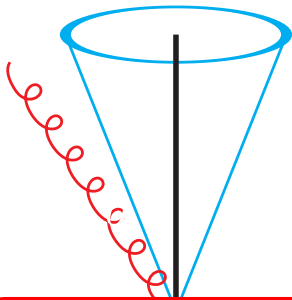
$$\langle \delta p_t \rangle \sim \text{Soft contents} \propto \text{jet area} \sim R^2$$

the coefficients depend on the algorithm

The pQCD-vs-UE paradox

Competition between

- catching perturbative radiation

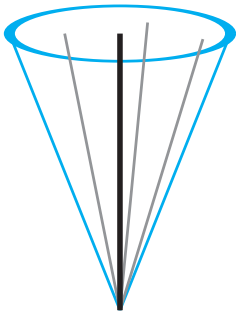


Out-of-cone radiation:

$$\langle \delta p_t \rangle \propto - \int_R \frac{d\theta}{\theta} \sim -\log(1/R)$$

What is the optimal jet definition (algo+ R)?

- not catching soft content



$$\langle \delta p_t \rangle \sim \text{Soft contents} \propto \text{jet area} \sim R^2$$

the coefficients depend on the algorithm

Optimisation: dijet reconstruction

Example process to illustrate various effects:

$$Z' \rightarrow q\bar{q} \rightarrow 2 \text{ jets}$$

- $M_{Z'}$ can be varied (between 100 GeV and 4 TeV)
- Also valid for $H \rightarrow gg$ to study gluon jets
- Reconstruction method:
 - get the 2 hardest jets: j_1 and j_2
 - reconstruct the Z' : $m_{Z'} = (j_1 + j_2)^2$

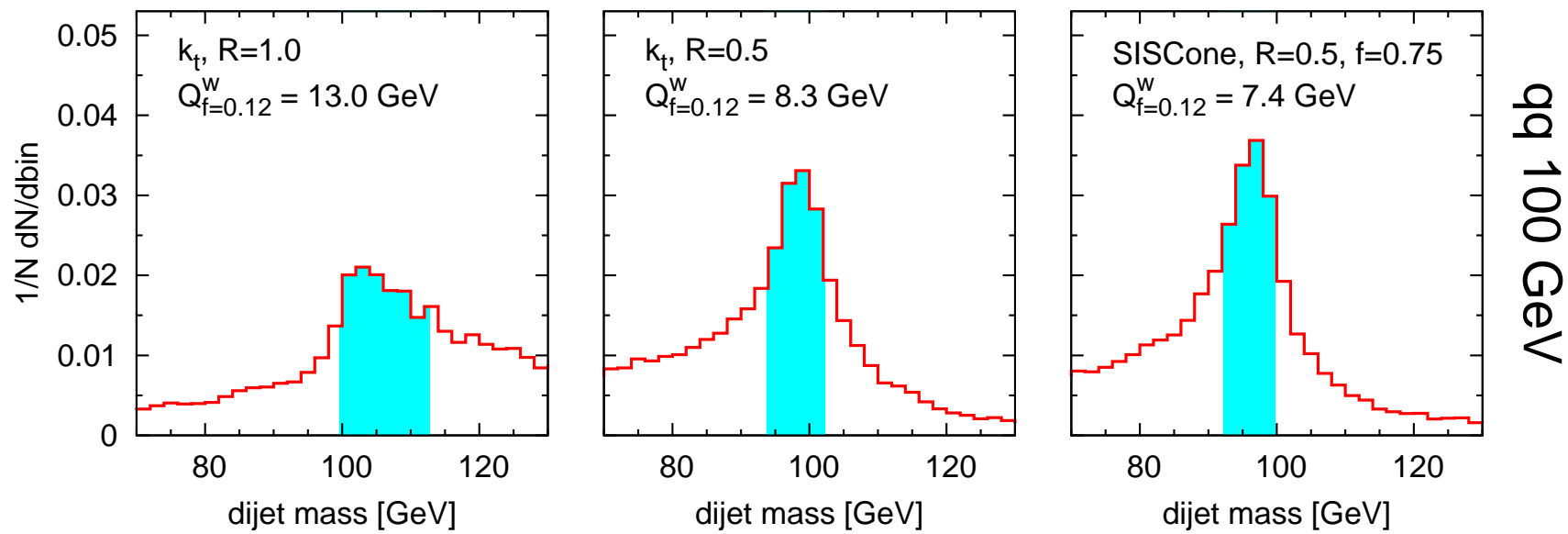
Look how the mass peak is reconstructed

- Also $t\bar{t}$ with full hadronic decay for multijet tests

Optimisation: quality measure

Measure of the jet reconstruction efficiency:

- Forget about measures related to parton-jet matching
 - Forget about fits depending on the shape of the peak
- ⇒ maximise the signal over background ratio (S/\sqrt{B})
a narrower peak is better.



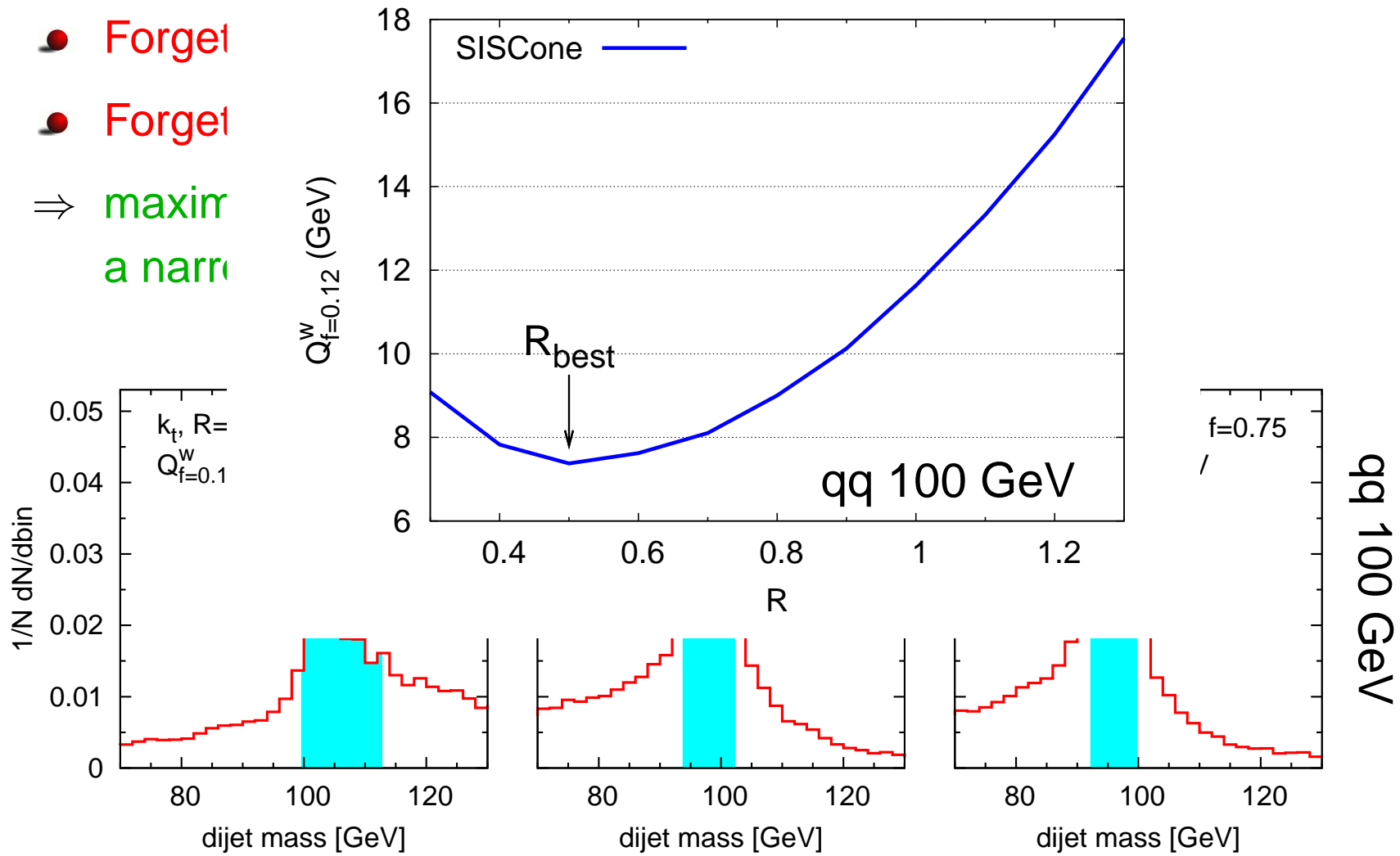
Optimisation: quality measure

Measure of the jet reconstruction efficiency:

● Forget

● Forget

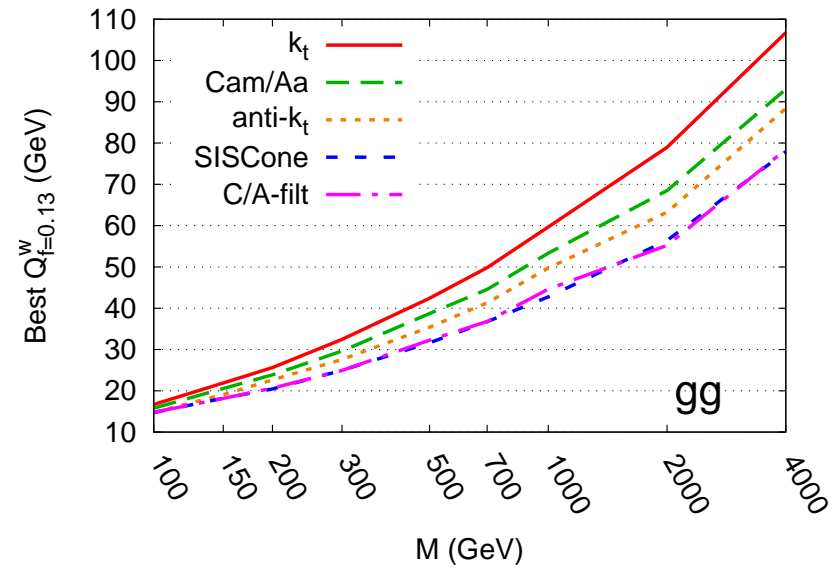
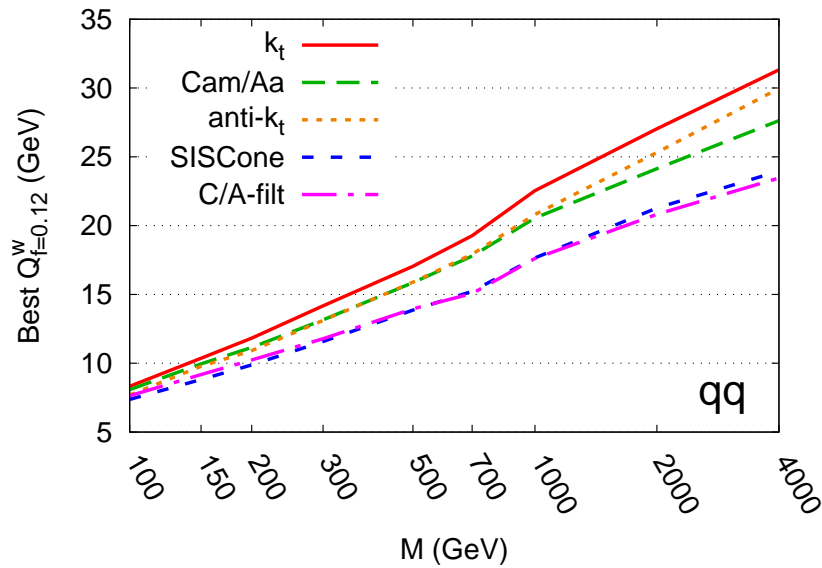
⇒ maximise
a narrow



Optimisation: best definition

[M.Cacciari, J.Rojo, G.Salam, GS, 08]

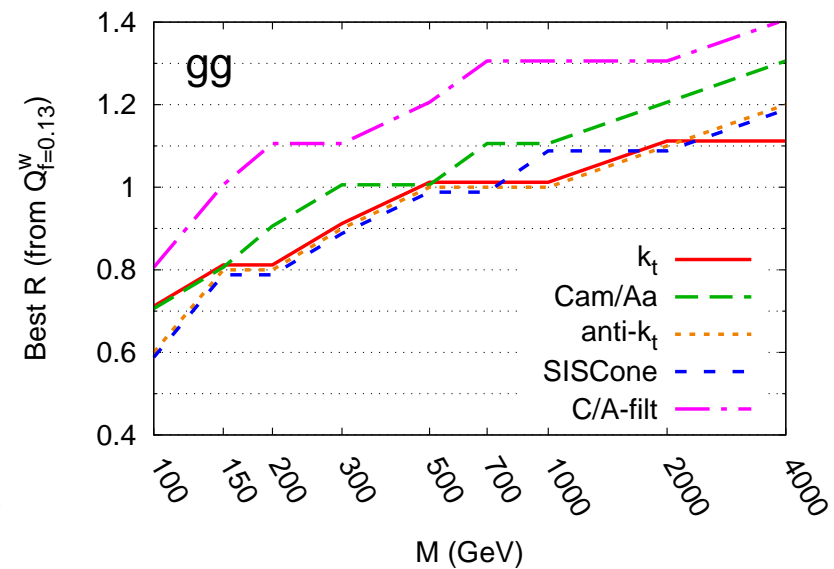
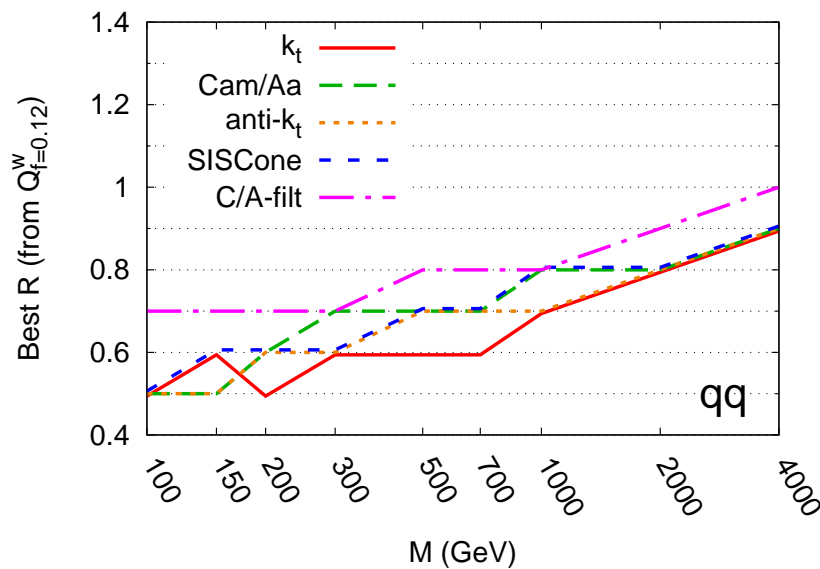
- **SISCone and C/A+filt.** do slightly better than k_t , C/A or anti- k_t



Optimisation: best definition

[M.Cacciari, J.Rojo, G.Salam, GS, 08]

- SIS Cone and C/A+filt. do slightly better than k_t , C/A or anti- k_t
- $M \nearrow \Rightarrow R_{\text{best}} \nearrow$ (and $R_{\text{best}}(g) > R_{\text{best}}(q)$)



Optimisation: quality measure \sim luminosity

Assuming a constant background,

quality measure \longrightarrow effective luminosity ratio

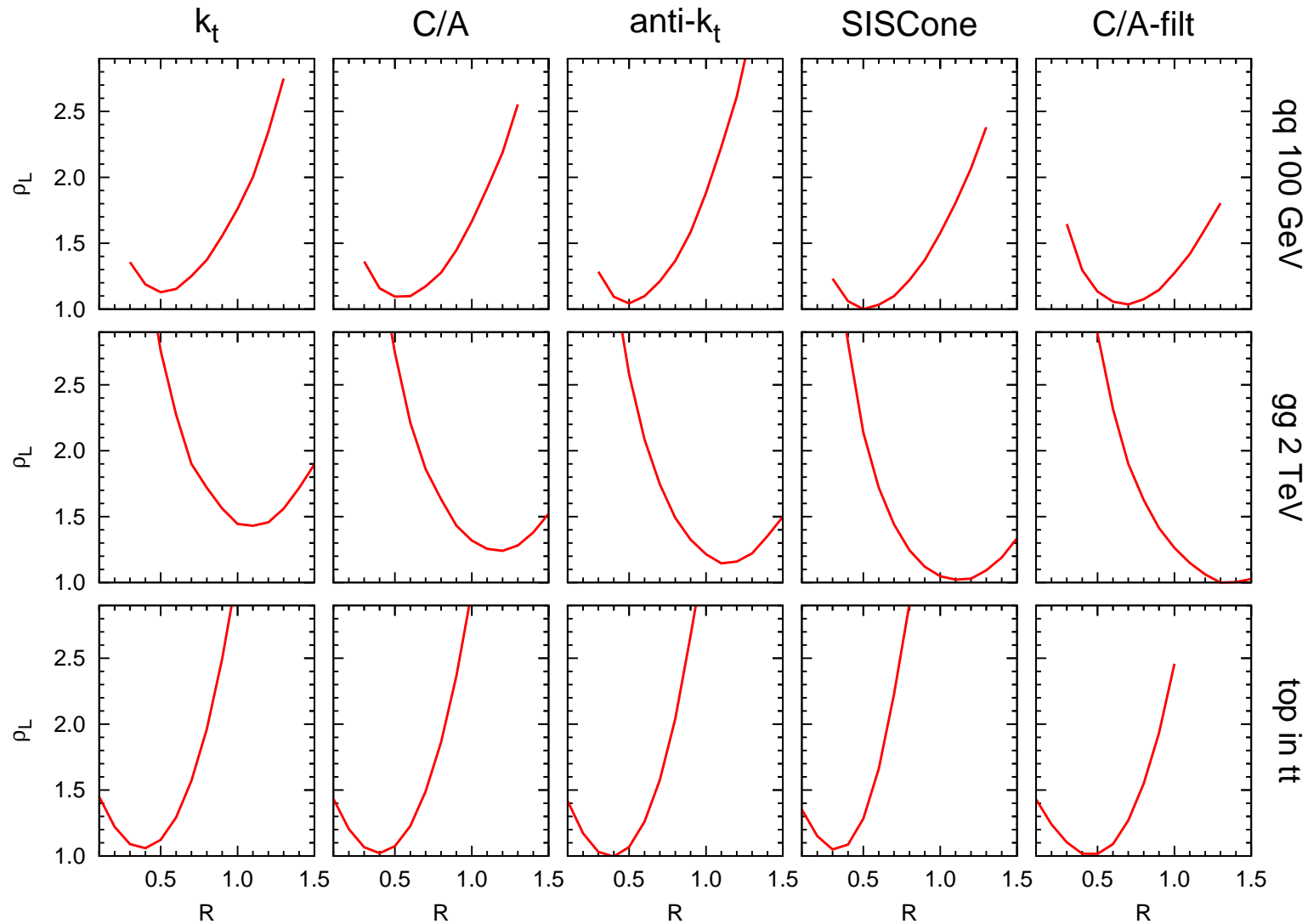
$$\rho_{\mathcal{L}}(\text{JD}_2/\text{JD}_1) = \frac{\mathcal{L} \text{ needed with JD}_2}{\mathcal{L} \text{ needed with JD}_1} = \frac{Q_{f=z}^w(\text{JD}_2)}{Q_{f=z}^w(\text{JD}_1)}$$

e.g. $\rho_{\mathcal{L}}(\text{JD}_2/\text{JD}_1) = 2$

\Leftrightarrow JD_2 requires 2 times the integrated luminosity of JD_1
to achieve the same discriminative power.

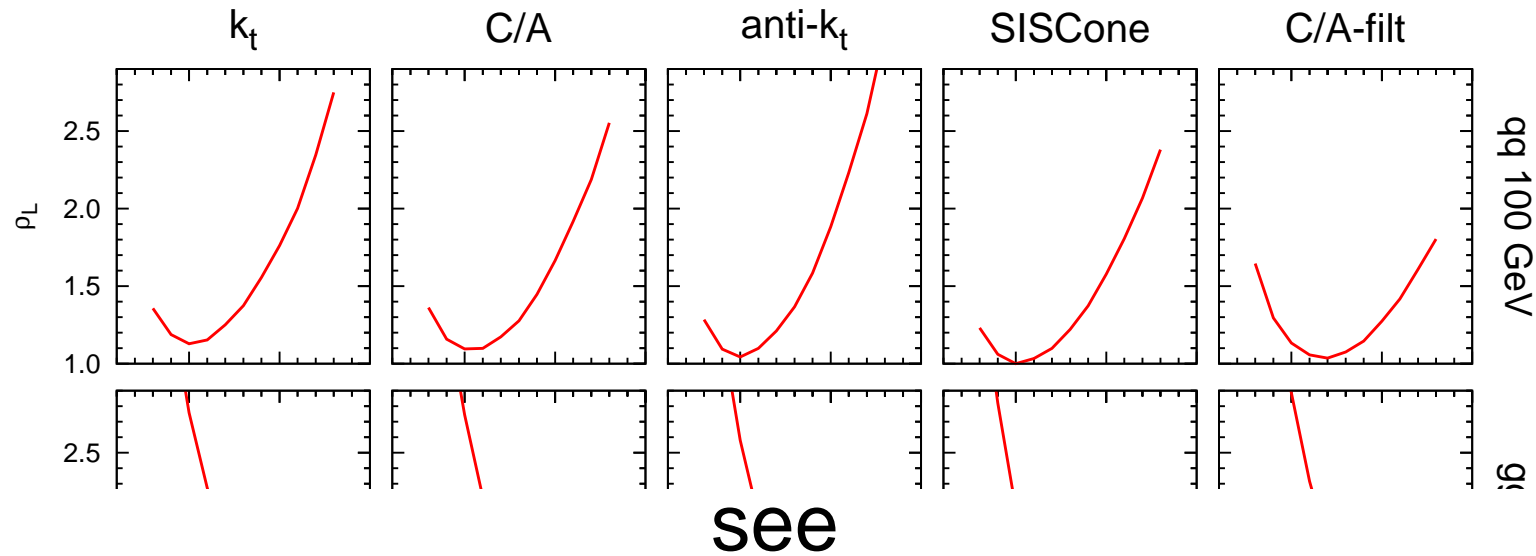
Note: results cross-checked with 2 different definitions of the quality measure

Optimisation: consequences



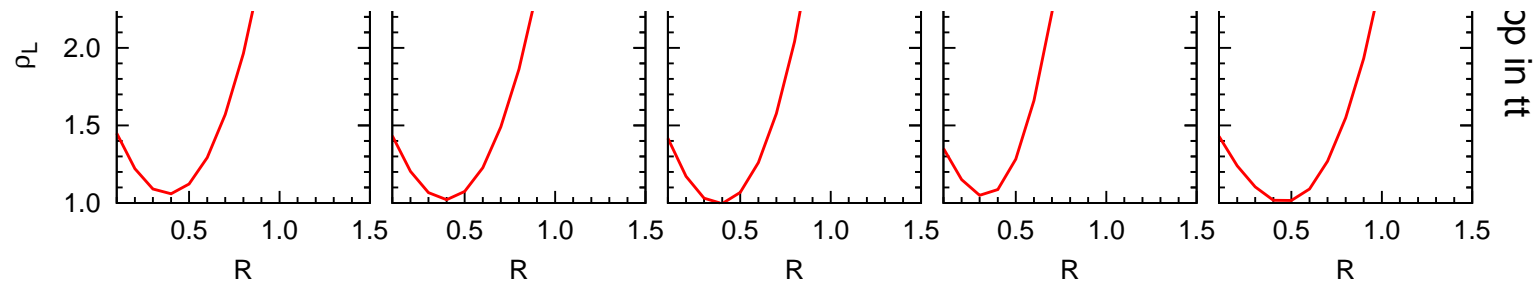
Using a single jet definition for all processes
may cost a factor ~ 2 in time for early discoveries at the LHC

Optimisation: consequences



<http://quality.fastjet.fr>

for more



Using a single jet definition for all processes
may cost a factor ~ 2 in time for early discoveries at the LHC

New idea #2: jet area and soft background subtraction

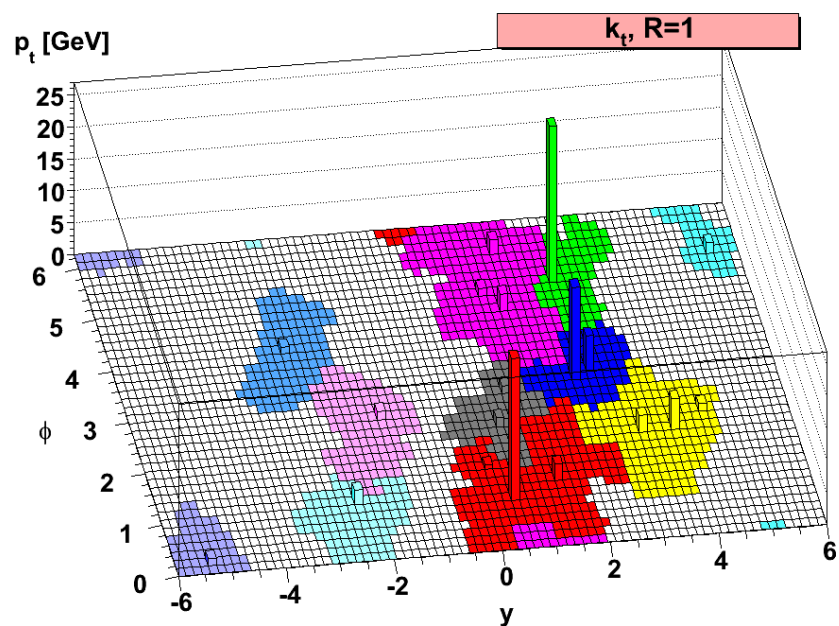
Area \equiv region where the jet catches soft particles

- Recipe: add infinitely soft particles (aka *ghosts*)
and see in which jet they are clustered
- 2 methods:
 - **Passive area**: add one ghost at a time and repeat many times
 - **Active area**: add a set of ghosts and cluster once
- Idea: ghost \approx background particle
 - \Rightarrow **active area \approx uniform background**
 - passive area \approx pointlike background**
- Notes:
 - passive = active for large multiplicities
 - require an IR-safe algorithm!
 - generic/universal definition (e.g. independent of a calorimeter)

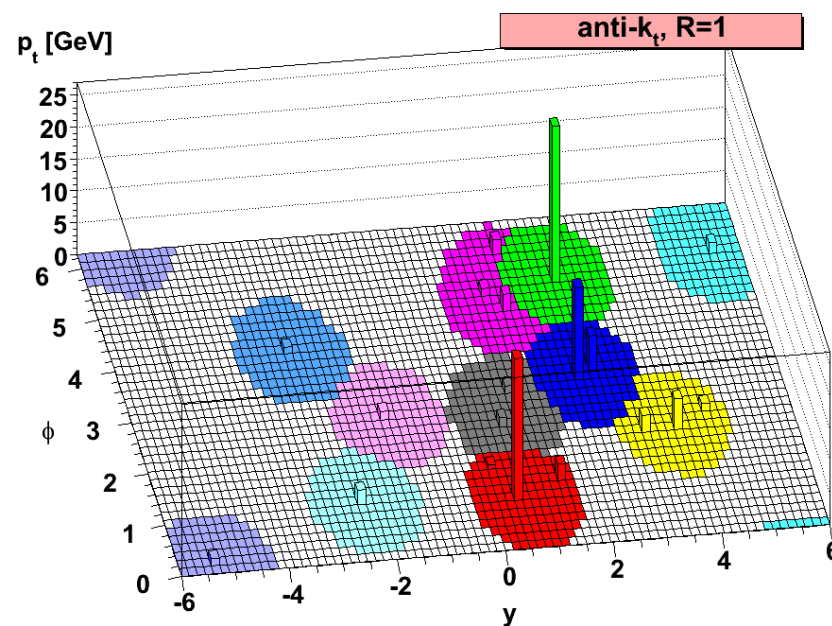
Jet area: examples

Example 1: active area for a simple event

k_t



anti- k_t



Note: analytic control

Example 2: perturbative expansion of areas (at order α_s)

$$\langle \mathcal{A}(p_t, R) \rangle = \mathcal{A}_0 + \frac{C_{F,A}}{b_0 \pi} \pi R^2 d \log \left(\frac{\alpha_s(Q_0)}{\alpha_s(Rp_t)} \right)$$

- area $\neq \pi R^2$, area \neq const.

- coefficients computable

	$\mathcal{A}_0/(\pi R^2)$		d	
	passive	active	passive	active
k_t	1	0.81	0.56	0.52
Cam/Aachen	1	0.81	0.08	0.08
anti- k_t	1	1	0	0
SISCone	1	1/4	-0.06	0.12

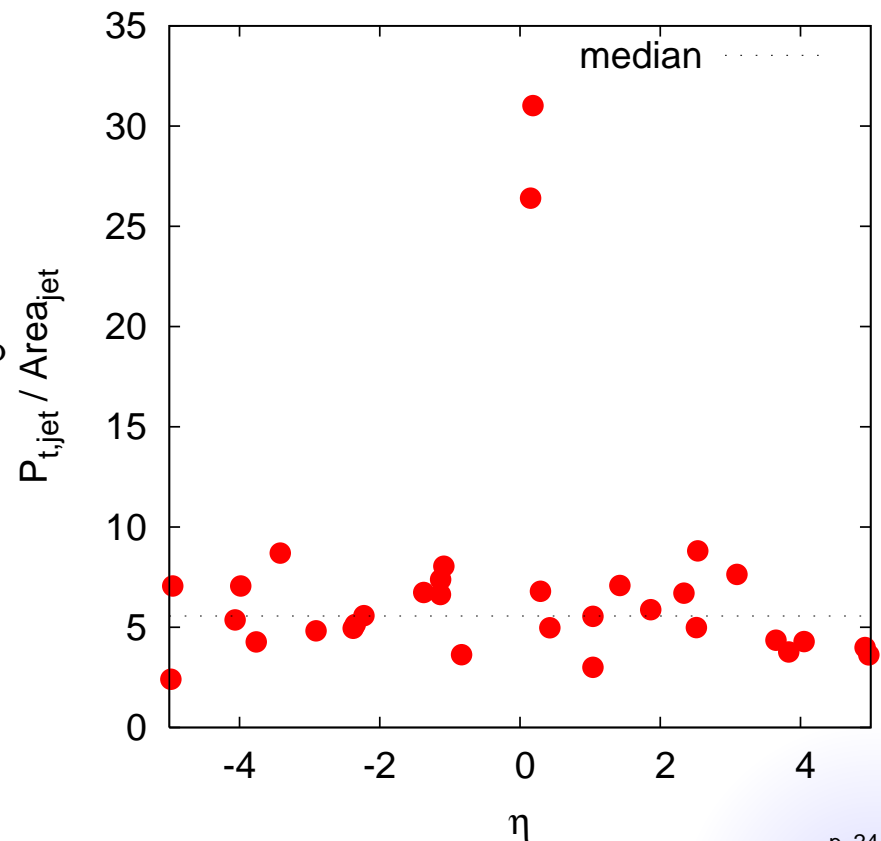
- $Q_0 \equiv$ IR regulator \propto background density

Pileup subtraction (for uniform backgrounds)

Basic idea: [M.Cacciari, G.Salam, 08]

$$p_{t,\text{subtracted}} = p_{t,\text{jet}} - \rho_{\text{pileup}} \times \text{Area}_{\text{jet}}$$

- Jet area: [M.Cacciari, G.Salam, G.S., 08]
 - region where the jet catches infinitely soft particles (active/passive)
 - analytic control and understanding in pQCD
- Pileup density per unit area: ρ_{pileup}
e.g. estimated from the median
of $p_{t,\text{jet}} / \text{Area}_{\text{jet}}$



Pileup subtraction (for uniform backgrounds)

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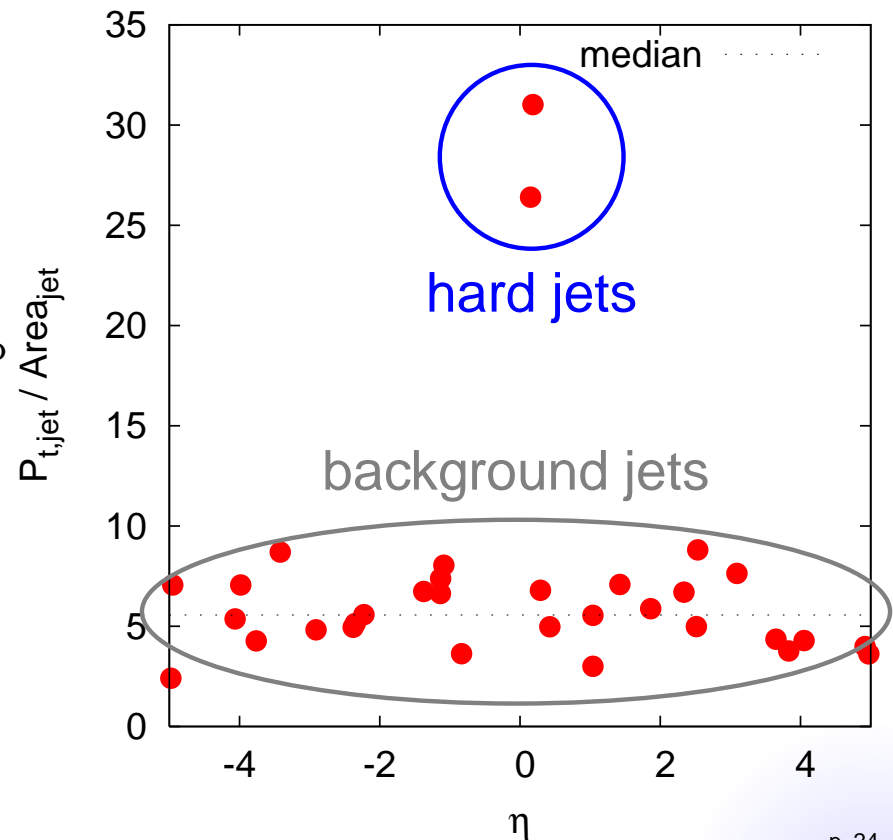
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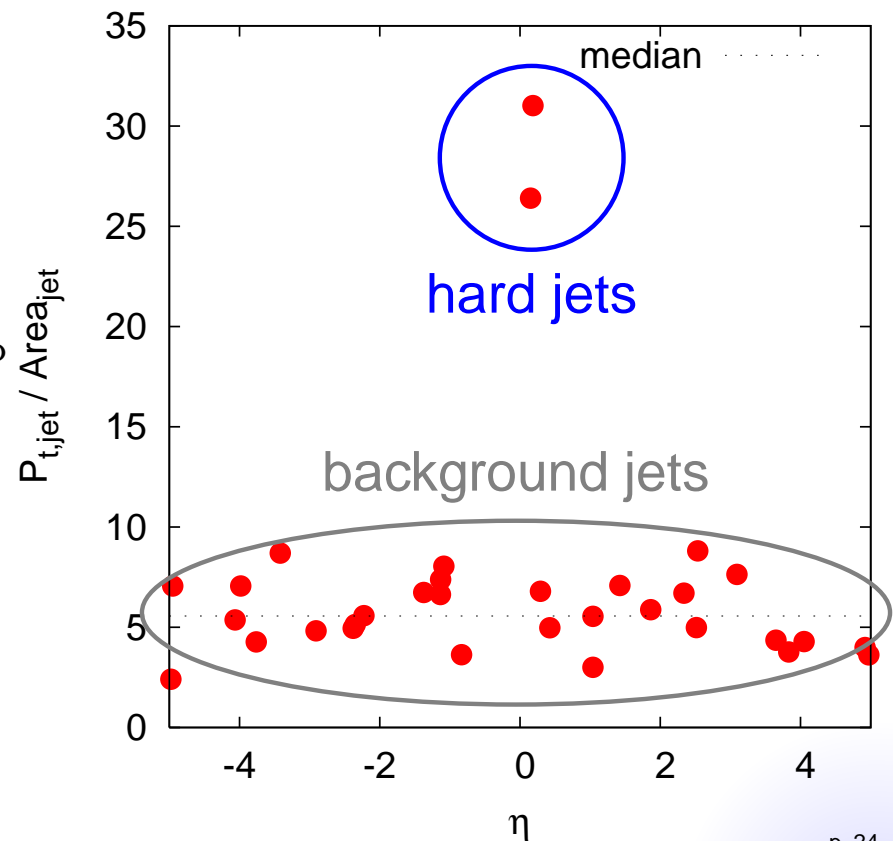
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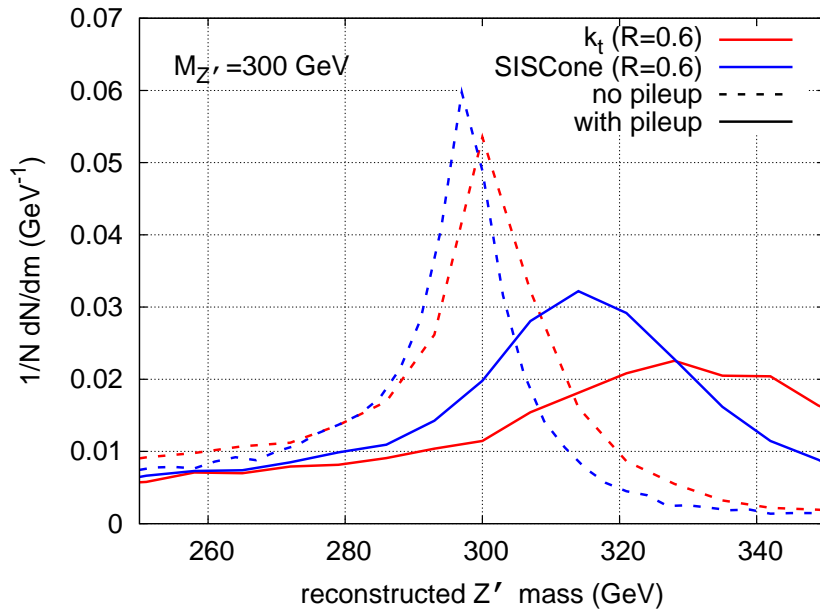
e.g. estimated from the median of $p_{t,\text{jet}} / \text{Area}_{\text{jet}}$

implemented in FastJet
on an event-by-event basis



Effect on dijet reconstruction

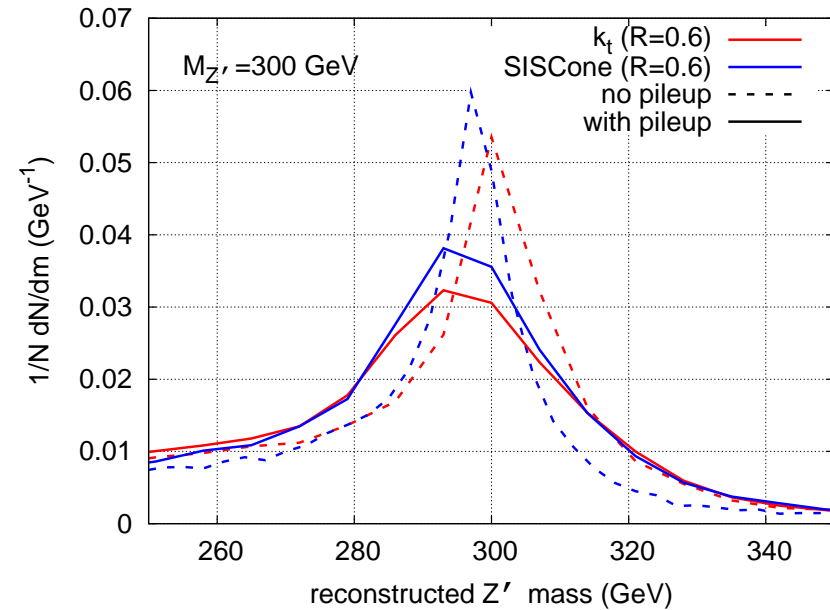
Pileup unsubtracted



width = 29.5 GeV

width = 21.0 GeV

pileup subtracted



width = 21.0 GeV

width = 17.7 GeV

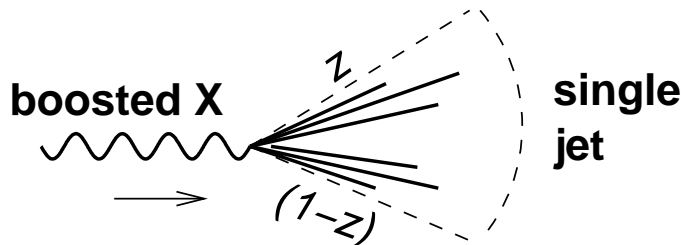
- ✓ position reasonable
- ✓ dispersion reduced (thanks to the event-by-event approach)
- ✓ used by STAR for the first jet analysis in heavy-ions

***New idea #3:
Subjets help tagging boosted objects***

Example: boosted Higgs

[J.Butterworth, A.Davison, M.Rubin, G.Salam,08]

- $H \rightarrow b\bar{b}$: dominant decay for small M_H but **large backgrounds**
- **boosted H (HW, HZ)**: many advantages (e.g. no $t\bar{t}$ background), main problem: small cross-section
- **boosted particle: decay products in the same jet**



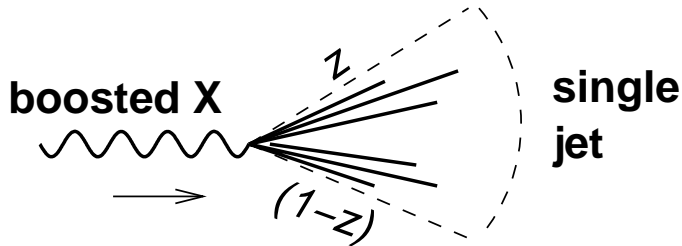
$$R \gtrsim \frac{m}{p_t} \frac{1}{\sqrt{z(1-z)}}$$

Note: other similar examples:

- boosted top
- $t\bar{t}H$
- $\tilde{\chi}^0 \rightarrow qqq$

Example: boosted Higgs

[J.Butterworth, A.Davison, M.Rubin, G.Salam,08]



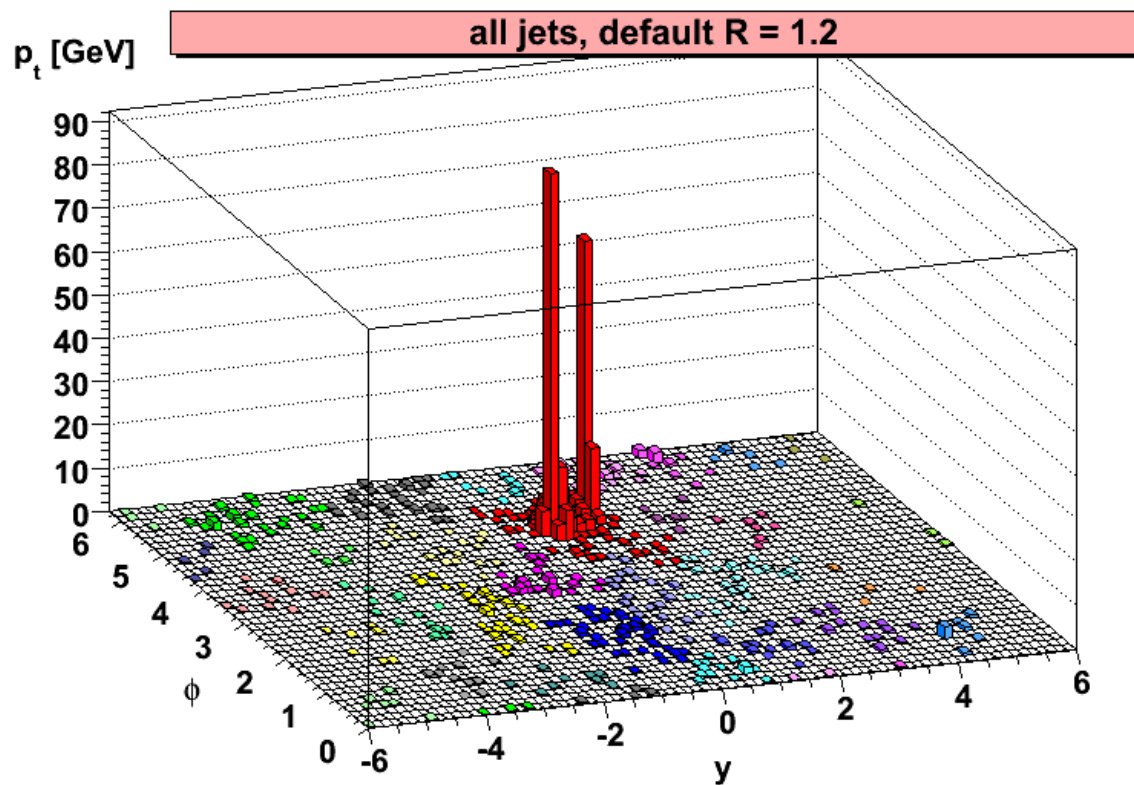
$$R \gtrsim \frac{m}{p_t} \frac{1}{\sqrt{z(1-z)}}$$

Method: start with a hard (C/A, radius R) jet j

- 1 Undo the last clustering $\rightarrow j_1, j_2$
- 2 If $\max(m_1, m_2) < 0.67m$, we have a mass drop, else back to 1
idea: find the 2 b -jets, dynamically find R_{bb}
- 3 Require symmetric splitting $y_{12} \approx \frac{\min(z_1, z_2)}{\max(z_1, z_2)} > 0.09$, else go to 1
idea: remove QCD asymmetric splittings
- 4 Require 2 b taggings
- 5 Filter *i.e.* uncluster down to $R_{\text{filt}} = R/3$, keep the 3 hardest subjects
idea: keep “hard” QCD radiations, reduce UE

Boosted Higgs: one event, effects on S/B

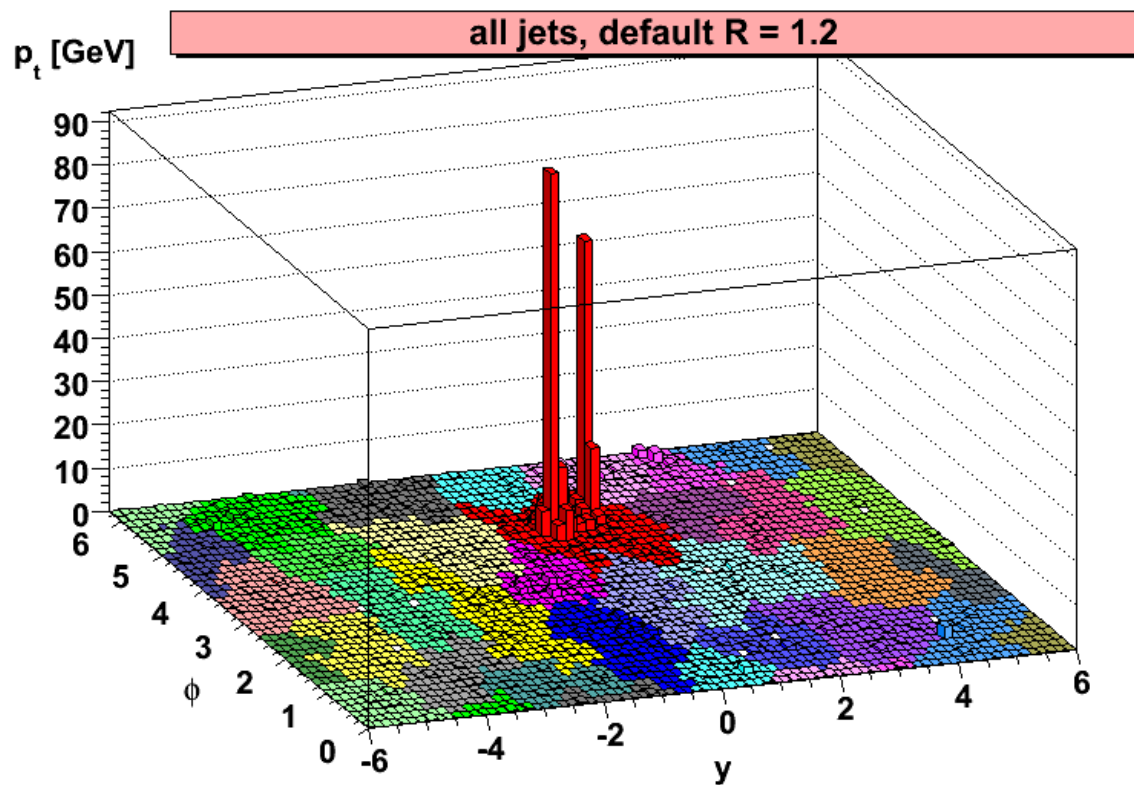
Herwig 6.510 + Jimmy 4.31 + FastJet 2.3



Cluster C/A, R=1.2

Boosted Higgs: one event, effects on S/B

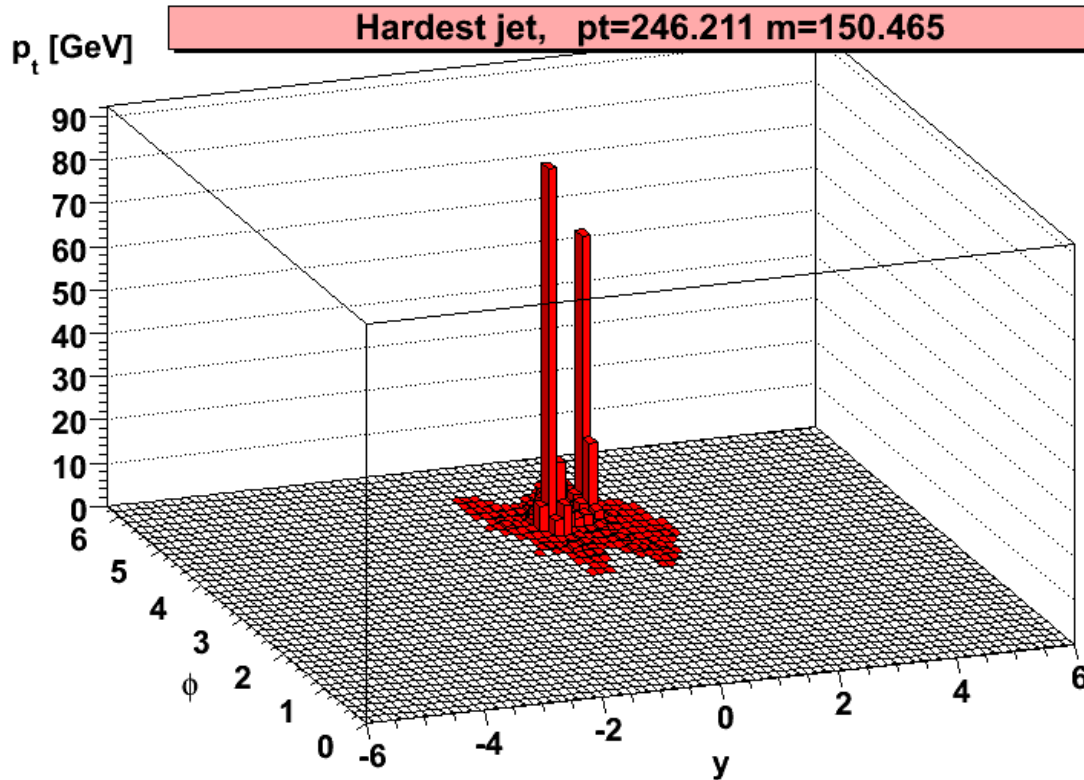
Herwig 6.510 + Jimmy 4.31 + FastJet 2.3



Show jets more clearly

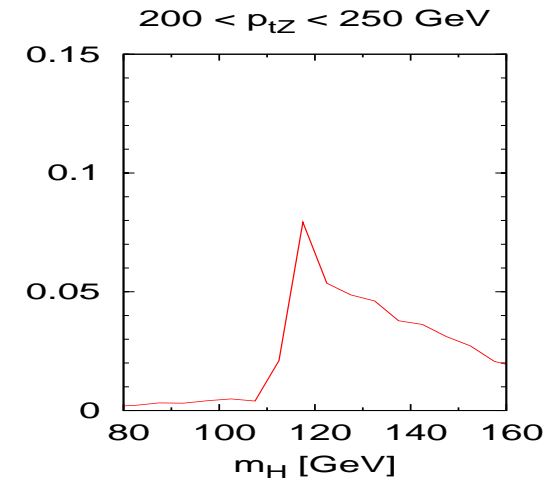
Boosted Higgs: one event, effects on S/B

Herwig 6.510 + Jimmy 4.31 + FastJet 2.3

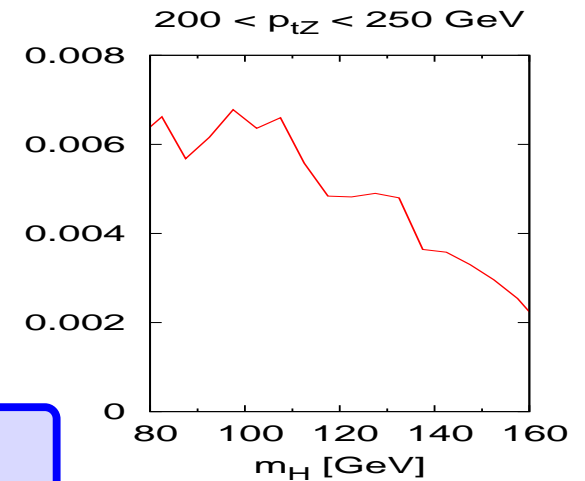


Hardest jet ($m = 150$ GeV)

HZ Signal

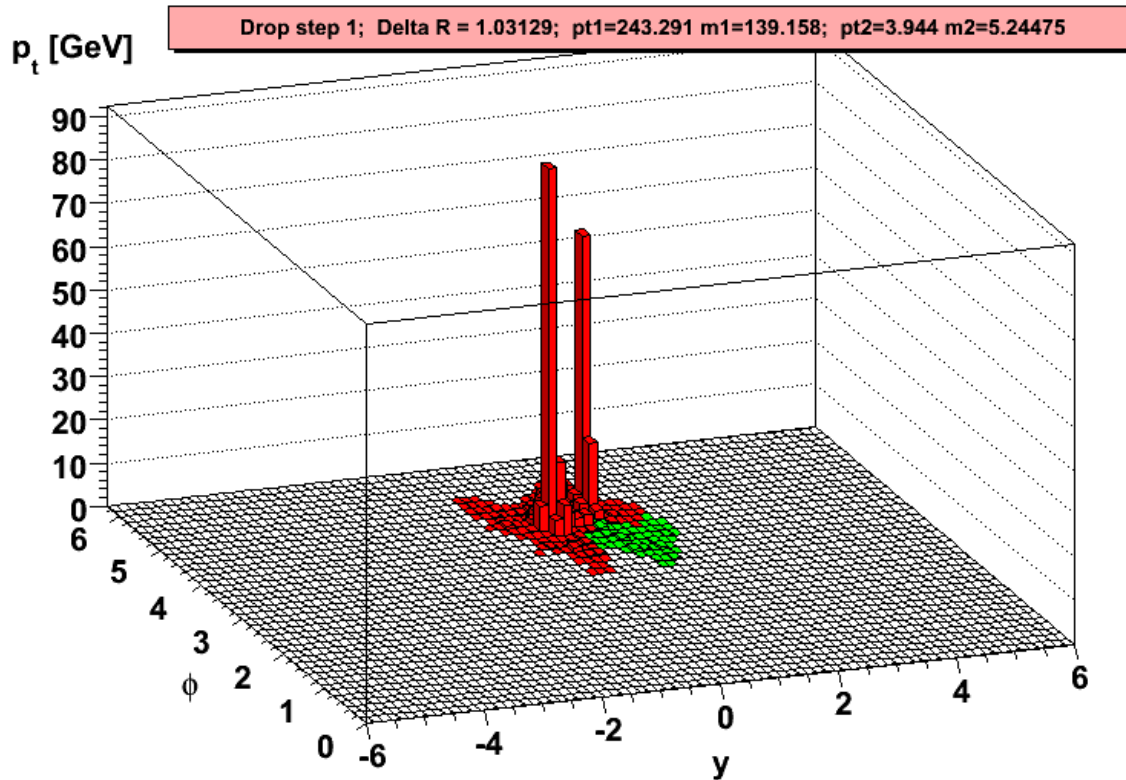


Zbb Background

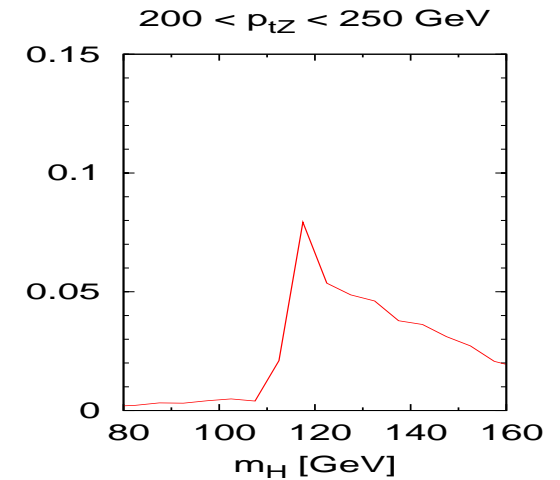


Boosted Higgs: one event, effects on S/B

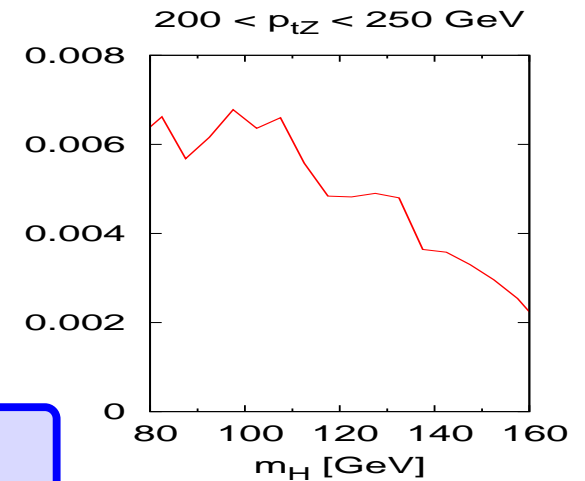
Herwig 6.510 + Jimmy 4.31 + FastJet 2.3



HZ Signal



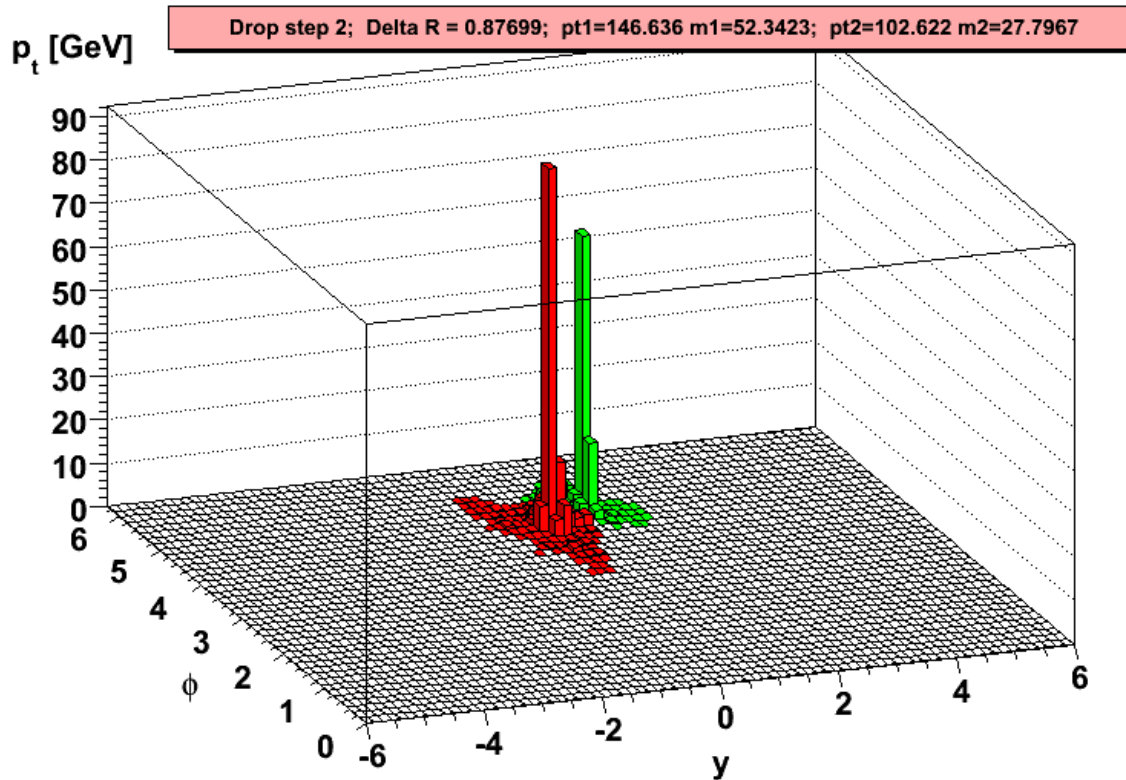
Zbb Background



Split: $\frac{\max(m_1, m_2)}{m} = 0.92$, repeat ($m = 150$ GeV)

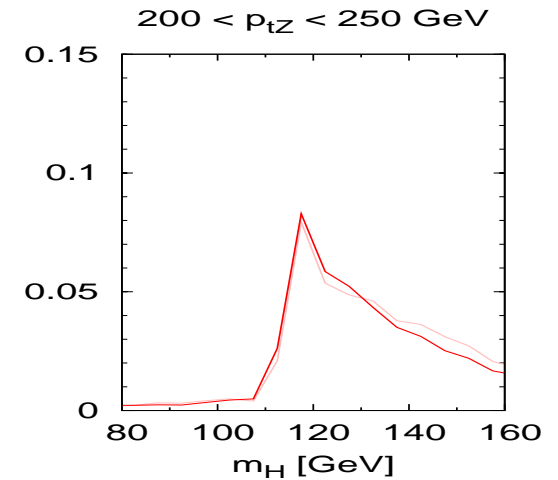
Boosted Higgs: one event, effects on S/B

Herwig 6.510 + Jimmy 4.31 + FastJet 2.3

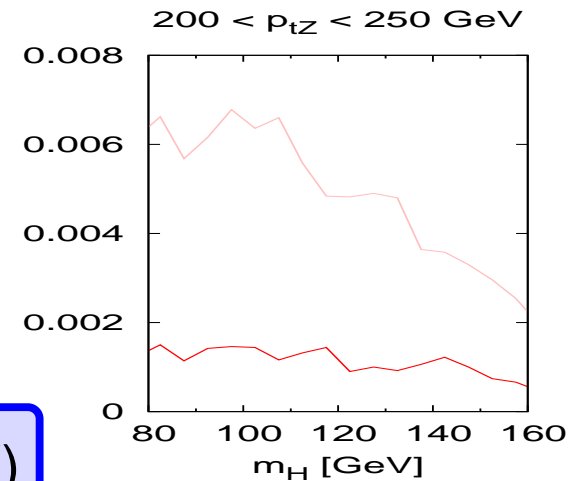


Split: $\frac{\max(m_1, m_2)}{m} = 0.37$, mass drop ($m = 139$ GeV)

HZ Signal

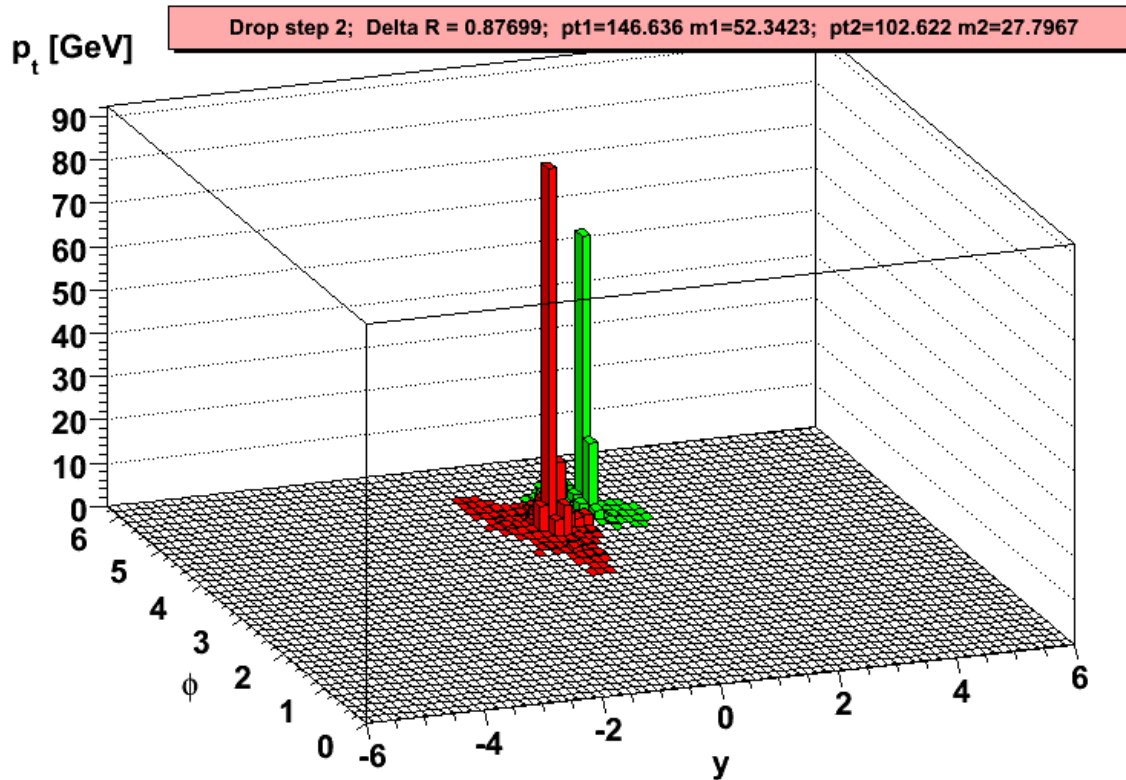


Zbb Background



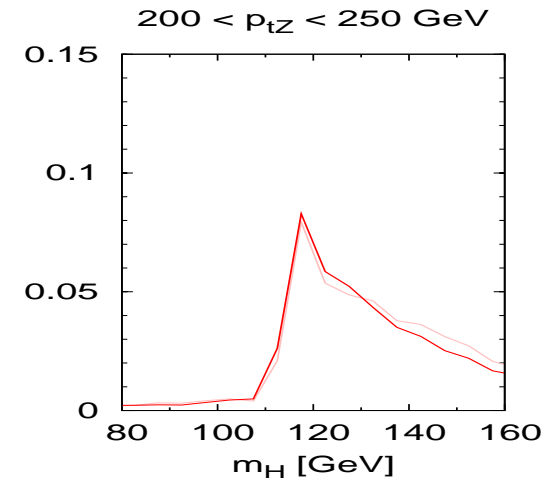
Boosted Higgs: one event, effects on S/B

Herwig 6.510 + Jimmy 4.31 + FastJet 2.3

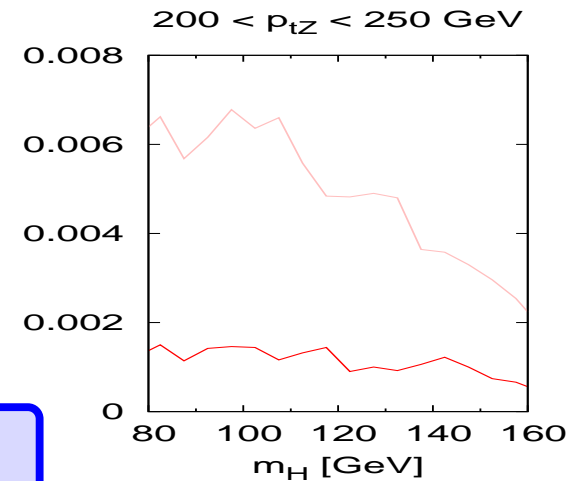


Split: $y_{12} = 0.7$, 2 b tags \Rightarrow OK ($m = 139$ GeV)

HZ Signal

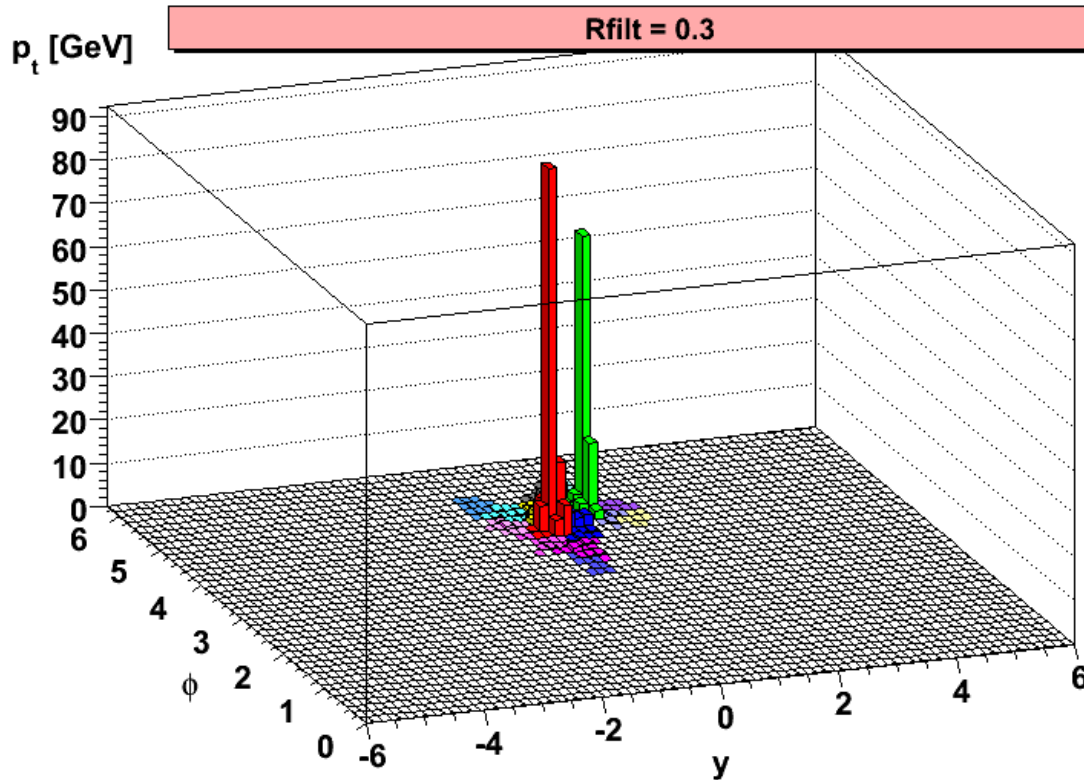


Zbb Background



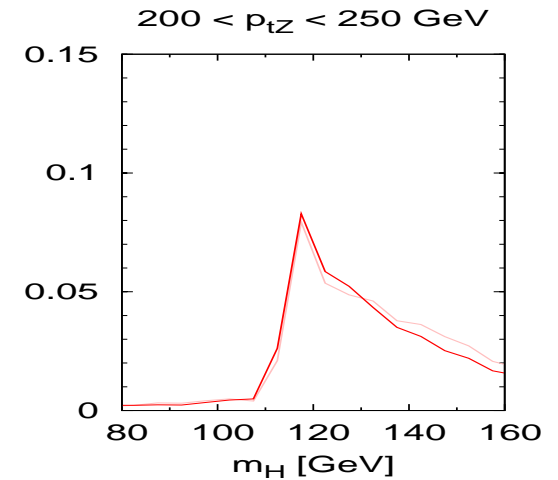
Boosted Higgs: one event, effects on S/B

Herwig 6.510 + Jimmy 4.31 + FastJet 2.3

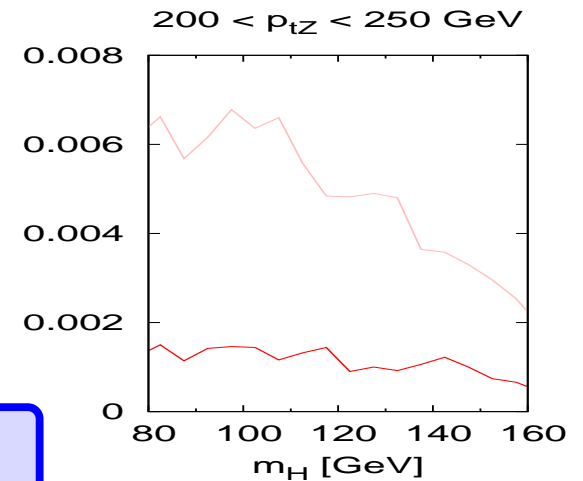


Re-cluster: $R_{\text{filt}} = 0.3$

HZ Signal

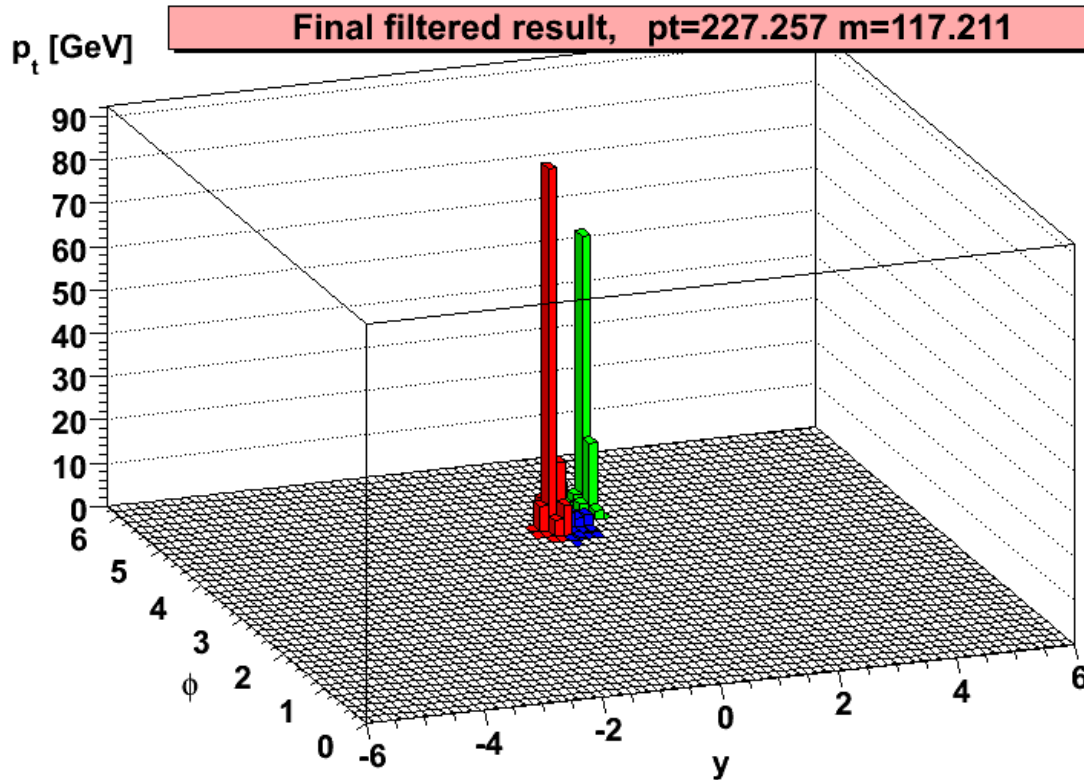


Zbb Background



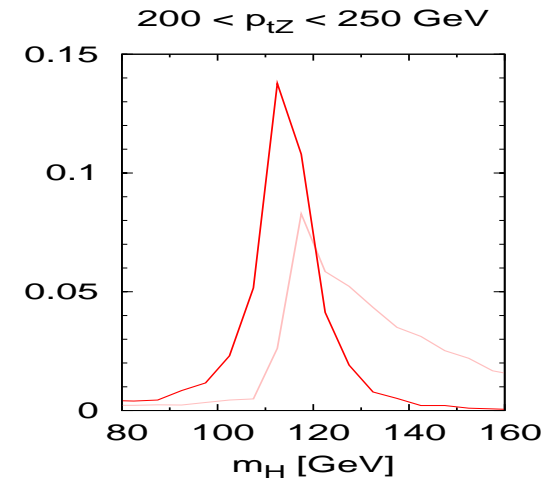
Boosted Higgs: one event, effects on S/B

Herwig 6.510 + Jimmy 4.31 + FastJet 2.3

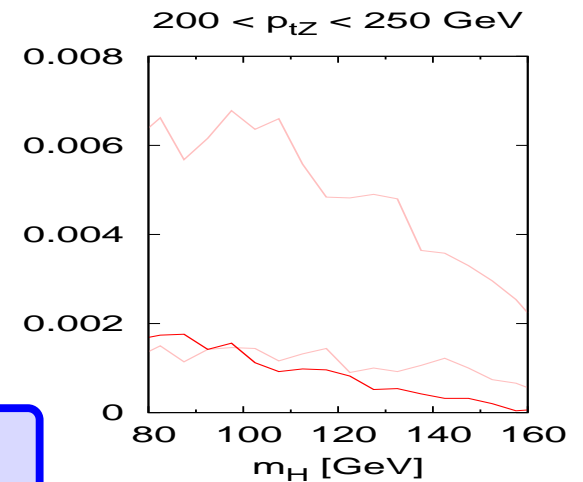


Filter: keep 3 hardest ($m = 117$ GeV)

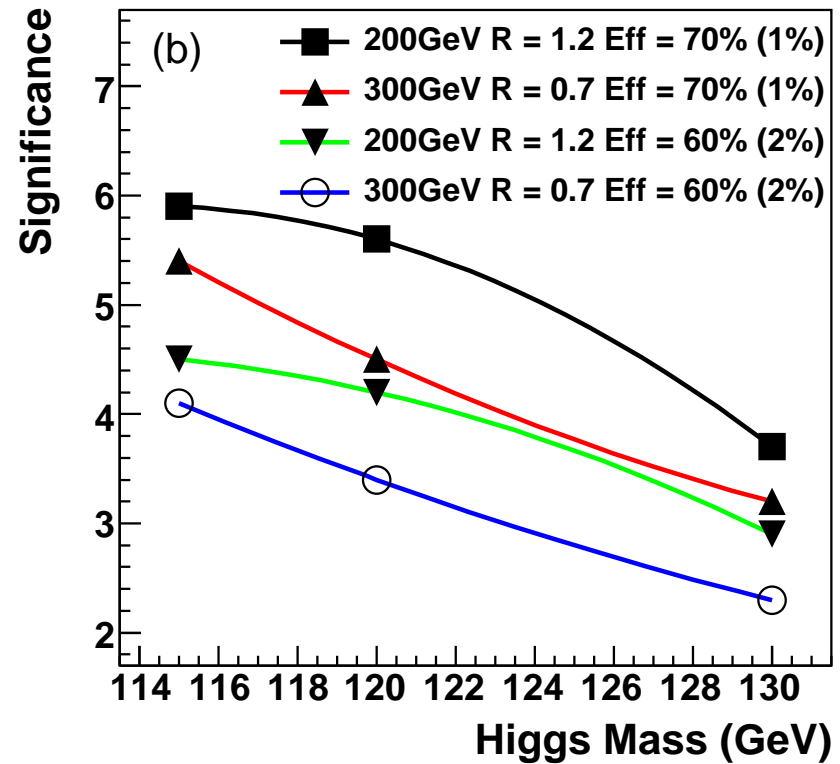
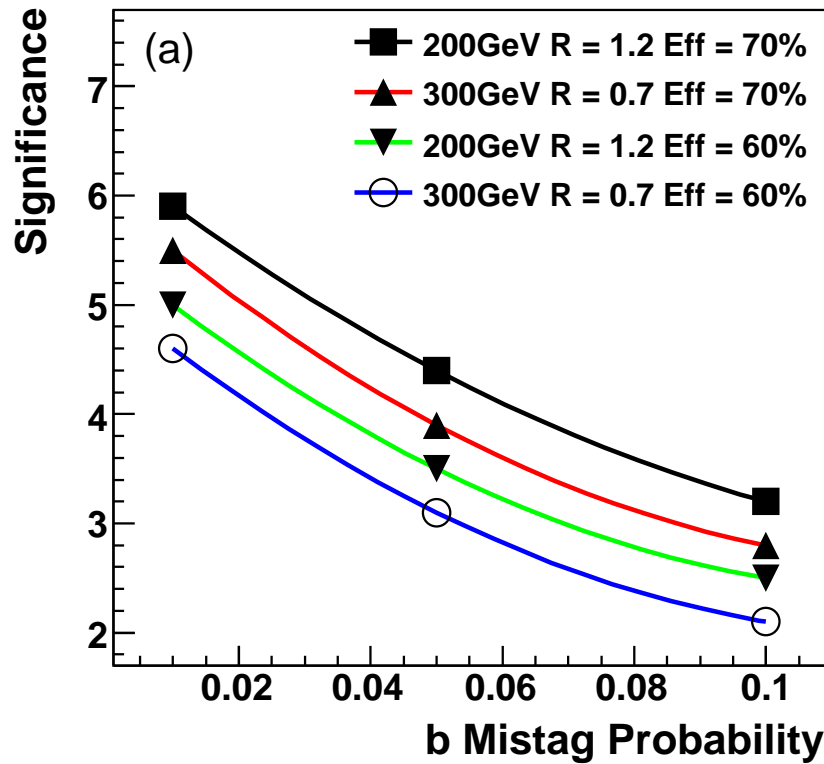
HZ Signal



Zbb Background



Boosted Higgs: one event, effects on S/B



More than 3σ for most scenarios (30 fb^{-1})

Filter: keep 5 hardest ($m = 117 \text{ GeV}$)

m_H [GeV]

160

Summary (1)

Message #1:

Use infrared-and-collinear-safe algorithms

ATLAS Cone CDF/D0 MidPoint	} →	SISCone	✓ fast ✓ safe
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CMS It. Cone	→	anti- k_t	✓ fast ✓ safe
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Mandatory to benefit fully from pQCD multileg/loop calculations at the LHC

Summary (2)

Message #2:

correct tools \Rightarrow new ideas, new concepts
 \Rightarrow new generation of jet definitions

- jet areas \longrightarrow pileup and HI background subtraction
- jet substructure improves reconstruction (Higgs, top, SUSY, ...)

Message #3:

keep some flexibility in the jet definition choice

- optimisation \longrightarrow luminosity gains for LHC searches

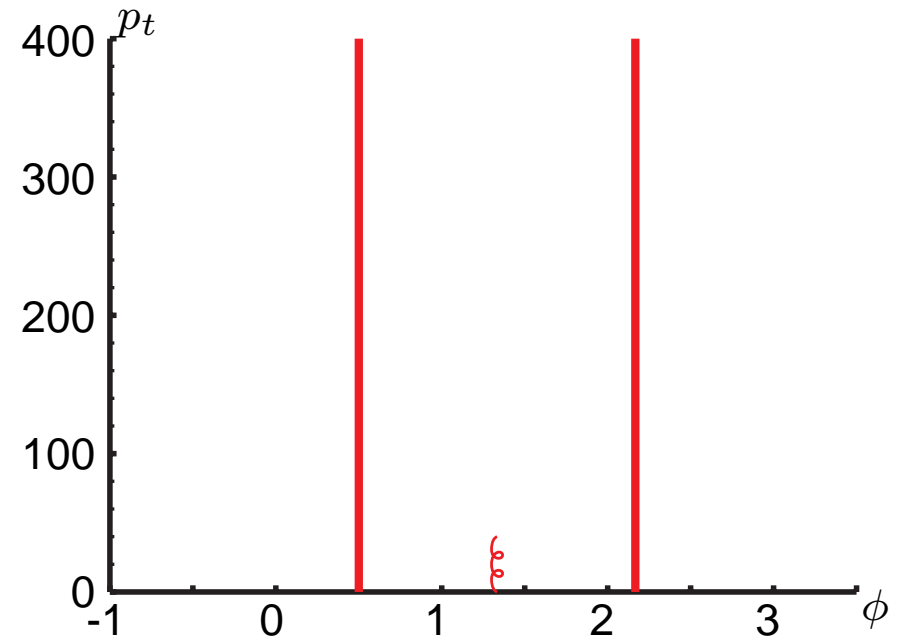
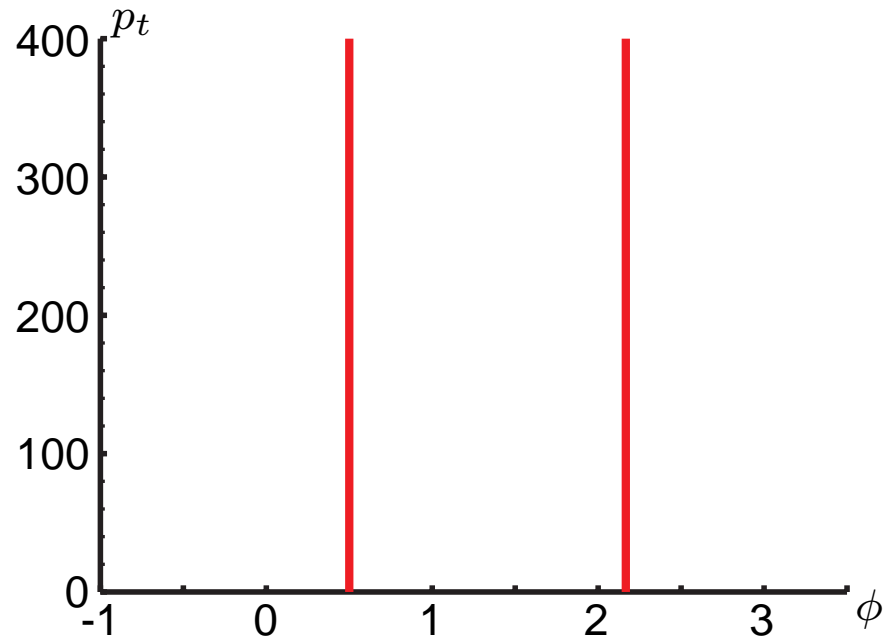
Future perspectives

This is not the end of the story:

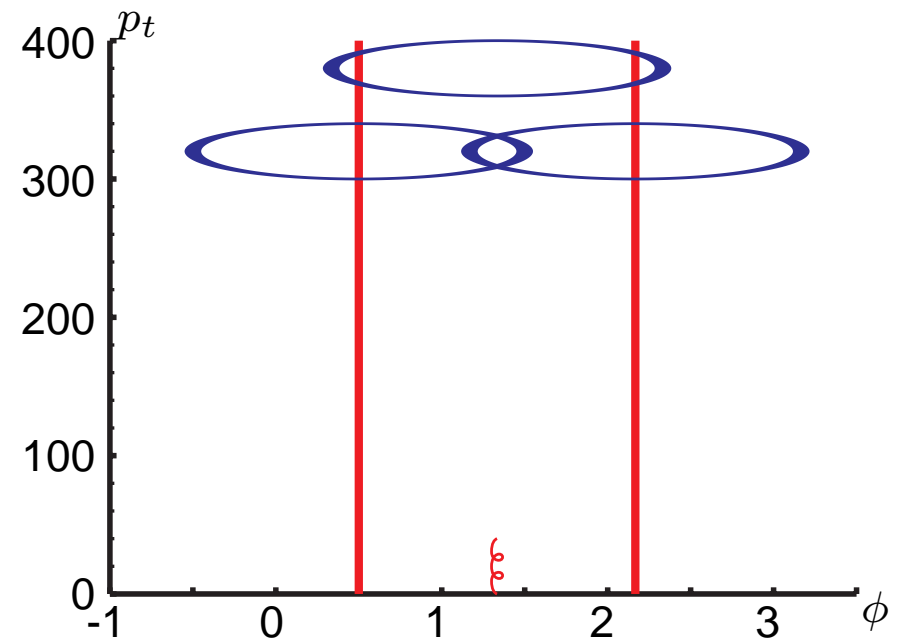
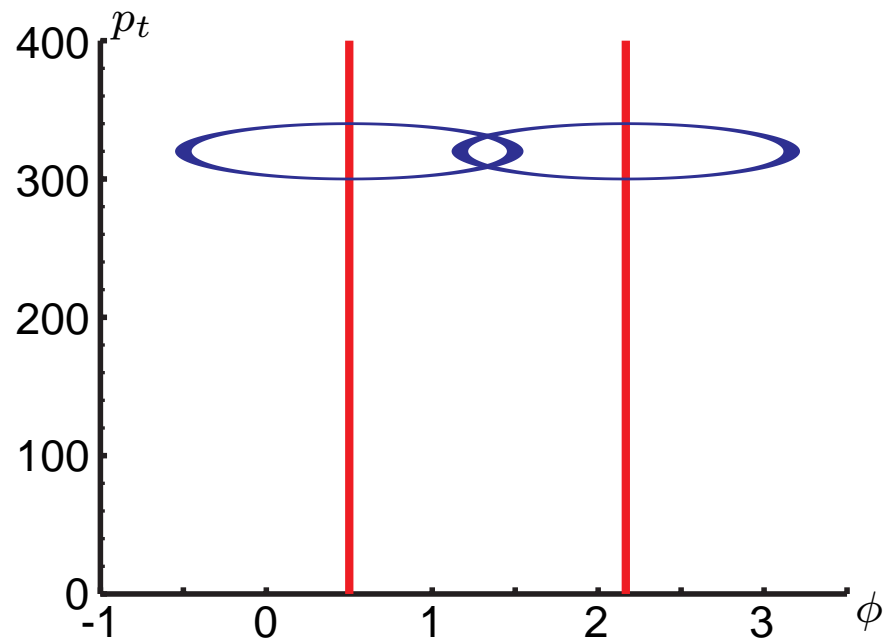
- Study the parameters used when filtering
- Analytic understanding of the optimisation of R
- Optimisation for other processes

backup slides

IR (un)safety? JetClu and Atlas Cone

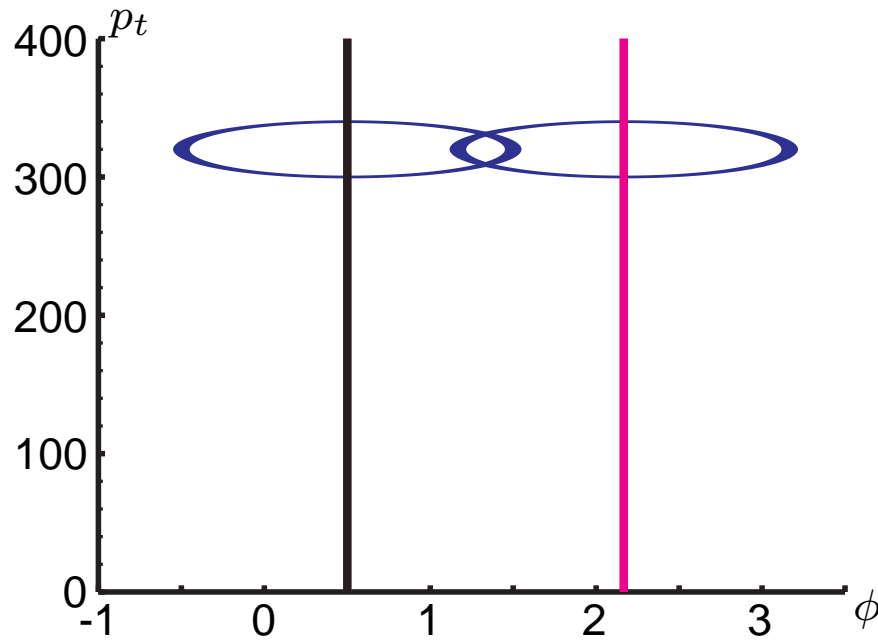


IR (un)safety? JetClu and Atlas Cone

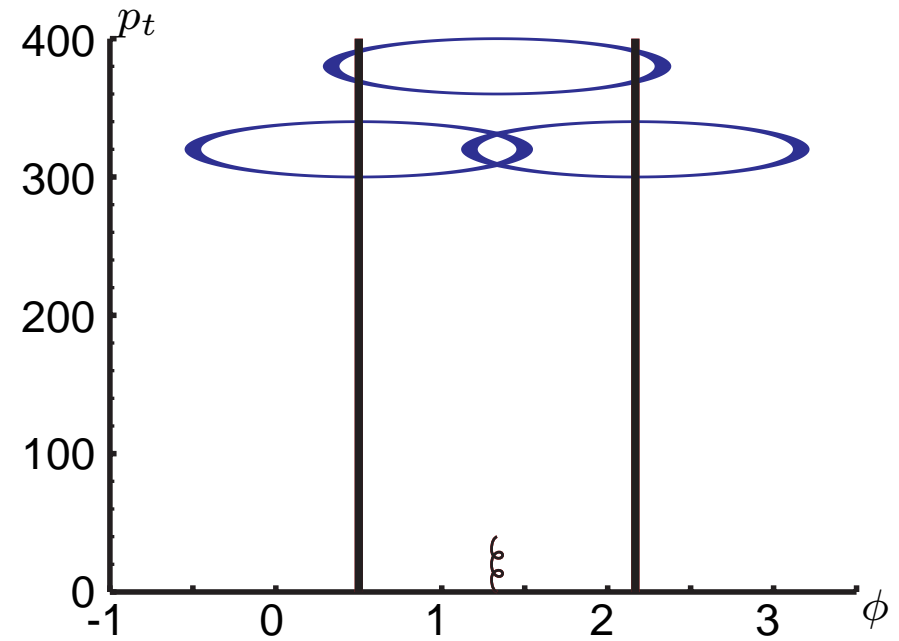


Stable cones found

IR (un)safety? JetClu and Atlas Cone



2 jets

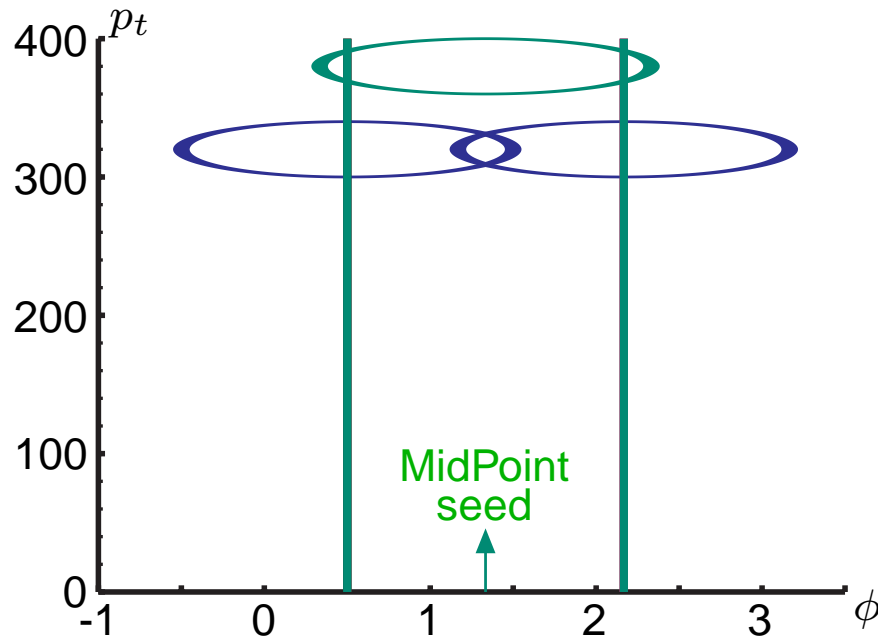


1 jet

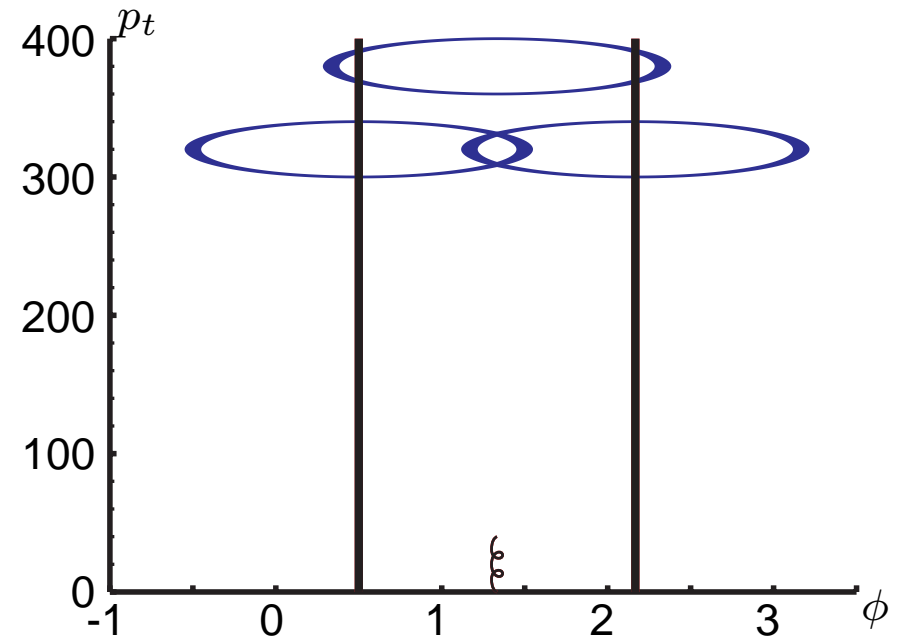
A soft gluon changed the number of jets

⇒ IR unsafety of JetClu and the ATLAS Cone

IR (un)safety? JetClu and Atlas Cone



2 jets



1 jet

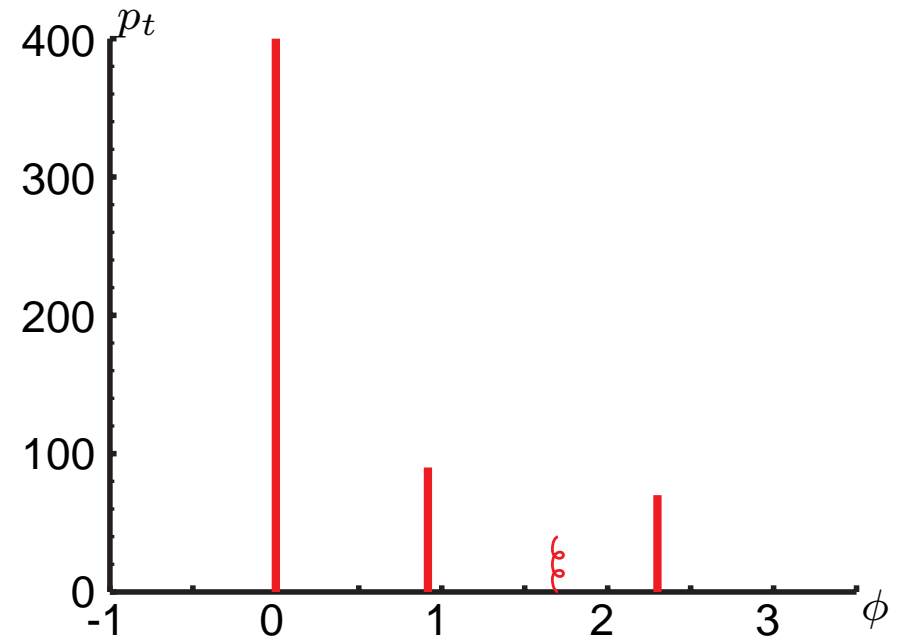
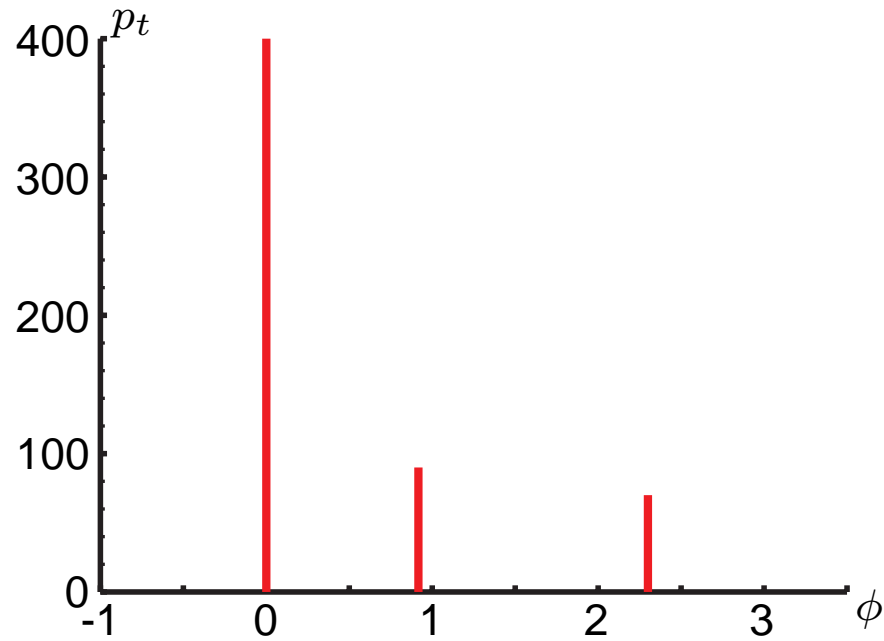
A soft gluon changed the number of jets

⇒ IR unsafety of JetClu and the ATLAS Cone

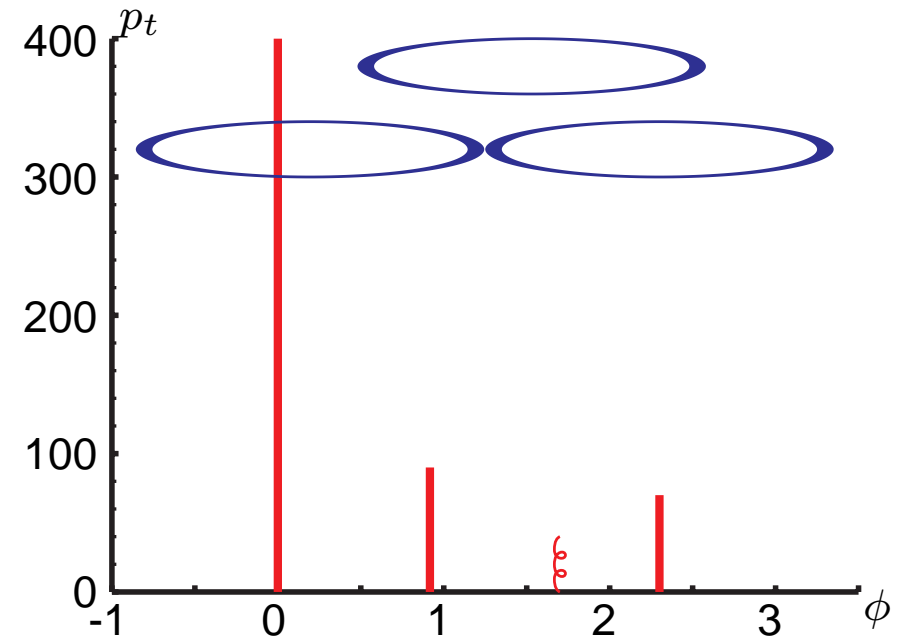
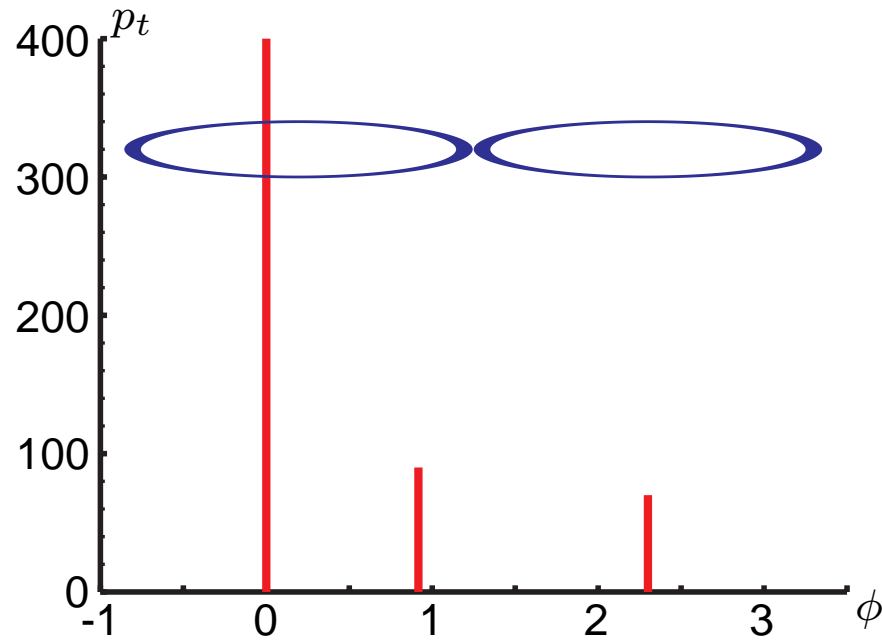
Fixed by MidPoint

[Blazey *et al.*, 00]

IR (un)safety? MidPoint

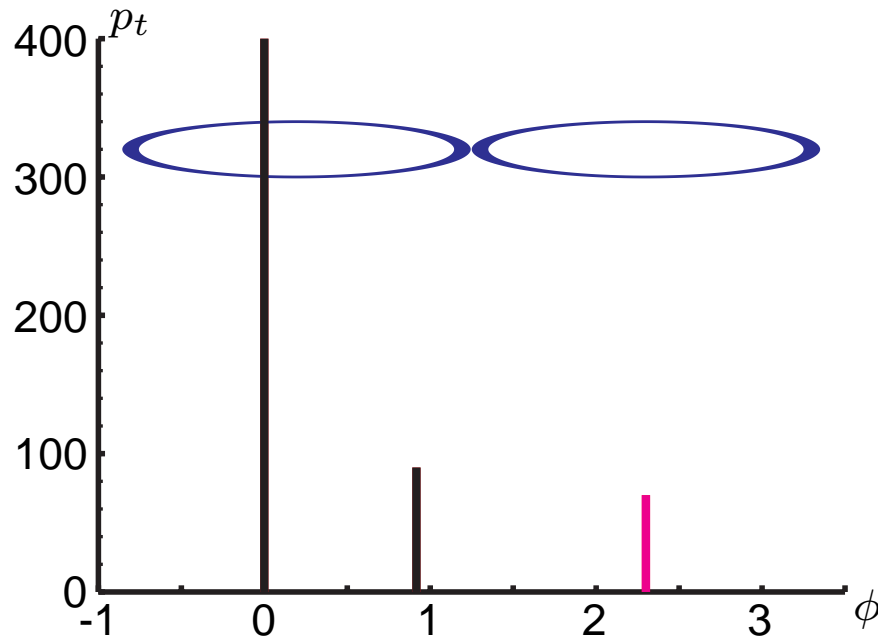


IR (un)safety? MidPoint

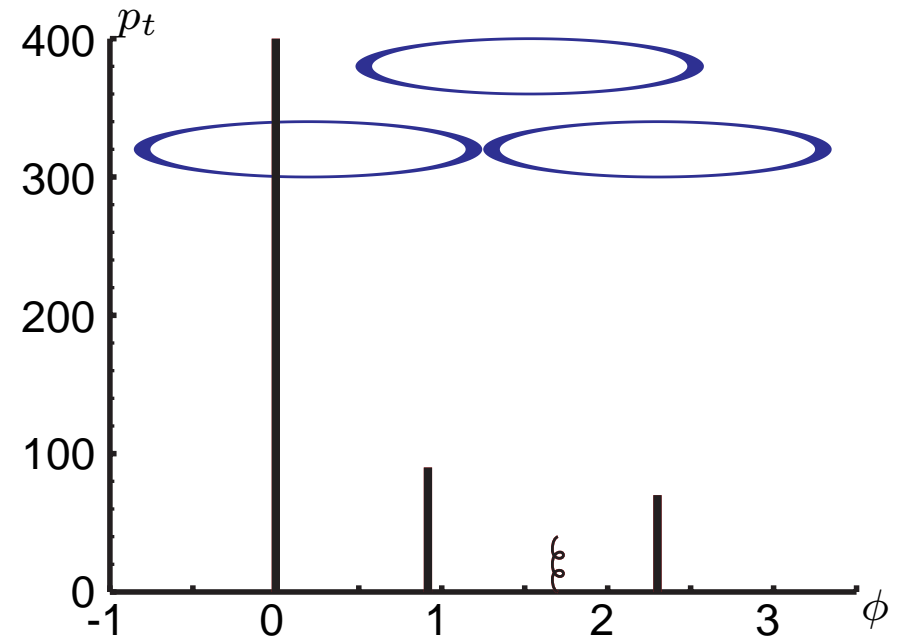


Stable cones found

IR (un)safety? MidPoint



2 jets

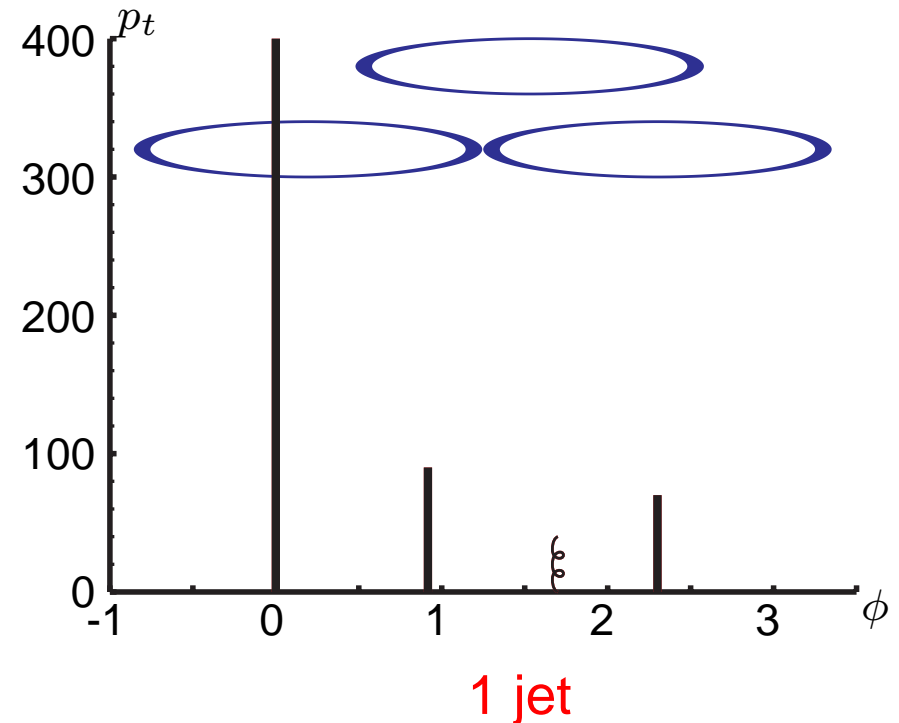
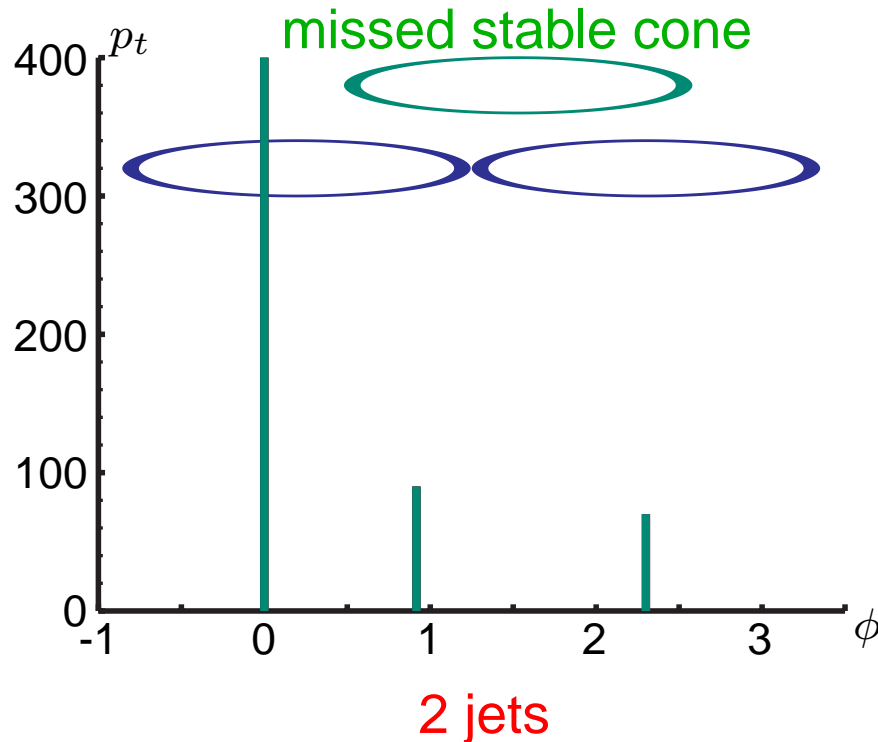


1 jet

A soft gluon changed the number of jets

⇒ **IR unsafety of MidPoint** (1 order in α_s later than JetClu)

IR (un)safety? MidPoint



Solution: be sure to find all stable cones

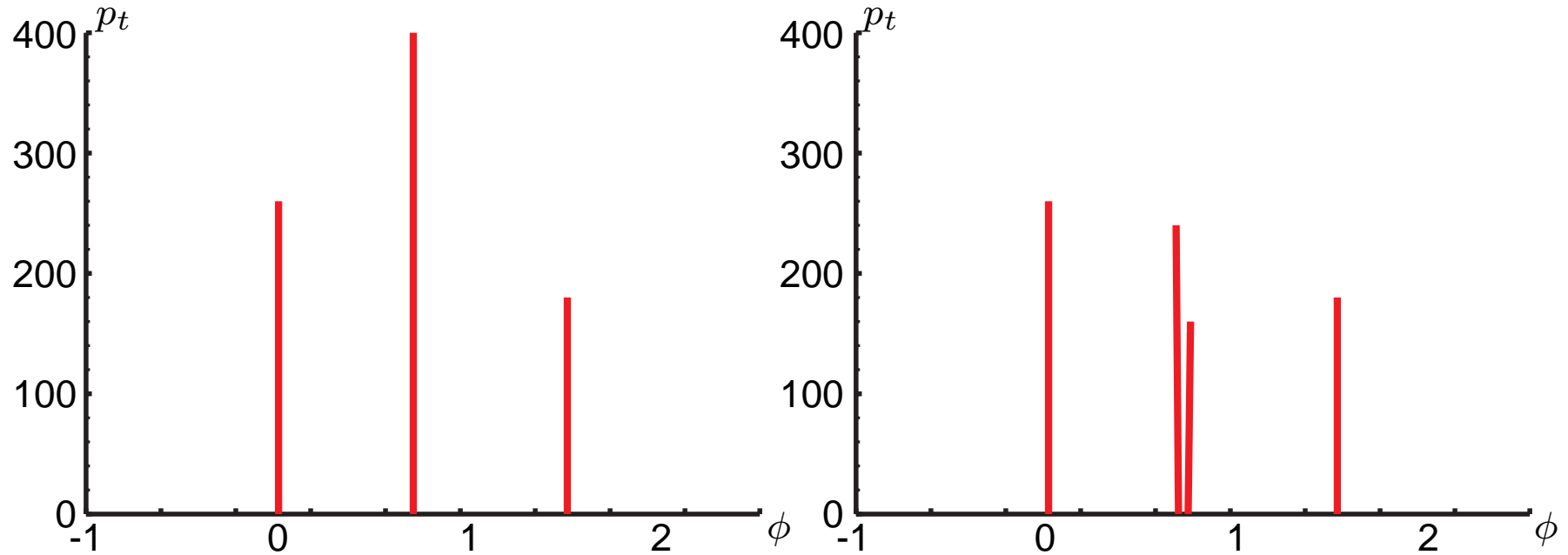
SISCone: Seedless Infrared-Safe Cone algorithm

<http://projects.hepforge.org/siscone>

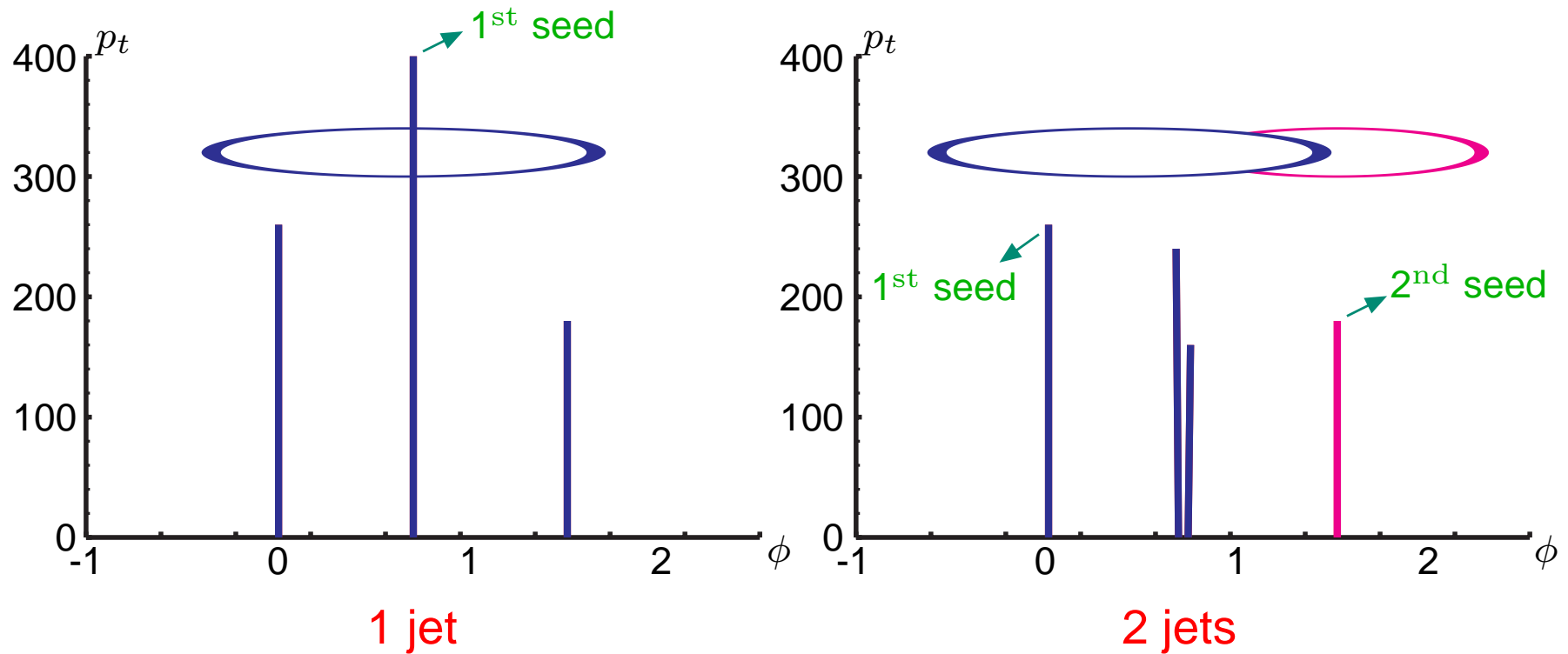
[G.Salam, G.S., 07]

Idea: enumerate enclosures by enumerating pairs of particles

Collinear (un)safety? the CMS iterative cone



Collinear (un)safety? the CMS iterative cone



A collinear splitting changed the number of jets

⇒ **Collinear unsafety of the CMS iterative cone**

Come back to recombination-type algorithms:

$$d_{ij} = \min(k_{t,i}^{2p}, k_{t,j}^{2p}) (\Delta\phi_{ij}^2 + \Delta\eta_{ij}^2)$$

- $p = 1$: k_t algorithm
- $p = 0$: Aachen/Cambridge algorithm

Come back to recombination-type algorithms:

$$d_{ij} = \min(k_{t,i}^{2p}, k_{t,j}^{2p}) (\Delta\phi_{ij}^2 + \Delta\eta_{ij}^2)$$

- $p = 1$: k_t algorithm
- $p = 0$: Aachen/Cambridge algorithm
- $p = -1$: anti- k_t algorithm [M.Cacciari, G.Salam, G.S., 08]

Why should that be related to the iterative cone ?!?

- “large $k_t \Rightarrow$ small distance”
i.e. hard partons “eat” everything up to a distance R
i.e. circular/regular jets, jet borders unmodified by soft radiation
- infrared and collinear safe

When does IRC safety matters?

Take e.g. the MidPoint cone

$$\underbrace{\alpha_s^2 \times \dots}_{2 \text{ particles}} + \underbrace{\alpha_s^3 \times \dots}_{3 \text{ particles}} + \underbrace{\alpha_s^4 \times \dots}_{4 \text{ particles}} + \underbrace{\alpha_s^5 \times \dots}_{4 \text{ particles} + 1 \text{ soft}} + \dots$$

- QCD expansion (one α_s can be replaced by α_{EW})

When does IRC safety matters?

Take e.g. the MidPoint cone

$$\underbrace{\alpha_s^2 \times \dots}_{\text{2 particles}} + \underbrace{\alpha_s^3 \times \dots}_{\text{3 particles}} + \underbrace{\alpha_s^4 \times \dots}_{\text{4 particles}} + \underbrace{\alpha_s^5 \times \log(p_t/\Lambda_{\text{QCD}}) \dots}_{\text{4 particles + 1 soft}} + \dots$$

- QCD expansion (one α_s can be replaced by α_{EW})
- IRC unsafety (regulated at the hadronic scale $\sim \Lambda_{\text{QCD}}$)

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cannot be trusted

- QCD expansion (one α_s can be replaced by α_{EW})
- IRC unsafety (regulated at the hadronic scale $\sim \Lambda_{\text{QCD}}$)
- $\alpha_s \log(p_t/\Lambda_{\text{QCD}}) \sim 1$
- **last meaningful order = α_s^3 or $\alpha_{\text{EW}}\alpha_s^2$**

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$$\underbrace{\alpha_s^2 \times \dots}_{\text{2 particles}} + \underbrace{\alpha_s^3 \times \dots}_{\text{3 particles}} + \underbrace{\alpha_s^4 \times \dots}_{\text{4 particles}} + \underbrace{\alpha_s^5 \times \log(p_t/\Lambda_{\text{QCD}}) \dots + \dots}_{\text{4 particles + 1 soft}}$$

cannot be trusted

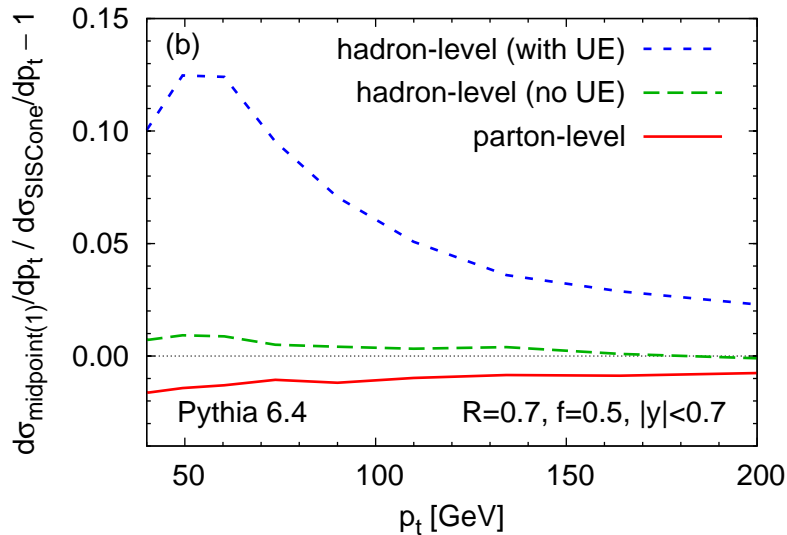
- QCD expansion (one α_s can be replaced by α_{EW})
- IRC unsafety (regulated at the hadronic scale $\sim \Lambda_{\text{QCD}}$)
- $\alpha_s \log(p_t/\Lambda_{\text{QCD}}) \sim 1$
- **last meaningful order = α_s^3 or $\alpha_{\text{EW}}\alpha_s^2$**
- same argument for the Iterative Cone
- 1 order worse for JetClu or the ATLAS cone

Physical impact

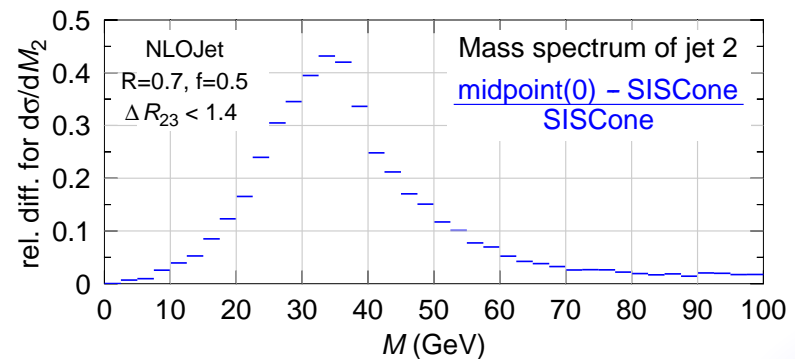
Observable	Last meaningful order	
	MidPoint/CMS	JetClu/ATLAS
Inclusive jet cross sect.	NLO	LO (NLOJet: NLO)
3 jet cross section	LO	none (NLOJet: NLO)
$W/Z/H + 2$ jet x-sect.	LO	none (MCFM: NLO)
jet masses in 3 jets	none	none (NLOJet: LO)

Example: (Midpoint-SISCone)/SISCone

pp $\sqrt{s} = 14$ TeV



- Incl. cross-section: a few %
- Masses in 3-jet events: $\sim 45\%$



Physical impact

Observable	Last meaningful order	
	MidPoint/CMS	JetClu/ATLAS
Inclusive jet cross sect.	NLO	LO (NLOJet: NLO)
3 jet cross section	LO	none (NLOJet: NLO)
$W/Z/H + 2$ jet x-sect.	LO	none (MCFM: NLO)
jet masses in 3 jets	none	none (NLOJet: LO)



Huge effort (~ 50 M€) to compute processes in pQCD

The SIScone search for stable cones

- Solution: use a seedless approach, find **ALL** stable cones
- Naive approach: check stability of each subset of particle

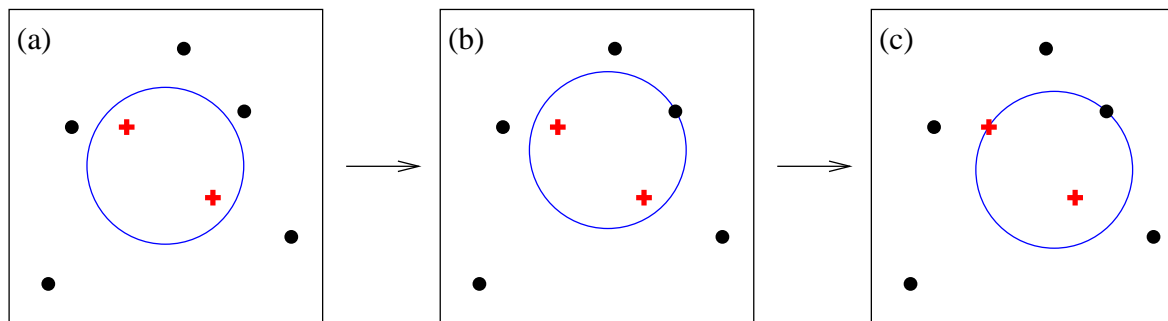
The SIScone search for stable cones

- Solution: use a seedless approach, find **ALL** stable cones
- Naive approach: check stability of each subset of particle
Complexity is $\mathcal{O}(N2^N)$
 \Rightarrow **definitely unrealistic: 10^{17} years for $N = 100$**
- Midpoint complexity: $\mathcal{O}(N^3)$

The SIScone search for stable cones

- Solution: use a seedless approach, find **ALL** stable cones
- Midpoint complexity: $\mathcal{O}(N^3)$

Idea: use geometric arguments



- Each enclosure can be moved (in any dir.) until it touches a point
- ... then rotated until it touches a second one

⇒ Enumerate all pairs of particles
with 2 circle orientations and 4 possible inclusion/exclusion
→ find all enclosures

The SIScone search for stable cones

- Solution: use a seedless approach, find **ALL** stable cones
- Midpoint complexity: $\mathcal{O}(N^3)$

Idea: use geometric arguments

⇒ Enumerate all pairs of particles
with 2 circle orientations and 4 possible inclusion/exclusion
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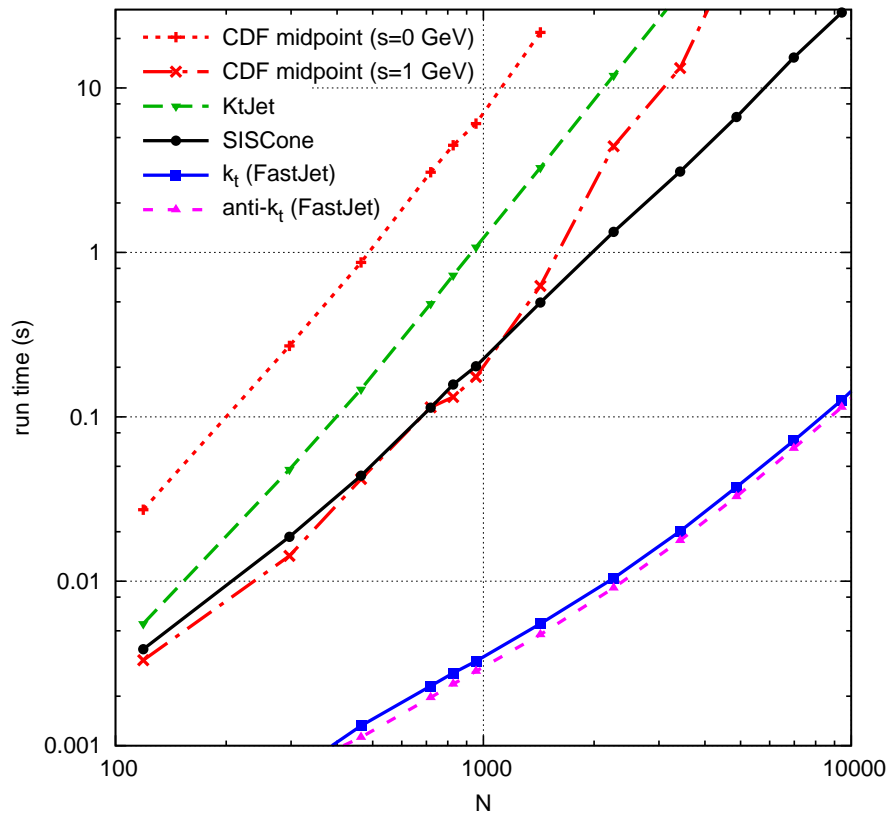
- Complexity: $\mathcal{O}(N^3)$, with improvements: $\mathcal{O}(N^2 \log(N))$

→ C++ implementation: Seedless Infrared-Safe Cone algorithm (SIScone)
G.Salam, G.S., JHEP 04 (2007) 086; <http://projects.hepforge.org/siscone>

NB.: also available from FastJet

[M.Cacciari, G.Salam, G.S.]; <http://www.fastjet.fr>

Algorithm timings



- Recombination algorithms very fast

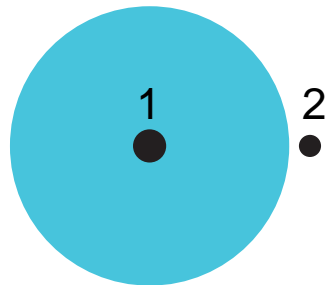
[M. Cacciari, G. Salam, 06]

- SISCone not slower than Midpoint (even with a 1 GeV seed threshold)

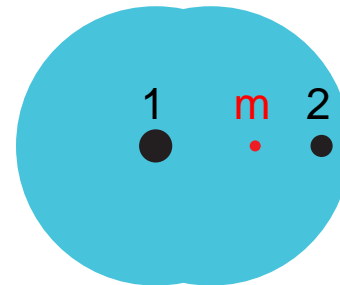
A technical point: Back-reaction

Additional soft background has 2 effects:

- **Throw soft particles in the hard jet:** dealt with by subtraction
- **Modify the hard scattering (back-reaction)**
 - can be pointlike or diffuse
 - **gain:**

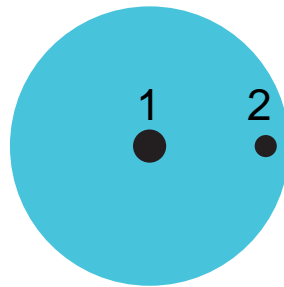


no medium: $p_t = p_{t1}$

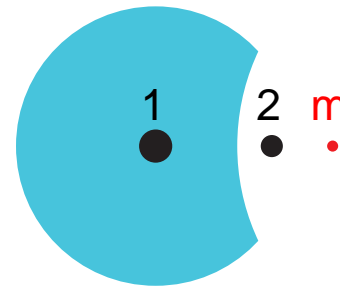


medium: $p_t = p_{t1} + p_{t2} + p_{tm}$

- **loss:**



no medium: $p_t = p_{t1} + p_{t2}$



medium: $p_t = p_{t1} + p_{tm}$

A technical point: Back-reaction

Additional soft background has 2 effects:

- **Throw soft particles in the hard jet:** dealt with by subtraction
- **Modify the hard scattering (back-reaction)**
 - can be pointlike or diffuse
 - tractable analytically (similar to areas)
 - $k_t \gtrsim$ Cambridge $>$ SIScone \gg anti- k_t

