Phenomenology of hadronic collisions

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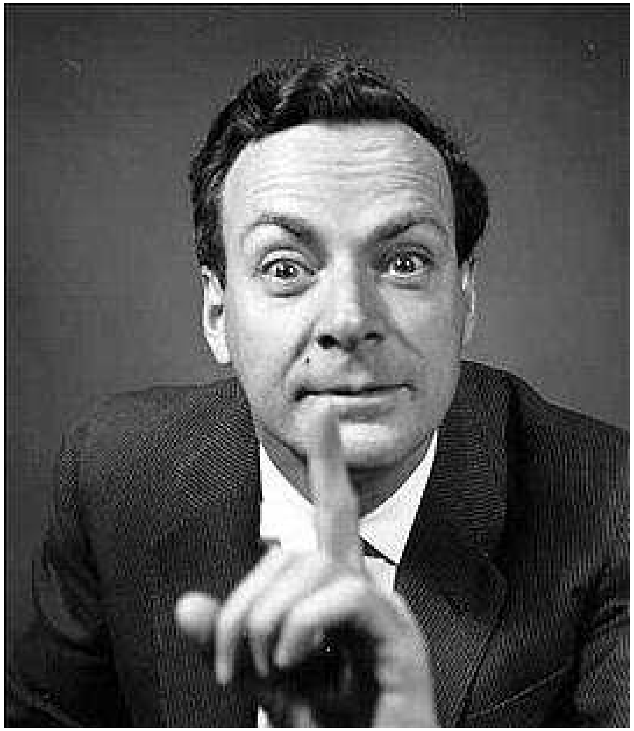
Plan

You are now experts in computing Feynman diagrams

You (hopefully) want to know how to compute things at hadronic colliders

(the LHC in particular)



Disclaimer

The physics of hadronic colliders is a very vast topic:

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- *ATLAS TDR (Detector and Physics Performance)*: 1852 pages
- *CMS TDR (2 volumes)*: 1317 pages

A good coverage of “basic” topics in collider physics:

- *QCD and Collider Physics*, R. K. Ellis, W. J. Stirling and B. R. Webber (447 pages)

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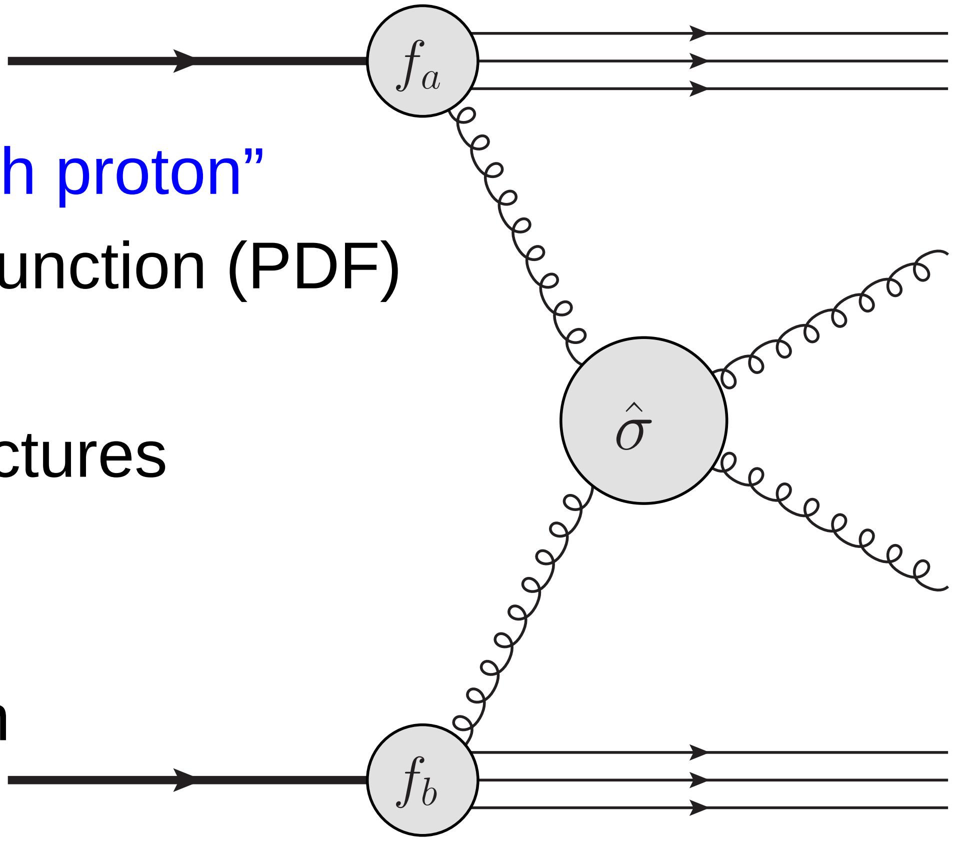
I won’t be able to cover all that in 6+2 hours!

Plan #2

How to describe a collision between 2 hadrons?

The very fundamental collision

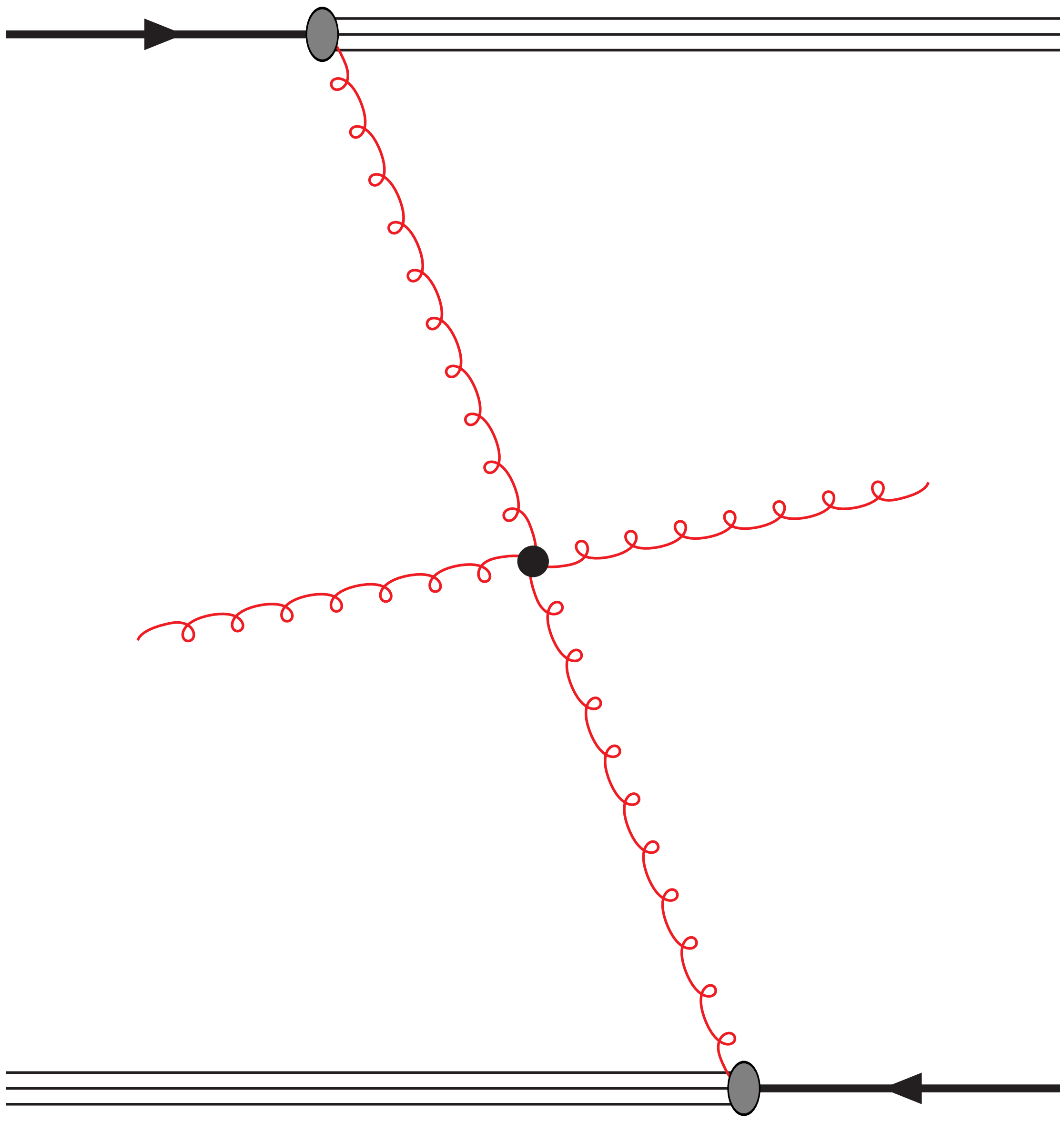
$$\sigma = f_a \otimes f_b \otimes \hat{\sigma}$$



- “take a parton out of each proton”
  $f_a \equiv$ parton distribution function (PDF)
  for quark and gluons
  a big chapter of these lectures

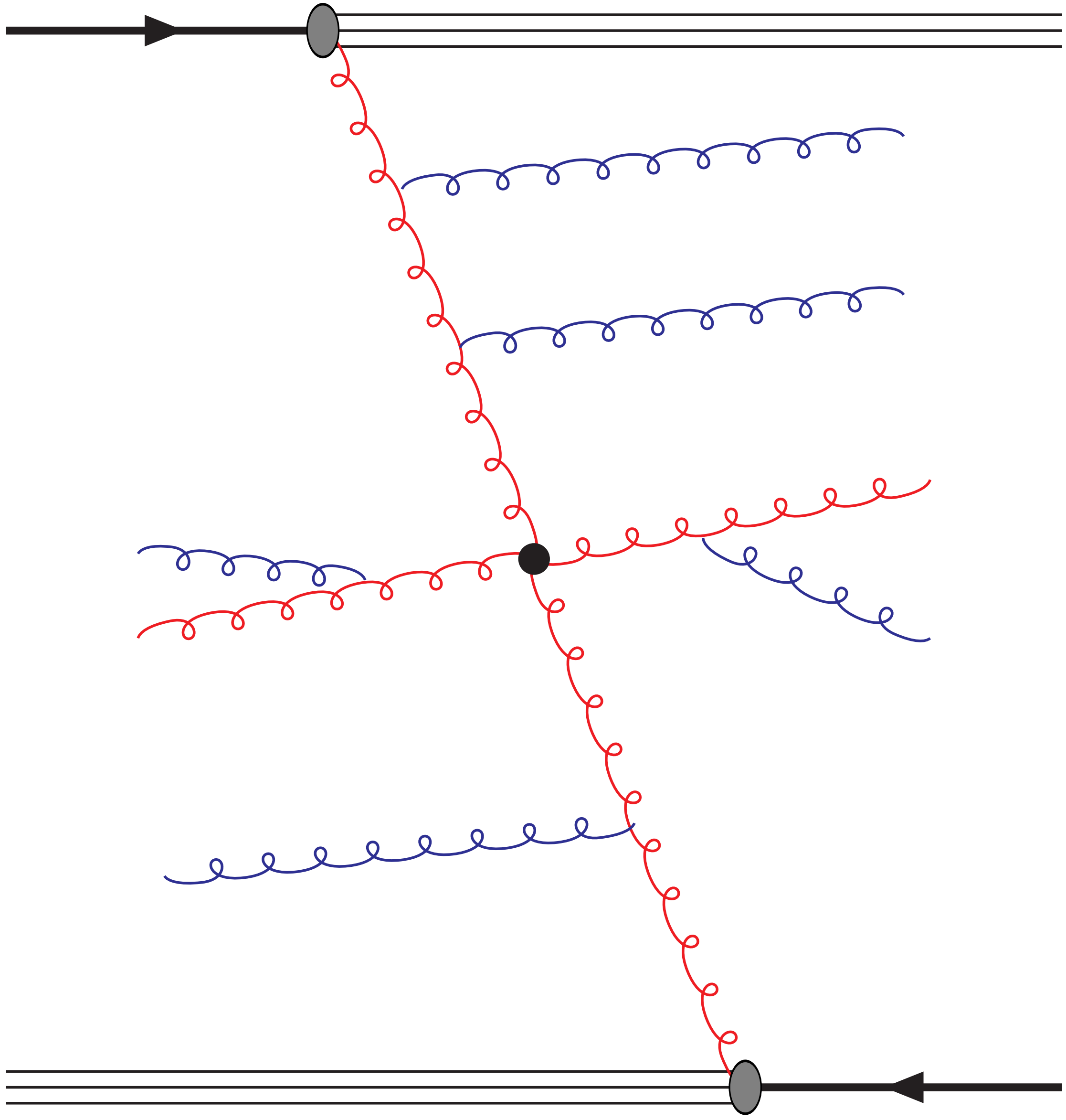
- hard matrix element
  perturbative computation
  Forde-Feynman rules

The more realistic version



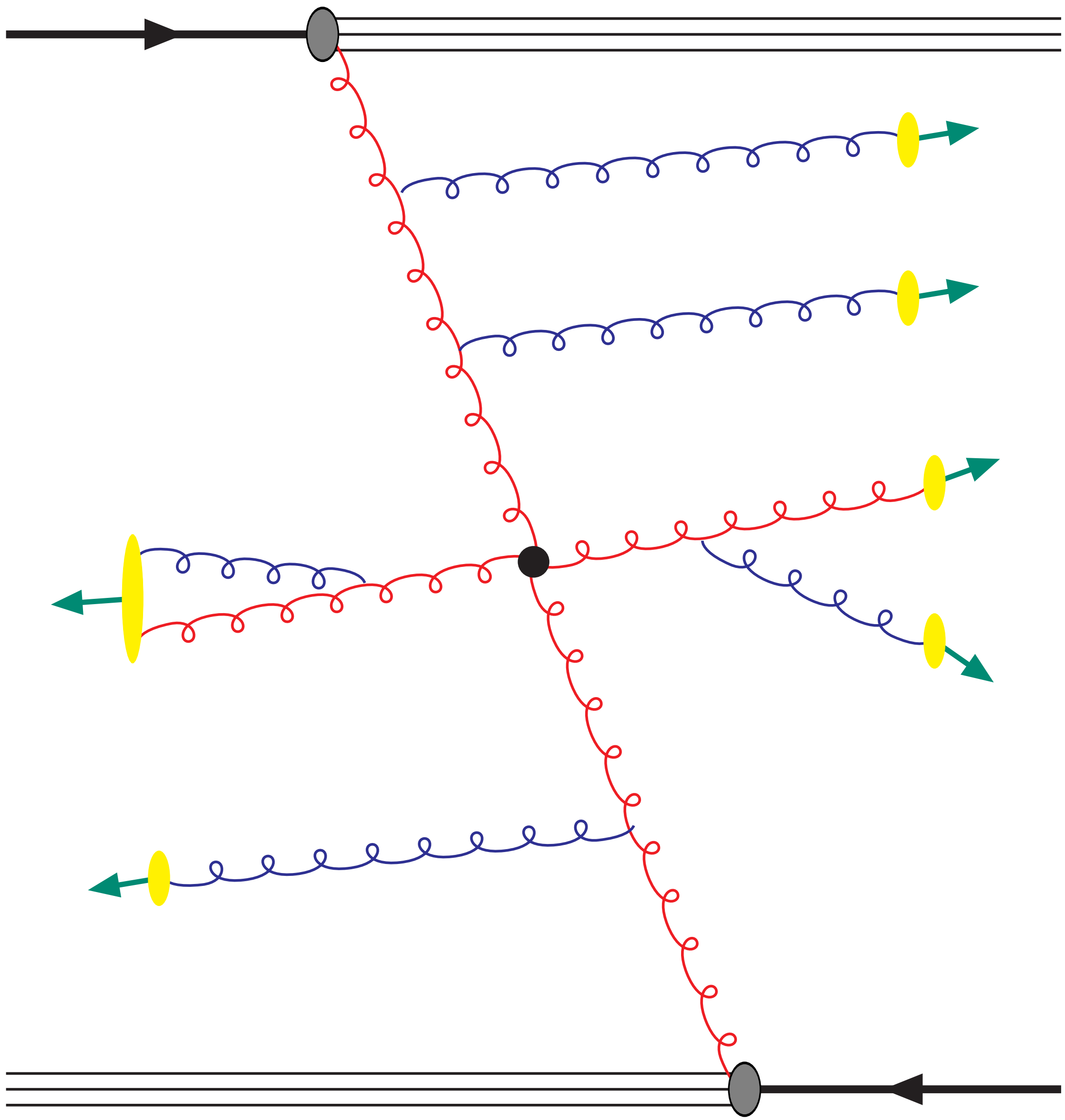
- Hard ME
  perturbative

The more realistic version



- Hard ME
  perturbative
- Parton branching
  initial+final state radiation

The more realistic version



- Hard ME

  perturbative

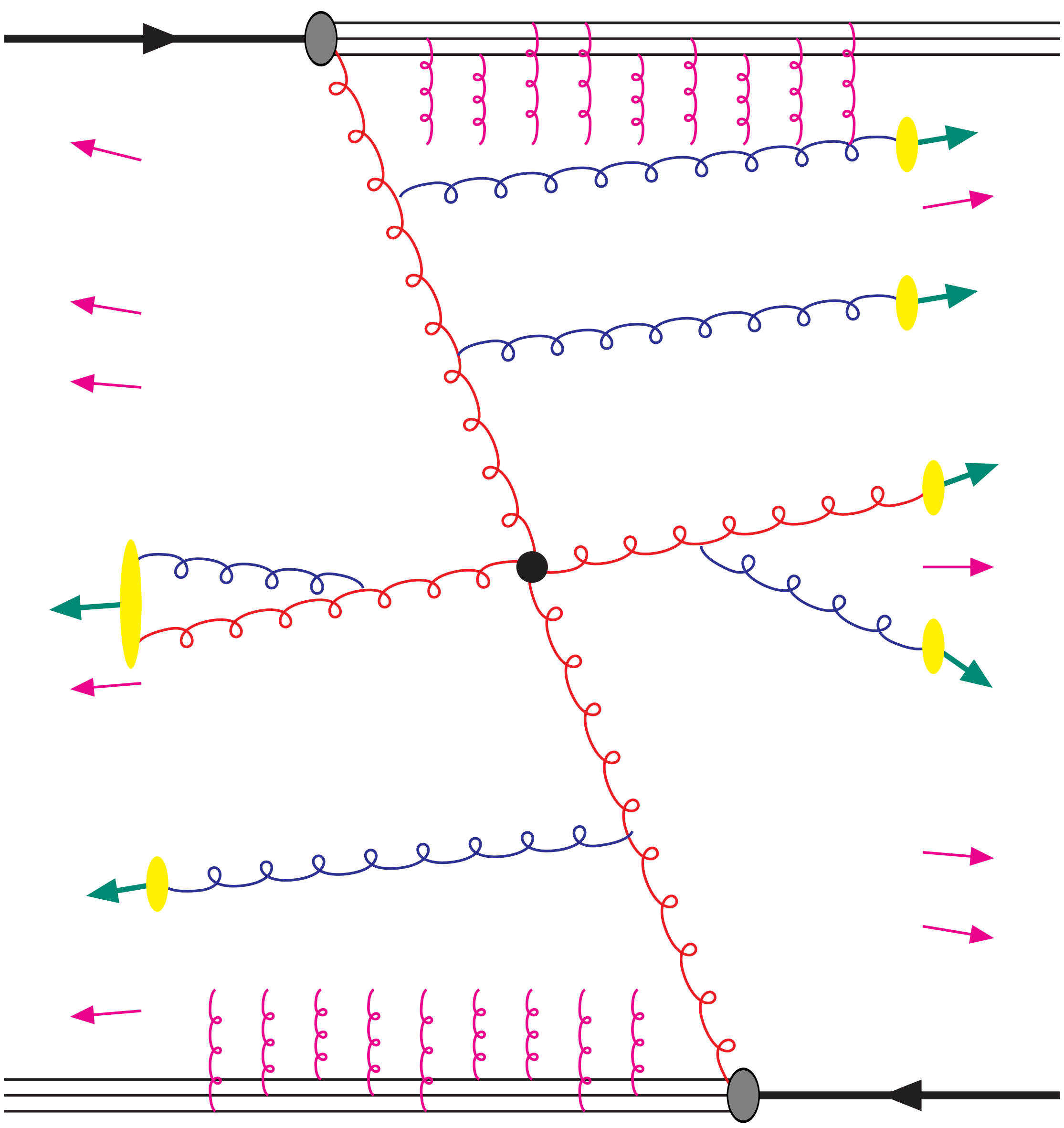
- Parton branching

  initial+final state radiation

- Hadronisation

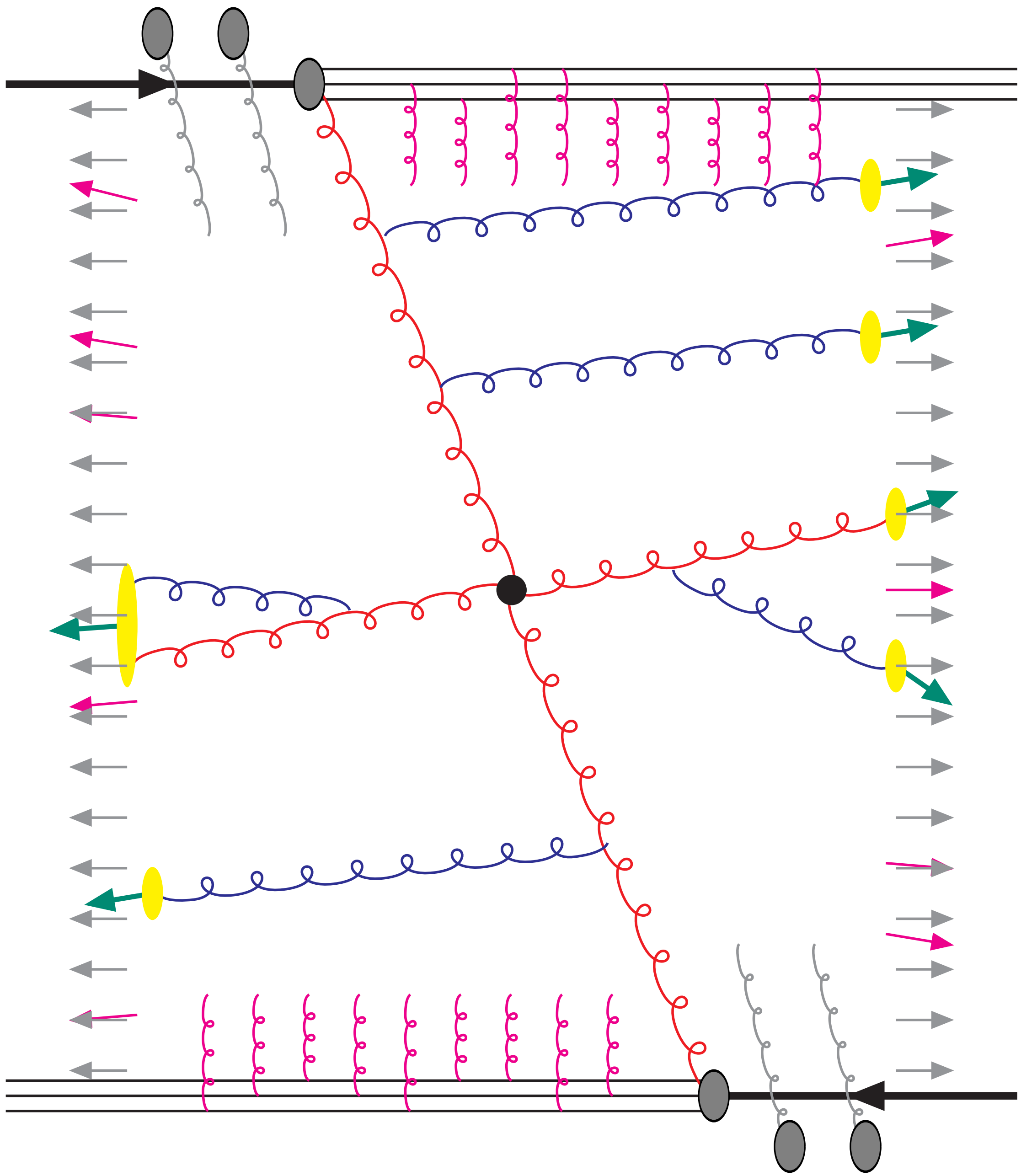
  $q, g \rightarrow$ hadrons

The more realistic version



- Hard ME
  perturbative
- Parton branching
  initial+final state radiation
- Hadronisation
  $q, g \to$ hadrons
- Multiple interactions
  Underlying event (UE)

The more realistic version



- Hard ME
  perturbative
- Parton branching
  initial+final state radiation
- Hadronisation
  $q, g \to$ hadrons
- Multiple interactions
  Underlying event (UE)
- Pile-up
  $\lesssim$ 25 pp at the LHC

Step by step...

We shall investigate those effects one by one:

- e^+e^- collisions for QCD final state (and hadronisation)
- ep collisions *aka* Deep Inelastic scattering (DIS) for the Parton Distribution Functions
- pp collisions: put everything together
  - kinematics
  - Monte-Carlo
  - jets + various processes (W/Z, Higgs, top, ...)

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Tutorial

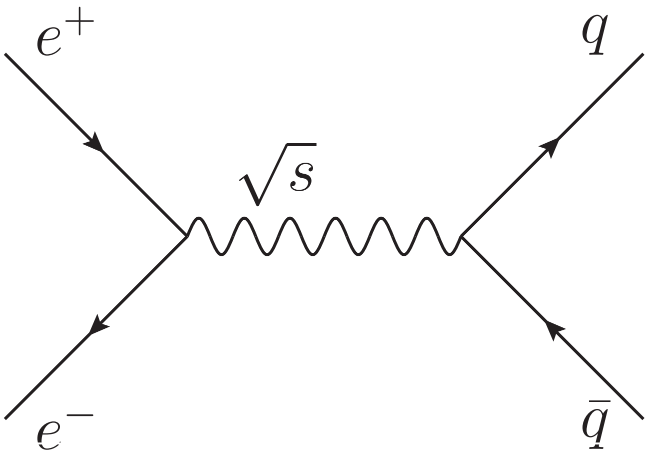
The plan is to play with Pythia 8 (the C++ version) and FastJet.
You can get them (and a few sample codes) from the link at

`http://soyez.fastjet.fr`

e^+e^- ***collisions***

QCD final state

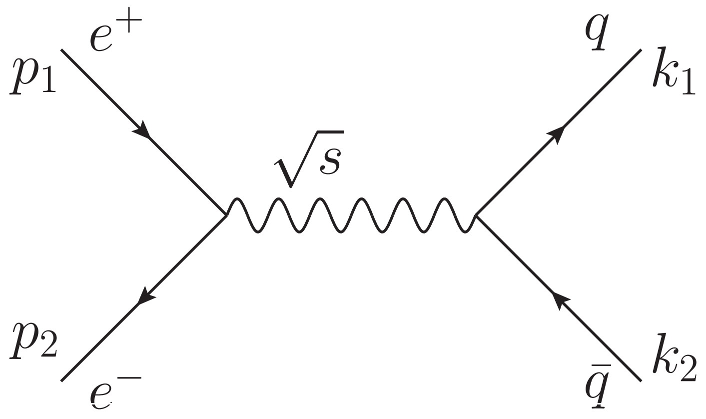
e^+e^- collisions give QCD final state without initial-state/beam contamination



Useful for many QCD studies

Intermediate state can be γ or Z, we only consider γ for simplicity

QCD final state: basic QCD



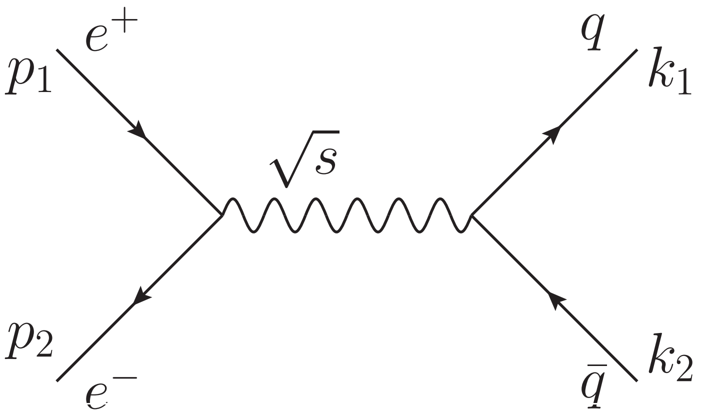
$$p_1 \equiv \frac{\sqrt{s}}{2}(0, 0, 1, 1)$$

$$p_2 \equiv \frac{\sqrt{s}}{2}(0, 0, -1, 1)$$

$$k_1 \equiv \frac{\sqrt{s}}{2}(\sin(\theta), 0, \cos(\theta), 1)$$

$$k_2 \equiv \frac{\sqrt{s}}{2}(-\sin(\theta), 0, -\cos(\theta), 1)$$

QCD final state: basic QCD



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$$k_1 \equiv \frac{\sqrt{s}}{2}(\sin(\theta),0,\cos(\theta),1)$$

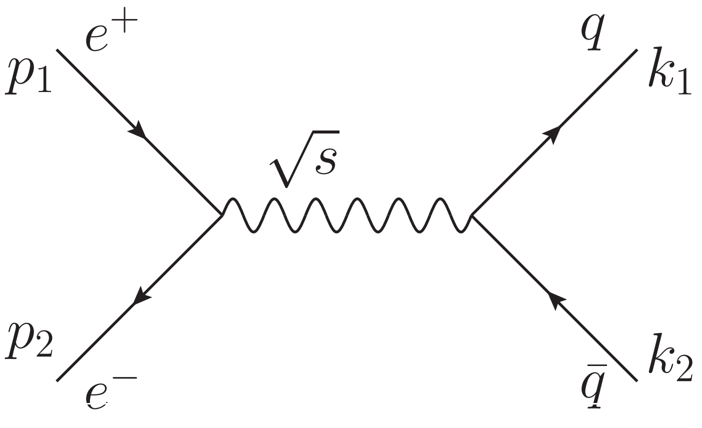
$$k_2 \equiv \frac{\sqrt{s}}{2}(-\sin(\theta),0,-\cos(\theta),1)$$

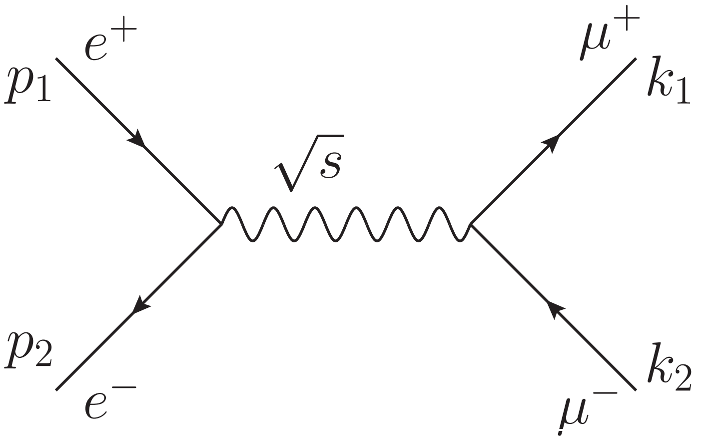
$$\frac{d\sigma}{d\cos(\theta)} = e_q^2 N_c \frac{\pi\alpha_e^2}{2s}[1+\cos^2(\theta)]$$

$$\sigma(e^+e^- \to q\bar{q}) = N_c \left(\sum_q e_q^2\right) \sigma_0$$

$$\sigma_0 = \frac{4\pi\alpha_e^2}{3s}$$

QCD final state: basic QCD





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$$\sigma(e^+e^- \to q\bar{q}) = N_c \left(\sum_q e_q^2\right)\sigma_0 \qquad \sigma(e^+e^- \to \mu^+\mu^-) = \sigma_0$$

$$\sigma_0 = \frac{4\pi\alpha_e^2}{3s}$$

QCD final state: basic QCD

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \approx N_c \left(\sum_q e_q^2 \right)$$

- u, d, s: $R = 3\left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9}\right) = 2$
- u, d, s, c: $R = 3\left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9} + \frac{4}{9}\right) = \frac{10}{3}$
- u, d, s, c, b: $R = 3\left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9} + \frac{4}{9} + \frac{4}{9}\right) = \frac{14}{3}$

Test of

- The 3 colours in QCD ($N_c = 3$)
- The number of quark flavours

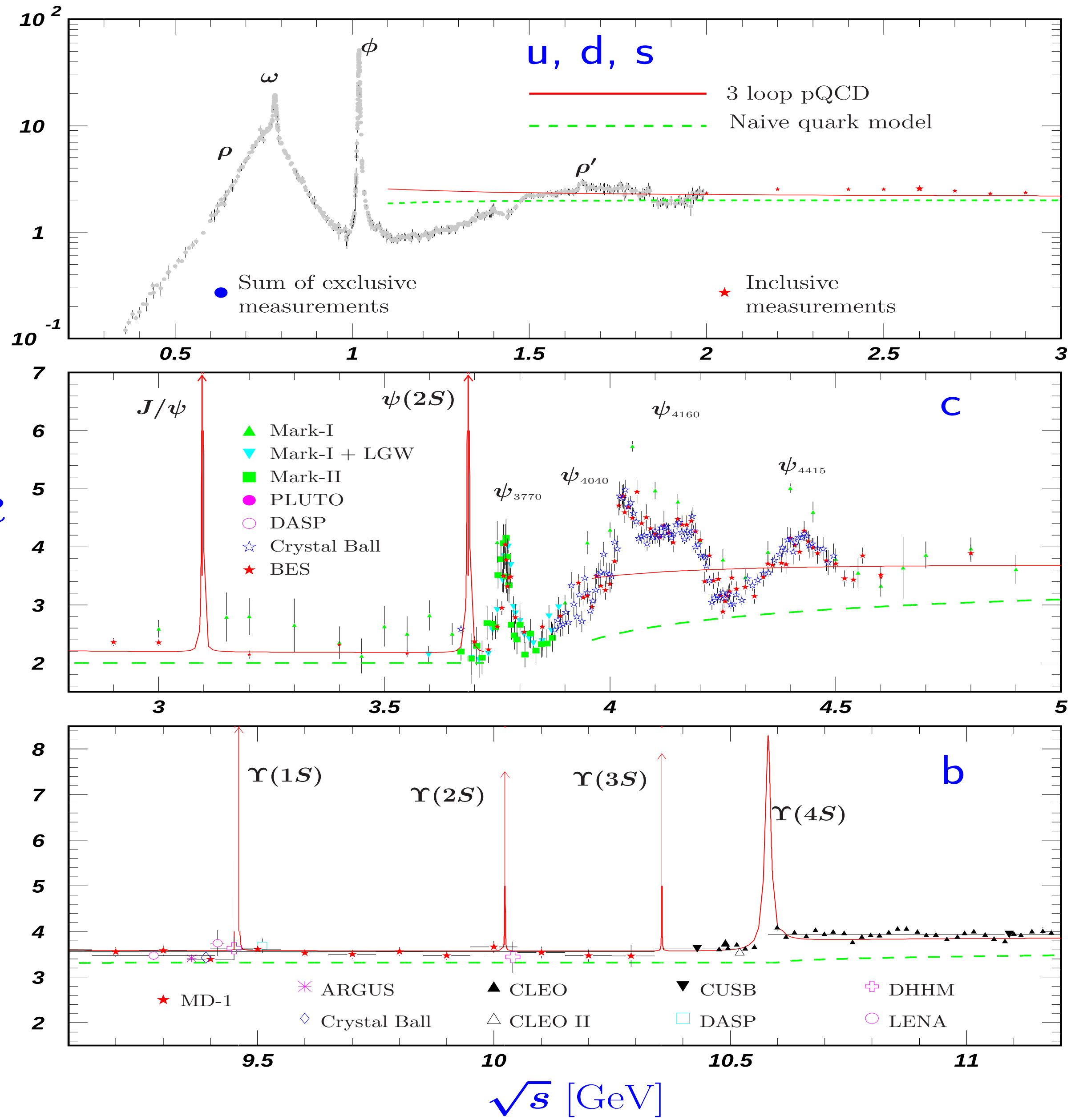
QCD final state: basic QCD

R

- u, d, s:
- u, d, s, c
- u, d, s, c

Test of

- The 3
- The nu



QCD final state: basic QCD

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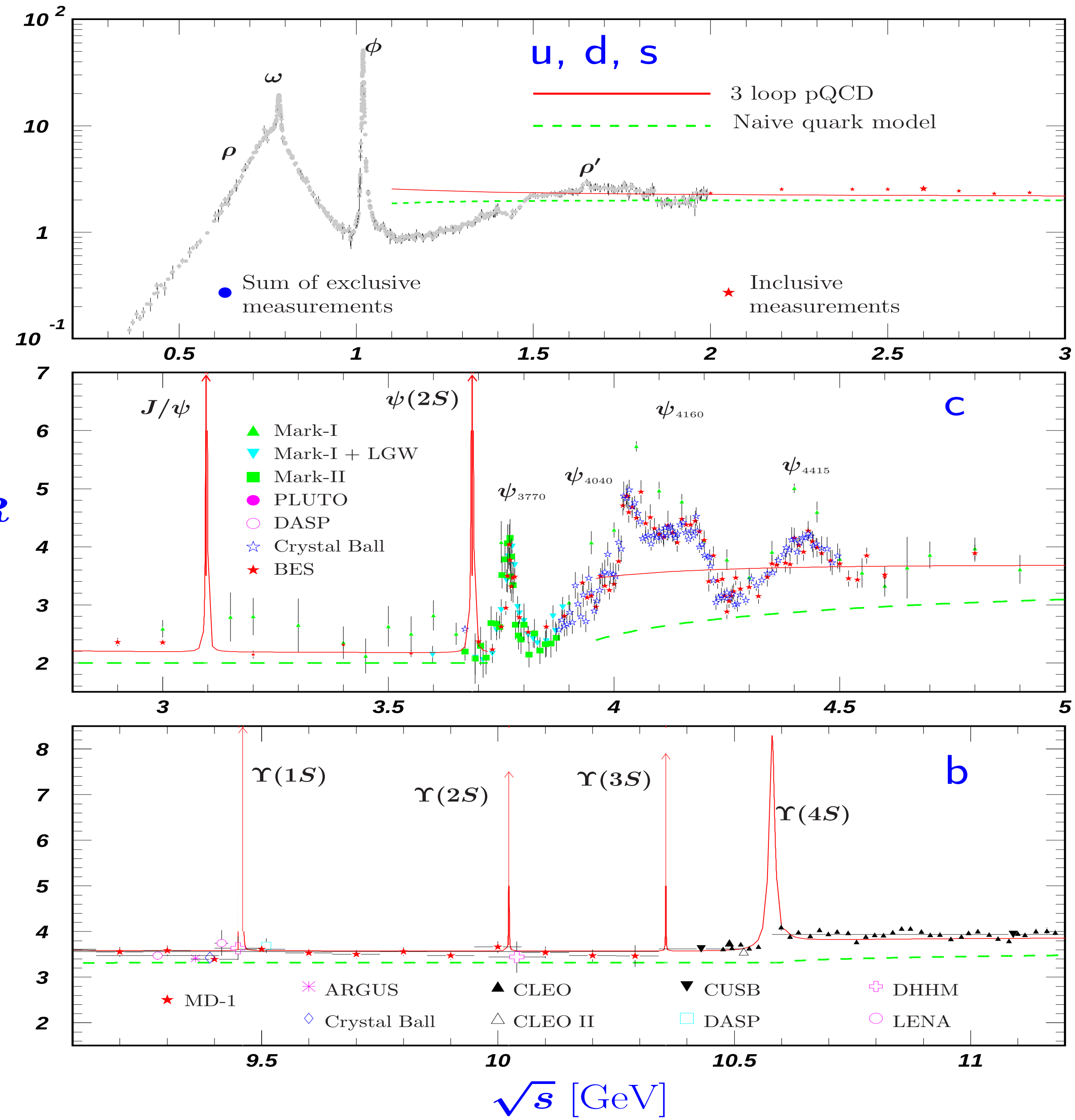


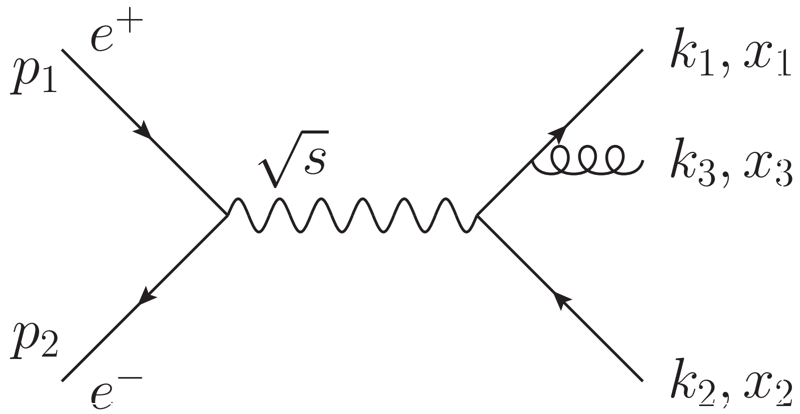
- u, d, s:
- u, d, s, c
- u, d, s, c

Test of

- The 3
- The nu

Q: why $\sigma(e^+e^- \to \mu^+\mu^-)$ and not $\sigma(e^+e^- \to e^+e^-)$?

QCD final state: QCD dynamics



$3 \times (4-1) - 4 = 5$ d.o.f.

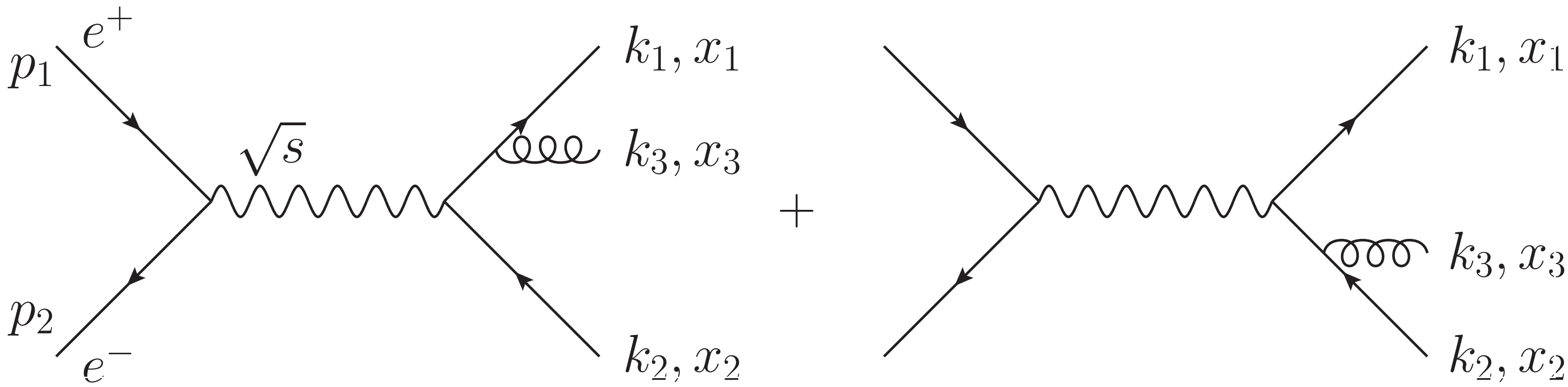
- 3 Euler angles
- $x_i = 2E_i/\sqrt{s}$, $x_1 + x_2 + x_3 = 2$
- or θ_{13}, θ_{23}

$$\int d\Phi_3 = \prod_{i=1}^{3} \frac{d^3 k_i}{(2\pi)^3 2E_i} (2\pi)^4 \, \delta^{(4)}(p_1 + p_2 - k_1 - k_2 - k_3)$$

$$= \frac{s}{32(2\pi)^5} \int d\alpha \, d\cos\beta \, d\gamma \, dx_1 \, dx_2$$

$$\cos(\theta_{13}) = -\frac{x_1^2 + x_3^2 - x_2^2}{2x_1 x_3} \qquad \cos(\theta_{23}) = -\frac{x_2^2 + x_3^2 - x_1^2}{2x_2 x_3}$$

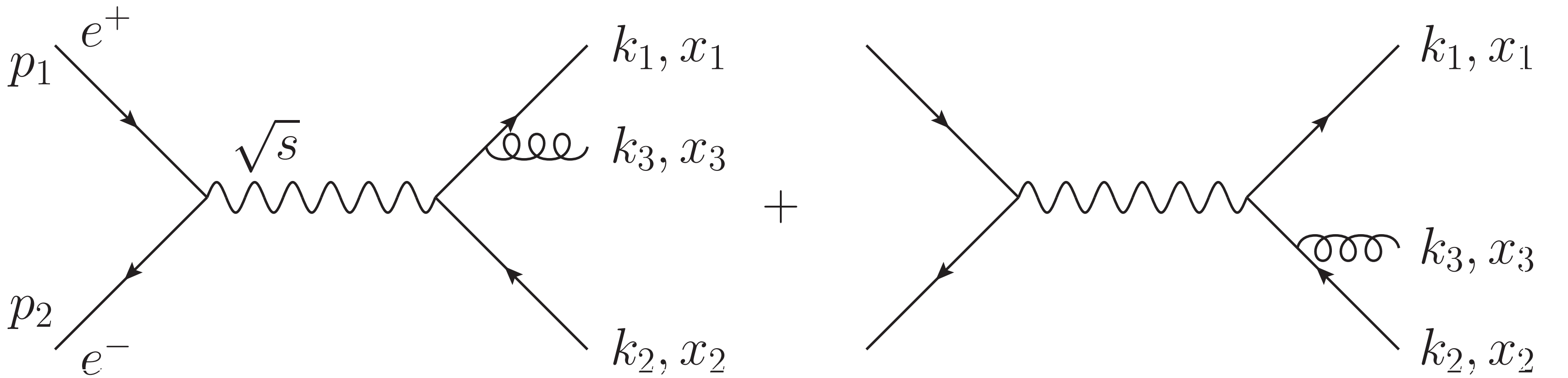
QCD final state: QCD dynamics



$$\bar{\sum} |\mathcal{M}|^2 = 4(4\pi)^3 \alpha_e^2 \alpha_s C_F N_c \frac{(p_1.k_1)^2 + (p_1.k_2)^2 + (p_2.k_1)^2 + (p_2.k_2)^2}{s(k_1.k_3)(k_2.k_3)}$$

$$\frac{d^2\sigma}{dx_1\, dx_2} = e_q^2\, N_c\, \sigma_0\, \frac{\alpha_s C_F}{2\pi}\, \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

QCD final state: QCD dynamics



$e^+e^- \to q\bar{q}$

QCD coupling

QCD colour

QCD dynamics

$$\frac{d^2\sigma}{dx_1\,dx_2} = e_q^2\,N_c\,\sigma_0\,\frac{\alpha_s C_F}{2\pi}\,\frac{x_1^2 + x_2^2}{(1 - x_1)(1 - x_2)}$$

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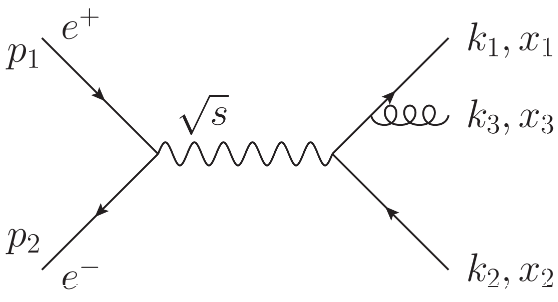

$$\bar{\sum} |\mathcal{M}|^2 \propto \frac{(p_1.k_1)^2+(p_1.k_2)^2+(p_2.k_1)^2+(p_2.k_2)^2}{s(k_1.k_3)(k_2.k_3)}$$

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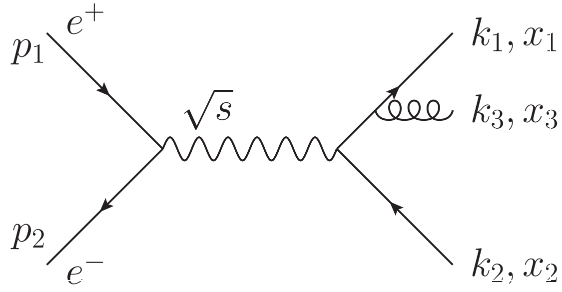
$$x_i = \frac{2E_i}{\sqrt{s}} \qquad x_1+x_2+x_3=2 \qquad c_{\theta_{13}} = -\frac{x_1^2+x_3^2-x_2^2}{2x_1x_3}$$

QCD final state: QCD dynamics



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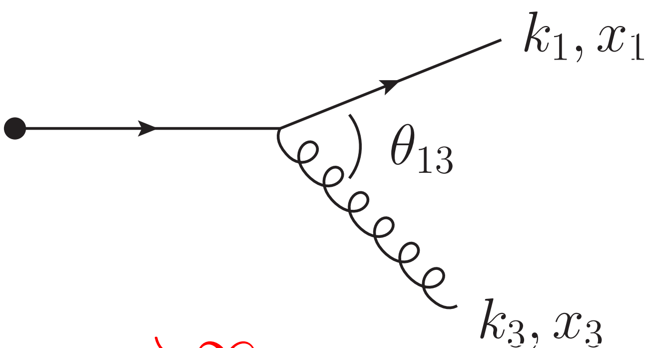
Divergent when $k_1.k_3 \to 0$ or $k_2.k_3 \to 0$



$k_1.k_3 \to 0 \Rightarrow (k_1+k_3)^2 \to 0$ *i.e.*

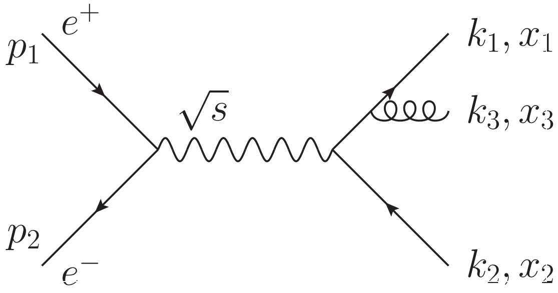
parent quark propag $= \dfrac{1}{(k_1+k_3)^2} \to \infty$

Physical origin of the divergence!

They are **infrared divergences** ($(k_1+k_3)^2 \to 0$, not ∞)

(one power cancelled by phase-space $\Rightarrow$ log divergence)

QCD final state: QCD dynamics



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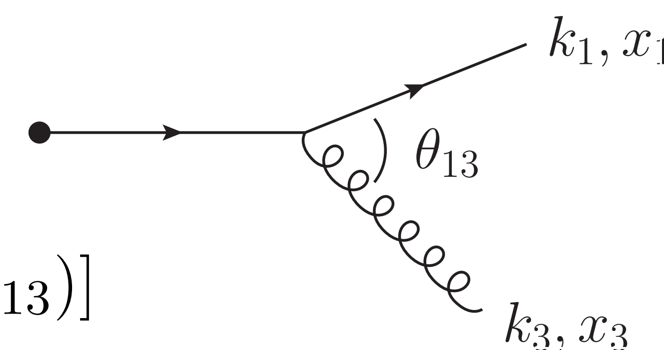
Divergent when x_1 (or x_2) $\to 1$



$$1 - x_2 = \frac{1}{2} x_1 x_3 \left[1 - \cos(\theta_{13})\right]$$

- $\theta_{13} \to 0$ (or θ_{23}): collinear divergence divergence
- $x_3 \to 0$ (*i.e.* $E_g \to 0$): soft divergence

QCD final state: coll and soft divergences

Collinear and soft divergences

- fundamental/omnipresent in QCD! (also in QED)
  we will meet them often through these lectures

QCD final state: coll and soft divergences

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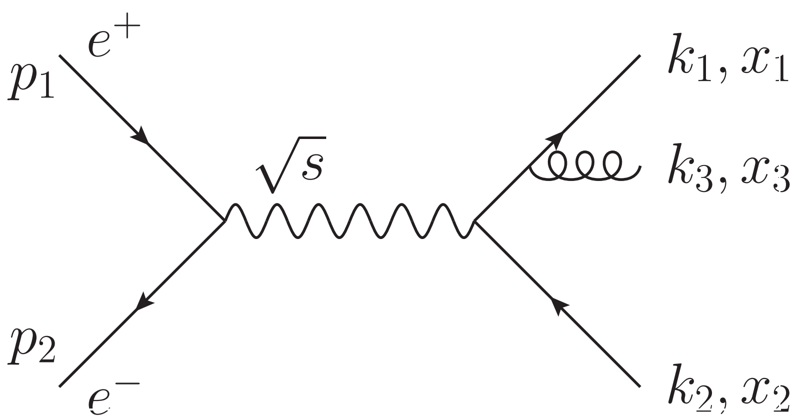
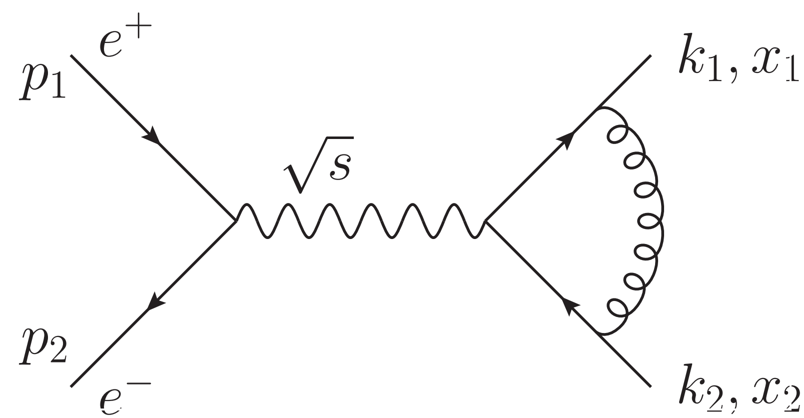
- also present for $g \to gg$ ($\neq$ QED; $C_F \to C_A$)
- cancelled by virtual corrections



Real



Virtual

QCD final state: coll and soft divergences

Collinear and soft divergences

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- also present for $g \to gg$ ($\neq$ QED; $C_F \to C_A$)
- cancelled by virtual corrections
  Dimensional regularisation $d = 4 - 2\varepsilon$:

$$\sigma^{(q\bar{q}g)}_{\text{real}} = e_q^2 N_c \sigma_0 \frac{\alpha_s C_F}{2\pi} T(\epsilon) \left[\frac{2}{\varepsilon^2} + \frac{3}{\varepsilon} + \frac{19}{2} + \mathcal{O}(\varepsilon) \right]$$

$$\sigma^{(q\bar{q}g)}_{\text{virt}} = e_q^2 N_c \sigma_0 \frac{\alpha_s C_F}{2\pi} T(\epsilon) \left[\frac{-2}{\varepsilon^2} - \frac{3}{\varepsilon} - 8 + \mathcal{O}(\varepsilon) \right]$$

$$\sigma^{(q\bar{q}g)}_{\mathcal{O}(\alpha_s)} = e_q^2 N_c \sigma_0 \frac{3\alpha_s C_F}{4\pi} = e_q^2 N_c \sigma_0 \frac{\alpha_s}{\pi}$$

QCD final state: coll and soft divergences

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- cancellation order-by-order in perturbation theory

Block-Nordsieck, Kinoshita-Lee-Nauenberg theorems

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Block-Nordsieck, Kinoshita-Lee-Nauenberg theorems

- Terminology issue: 'soft' divergence sometimes called 'infrared' divergence (though both soft and coll are infrared)

QCD final state: IRC safety

Cancellation of divergence not true for any observable

Example: "number of partons in the final state", dP/dn

- LO ($\mathcal{O}(\alpha_s^0)$): $dP/dn = \delta(n-2)$
- NLO ($\mathcal{O}(\alpha_s^1)$):
  - (i) real emission: $n = 3$
  - (ii) virtual correction: $n = 2$

  $\Rightarrow$ $dP/dn = [1 - \infty\alpha_s]\delta(n-2) + \infty\alpha_s\delta(n-3)$

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Observables for which cancellation happens
are called **INFRARED-AND-COLLINEAR SAFE**
Necessary for perturbative QCD computation
to make sense!!

QCD final state: IRC safety

Observable $\mathcal{O}$:

$$\mathcal{O} = \sum_{n=0}^{\infty} \int \underbrace{d\Psi_n(k_1, \dots, k_n)}_{\text{phasespace}} \underbrace{\frac{d\sigma}{d\Psi_n}(k_1, \dots, k_n)}_{\text{matrix element}} \underbrace{\mathcal{O}_n(k_1, \dots, k_n)}_{\text{observable}}$$

- IR safety: "adding a soft particle does not change $\mathcal{O}$"

$$\mathcal{O}_{n+1}(k_1, \dots, k_n, k_{n+1}) \stackrel{k_{n+1} \to 0}{=} \mathcal{O}_n(k_1, \dots, k_n)$$

- Collinear safety: "a collinear splitting does not change $\mathcal{O}$"

$$\mathcal{O}_{n+1}(k_1, \dots, \lambda k_n, (1-\lambda) k_n) = \mathcal{O}_n(k_1, \dots, k_n)$$

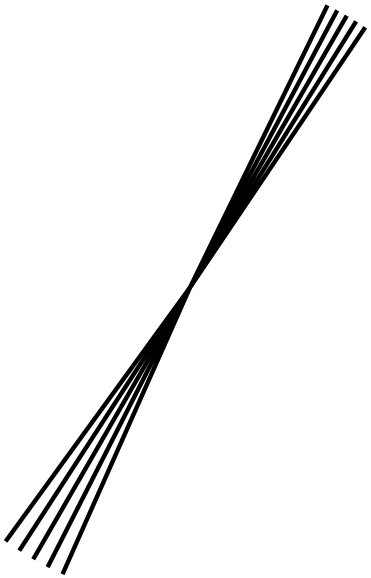
for $0 < \lambda < 1$

QCD final state: IRC safety

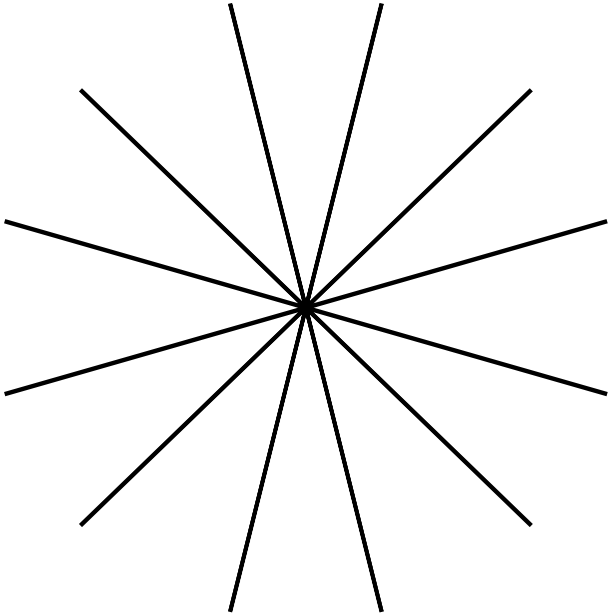
Example #1: event-shapes in e^+e^-

thrust, sphericity, thrust major, thrust minor, ...

Thrust: $$T_n = \max_{|\vec{u}|=1} \frac{\sum_{i=0}^{n} |\vec{k}_i.\vec{u}|}{\sum_{i=0}^{n} |\vec{k}_i|}$$



pencil-like: $T \lesssim 1$



spherical: $T \gtrsim 1/2$

QCD final state: IRC safety

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- the thrust is infrared safe: for $k_{n+1} \to 0$

$$T_{n+1} = \max_{|\vec{u}|=1} \frac{\sum_{i=0}^{n+1} |\vec{k}_i.\vec{u}|}{\sum_{i=0}^{n+1} |\vec{k}_i|} = \max_{|\vec{u}|=1} \frac{\sum_{i=0}^{n} |\vec{k}_i.\vec{u}|}{\sum_{i=0}^{n} |\vec{k}_i|} = T_n$$

- the thrust is collinear safe

$$0 < \lambda < 1 \Rightarrow \begin{cases} |\vec{u}.(\lambda\vec{k} + (1-\lambda)\vec{k})| = |\vec{u}.\vec{k}| \\ |\lambda\vec{k} + (1-\lambda)\vec{k}| = |\vec{k}| \end{cases}$$

QCD final state: IRC safety

Example #1: event-shapes in e^+e^-

thrust, sphericity, thrust major, thrust minor, ...

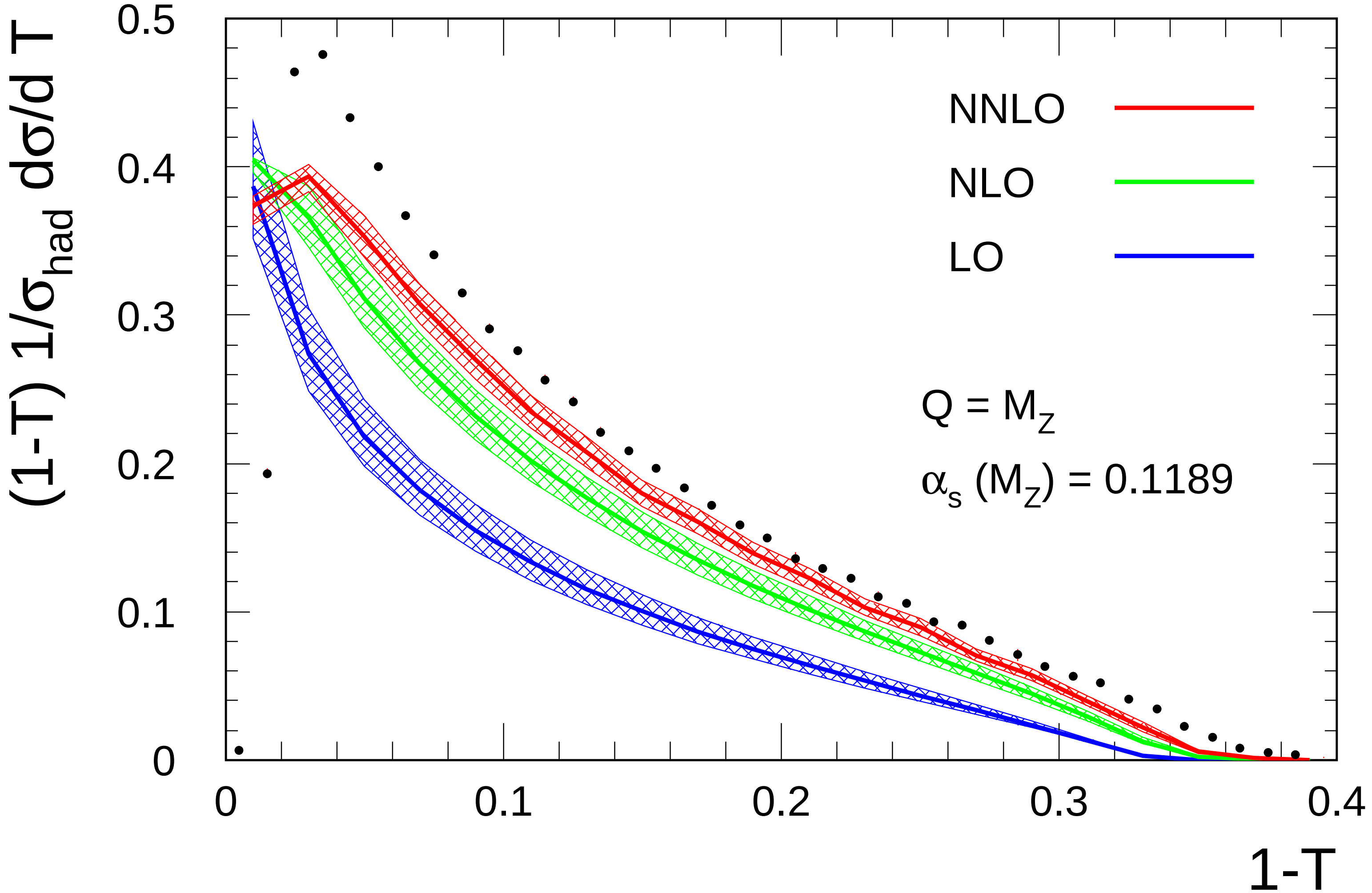
Thrust: $$T_n = \max_{|\vec{u}|=1} \frac{\sum_{i=0}^{n} |\vec{k}_i.\vec{u}|}{\sum_{i=0}^{n} |\vec{k}_i|}$$

Computation in perturbative QCD (from the matrix element given earlier)

$$\frac{1}{\sigma}\frac{d\sigma}{dT} = \frac{\alpha_s C_F}{2\pi}\left[\frac{2(2-3T+3T^2)}{T(1-T)}\log\left(\frac{2T-1}{1-T}\right) - \frac{3(2-T)(3T-2)}{1-T}\right]$$

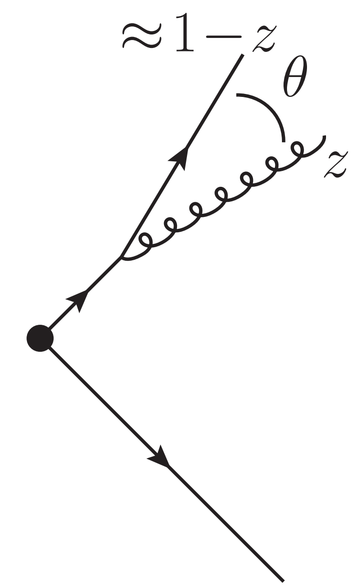
- Allows for test of QCD (*e.g.* at LEP)
- “log” is a reminiscence from the soft and collinear divergence

Thrust



comparison with LEP data: peaked at $T = 1$

Typical behaviour of divergences:

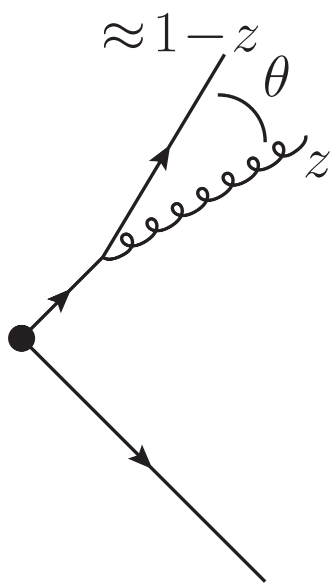


- Collinear limit:

$$\frac{1}{\sigma_0}d\sigma \approx \underbrace{\frac{\alpha_s}{2\pi}\frac{1+(1-z)^2}{z}}_{\text{splitting proba}} \underbrace{\frac{d\theta^2}{\theta^2}}_{\text{coll.div}}$$

For different situations (different parton types), the branching probability changes but the $d\theta/\theta$ is generic!

Typical behaviour of divergences:



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- Soft limit:

$$d\sigma_{q\bar{q}g} = d\sigma_{q\bar{q}}\frac{\alpha_s C_F}{\pi^2}\frac{(k_1.k_2)}{(k_1.k_3)(k_2.k_3)}d^4k_3\,\delta(k^2) \propto \frac{dE_3}{E_3} \propto \frac{dz}{z}$$

Antenna formula — soft-gluon emission

e^+e^-: QCD divergences

Frequent appearance in computations:

Both soft and collinear divergences are logarithmic
$\Rightarrow$ the emission of a gluon comes with a factor $\alpha_s \log$

Example:

soft emissions for the thrust : $\alpha_s \log(1-T)$

At some point, $\alpha_s \log \sim 1$ *i.e.* NLO$\sim$LO in the perturbative series

$\Rightarrow$ At order n, we will have $\alpha_s^n \log^n$ all of the same order

$\Rightarrow$ ALL have to be considered: **resummation**

Other interests in e^+e^- *collisions*

- Fragmentation functions

  "parton $\rightarrow$ hadron transition", $D_{p/\pi}(z, p_t)$

- Hadronisation

  e.g. Lund strings

- Jets

  Collinear divergence $\longrightarrow$ a parton develops into a bunch of collimated particles

We will postpone (part of) this to the "hadronic collisions" chapter

e^+e^-: Summary

- e^+e^- collisions: good framework to test QCD (final state)
- emission of a gluon has 2 divergences: soft and collinear
  - cancel between “real” and “virtual” daigrams
  - ... provided the observable is IRC safe
  - give rise to “logarithms” in perturbative computations
  - ... resummed to all orders when $\alpha_s \log \sim 1$
  - ... done analytically or by parton cascade MC
- collinear divergence+parton branching $\to$ jets

Time for questions!

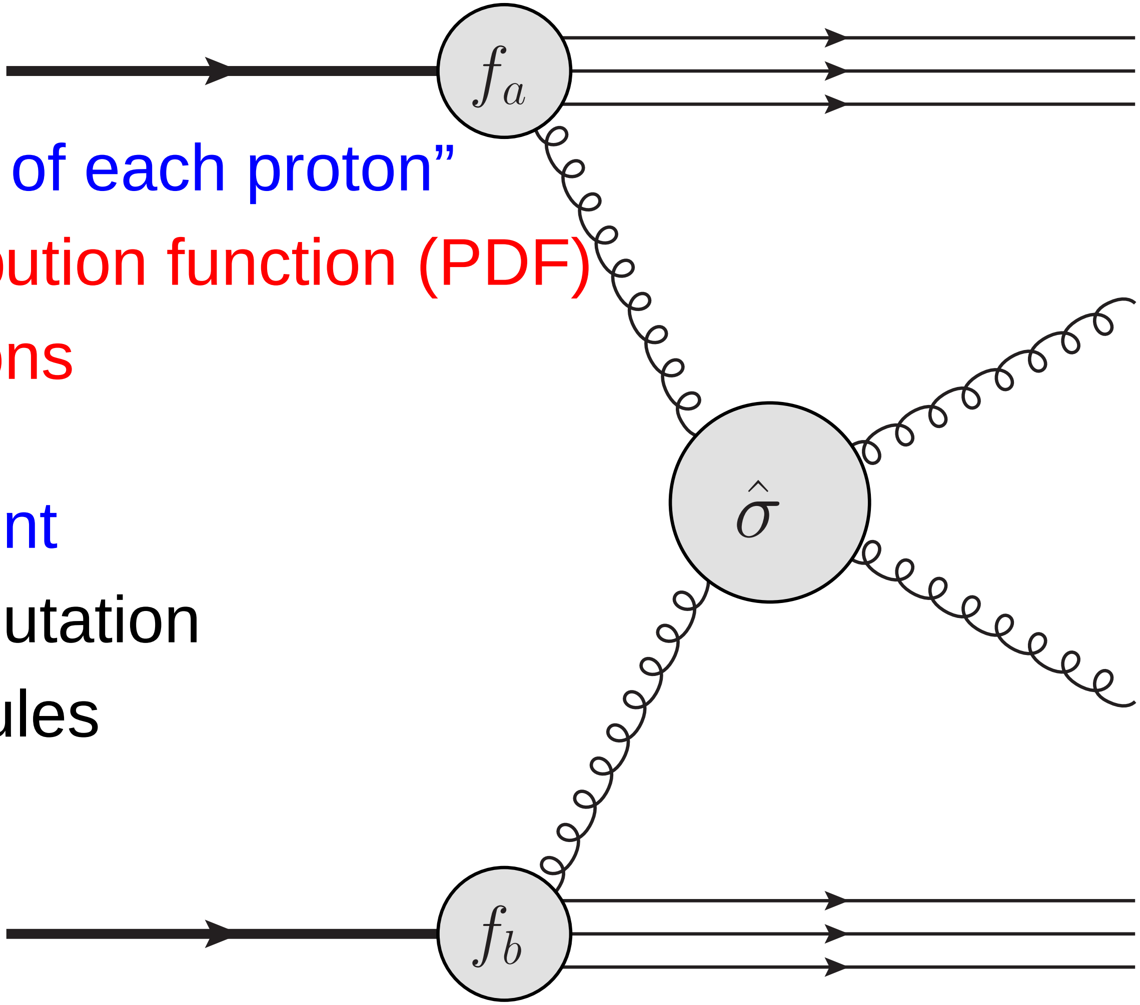
<interlude hadronic collisions>

kinematics

jets

The very fundamental collision

$$\sigma = f_a \otimes f_b \otimes \hat{\sigma}$$



- “take a parton out of each proton”
  $f_a \equiv$ parton distribution function (PDF)
  for quark and gluons

- hard matrix element
  perturbative computation
  Forde-Feynman rules

Kinematics

Incoming partons:

$$
\begin{aligned}
p_1 &\equiv x_1 \frac{\sqrt{s}}{2}(0,0,\;\;\;1,1) \\
p_2 &\equiv x_2 \frac{\sqrt{s}}{2}(0,0,-1,1)
\end{aligned}
$$

- carry a fraction of the beam's (longitudinal) momentum
- Energy^2 in the hard collision: $(p_1 + p_2)^2 = x_1 x_2 s \leq s$
- the partonic centre-of-mass is shifted/boosted compared to the lab/pp centre-of-mass
  $\Rightarrow$ need variables (longitudinally) boost-invariant

Kinematics

Final-state particles: commonly-used variables

$$k \equiv (k_x, k_y, k_z, E) \equiv E(\sin(\theta)\cos(\phi), \sin(\theta)\sin(\phi), \cos(\theta), 1)$$

- E and θ are not suited!

Kinematics

Final-state particles: commonly-used variables

- Transverse plane
  - azimuthal angle ϕ
  - transverse momentum $p_t = \sqrt{p_x^2 + p_y^2}$

Kinematics

Final-state particles: commonly-used variables

- Transverse plane
  - azimuthal angle ϕ
  - transverse momentum $p_t = \sqrt{p_x^2 + p_y^2}$
- Longitudinal variable
  - Rapidity: $y = \frac{1}{2}\log\left(\frac{E+p_z}{E-p_z}\right)$

Boost: $$y \;\rightarrow\; \frac{1}{2}\log\left(\frac{\gamma(E-\beta p_z)+\gamma(p_z-\beta E)}{\gamma(E-\beta p_z)-\gamma(p_z-\beta E)}\right)$$
$$= \frac{1}{2}\log\left(\frac{\gamma(1-\beta)(E+p_z)}{\gamma(1+\beta)(E-p_z)}\right) = y + \frac{1}{2}\log\left(\frac{(1-\beta)}{(1+\beta)}\right)$$

not boost-invariant itself but $\Delta y = y_2 - y_1$ is ($\Delta\theta$ is not)

Kinematics

Final-state particles: commonly-used variables

- Transverse plane
  - azimuthal angle ϕ
  - transverse momentum $p_t = \sqrt{p_x^2 + p_y^2}$
- Longitudinal variable
  - Rapidity: $y = \frac{1}{2} \log \left(\frac{E+p_z}{E-p_z} \right)$

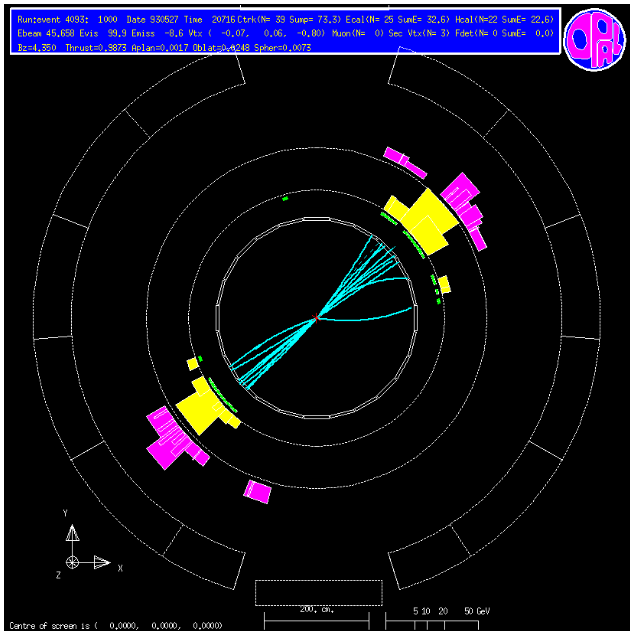
    $k \equiv (k_t \cos(\phi), k_t \sin(\phi), m_t \sinh(y), m_t \cosh(y))$

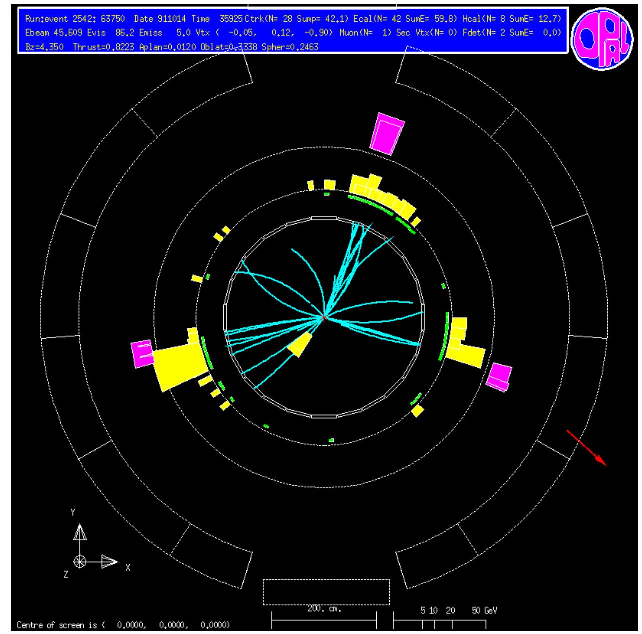
    Transverse mass: $m_t^2 = k_t^2 + m^2$
  - Pseudo-rapidity: $\eta = \frac{1}{2} \log (\tan(\theta/2))$

    $\Delta\eta$ boost-invariant if massless
  - For massless particles: $y = \eta$

Jets

- We have seen in the e^+e^- studies (thrust) that the final state is pencil-like





- Consequence of the collinear divergence
  QCD branchings are most likely collinear
  ($dP/d\theta \propto \alpha_s/\theta$)

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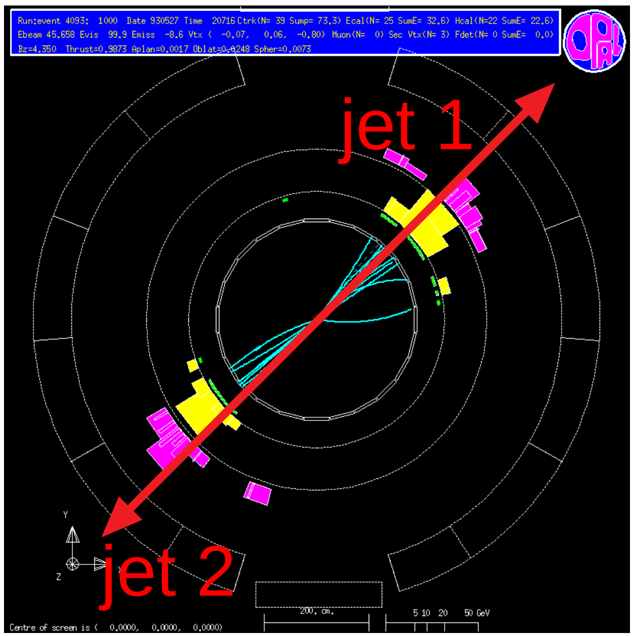

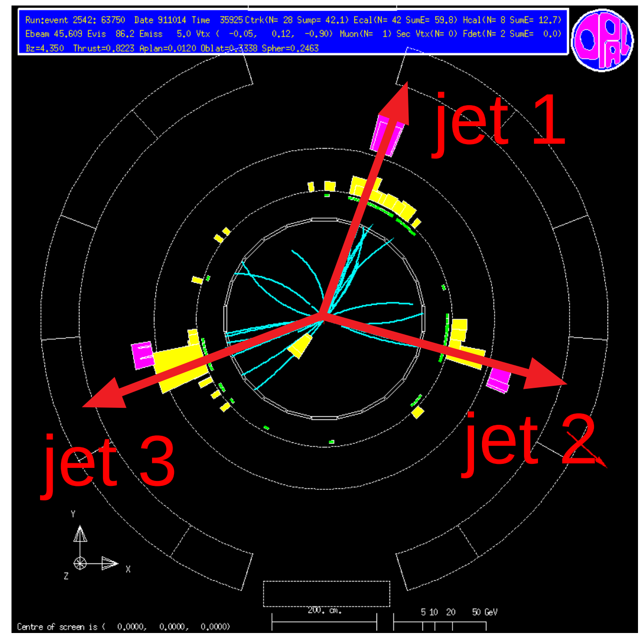

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"Jets" $\equiv$ bunch of collimated particles $\cong$ hard partons

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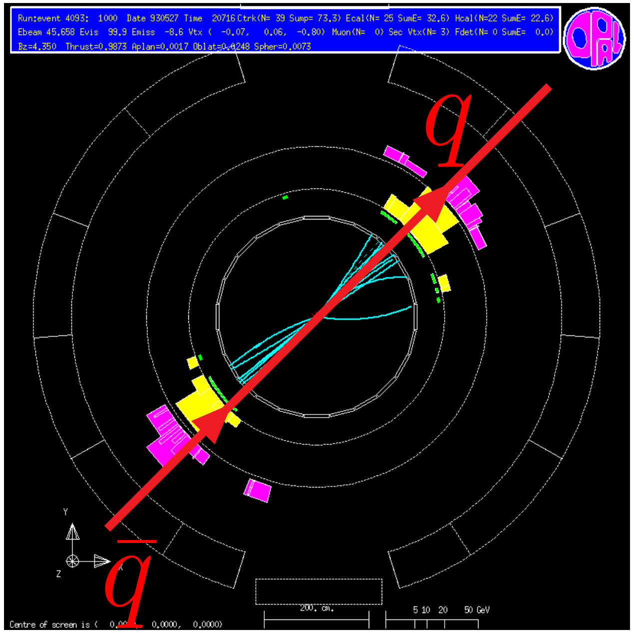

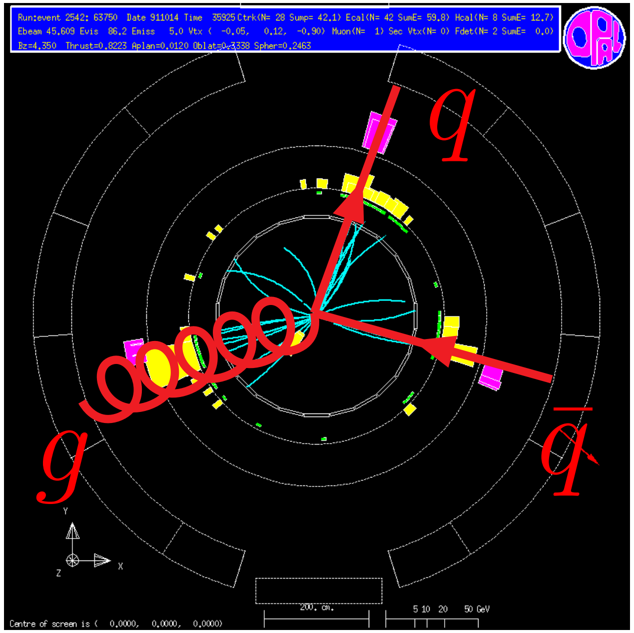

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Jets

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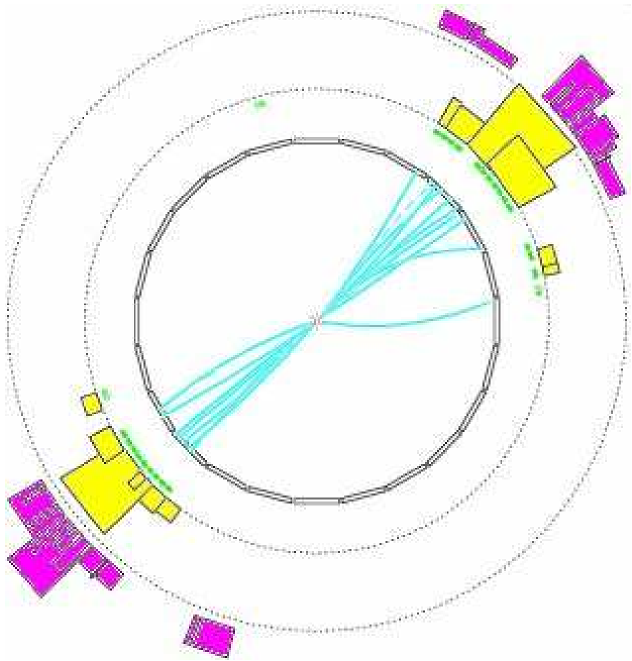
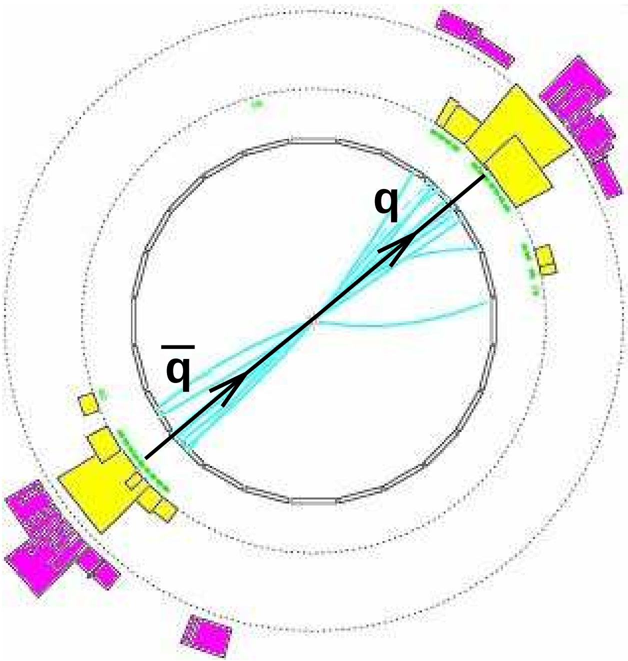
obviously 2 jets



$\rightarrow$



Jets

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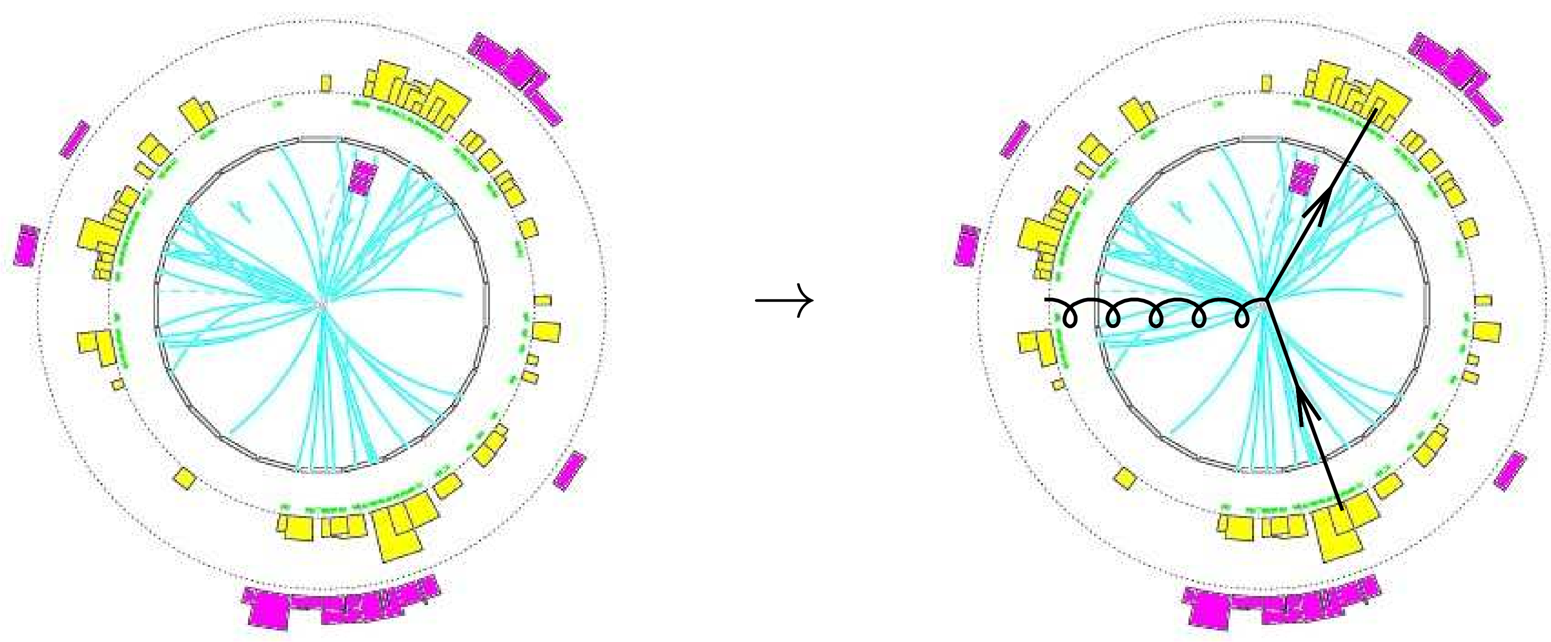
3 jets



Jets

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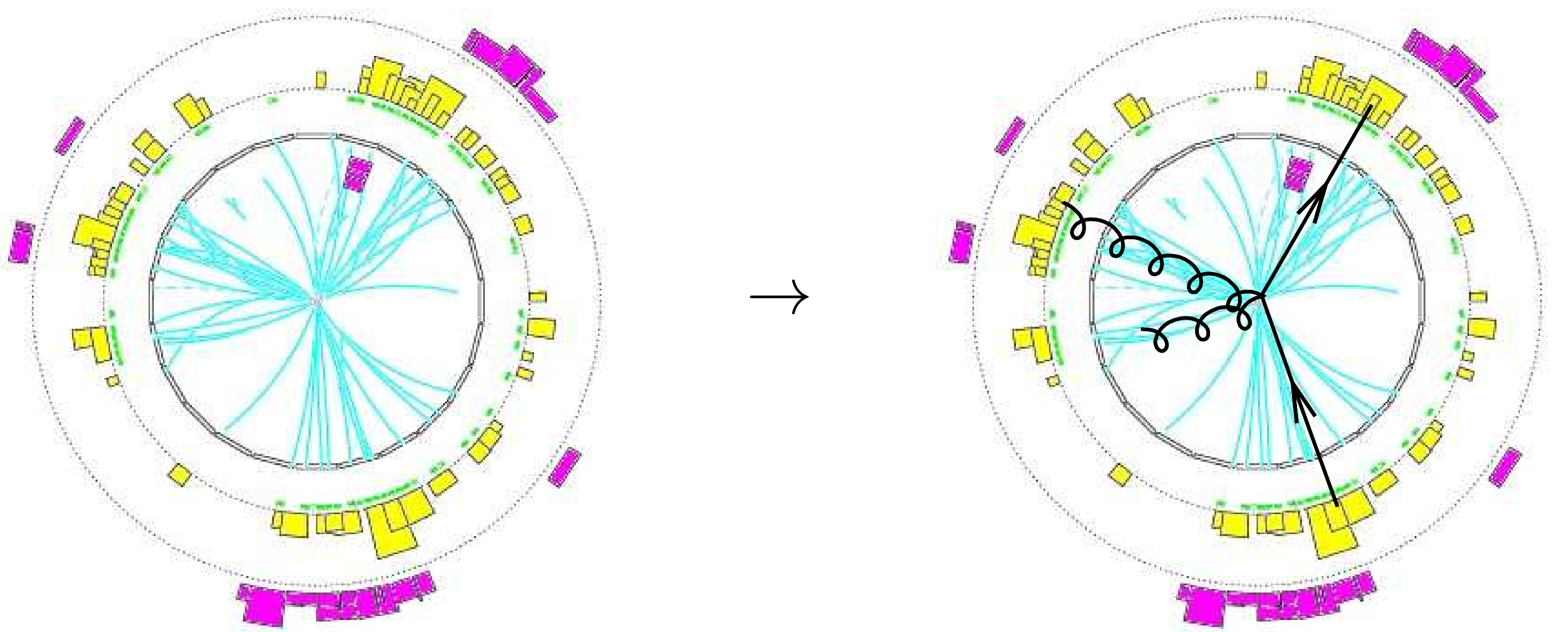
3 jets... or 4?



- “collinear” is arbitrary

Jets

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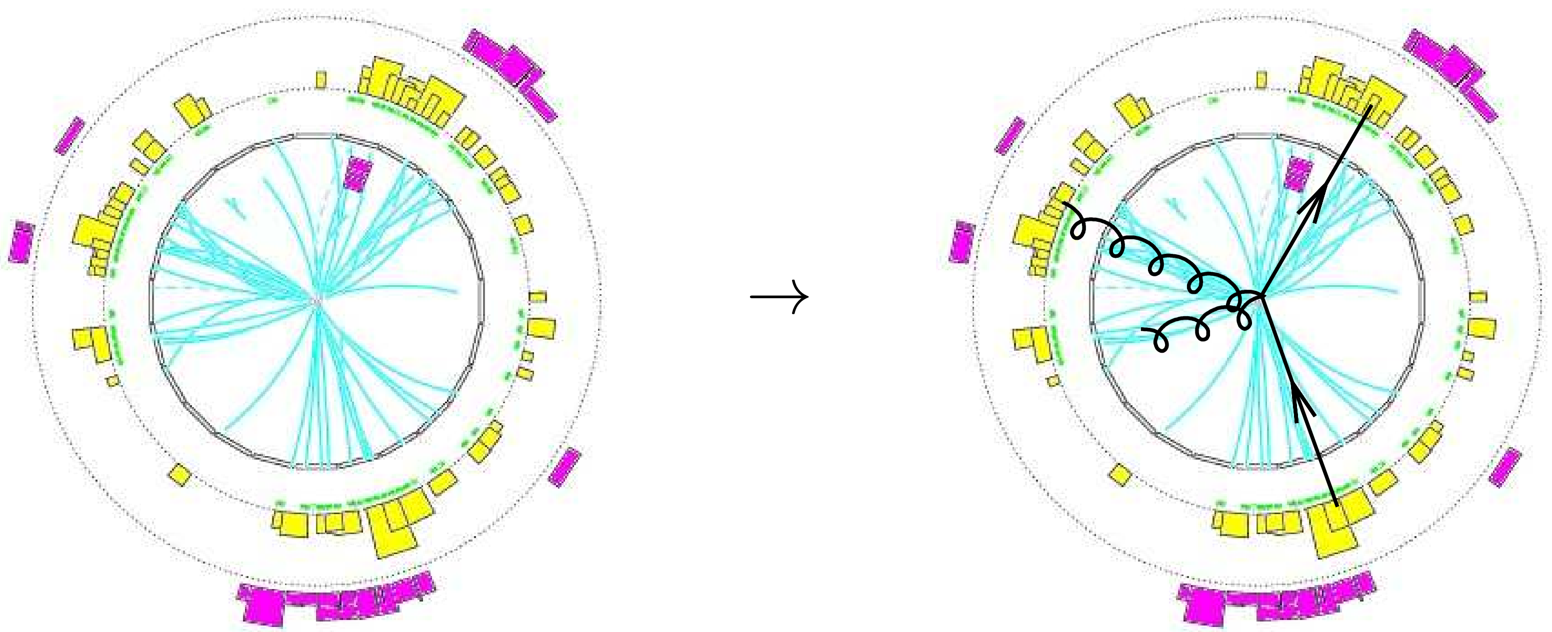
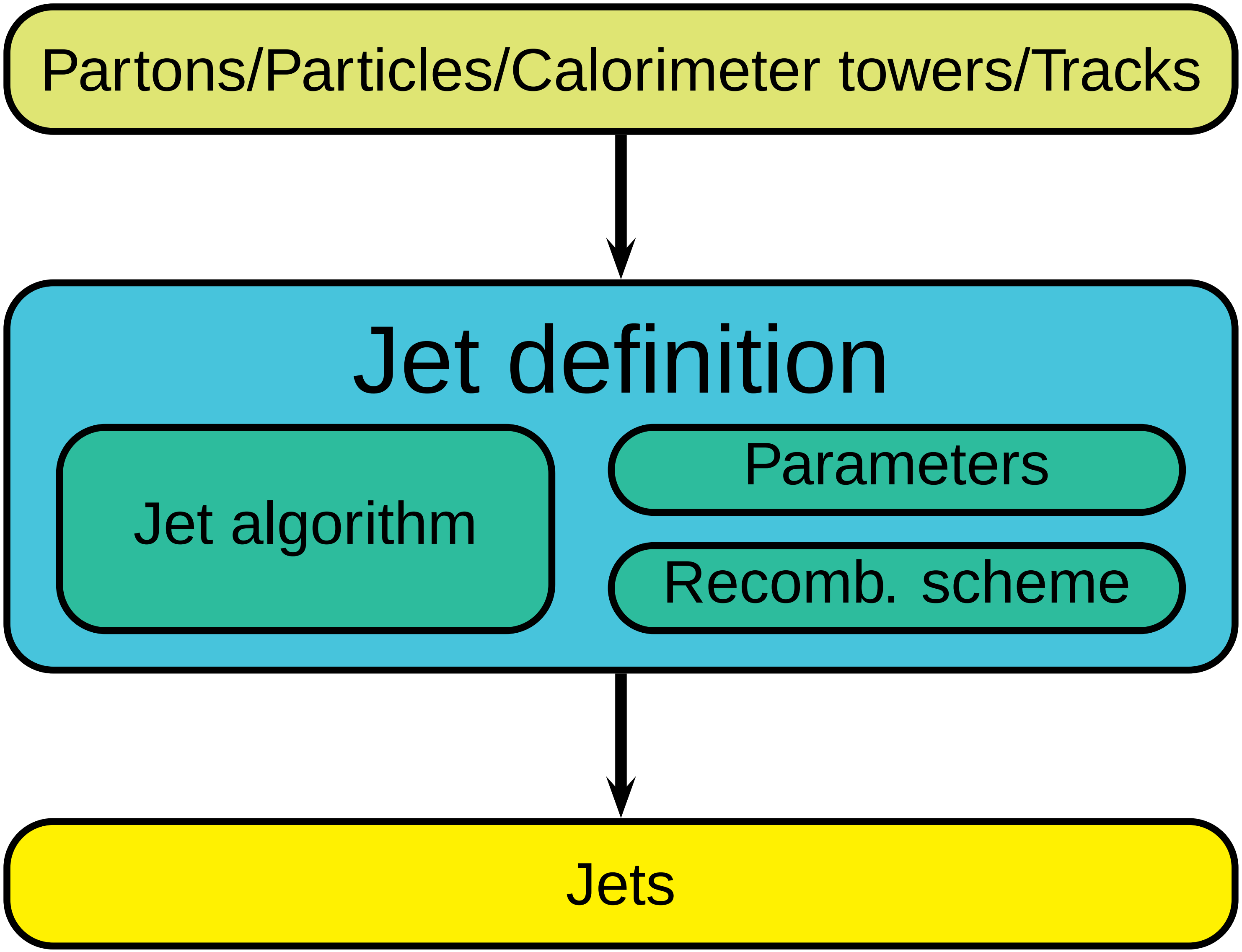


- “collinear” is arbitrary
- “parton” concept strictly valid only at LO

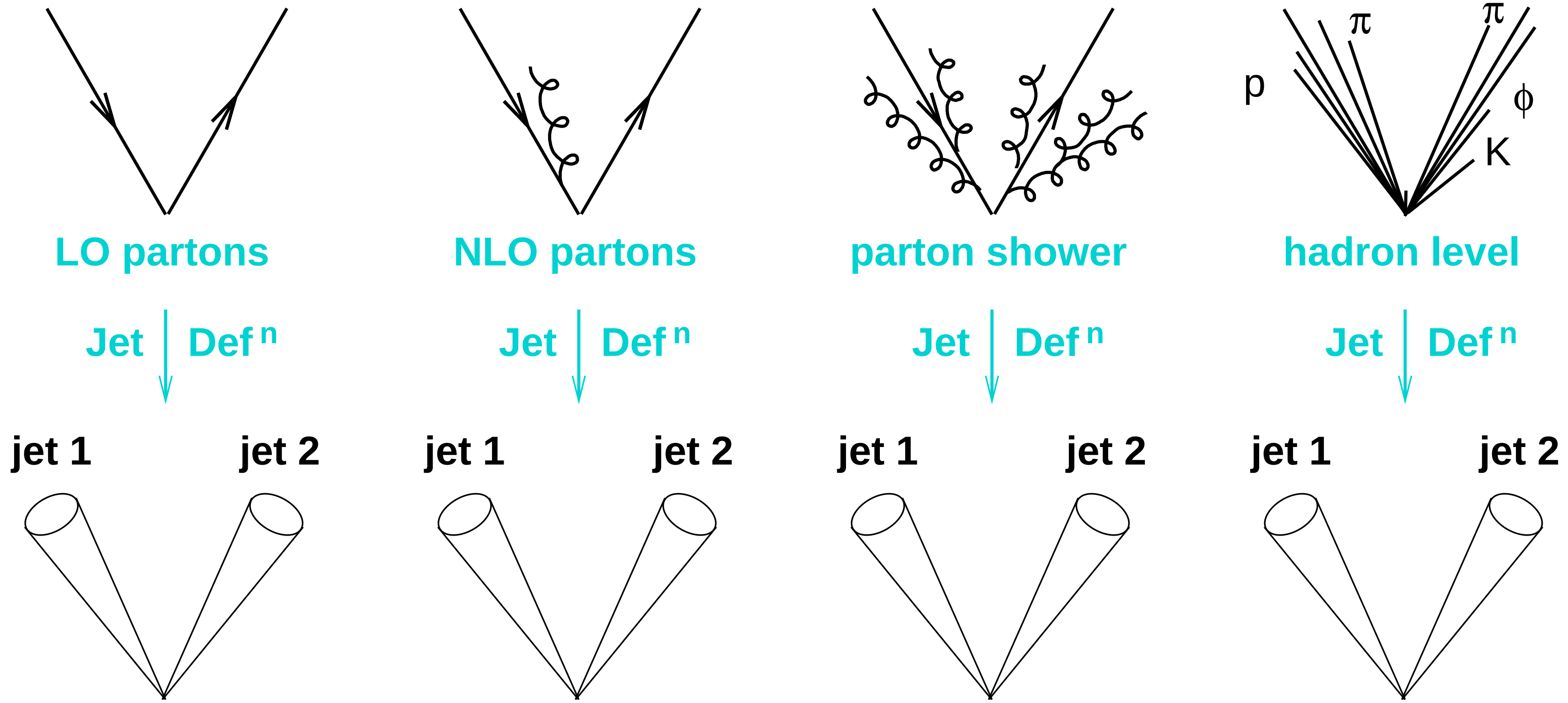
Jets



Partons/Particles/Calorimeter towers/Tracks
Jet definition
Jet algorithm
Parameters
Recomb. scheme
Jets

Jets

A jet definiton is supposed to be (as) consistent (as possible) across different view of an event



Jet definitions: constraints

SNOWMASS accords (FermiLab, 1990)

Several important properties that should be met by a jet definition are [3]:

1. Simple to implement in an experimental analysis;
2. Simple to implement in the theoretical calculation;
3. Defined at any order of perturbation theory;
4. Yields finite cross section at any order of perturbation theory;
5. Yields a cross section that is relatively insensitive to hadronization.

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5. Yields a cross section that is relatively insensitive to hadronization.

30 years later, these are only recently satisfied!!!

Jet definitions: cone

Cone algorithm

- Concept of *stable cone* as a direction of energy flow
  - *"cone"*: circle of fixed radius R in the (y, ϕ) plane
  - *"stable"*: sum of the particles (4-mom.) inside the cone points in the direction of its centre

Jet definitions: cone

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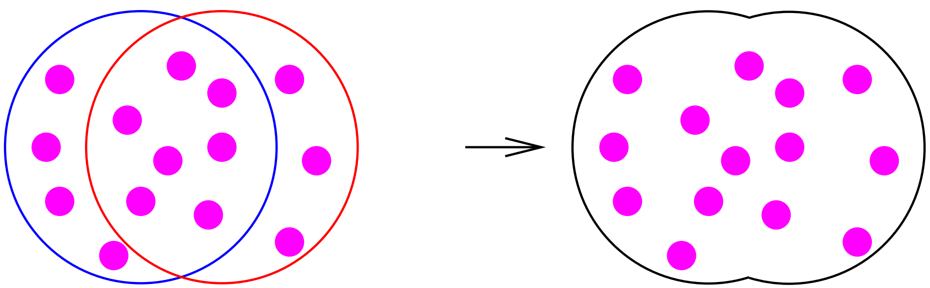
  - the sum of particles in the cone gives a new direction
  - iterate until stable
- Stable cones $\equiv$ jets ... up to overlaps!

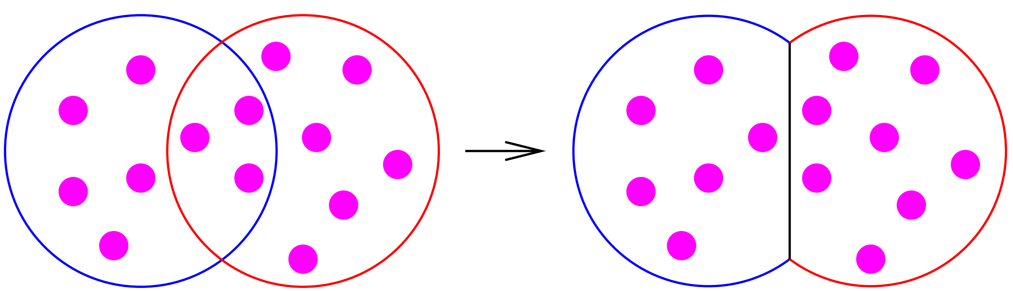
Jet definitions: cone with SM

Cone algorithm: (1) cone with split–merge

- Step 1: find the stable cones with the seeds
  1. input particles (over a seed threshold)
  2. midpoints of the stable cones found above
- Step 2: split–merge (with threshold f)



$$p_{t,\text{common}} > f\, p_{t,\text{hard}}$$



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Examples: main algorithm at the Tevatron

- CDF JetClu (1)
- CDF MidPoint (1+2)
- D0 Run II Cone (1+2)
- ATLAS Cone (1)

Jet definitions: cone with SM

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Examples: main algorithm at the Tevatron

- CDF JetClu (1) IR unsafe (2 hard+1 soft)
- CDF MidPoint (1+2) IR unsafe (3 hard+1 soft)
- D0 Run II Cone (1+2) IR unsafe (3 hard+1 soft)
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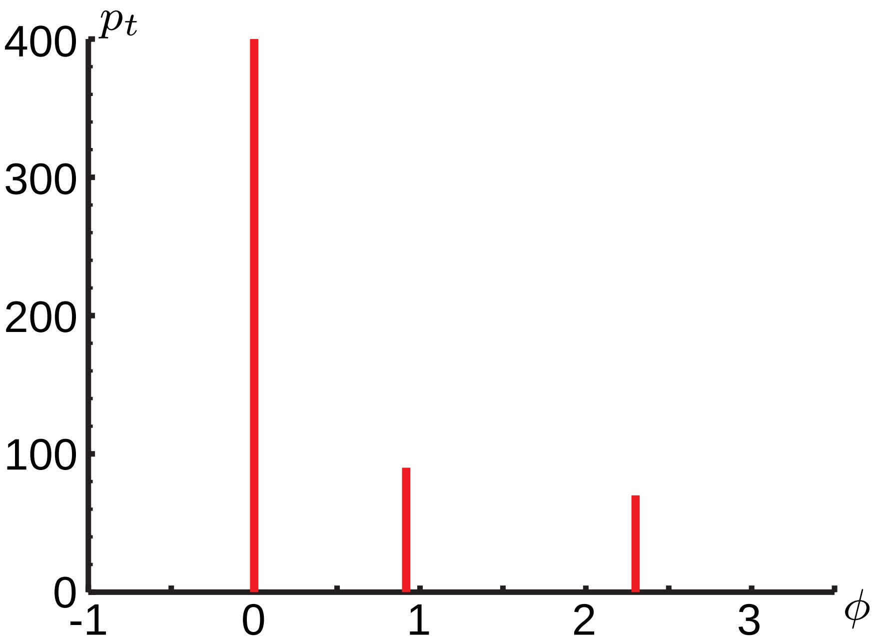
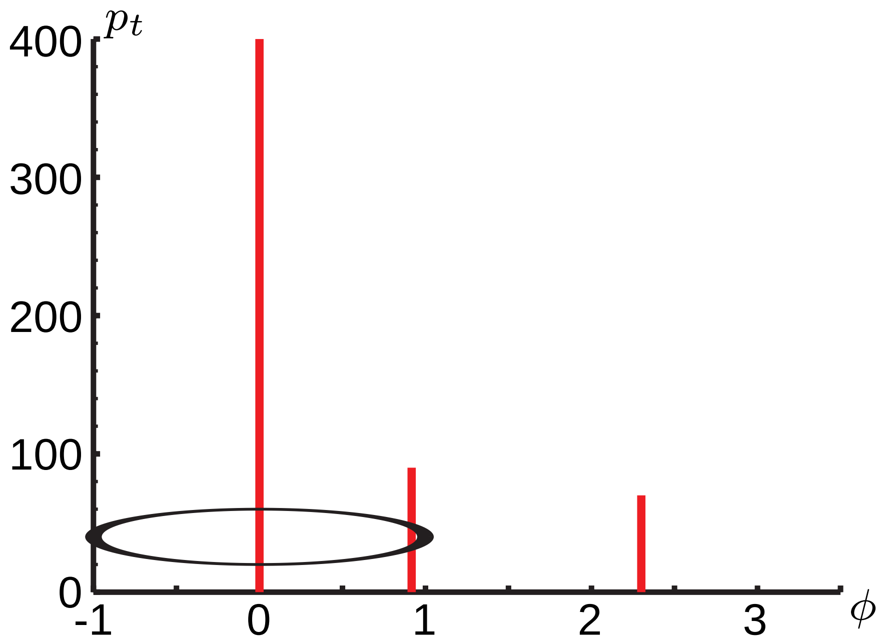
IR unsafety of the Midpoint alg



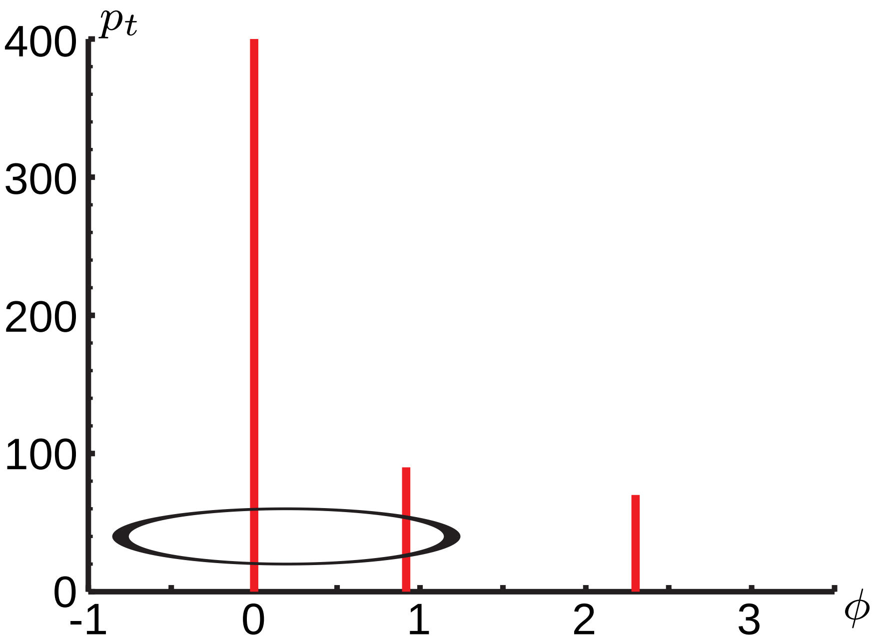
3-particle event — MidPoint clustering

IR unsafety of the Midpoint alg



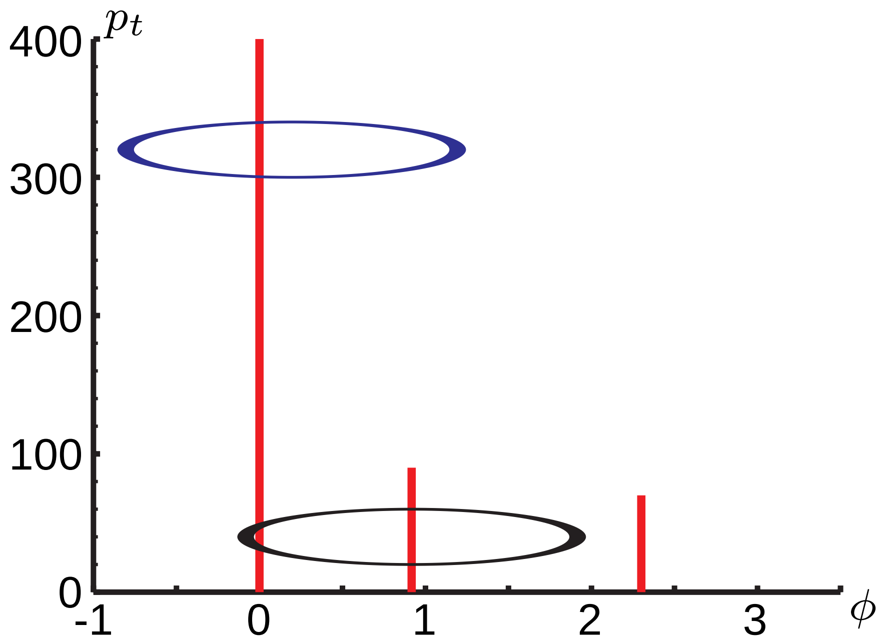
1st seed

IR unsafety of the Midpoint alg



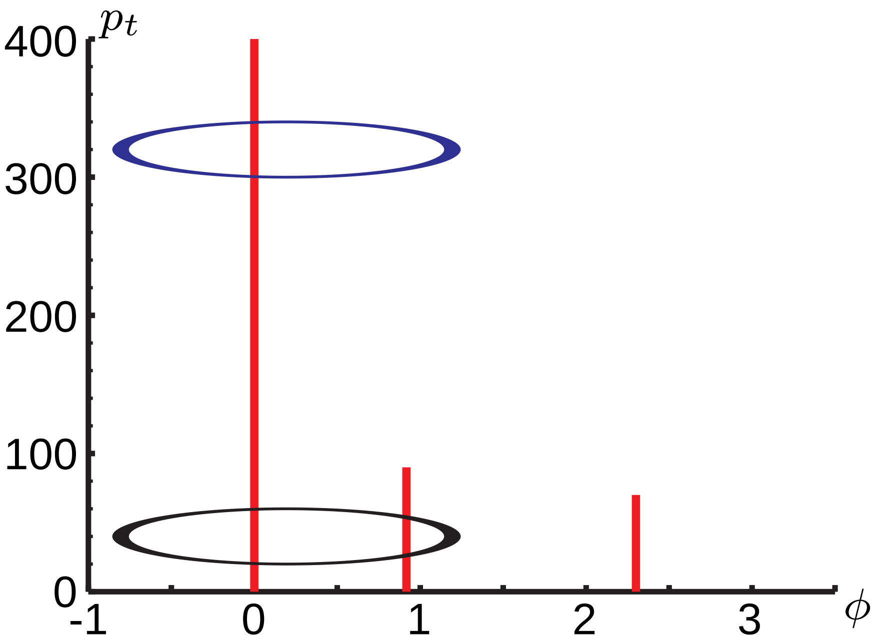
iterate

IR unsafety of the Midpoint alg



stable; 2nd seed

IR unsafety of the Midpoint alg



iterate

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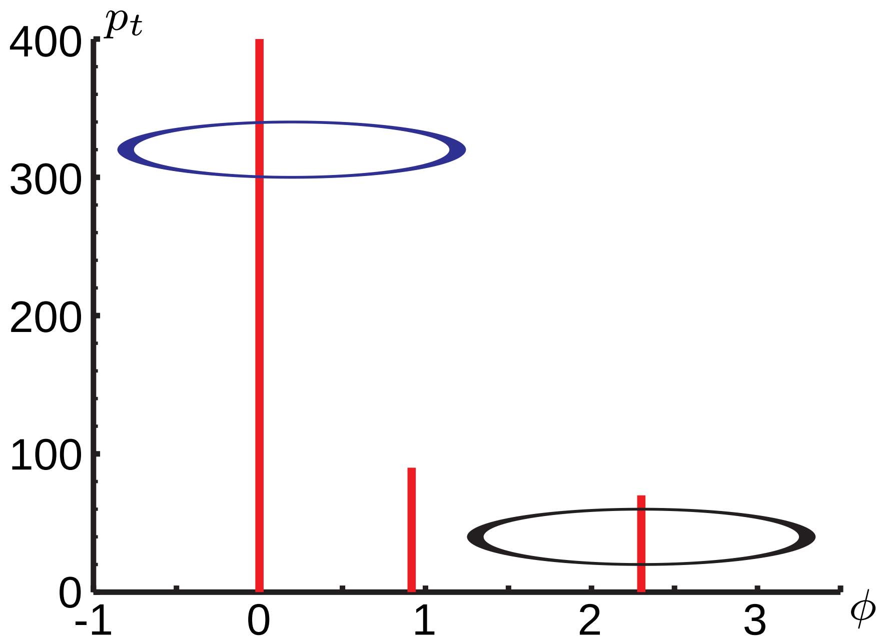
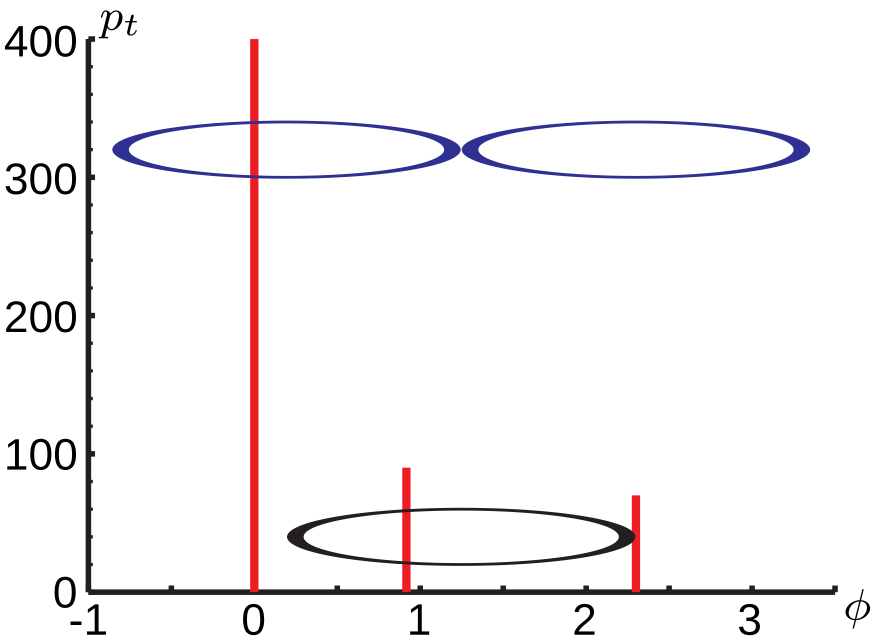


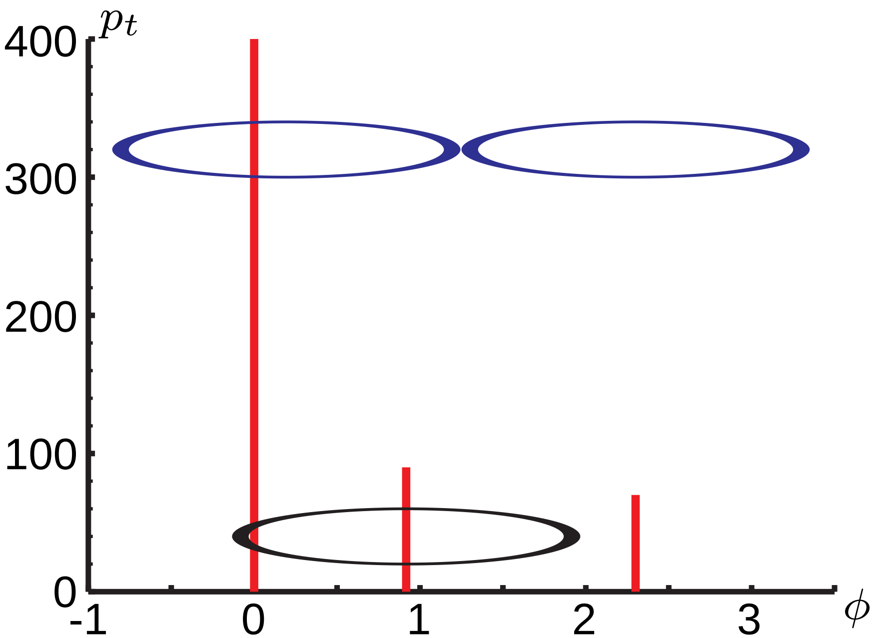
stable; 3rd seed

IR unsafety of the Midpoint alg



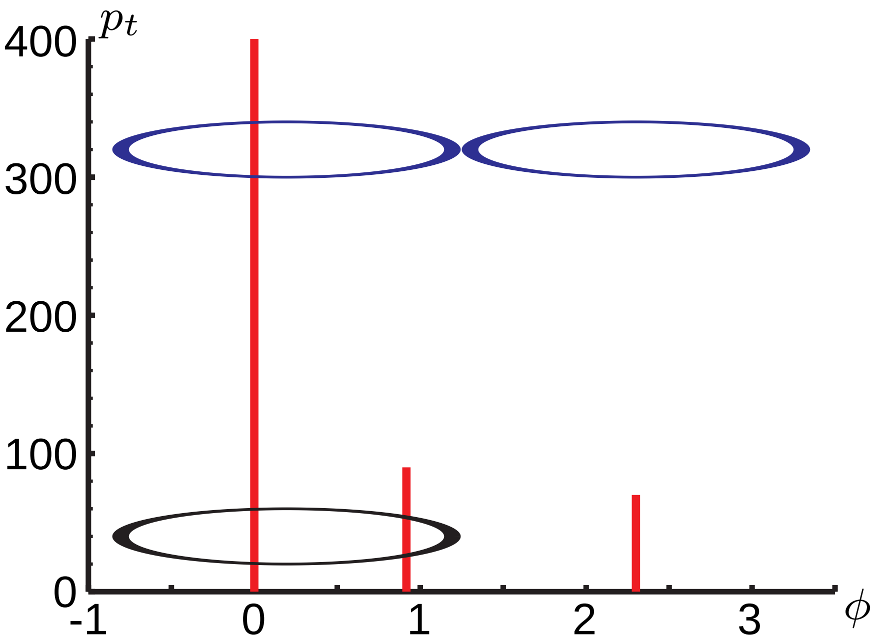
stable; midpoint seed

IR unsafety of the Midpoint alg



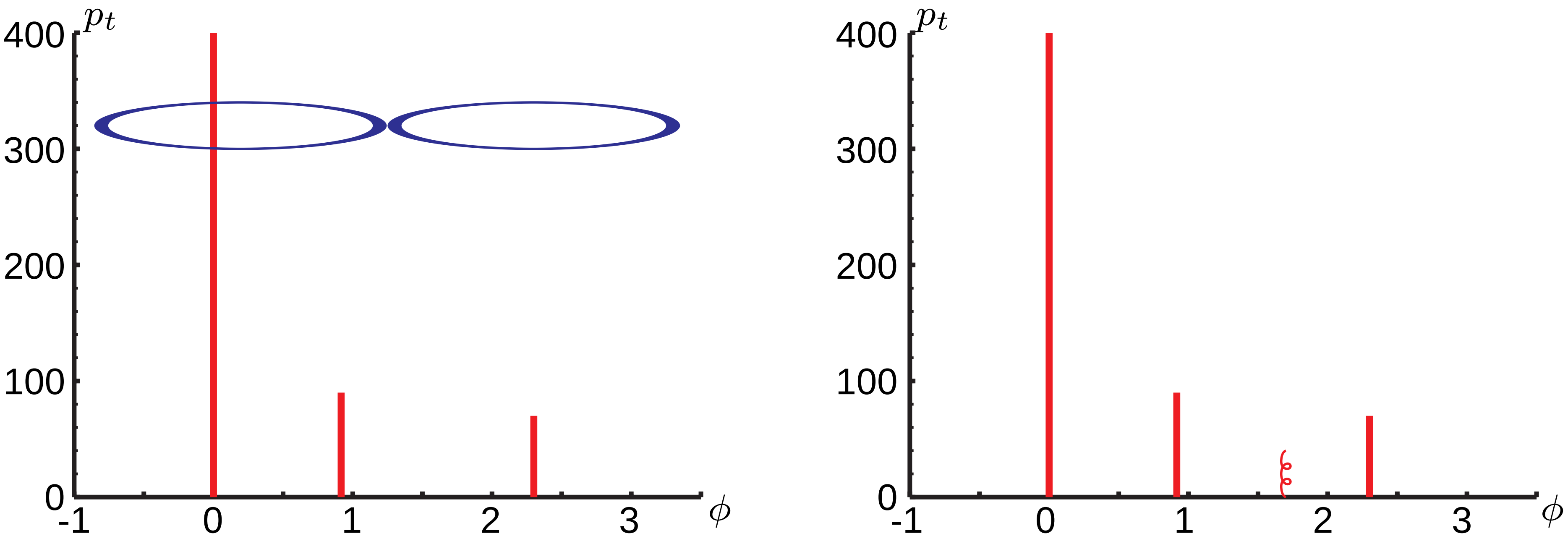
iterate

IR unsafety of the Midpoint alg



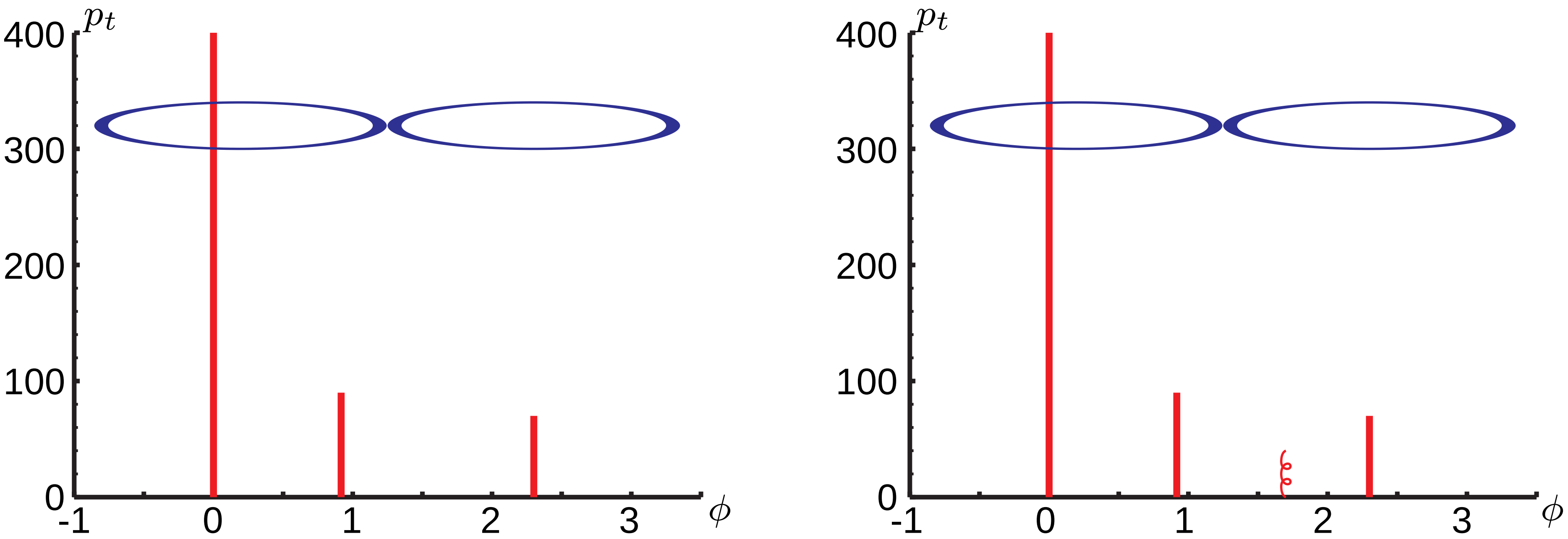
iterate

IR unsafety of the Midpoint alg



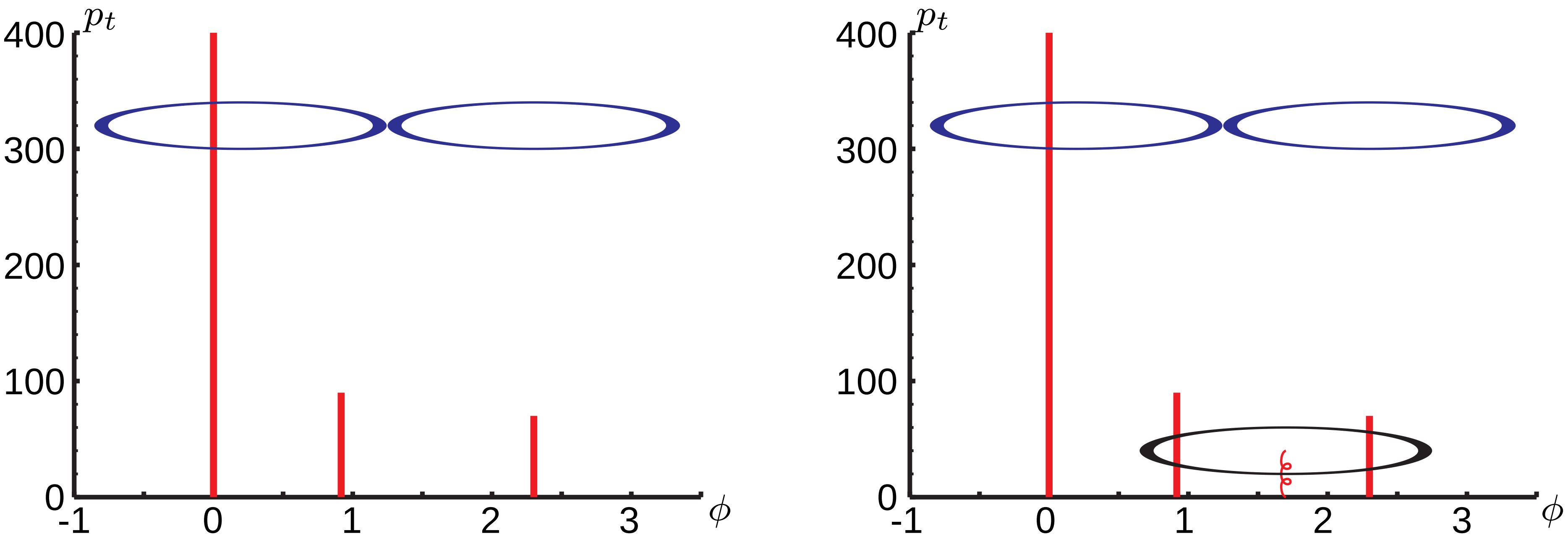
add an infinitely soft particle

IR unsafety of the Midpoint alg



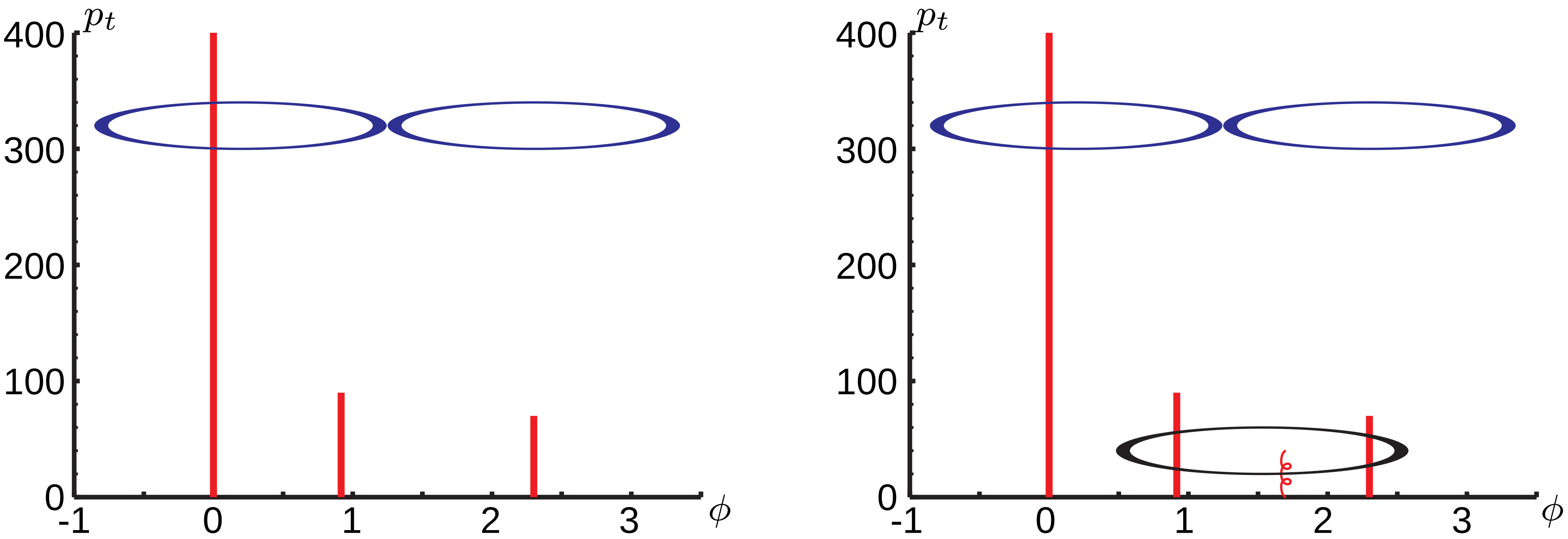
3 hard seeds + midpoint seed $\rightarrow$ 2 stable cones

IR unsafety of the Midpoint alg



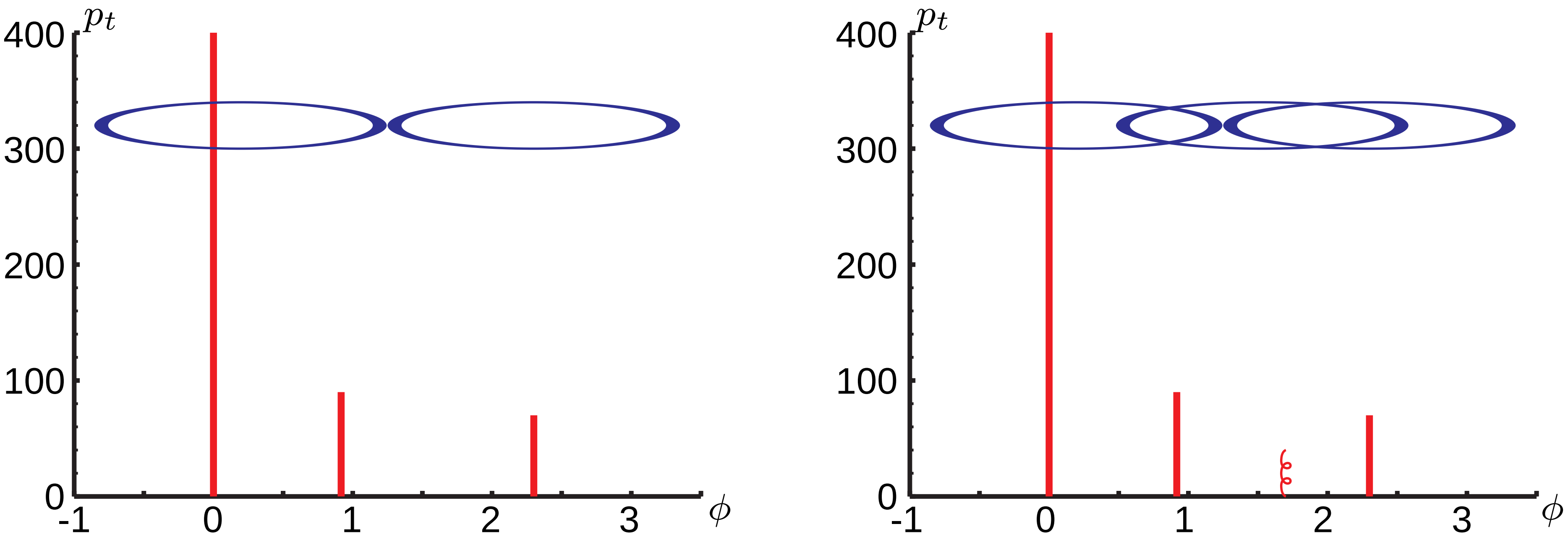
new seed!

IR unsafety of the Midpoint alg



iterate

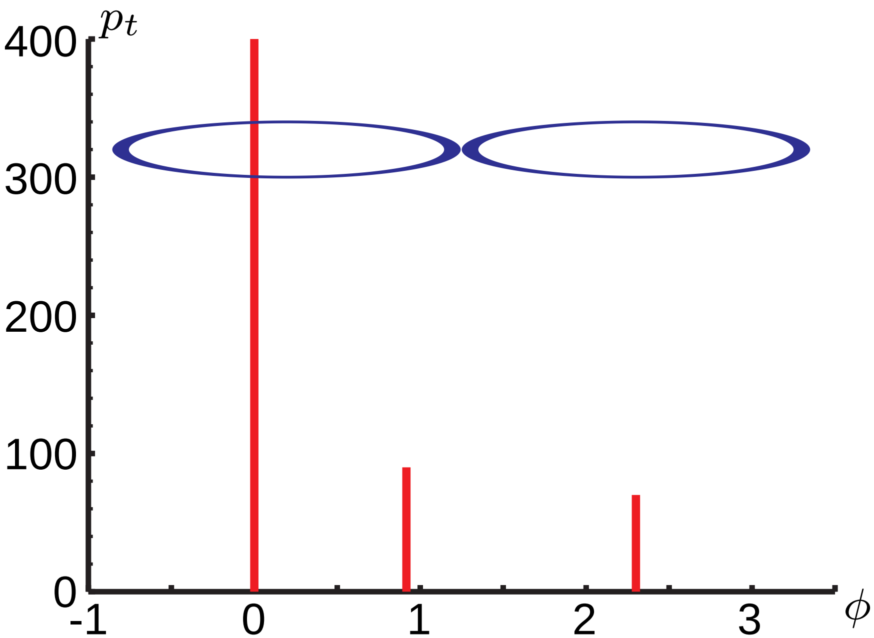
IR unsafety of the Midpoint alg

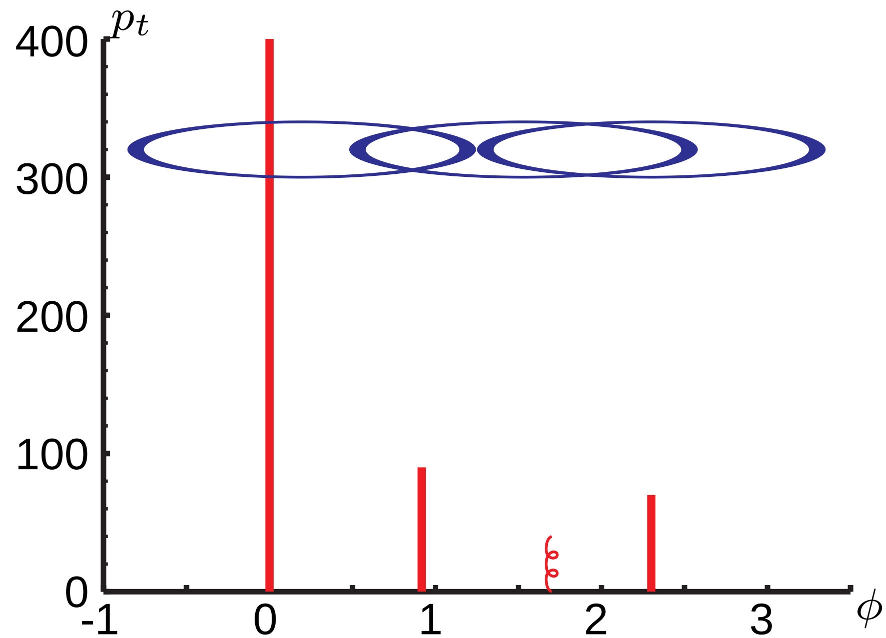


Stable cones:

Midpoint: {1,2} & {3} {1,2} & {3} & {2,3}

IR unsafety of the Midpoint alg





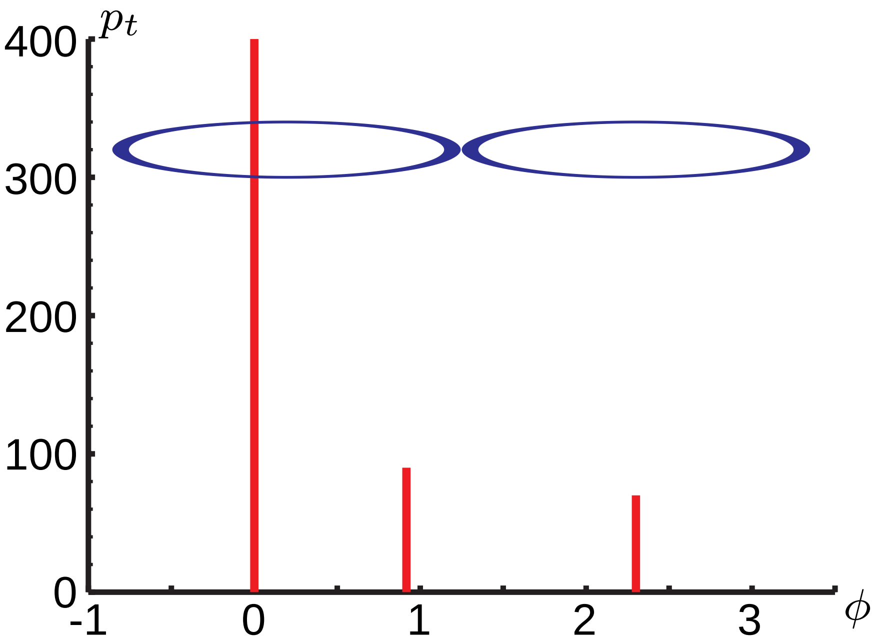
Stable cones:

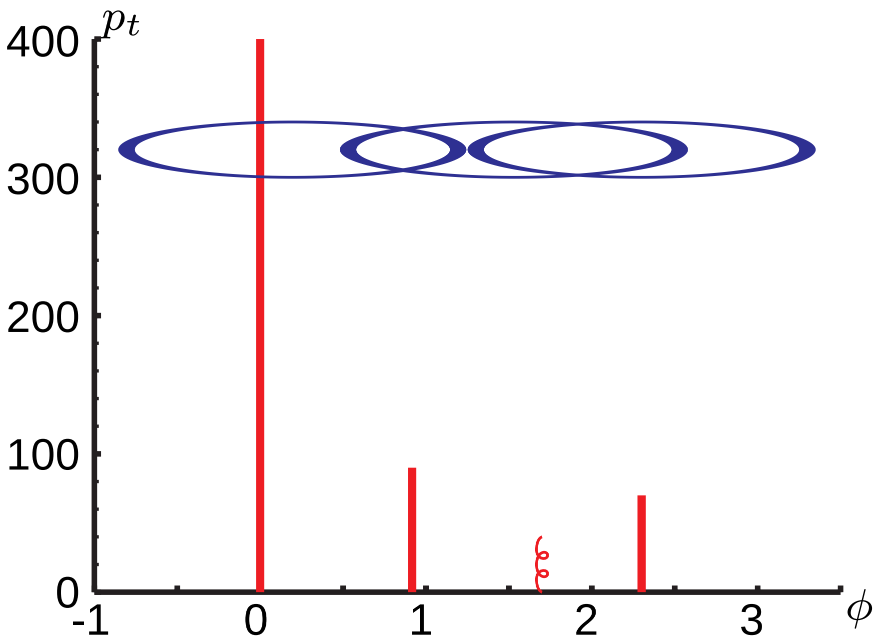
Midpoint:	{1,2} & {3}	{1,2} & {3} & {2,3}

Jets: ($f = 0.5$)

Midpoint:	{1,2} & {3}	{1,2,3}

IR unsafety of the Midpoint alg





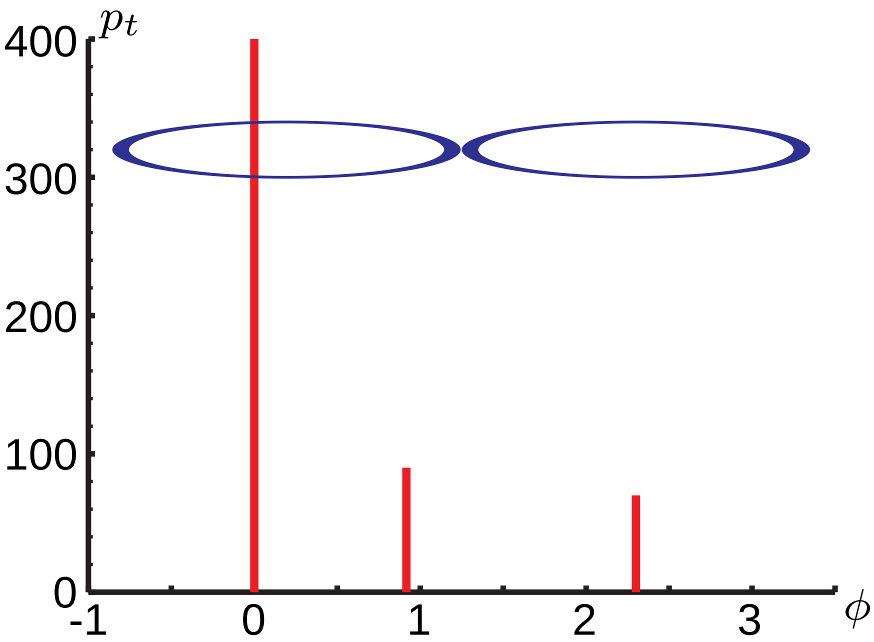
Stable cones:

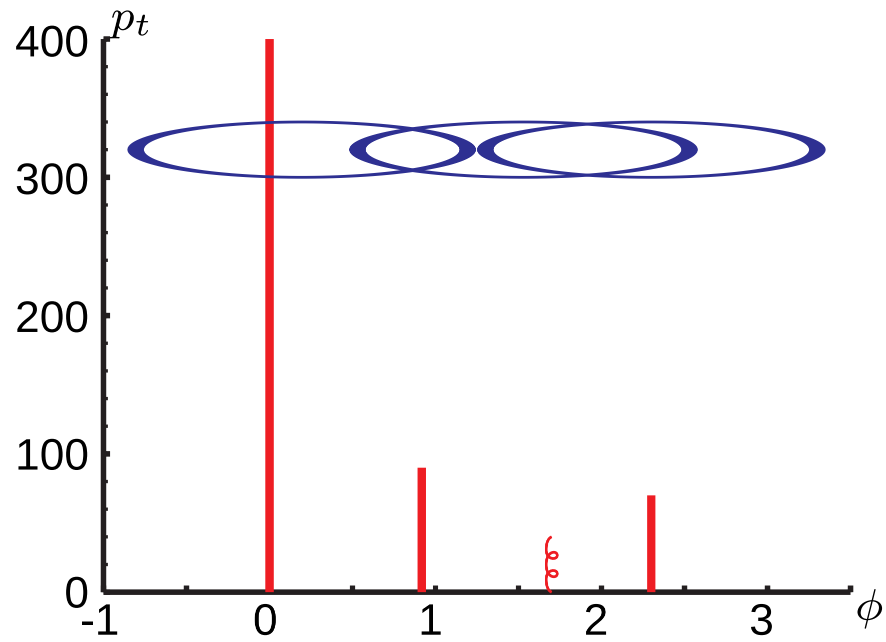
Midpoint:	{1,2} & {3}	{1,2} & {3} & {2,3}
Seedless:	{1,2} & {3} & {2,3}	{1,2} & {3} & {2,3}

Jets: ($f = 0.5$)

Midpoint:	{1,2} & {3}	{1,2,3}
Seedless:	{1,2,3}	{1,2,3}

IR unsafety of the Midpoint alg





Stable cones:

Midpoint:	{1,2} & {3}	{1,2} & {3} & {2,3}
Seedless:	{1,2} & {3} & {2,3}	{1,2} & {3} & {2,3}

Jets: ($f = 0.5$)

Midpoint:	{1,2} & {3}	{1,2,3}
Seedless:	{1,2,3}	{1,2,3}

Stable cone missed $\longrightarrow$ MidPoint is IR unsafe

Jet definitions

Cone algorithm: (1) cone with split–merge

- Step 1: find ALL stable cones in a reasonable time
  - MidPoint: time $\propto N^3$
  - All-Naive: time $\propto 2^N$
  - SISCone: time $\propto N^2 \log(N)$
- Step 2: split–merge (with threshold f)

Example: SISCone Seedless Infrared-Safe Cone

2007!!!

Jet definition: cone with PR

Cone algorithm: (2) cone with progressive removal

- Recipe:
  - start with the hardest particle as a seed
  - iterate to find a stable cone
  - stable cone $\to$ 1^{st} jet
  - remove its constituents
  - continue with the next hardest particle left

Jet definition: cone with PR

Cone algorithm: (2) cone with progressive removal

- Recipe:
  - start with the hardest particle as a seed
  - iterate to find a stable cone
  - stable cone $\rightarrow$ 1^{st} jet
  - remove its constituents
  - continue with the next hardest particle left
- Benchmark: circular/soft-resilient hard jets

Jet definition: cone with PR

Cone algorithm: (2) cone with progressive removal

- Recipe:
  - start with the hardest particle as a seed
  - iterate to find a stable cone
  - stable cone $\rightarrow 1^{\rm st}$ jet
  - remove its constituents
  - continue with the next hardest particle left
- Benchmark: circular/soft-resilient hard jets
- Example: CMS Iterative Cone

Jet definition: cone with PR

Cone algorithm: (2) cone with progressive removal

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  - stable cone $\rightarrow 1^{\mathrm{st}}$ jet
  - remove its constituents
  - continue with the next hardest particle left
- Benchmark: circular/soft-resilient hard jets
- Example: CMS Iterative Cone
  BUT Collinear unsafe (3 hard+1 coll.splitting) !!

Jet definition: successive recombinations

Idea: Undo the QCD cascade

- Define an inter-particle distance d_{ij} and a beam distance d_{iB}
- Successively
  - Find the minimum of all d_{ij}, d_{iB}
  - If d_{ij}, recombine $i + j \to k$ (remove i, j; add k)
  - If d_{iB}, call i a jet (remove i)
- Until all particles have been clustered

Jet definition: successive recombinations

Typical choice of distances:

$$
\begin{aligned}
d_{ij}^2 &= \min(k_{t,i}^{2p}, k_{t,j}^{2p})(\Delta y_{ij}^2 + \Delta\phi_{ij}^2) \\
d_{iB}^2 &= k_{t,i}^{2p} R^2
\end{aligned}
$$

- $p = 1$: k_t algorithm (1993)
- $p = 0$: Cambridge-Aachen algorithm (1997)
- $p = -1$: anti-k_t algorithm (2008)
- parameter R (jet separation)
- trivially IRC-safe

Jet definition: successive recombinations

Typical choice of distances:

$$d_{ij}^2 = \min(k_{t,i}^{2p}, k_{t,j}^{2p})(\Delta y_{ij}^2 + \Delta\phi_{ij}^2)$$
$$d_{iB}^2 = k_{t,i}^{2p} R^2$$

- $p = 1$: k_t algorithm (1993)
  (as close as possible to pQCD)
- $p = 0$: Cambridge-Aachen algorithm (1997)
  (close to pQCD; useful for substructure)
- $p = -1$: anti-k_t algorithm (2008)
  (circular/soft-resilient jets; replaces it. cone)

Variants for e^+e^- collisions (+JADE)

Jet definitions: IRC safety matters

As said in e^+e^-: **IRC safety matters** if you want to compare to QCD computations

Process	Last OK order IR$_{2+1}$	IR/Coll$_{3+1}$	safe	today's pQCD
Incl. jet x-sect	LO	NLO	any	NLO
W/Z/H+1 jet	LO	NLO	any	NLO
3-jet x-sect	none	LO	any	NLO
W/Z/H+2 jet	none	LO	any	NLO
jet mass in 3-jet	none	none	any	LO

Jet definitions: IRC safety matters

As said in e^+e^-: **IRC safety matters** if you want to compare to QCD computations

	Last OK order			today's
Process	IR_{2+1}	$IR/Coll_{3+1}$	safe	pQCD
Incl. jet x-sect	LO	NLO	any	NLO
W/Z/H+1 jet	LO	NLO	any	NLO
3-jet x-sect	none	LO	any	NLO
W/Z/H+2 jet	none	LO	any	NLO
jet mass in 3-jet	none	none	any	LO

$\Rightarrow$ Use an IRC-safe algorithm like k_t, C/A, anti-k_t or SISCone

Jet definitions: comparison

Quick comparison of the algorithms

	k_t	C/A	anti-k_t	SISCone
pQCD	✓✓✓	✓✓✓	✓✓	✓✓
soft (UE)	✗	$\sim$ OK	✓✓	✓✓✓
speed	✓✓✓	✓✓✓	✓✓✓	✓
substruct	✓✓	✓✓✓	✗	✗
calibr.	✓	✓	✓✓✓	✓✓

Jet clustering: usage/access

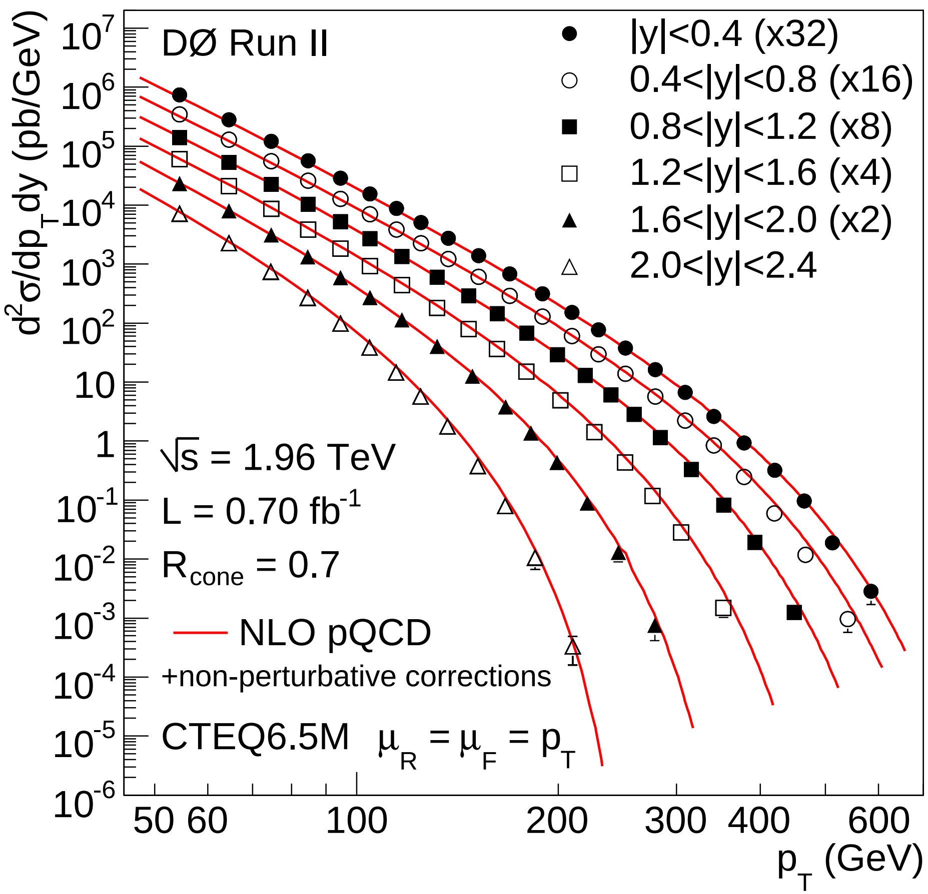
FastJet

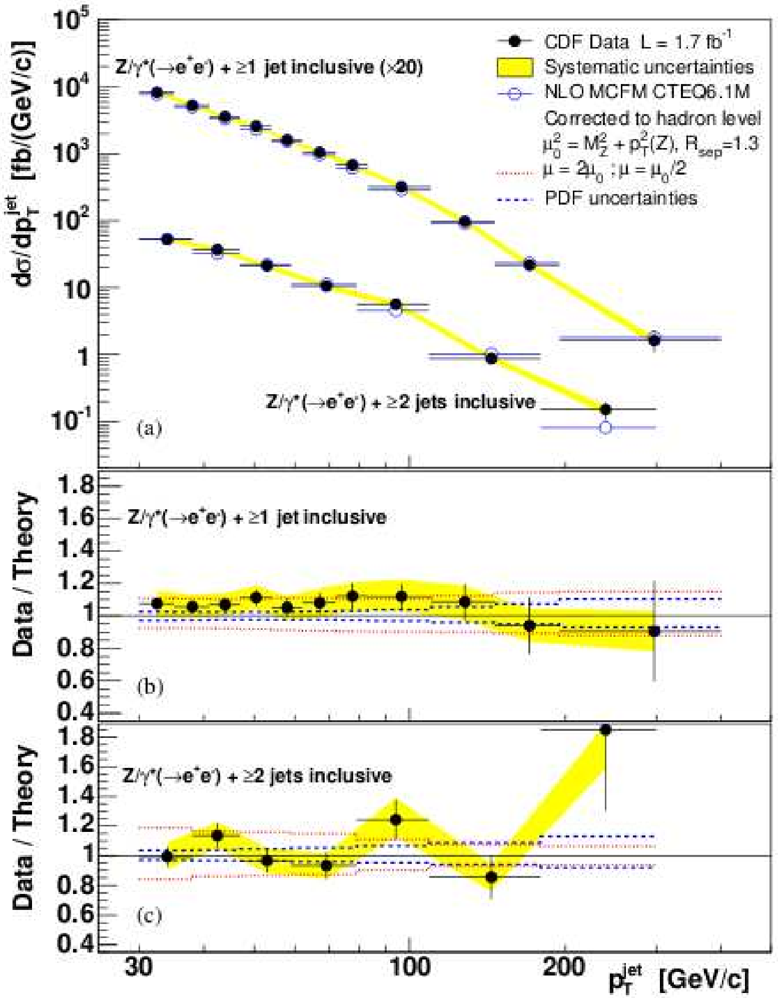
[M.Cacciari, G.Salam, GS]

- Fast implementation of recomb. algs ($N \log(N)$)
- Plugins for all common algs
  (SISCone; CDF, D0, ATLAS, CMS algs; e^+e^- algs)
- Other tools (like jet areas)
- More in the tutorial part!

Jets: experimentally

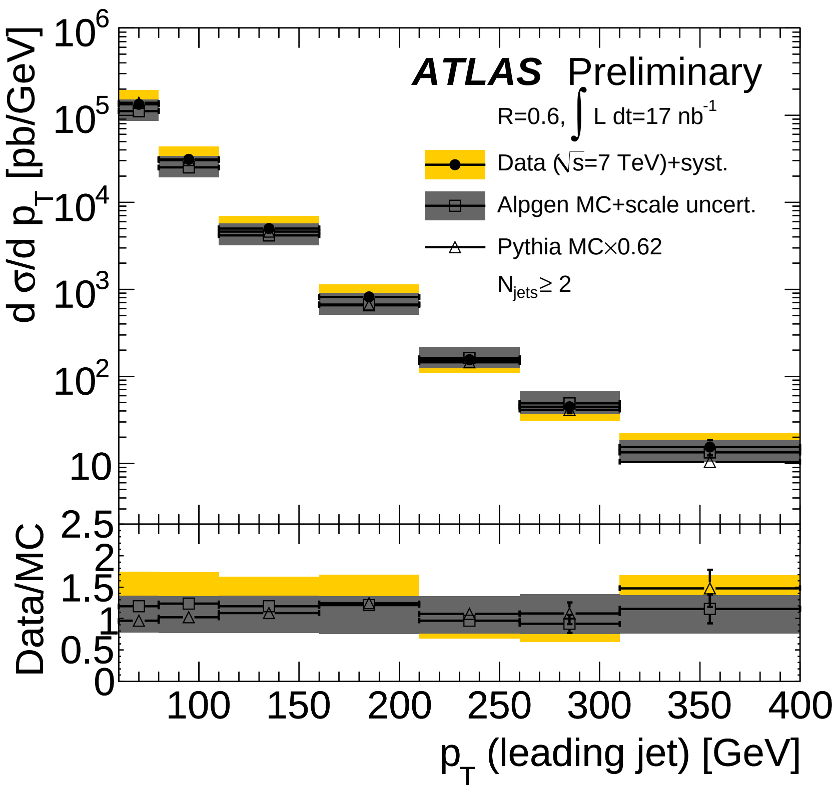
- Tevatron
  Use of IR-unsafe JetClu or MidPoint and sometimes k_t

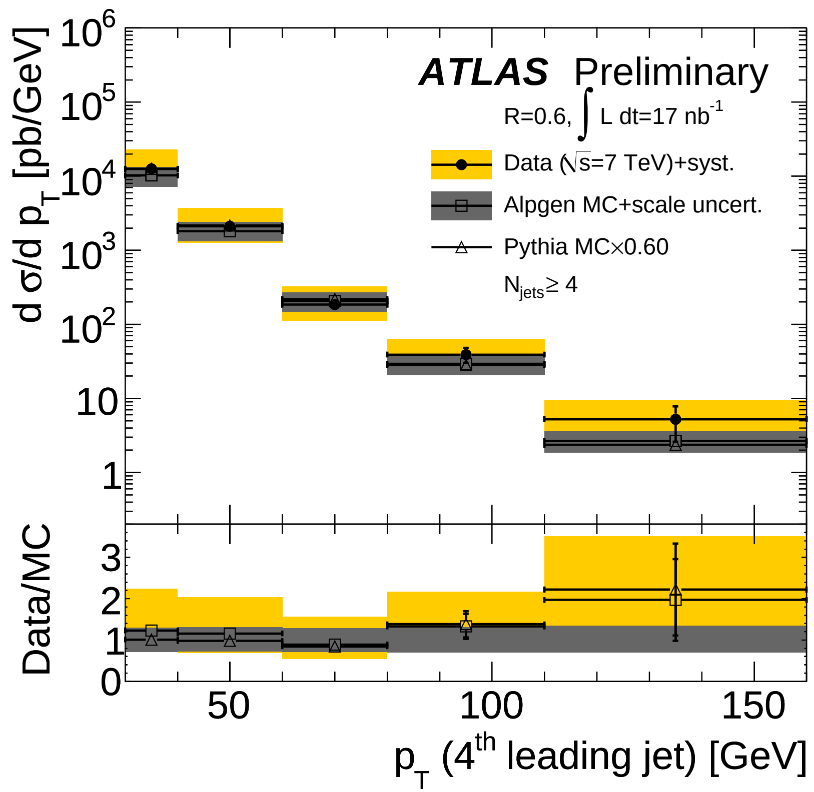




Jets: experimentally

- Tevatron
  Use of IR-unsafe JetClu or MidPoint and sometimes k_t
- LHC: anti-k_t by default





Jets: hadronic colliders

At hadronic colliders, many “contaminations” to a jet:

- radiation from partons in the initial state
- Underlying event/Multiple interactions
  - shift: UE $\approx$ uniform soft background *i.e.* contamination $\propto$ jet area $\propto R^2$
  - smearing: due to UE fluctuations
  - typical scale: a few GeV
- Pile-up: many pp interactions in 1 bunch-crossing:

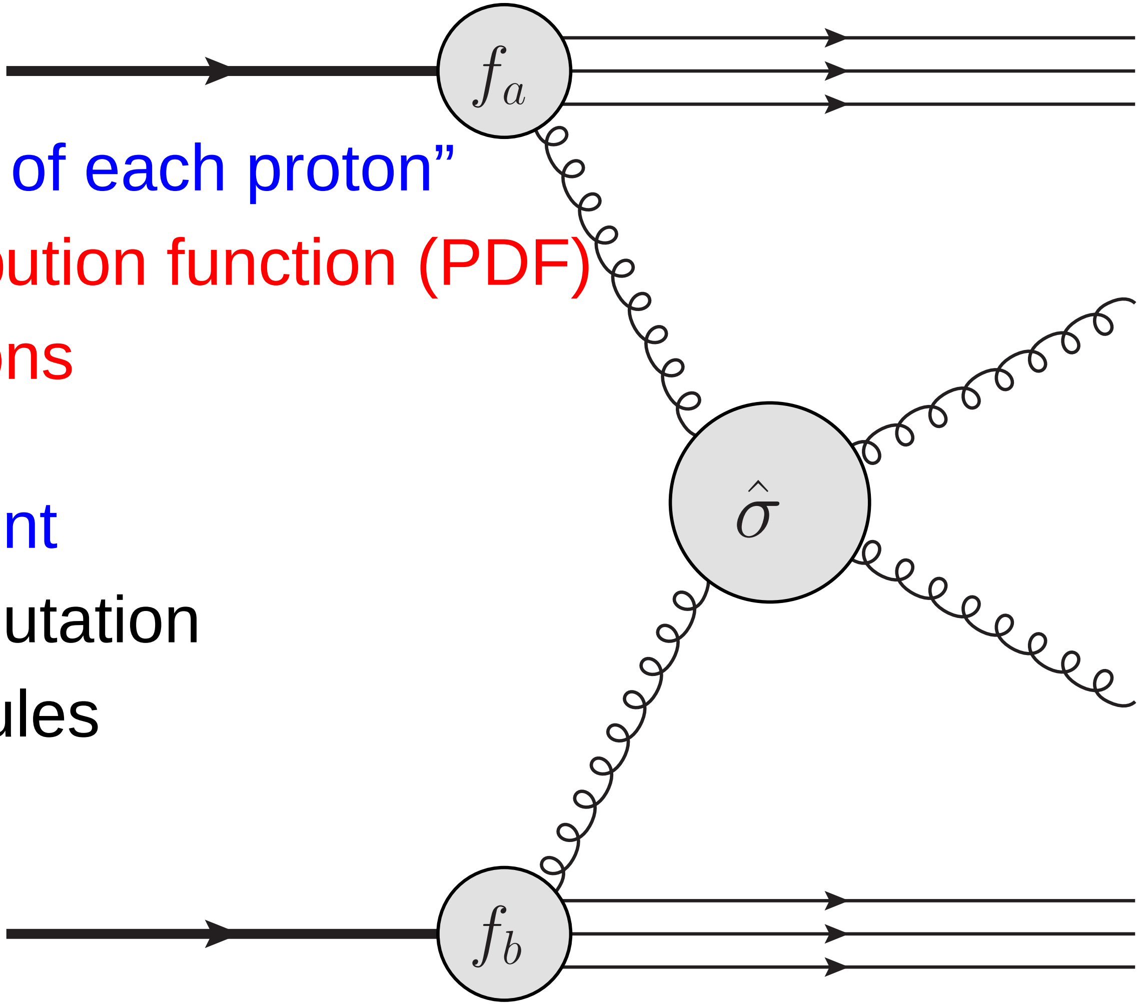
$$n \approx \mathcal{L}\, \Delta t_{\mathrm{bunch}}\, \sigma_{pp} \approx 10^{34}\, 25.10^{-9}\, 100.10^{-27} \approx 25$$

Again: shift + smearing
Typical scale: 20-30 GeV
Need for subtraction techniques

</interlude>

The very fundamental collision

$$\sigma = f_a \otimes f_b \otimes \hat{\sigma}$$



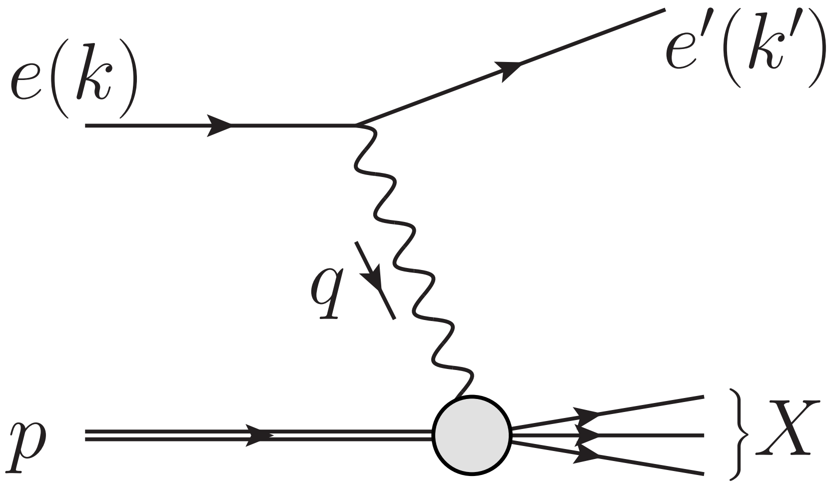
- "take a parton out of each proton"
  $f_a \equiv$ parton distribution function (PDF)
  for quark and gluons

- hard matrix element
  perturbative computation
  Forde-Feynman rules

Deep Inelastic Scattering

Introduce/Discuss/Study the PDFs

Process + kinematics



$$s = (e+p)^2$$

$$W^2 = (q+p)^2$$

$$Q^2 = -q^2 > 0$$

$$\nu = p.q = W^2 + Q^2$$

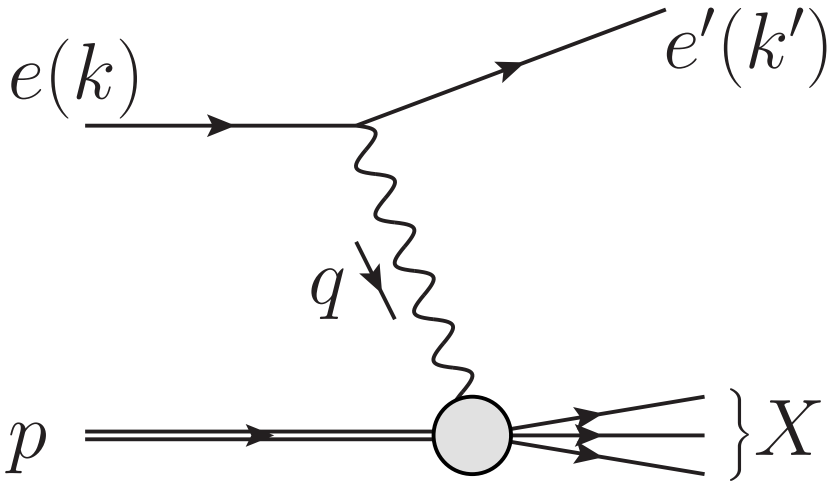
$$x = Q^2/(2\nu)$$

$$y = p.q/p.k = (W^2 + Q^2)/s$$

$ep \to eX$ with γ exchange

- Z and W also possible as well as ν instead of e
- also more exclusive meas.: $ep \to ep$, eXY, eYp, *e.g.* jets, charm, vector-mesons, photons

Process + kinematics



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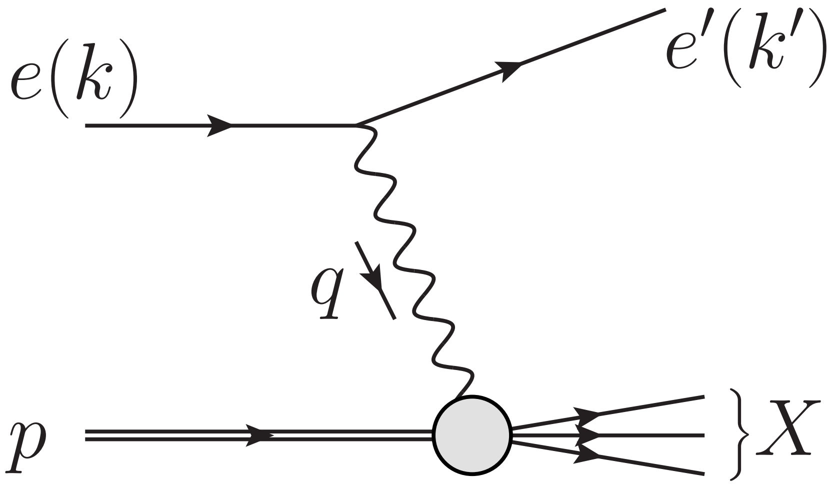
$$x = Q^2/(2\nu)$$

$$y = p.q/p.k = (W^2 + Q^2)/s$$

Experimentally: only the outgoing e is needed to reconstruct the kinematics

$$Q^2 = 4EE' \cos^2(\theta_e/2) \qquad x = \frac{EE' \cos^2(\theta_e/2)}{P[E - E' \sin^2(\theta_e/2)]}$$

Process + kinematics



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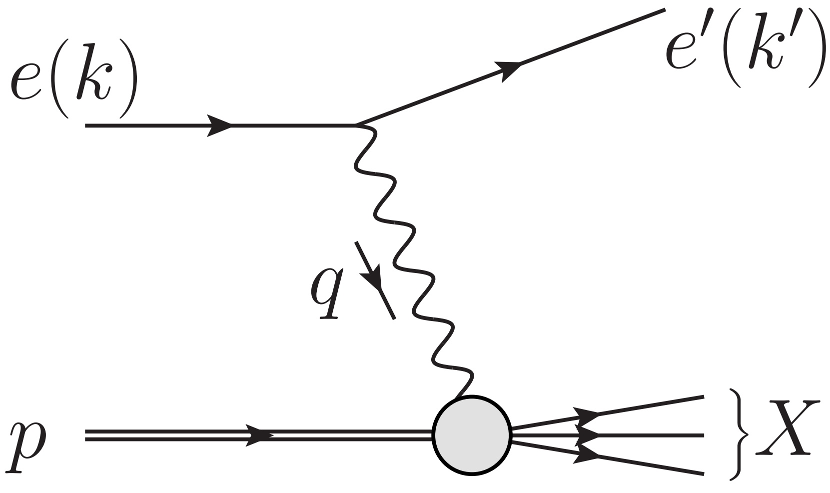
$$x = Q^2/(2\nu)$$
$$y = p.q/p.k = (W^2 + Q^2)/s$$

Idea:

use the photon to probe the proton structure
Q^2 large $\Rightarrow$ small distance $\sim 1/Q$

Process + kinematics



$$s = (e+p)^2$$

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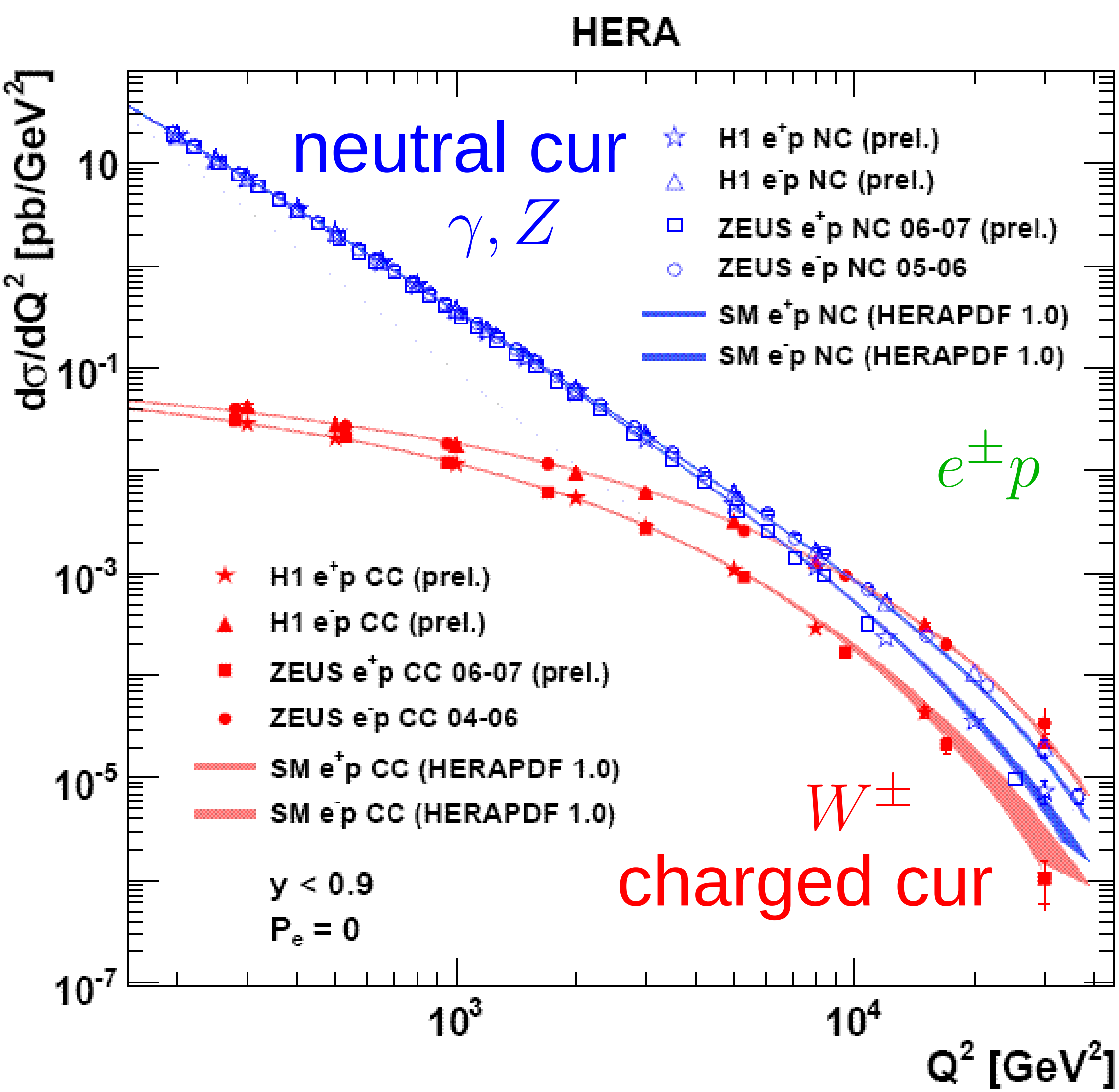
$$x = Q^2/(2\nu)$$

$$y = p.q/p.k = (W^2 + Q^2)/s$$

Experiments:
most important results recently from HERA at DESY
(H1 and ZEUS experiments)

A crystal-clear example

Electroweak unification

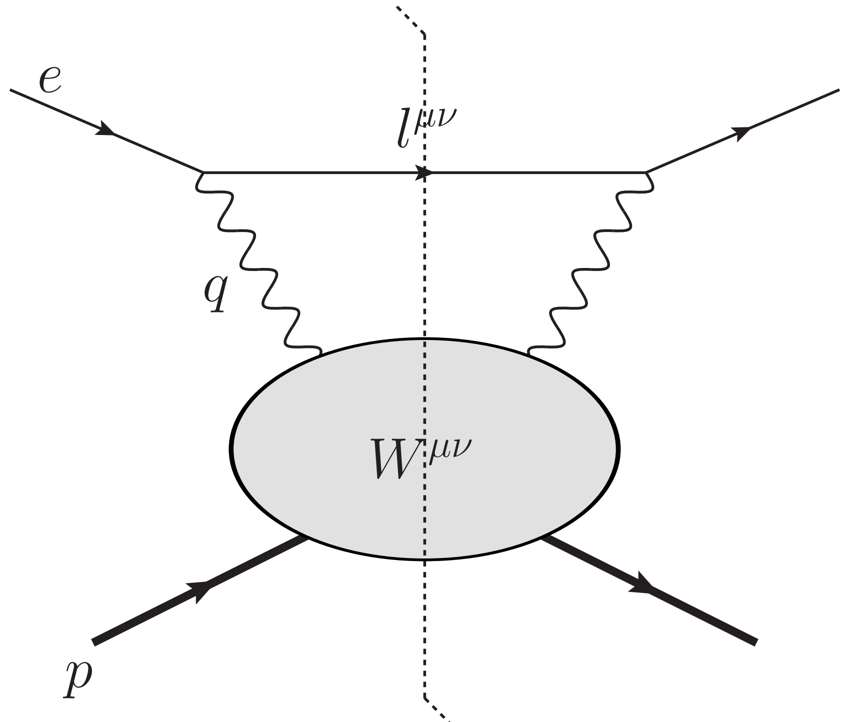


$e^{\pm}$ total x-sect differential in Q^2

Neutral currents
$ep \to eX$
via γ, Z

Charged currents
$ep \to \nu X$
via $W^{\pm}$

Process + kinematics



Factorisation in a leptonic and hadronic part:

$$|\mathcal{M}|^2 = l_{\mu\nu} W^{\mu\nu} \qquad l^{\mu\nu} = 4e^2(k^\mu k'^\nu + k^\nu k'^\mu - g^{\mu\nu} k.k')$$

$\longrightarrow$ study the hadronic tensor $W^{\mu\nu}(W^2, Q^2)$
(or $W^{\mu\nu}(x, Q^2)$)

Hadronic tensor

Most generic structure for $W^{\mu\nu}(x, Q^2)$

$$W^{\mu\nu} = Ag^{\mu\nu} + Bp^\mu p^\nu + Cq^\mu q^\nu + Dp^\mu q^\nu + Eq^\mu p^\nu.$$

Constraints:

$W^{\mu\nu} = W^{\nu\mu}$ and $q_\mu W^{\mu\nu} = 0$ (gauge inv.)

Implying

$$W^{\mu\nu} = -\left(g^{\mu\nu} + \frac{q^\mu q^\nu}{Q^2}\right) F_1 + \frac{2x}{Q^2}\left(p^\mu + \frac{q^\mu}{2x}\right)\left(p^\nu + \frac{q^\nu}{2x}\right) F_2$$

$F_1, F_2(x, Q^2)$: proton structure functions

Structure functions

(inclusive) proton interaction fully parametrised by the 2 structure functions F_1 and $F_2(x, Q^2)$

- dimensionless
- $F_L = F_2 - 2xF_1$ (longitudinally-polarized γ^*)
- For charged currents: additional $F_3(x, Q^2)$

Parton model

Useful to consider a frame where the proton is highly boosted ($P \gg 1$, p looks like a pancake)

$$p^\mu \equiv (0, 0, P, P)$$

$$n^\mu \equiv (0, 0, \frac{-1}{2P}, \frac{1}{2P}) \qquad (n^2 = 0,\ n.p = 1)$$

$$q^\mu \equiv q^\mu_\perp + \nu n^\mu \qquad (n.q = 0, \vec{q}_\perp^{\,2} = Q^2)$$

We obtain

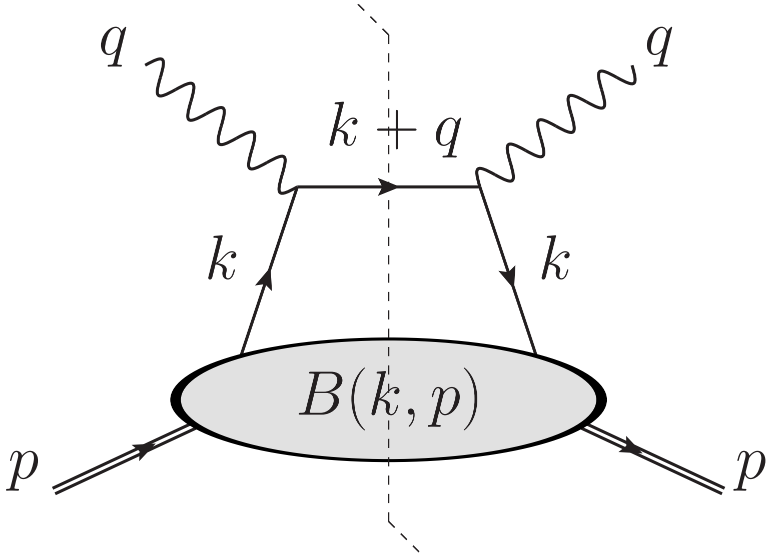
$$F_2 = \nu n^\mu n^\nu W_{\mu\nu}$$

$$F_L = \frac{4x^2}{\nu} p^\mu p^\nu W_{\mu\nu}$$

Parton model

Bag model

The photon resolves
a quark inside the proton



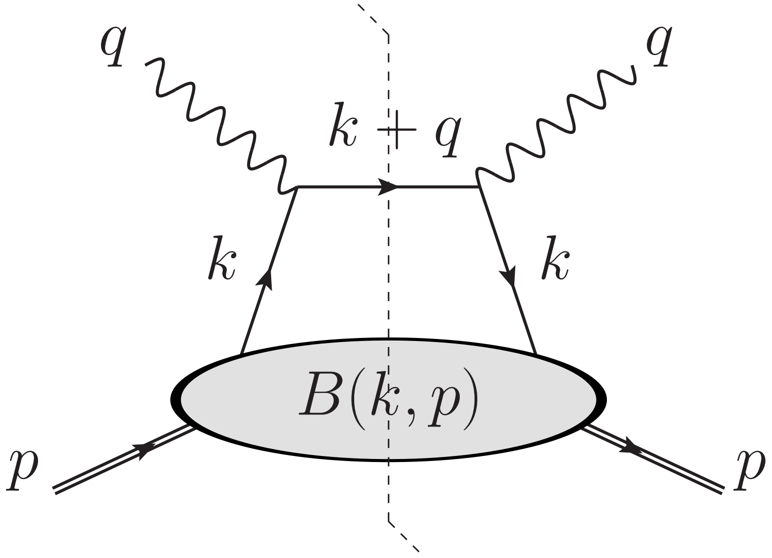
$$k^\mu = \xi p^\mu + \frac{k^2 + k_\perp^2}{2\xi} n^\mu + k_\perp^\mu$$

$$W^{\mu\nu} = e_q^2 \int \frac{d^4k}{(2\pi)^4} \mathrm{tr}\left(\gamma^\mu (\not{k} + \not{q}) \gamma^\nu B(k,p)\right) \delta\left((k+q)^2\right)$$

Parton model

Bag model

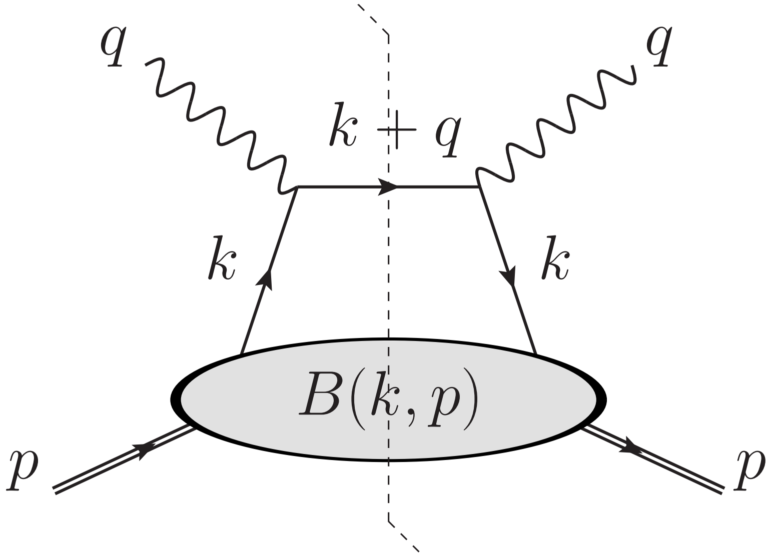
The photon resolves
a quark inside the proton



$$k^\mu = \xi p^\mu + \frac{k^2 + k_\perp^2}{2\xi} n^\mu + k_\perp^\mu$$

$$F_2 = \nu e_q^2 \int \frac{d^4k}{(2\pi)^4} \mathrm{tr}\left(\not{n}\left(\not{k} + \not{q}\right)\not{n}\, B(k,p)\right) \delta\left((k+q)^2\right)$$

Parton model



Bag model

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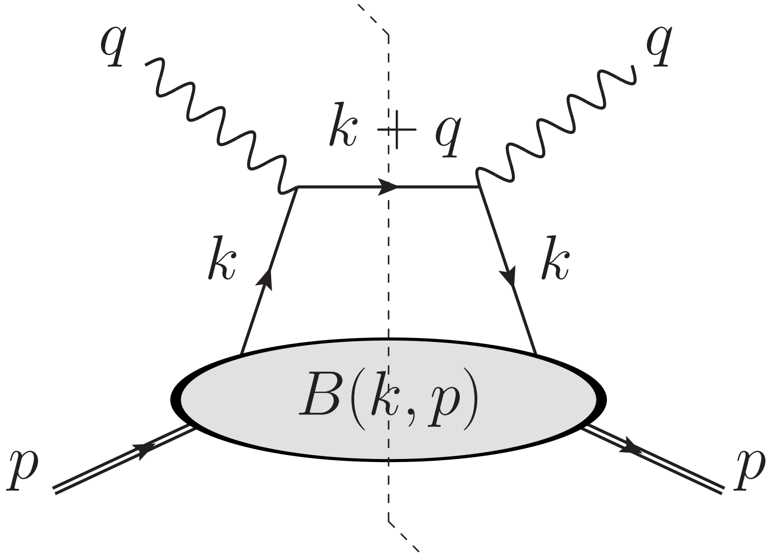
$$F_2 = \nu e_q^2 \int \frac{d^4k}{(2\pi)^4} \mathrm{tr}\left(\not{n}\,(\not{k} + \not{q}\,)\not{n}\,B(k,p)\right) \delta\left((k+q)^2\right)$$

$$\mathrm{tr}\left(\not{n}\,(\not{k} + \not{q}\,)\not{n}\,B(k,p)\right) = 2\xi\, \mathrm{tr}(\not{n}\,B(k,p))$$

Parton model

<u>Bag model</u>

The photon resolves
a quark inside the proton



$$k^\mu = \xi p^\mu + \frac{k^2 + k_\perp^2}{2\xi} n^\mu + k_\perp^\mu$$

$$F_2 = \nu e_q^2 \int \frac{d^4k}{(2\pi)^4} \mathrm{tr}\left(\not{n}\,(\not{k} + \not{q}\,)\not{n}\, B(k,p)\right) \delta\left((k+q)^2\right)$$

$$\delta\left((k+q)^2\right) = \delta\left(k^2 - Q^2 + 2\xi\nu - 2\vec{k}_\perp^{\,2}.\vec{q}_\perp^{\,2}\right)$$

$$\overset{Q^2 \gg}{\simeq} \delta(2\nu\xi - Q^2) \simeq \frac{1}{2\nu}\delta(\xi - x)$$

Parton model

Putting everything together:

$$F_2 = x e_q^2 \int \frac{d^4k}{(2\pi)^4} \mathrm{tr}\left(\not{q}\, B(k,p)\right) \delta(x - \xi)$$

i.e.

$$F_2 = x e_q^2 q(x) \qquad \text{with} \qquad q(x) = \int \frac{d^4k}{(2\pi)^4} \mathrm{tr}\left(\not{q}\, B(k,p)\right) \delta(x - \xi)$$

with a sum over flavours

$$\boxed{F_2 = \sum_q x\, e_q^2 [q(x) + \bar{q}(x)]}$$

$q(x)$: parton distribution function (PDF)

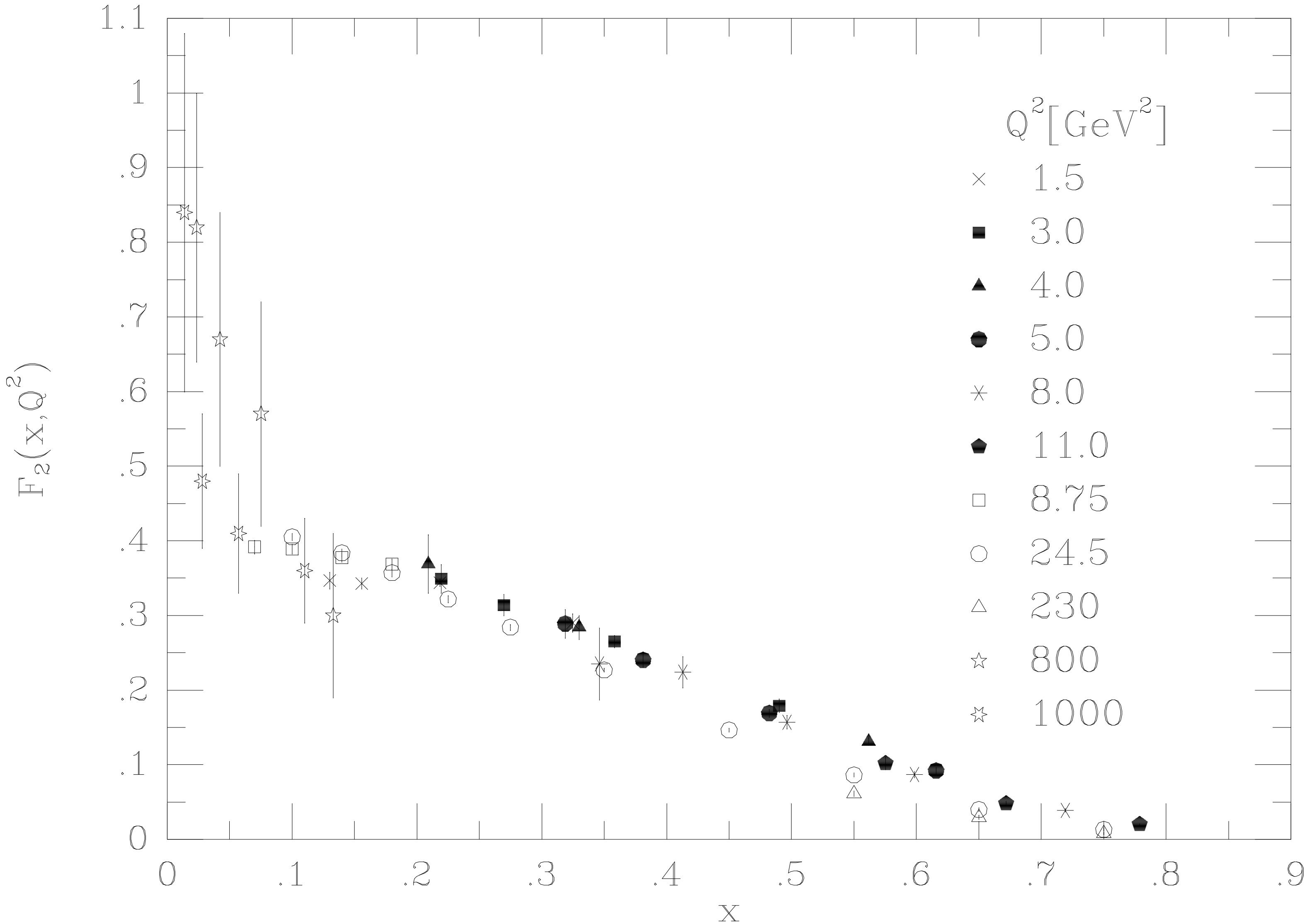
Parton model

$$F_2 = \sum_q x e_q^2 [q(x) + \bar{q}(x)] \qquad q(x) \equiv \text{ PDF}$$

- interpreted as the probability density to find a quark carrying a fraction x of the proton's momentum (universal!!)
- $F_2(x, Q^2) = F_2(x)$: Q^2-independent. Bjorken scaling
- F_L suppressed by $1/Q^2$ compared to F_2
  $F_2 = 2xF_1$. Calan-Gross relation: spin 1/2 for q
- charged currents: different quark combinations

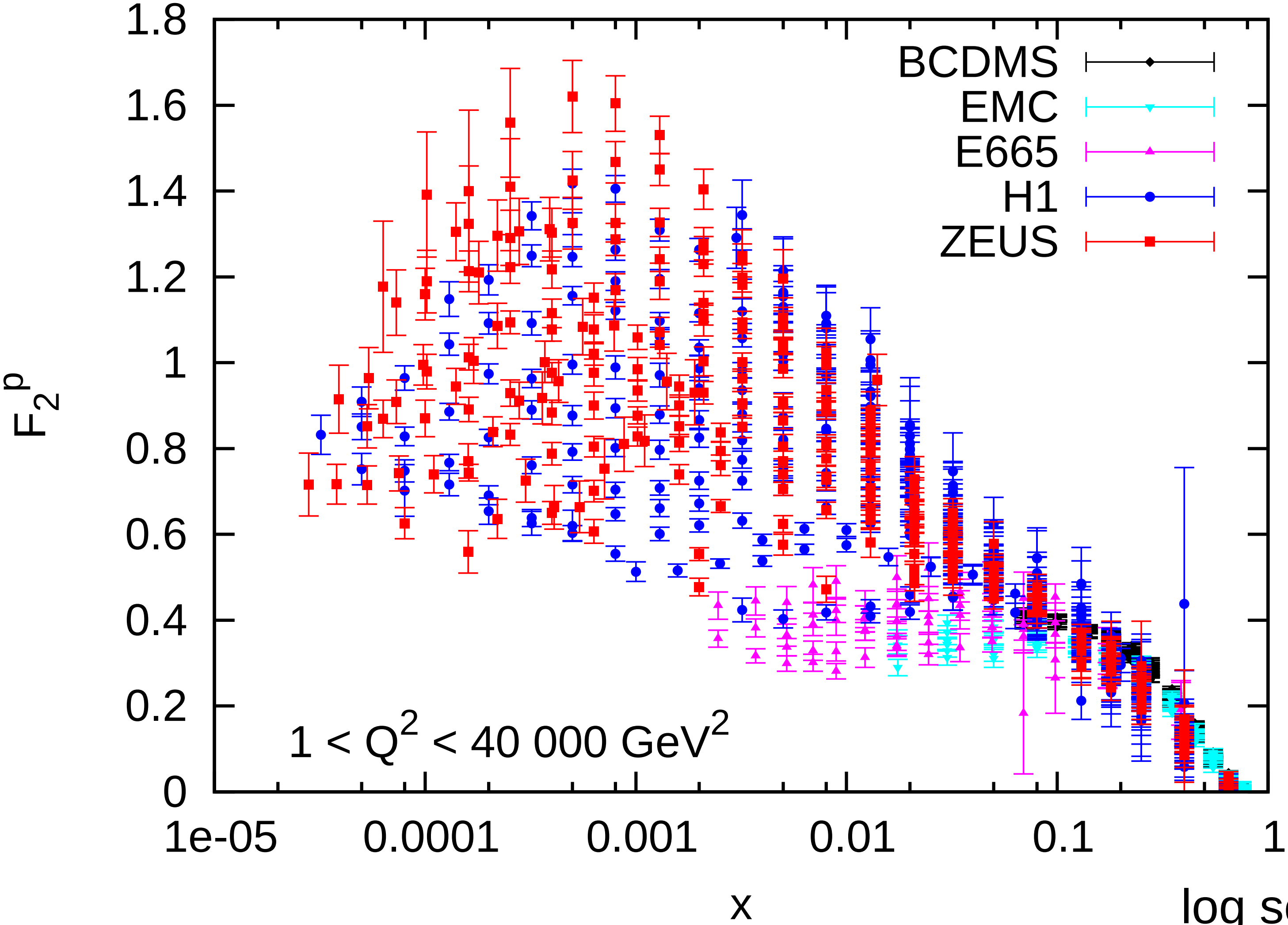
Bjorken scaling

F_2 from BCDMS, SLAC, NMC, H1 and ZEUS (~ 1990)



Bjorken scaling violations

HERA measurements ($\sim 1993 - 2007$)

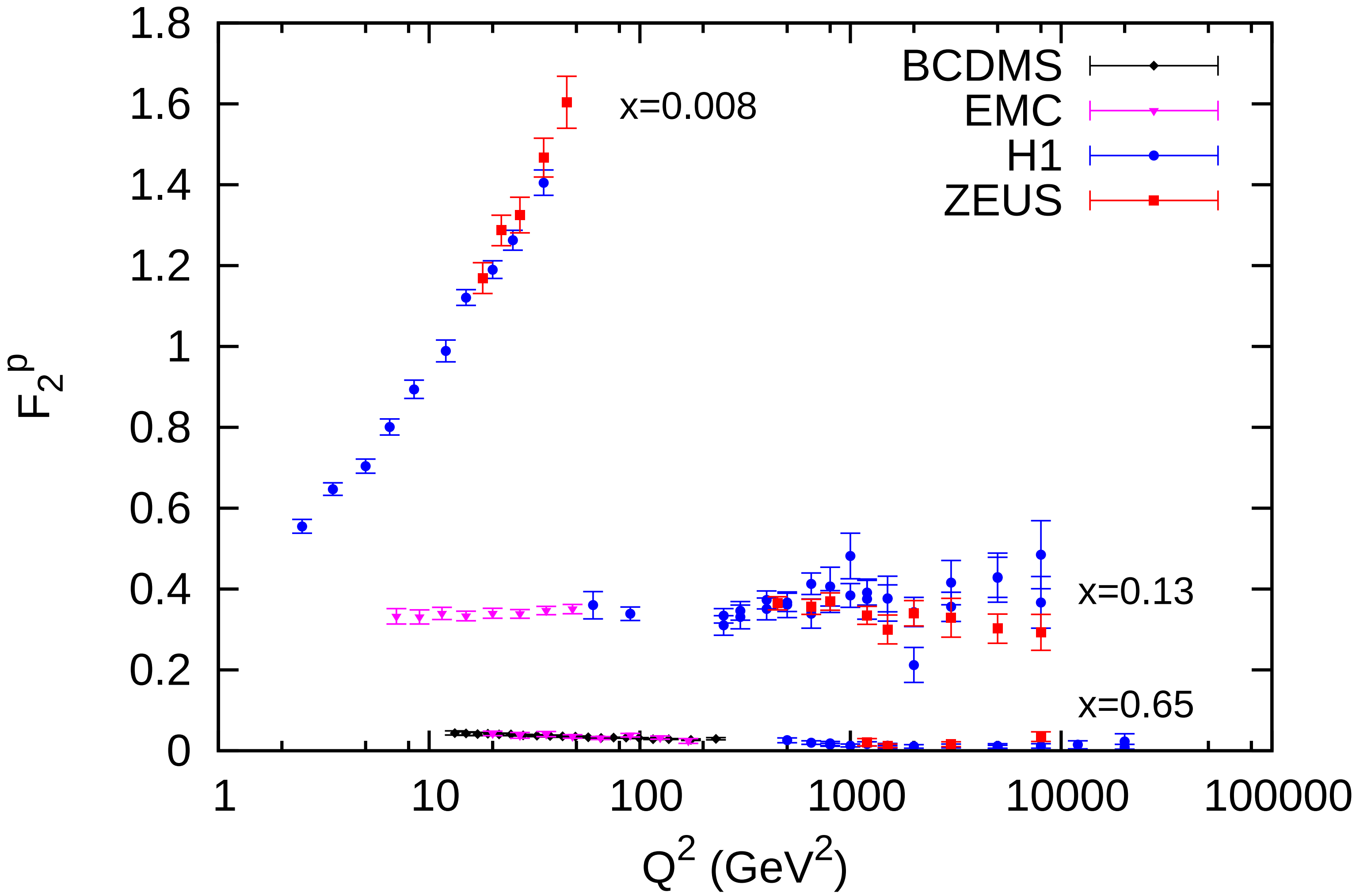


log scale!

Scaling violations!!!

Bjorken scaling violations

A closer look for 3 bins in x



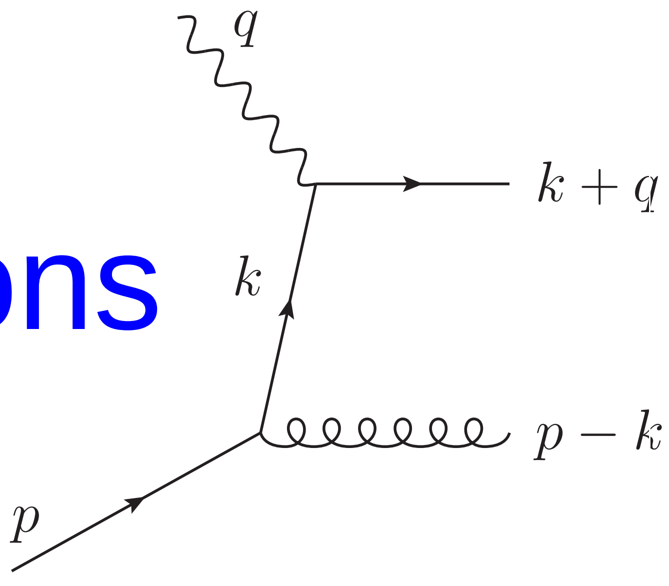
decrease at large x (strong) rise at small x

Can we describe the scaling violations in QCD?

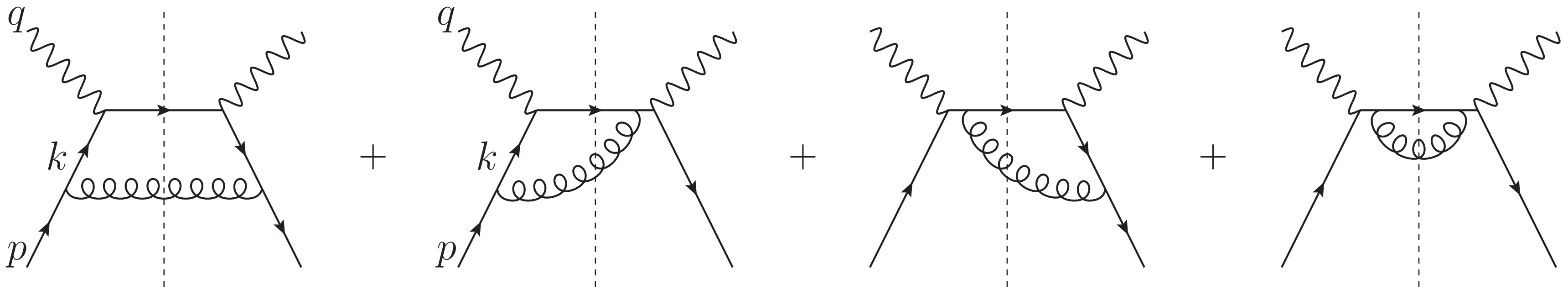
Bjorken scaling violations

Can we describe the scaling violations in QCD?

Idea: quarks can radiate gluons



One-gluon emission



4 graphs to compute

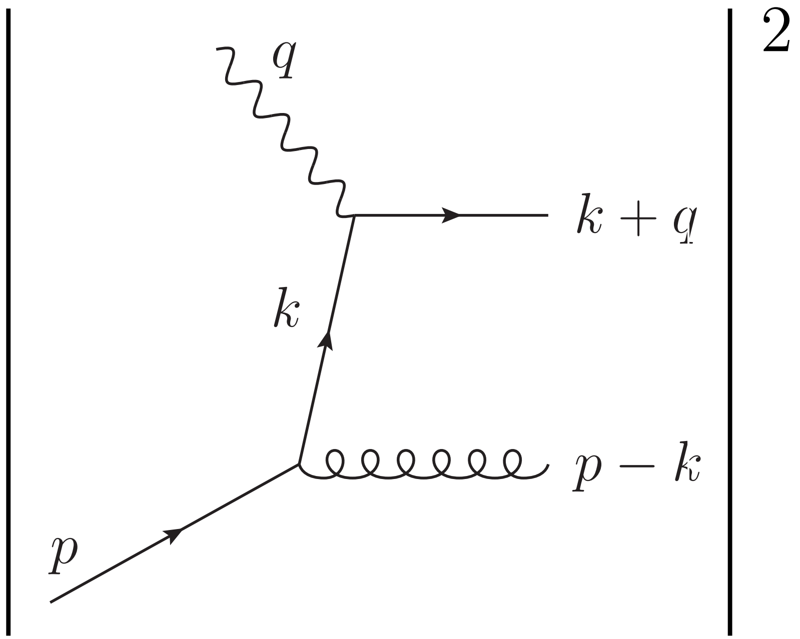
Work in an axial gauge $n.A = 0$ (recall $n^2 = 0$, $n.p = 1$, $n.q = 0$):
gluon of mom k^μ has propagator

$$d^{\mu\nu}(k) = \left(-g^{\mu\nu} + \frac{n^\mu k^\nu + k^\mu n^\nu}{n.k}\right)\frac{1}{k^2}$$

One-gluon emission

$$k^\mu = \xi p^\mu + \frac{k_\perp^2 - |k^2|}{2\xi} n^\nu + k_\perp^\mu$$

$$p \equiv (0, 0, P, P)$$



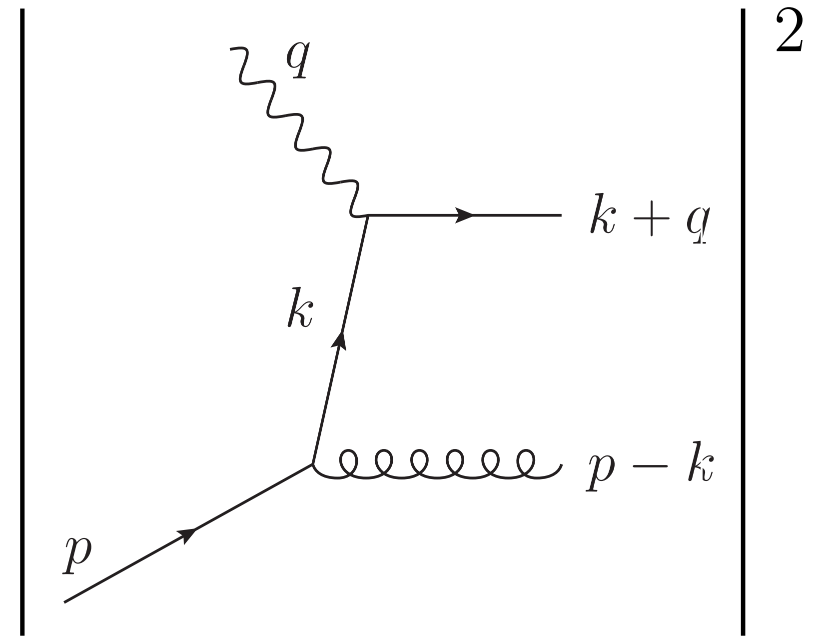
$$n^\mu n^\nu \overline{\sum} |\mathcal{M}|^2 = \frac{1}{2N_c} e_q^2 g^2 \,\mathrm{tr}(t_a t^a) \frac{1}{k^4} \mathrm{tr}\left(\not{n}\,(\not{k} + \not{q})\not{n}\,\not{k}\,\gamma^\alpha \not{p}\,\gamma^\beta \not{k}\right)$$

$$\left[-g^{\alpha\beta} + \frac{n^\alpha (p-k)^\beta + (p-k)^\alpha n^\beta}{n.(p-k)}\right]$$

$$= 32\pi\, e_q^2 \alpha_s \frac{\xi P(\xi)}{|k^2|} \qquad P(\xi) = C_F \frac{1+\xi^2}{1-\xi}$$

One-gluon emission

$$k^\mu = \xi p^\mu + \frac{k_\perp^2 - |k^2|}{2\xi} n^\nu + k_\perp^\mu$$



$$P(\xi) = C_F \frac{1+\xi^2}{1-\xi}$$

$$\hat{F}_2 \quad = \quad e_q^2 \frac{\alpha_s}{4\pi^2} \int d\xi\, \xi P(\xi) \int \frac{d|k^2|}{|k^2|}\, dk_\perp^2\, d\theta\, \delta\left((p-k)^2\right) \delta\left((k+q)^2\right)$$

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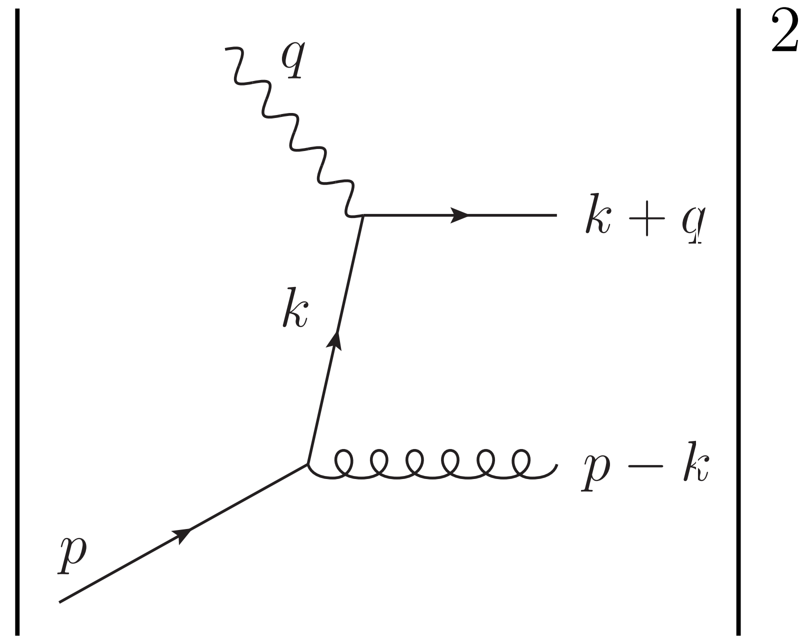


$$\hat{F}_2 = e_q^2 \frac{\alpha_s}{4\pi^2} \int d\xi\, \xi P(\xi) \int \frac{d|k^2|}{|k^2|}\, dk_\perp^2\, d\theta\, \delta\left((p-k)^2\right) \delta\left((k+q)^2\right)$$

$$= e_q^2 \frac{\alpha_s}{2\pi^2} \int_0^{2\nu} \frac{d|k^2|}{|k^2|} \int_{\xi_-}^{\xi_+} d\xi \frac{\xi P(\xi)}{\sqrt{(\xi_+ - \xi)(\xi - \xi_-)}}$$

with $\xi_\pm = x \pm \mathcal{O}(|k^2|/Q^2)$

One-gluon emission

$$\hat{F}_2 = e_q^2 \frac{\alpha_s}{2\pi^2} \, xP(x) \int_0^{2\nu} \frac{d|k^2|}{|k^2|} = e_q^2 \frac{\alpha_s}{2\pi^2} \, xP(x) \int_0^{Q^2} \frac{d|k^2|}{|k^2|}$$

- other diagrams suppressed by powers of Q
- only kept the leading terms in Q
- $|k^2|$ integration DIVERGENT!!

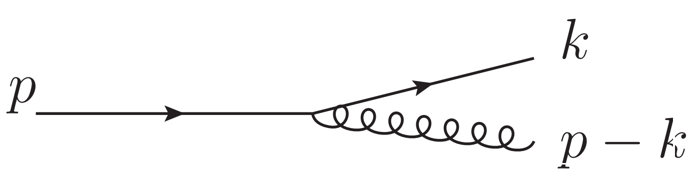
One-gluon emission

$$\hat{F}_2 = e_q^2 \frac{\alpha_s}{2\pi^2}\, xP(x) \int_0^{2\nu} \frac{d|k^2|}{|k^2|} = e_q^2 \frac{\alpha_s}{2\pi^2}\, xP(x) \int_0^{Q^2} \frac{d|k^2|}{|k^2|}$$

- other diagrams suppressed by powers of Q
- $|k^2|$ integration DIVERGENT!!

From $\delta((p-k)^2)$ we get $\vec{k}_\perp^{\,2} = (1-\xi)|k^2|$

Thus, $|k^2| \to 0 \Rightarrow \vec{k}_\perp \to 0$



This is thus a collinear divergence! The same as we already encountered in e^+e^- collisions.

One-gluon emission

$$\hat{F}_2 = e_q^2 \frac{\alpha_s}{2\pi^2} xP(x) \int_0^{2\nu} \frac{d|k^2|}{|k^2|} = e_q^2 \frac{\alpha_s}{2\pi^2} xP(x) \int_0^{Q^2} \frac{d|k^2|}{|k^2|}$$

- other diagrams suppressed by powers of Q
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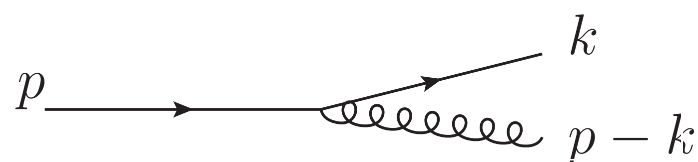
From $\delta((p-k)^2)$ we get $\vec{k}_\perp^{\,2} = (1-\xi)|k^2|$
Thus, $|k^2| \to 0 \Rightarrow \vec{k}_\perp \to 0$

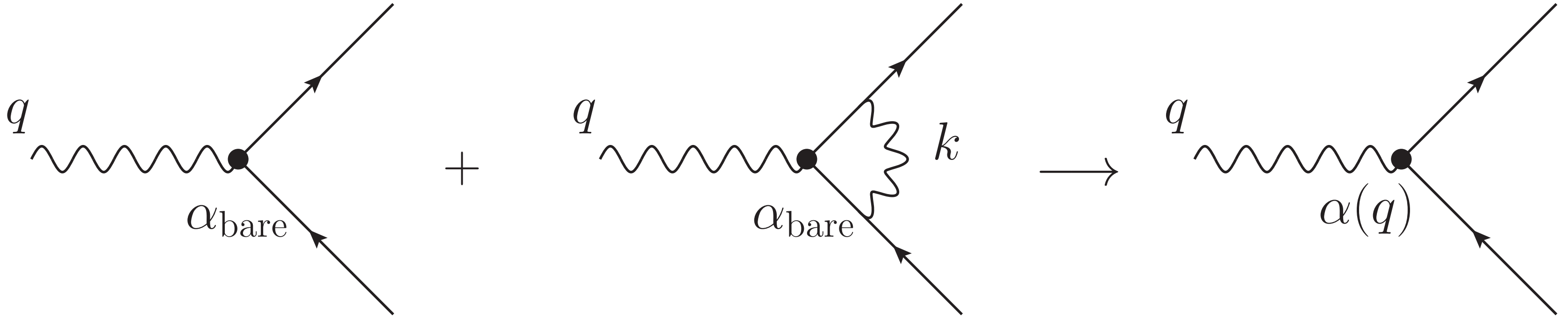


This is thus a collinear divergence! The same as we already encountered in e^+e^- collisions.

Not cancelled by virtual corrections
Here: technique similar to renormalisation

Recall: renormalisation

Vertex correction in QED



$$\alpha + \beta_0\,\alpha^2 \int_0^{q^2} \frac{dk^2}{k^2} \;=\; \alpha + \beta_0\,\alpha^2 \int_0^{\mu^2} \frac{dk^2}{k^2} + beta\,\alpha^2 \int_{\mu^2}^{q^2} \frac{dk^2}{k^2}$$

$$\to\; \alpha(\mu^2) + \beta_0\,\alpha^2 \int_{\mu^2}^{q^2} \frac{dk^2}{k^2}$$

$$\to\; \alpha(\mu^2) + \beta_0\,\alpha^2(\mu^2) \int_{\mu^2}^{q^2} \frac{dk^2}{k^2}$$

$$\to\; \alpha(q^2)$$

Recall: renormalisation

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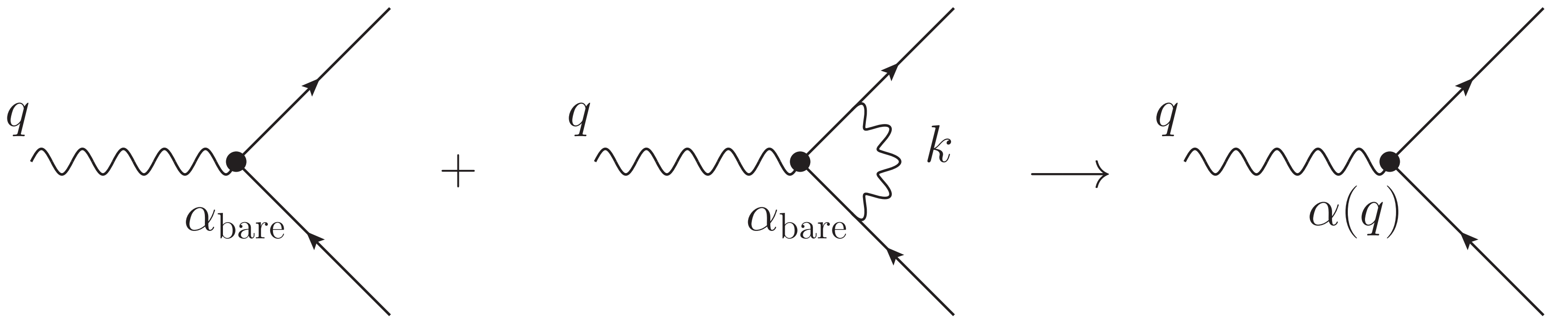
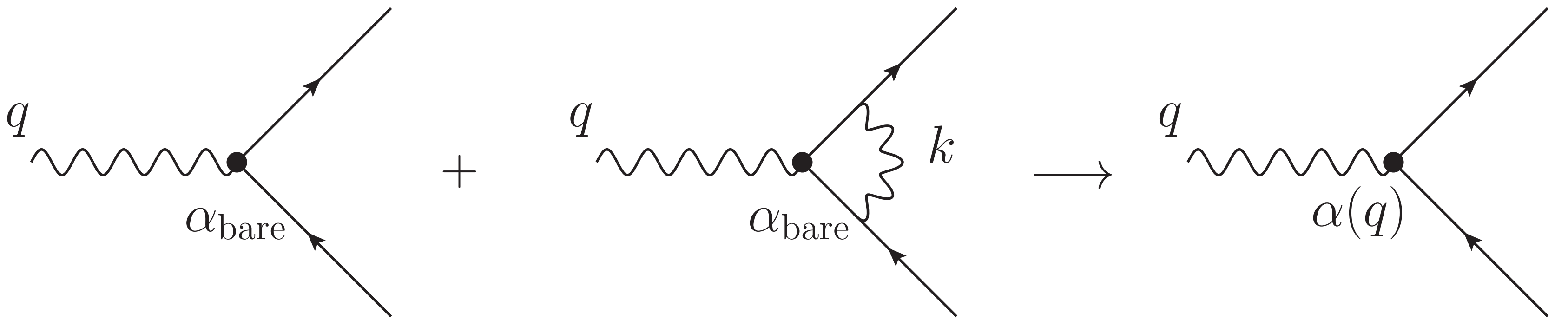


We have defined a scale-dependent coupling

$$\alpha(\mu^2) = \alpha + \beta_0 \,.\alpha^2 \int_0^{\mu^2} \frac{dk^2}{k^2}$$

Recall: renormalisation

Vertex correction in QED



We have defined a scale-dependent coupling

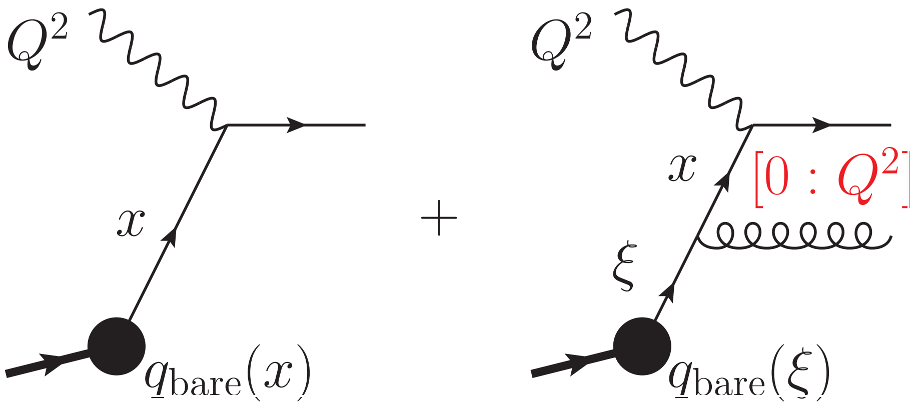
$$\alpha(\mu^2) = \alpha + \beta_0 \,.\alpha^2 \int_0^{\mu^2} \frac{dk^2}{k^2}$$

μ^2 is arbitrary *i.e.* physics should not depend on it

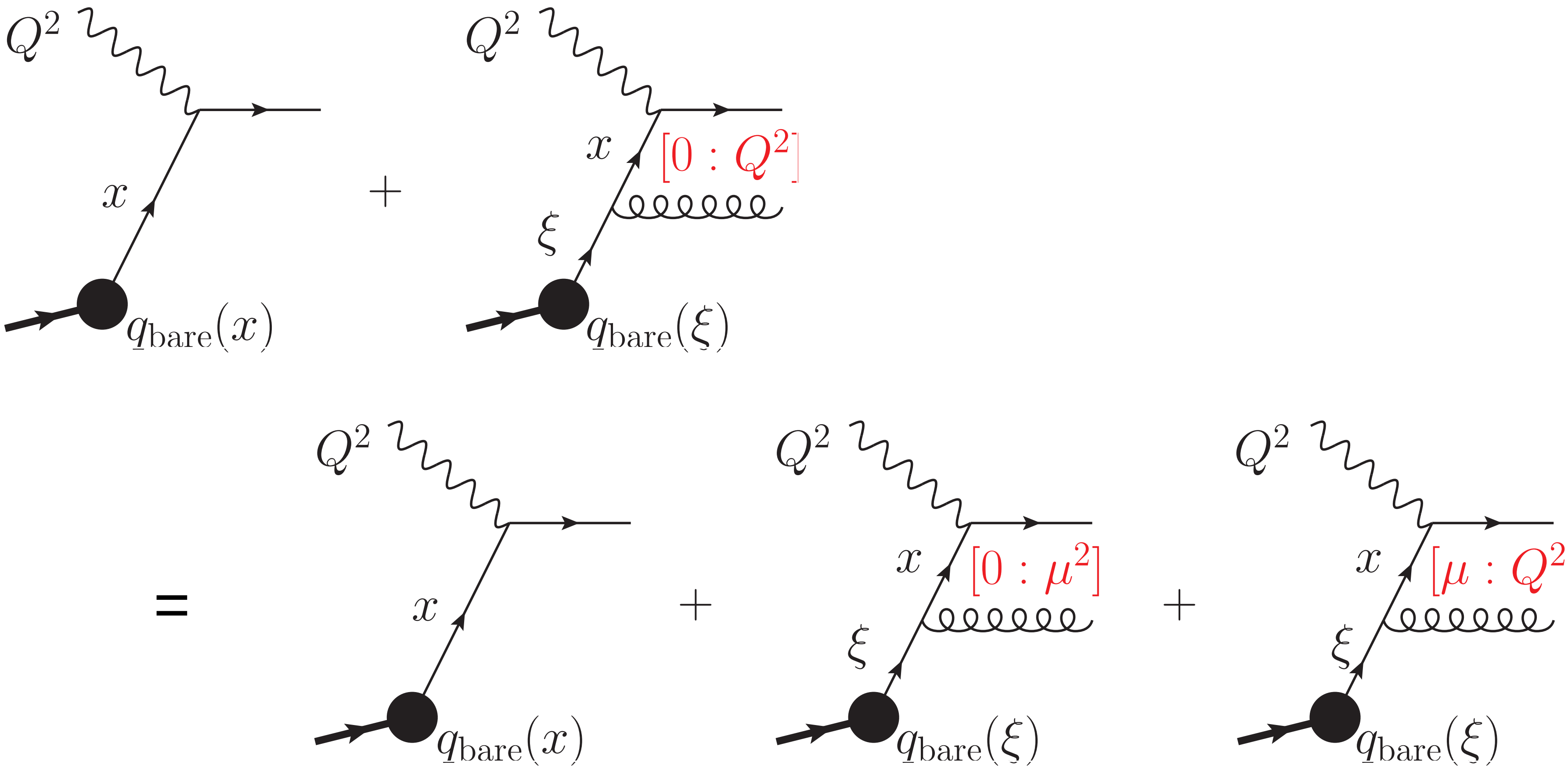
$$\boxed{\mu^2 \partial_{\mu^2} \alpha(\mu^2) = \beta_0 \alpha^2(\mu^2)}$$

renormalisation group equation

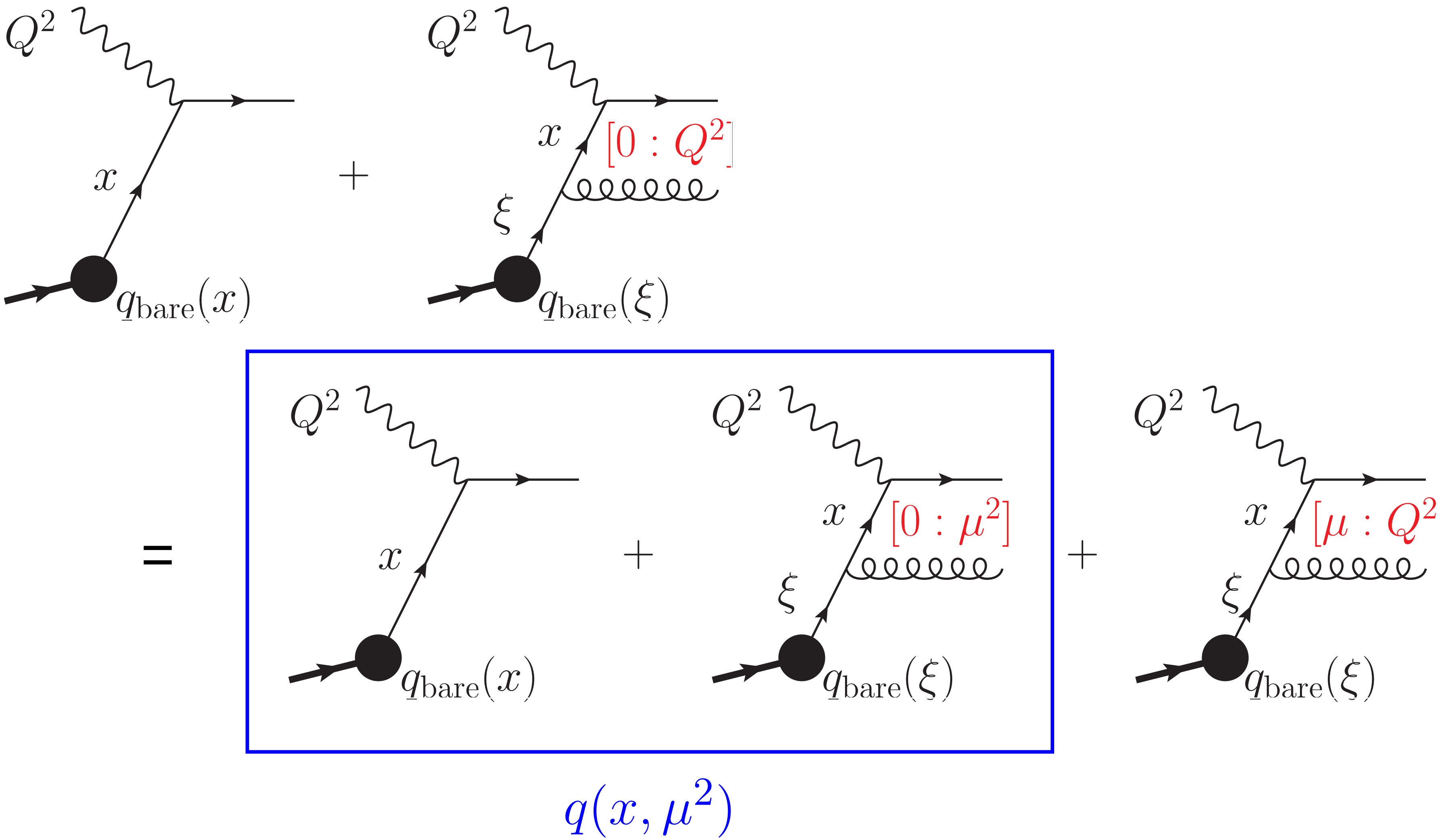
Reabsorption of the collinear divergence



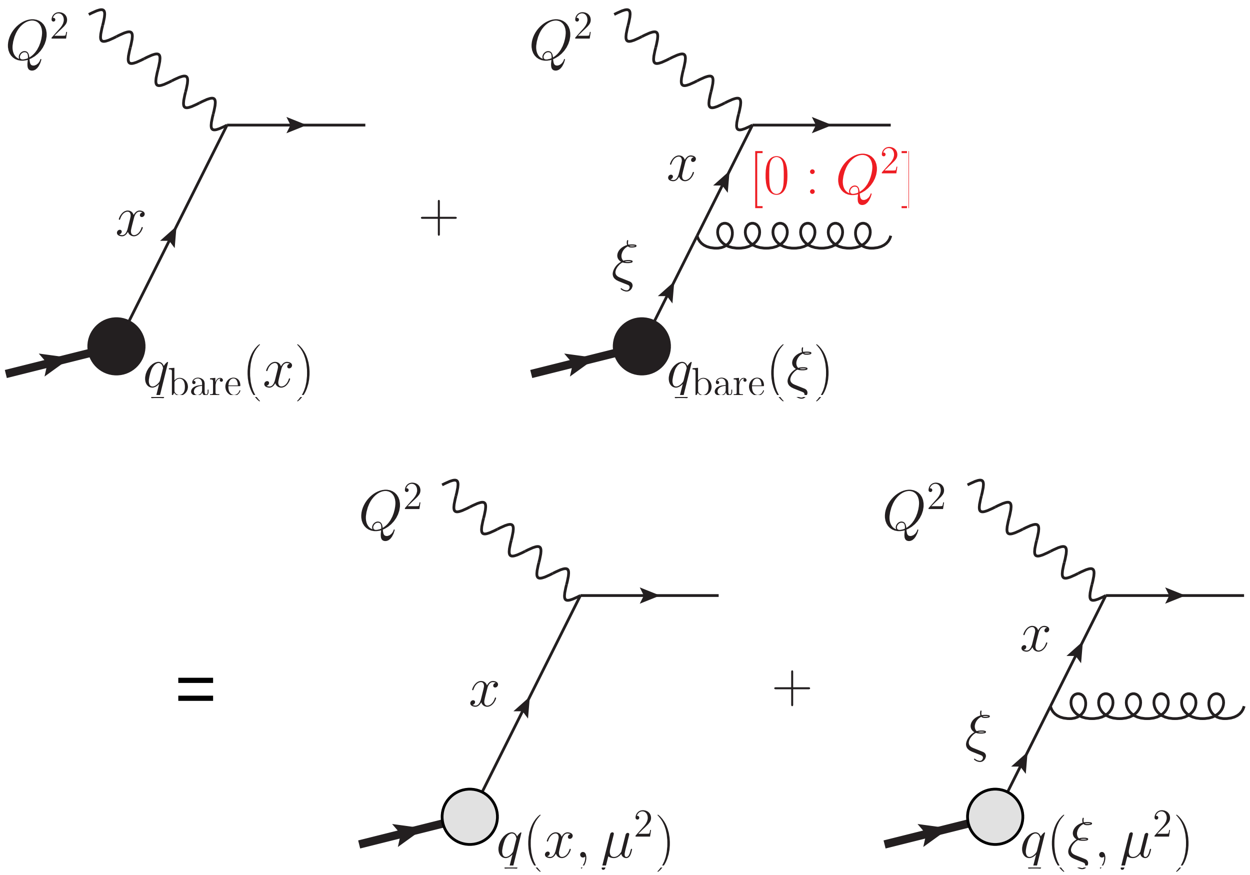
Reabsorption of the collinear divergence



Reabsorption of the collinear divergence



Reabsorption of the collinear divergence



Reabsorption of the collinear divergence

$$
\begin{aligned}
F_2(x,Q^2) &= x e_q^2 \int_x^1 \frac{d\xi}{\xi} \left[\delta\left(1-\frac{x}{\xi}\right) + P\left(\frac{x}{\xi}\right) \int_0^{Q^2} \frac{d|k^2|}{|k^2|} \right] q_{\text{bare}}(\xi) \\
&= x e_q^2 \int_x^1 \frac{d\xi}{\xi} \left[\delta\left(1-\frac{x}{\xi}\right) + P\left(\frac{x}{\xi}\right) \int_0^{\mu^2} \frac{d|k^2|}{|k^2|} \right] q_{\text{bare}}(\xi) \\
&\quad + x e_q^2 \int_x^1 \frac{d\xi}{\xi} P\left(\frac{x}{\xi}\right) \int_{\mu^2}^{Q^2} \frac{d|k^2|}{|k^2|} q_{\text{bare}}(\xi) \\
&= x e_q^2 \int_x^1 \frac{d\xi}{\xi} \left[\delta\left(1-\frac{x}{\xi}\right) + P\left(\frac{x}{\xi}\right) \int_{\mu^2}^{Q^2} \frac{d|k^2|}{|k^2|} \right] q(\xi,\mu^2) \\
&= x e_q^2 q(\xi, Q^2)
\end{aligned}
$$

$P(x) = \frac{\alpha_s}{2\pi} C_F \frac{1+x^2}{1-x}$

Reabsorption of the collinear divergence

We have defined

$$q(x,\mu^2) = q_{\mathrm{bare}}(x) + \int_x^1 \frac{d\xi}{\xi} P\left(\frac{x}{\xi}\right) \int_0^{\mu^2} \frac{d|k^2|}{|k^2|} q_{\mathrm{bare}}(\xi)$$

Reabsorption of the collinear divergence

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Physics independent of the choice for μ^2

$$\boxed{\mu^2 \partial_{\mu^2} q(x,\mu^2) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} P\left(\frac{x}{\xi}\right) q(\xi,\mu^2)}$$

DGLAP equation

The DGLAP equation

$$Q^2\partial_{Q^2} q(x,Q^2) = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi} P\left(\frac{x}{\xi}\right) q(\xi,Q^2)$$

- DGLAP: Dokshitzer-Gribov-Lipatov-Altarelli-Parisi

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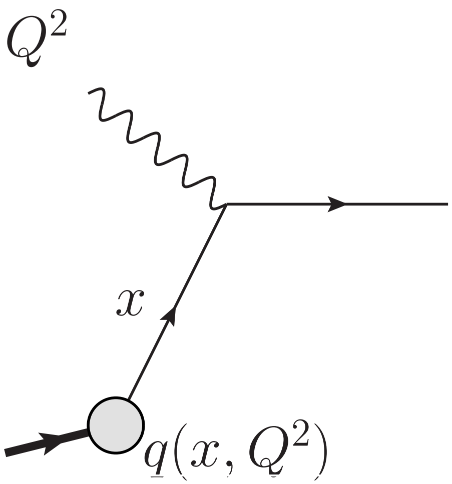
- the PDFs get some dependence on Q^2
- Bjorken scaling violations
- μ called the factorisation scale
- Leading order computation in $\alpha_s \log(Q^2/\mu^2)$
- Actually resums all terms $\alpha_s^n \log^n(Q^2/\mu^2)$
  (recall: $\alpha_s \log(Q^2/\mu^2) \sim 1 \Rightarrow$ compute at all orders)

The DGLAP equation: resummation

$$Q^2 \partial_{Q^2} q(x, Q^2) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} P\left(\frac{x}{\xi}\right) q(\xi, Q^2)$$



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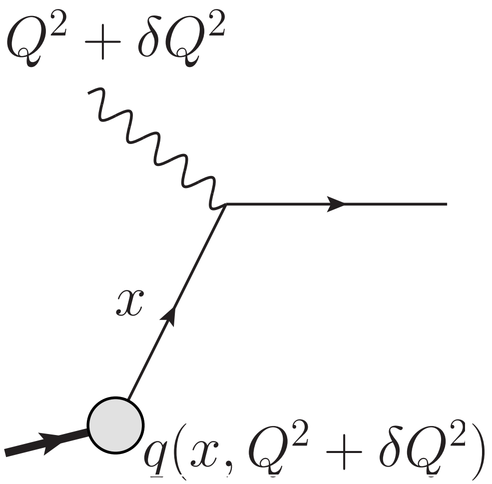

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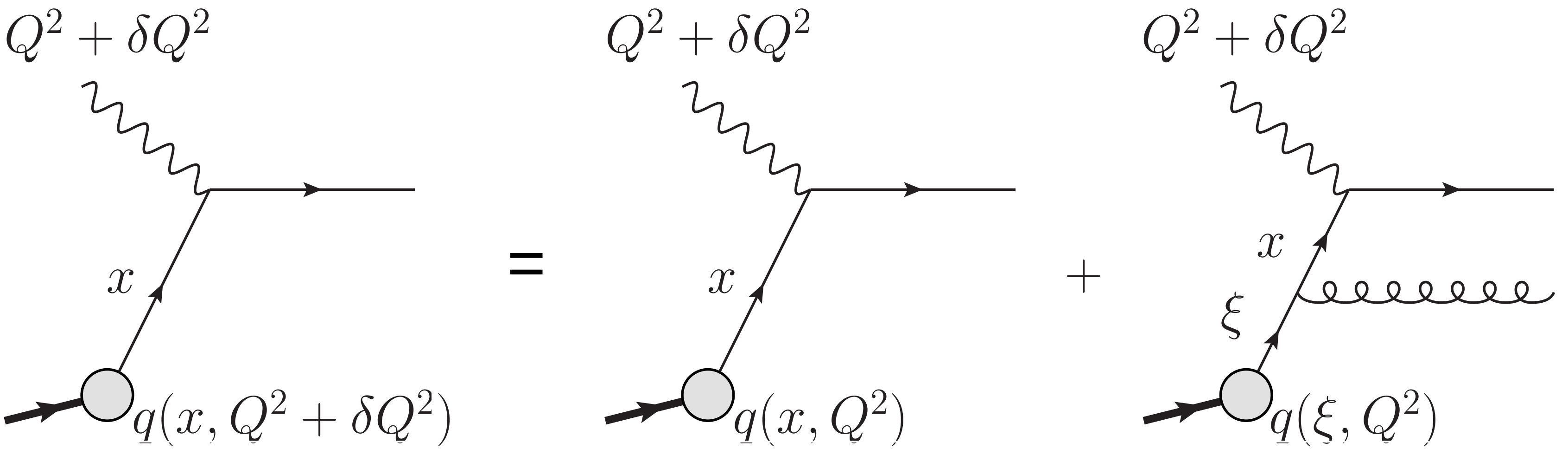

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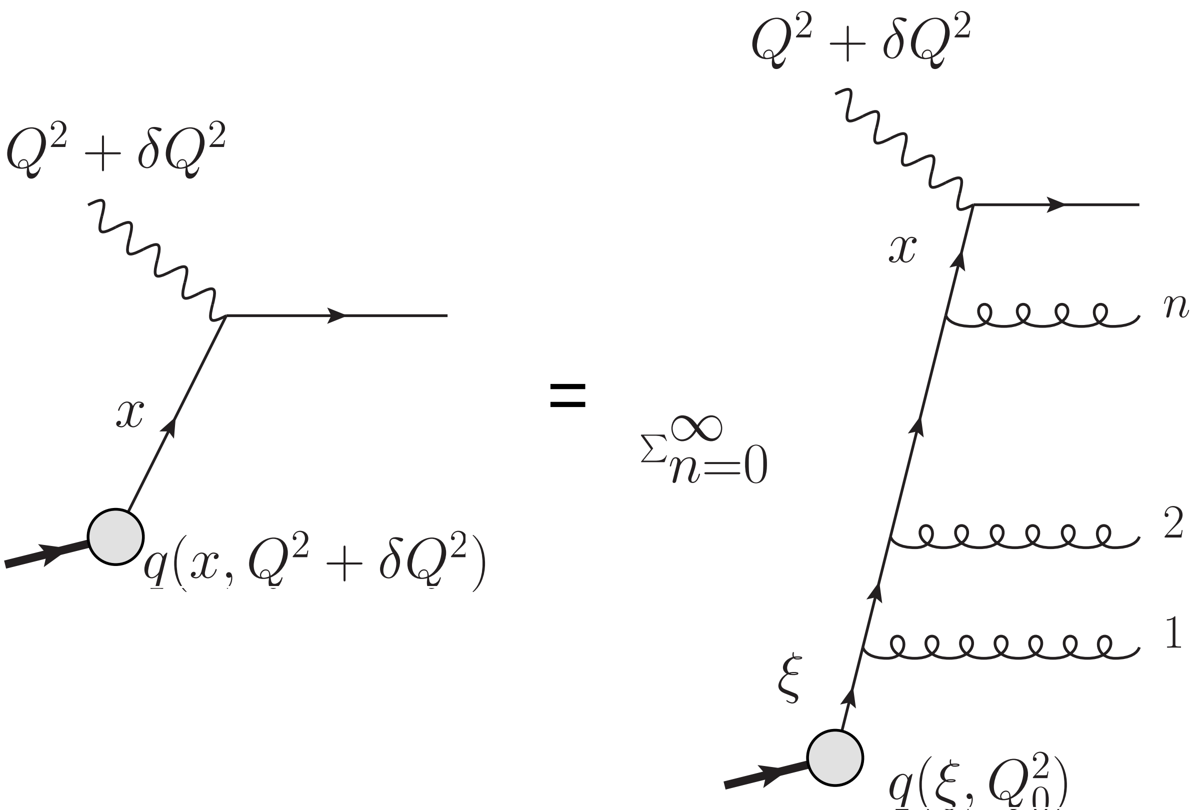

Resumming (leading) contributions $\alpha_s^n \log^n(Q^2/Q_0^2)$

The DGLAP equation: splitting function

$$\boxed{Q^2 \partial_{Q^2} q(x, Q^2) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} P\left(\frac{x}{\xi}\right) q(\xi, Q^2)}$$

$P(\xi)$ called the splitting function:

transition from a quark of longitudinal momentum xP to a quark of momentum $x\xi P$ with emission of a gluon

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Correction due to virtual-gluon emission:

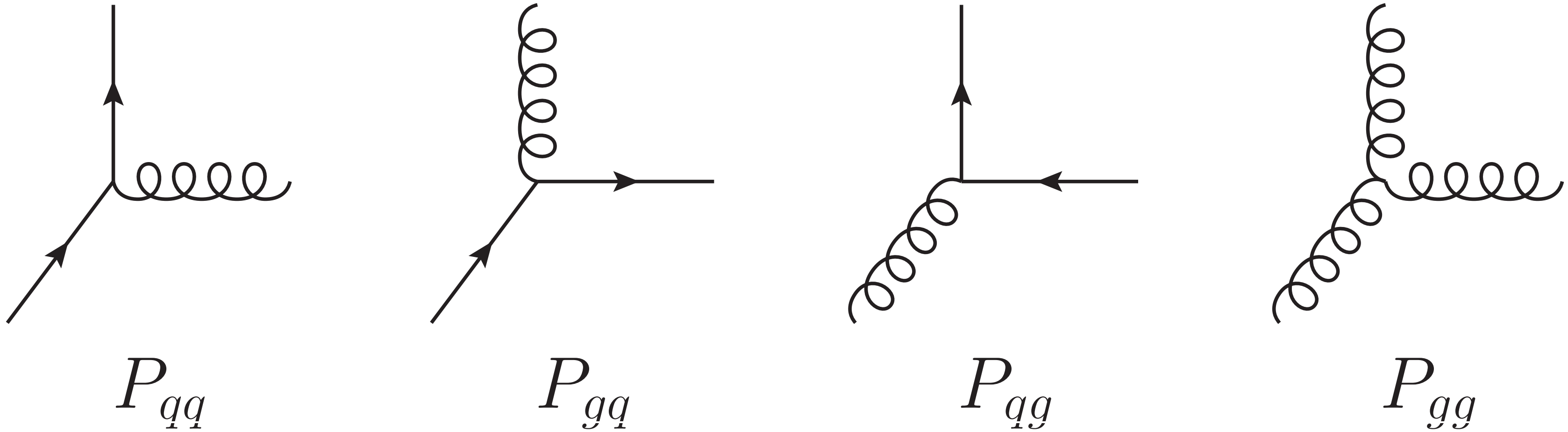
$$P(x) = C_F \left[\frac{1+x^2}{1-x}\right]_+$$

NB: the $1/(1-x)$ behaviour is the soft QCD divergence

The DGLAP equation: splitting function

$$Q^2 \partial_{Q^2} \begin{pmatrix} q(x,Q^2) \\ g(x,Q^2) \end{pmatrix} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} \begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix} \left(\frac{x}{\xi} \right) \begin{pmatrix} q(\xi,Q^2) \\ g(\xi,Q^2) \end{pmatrix}$$

$P_{ab}(\xi)$ called the splitting function:



$P_{ab}(x)$ is the probability to obtain a parton of type a carrying a fraction x of the longitudinal momentum of a parent parton of type b

DGLAP and the factorisation theorem

The result is more general: it holds at any order in perturbation theory

$$Q^2 \partial_{Q^2} q(x, Q^2) = \int_x^1 \frac{d\xi}{\xi} P\left(\frac{x}{\xi}\right) q(\xi, Q^2)$$

with

$$P(x) = \left(\frac{\alpha_s}{2\pi}\right) P^{(0)}(x) + \left(\frac{\alpha_s}{2\pi}\right)^2 P^{(1)}(x) + \left(\frac{\alpha_s}{2\pi}\right)^3 P^{(2x)}(x) + \ldots$$

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$$P(x) = \underbrace{\left(\frac{\alpha_s}{2\pi}\right) P^{(0)}(x)}_{\text{LO}} + \underbrace{\left(\frac{\alpha_s}{2\pi}\right)^2 P^{(1)}(x)}_{\text{NLO}} + \underbrace{\left(\frac{\alpha_s}{2\pi}\right)^3 P^{(2)}(x)}_{\text{NNLO}} + \ldots$$

- LO resums $\alpha_s^n \log^n(Q^2/\mu^2)$ (leading logarithms)
- NLO resums $\alpha_s^n \log^n(Q^2/\mu^2)$ and $\alpha_s^{n+1} \log^n(Q^2/\mu^2)$

Note: order refers to P; includes diagrams at all orders

Note: known up to NNLO since 2004 (Moch, Vermaseren, Vogt)

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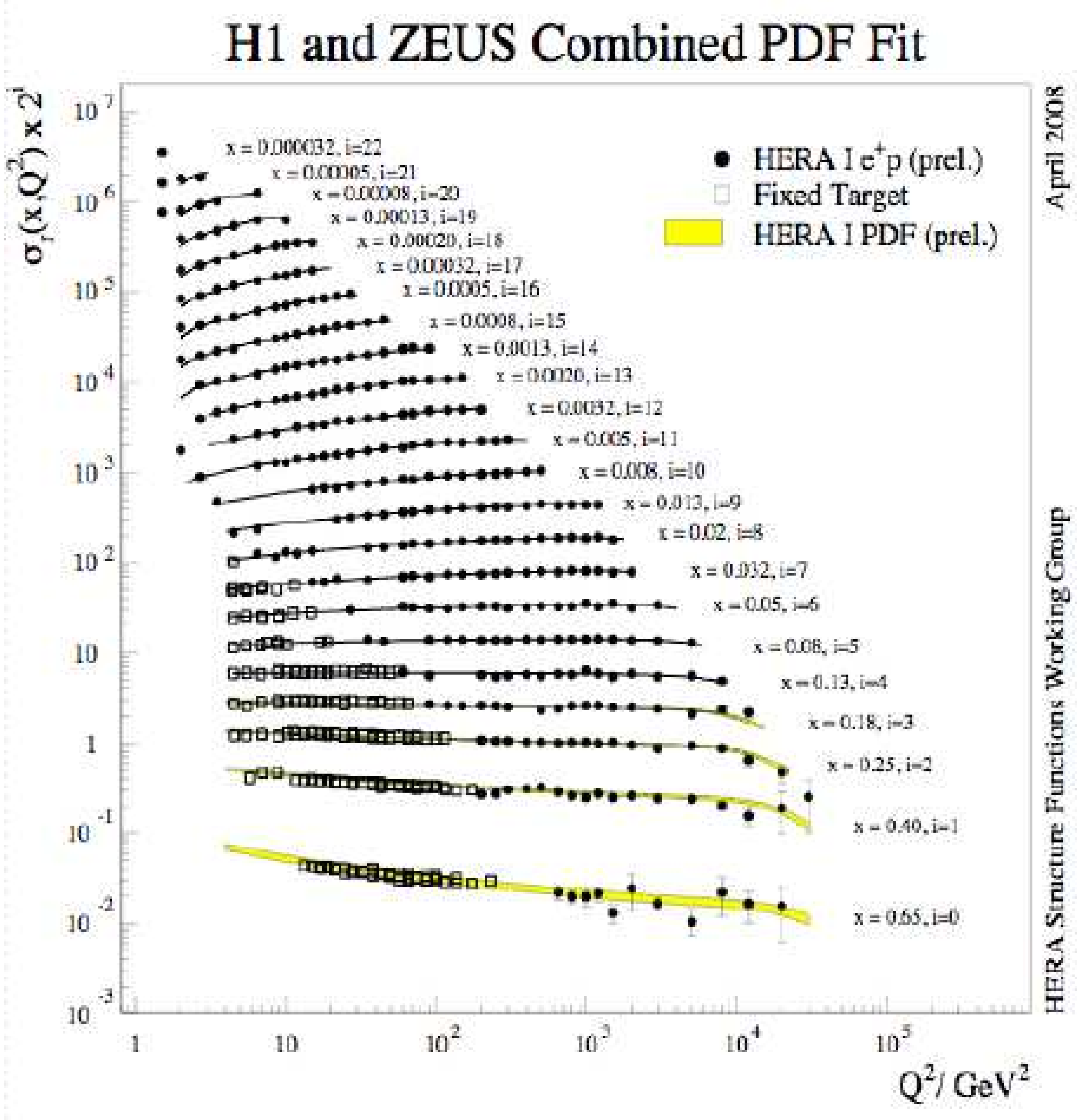
Fundamental result in QCD know as the factorisation theorem

Collinear divergences can be reabsorbed in the definition of the PDFs at all orders!

DGLAP vs. data

Very nice description of the Q^2-dependence observed in the data



DGLAP vs. data

DGLAP only gives the Q^2 evolution of the PDFs
One still needs an initial condition $f_a(x, \mu^2)$

Global PDF fit:

- Parametrise q and g at an initial scale μ^2
  e.g. $q(x, \mu^2) = x^\lambda (1-x)^\beta (A + B\sqrt{x} + Cx)$
- Obtain the PDFs $f_a(x, Q^2)$ at all Q^2 using DGLAP
- Compute a series of observables (*e.g.* F_2)
- Fit the experimental measurements (χ^2 minimisation)

DGLAP vs. data

- Many teams: MSTW/MRST, CTEQ, NNPDF, HERA, H1, ZEUS, Alekhin, GRV
- Each with many updates

  e.g. CTEQ4l, CTEQ4m, CTEQ5l, CTEQ5m, CTEQ6, CTEQ6l, CTEQ6m, CTEQ61, CTEQ65, CTEQ66

  MRST98, MRST2001, MRST2002, MRST2003, MRST2004, MRST2006, MRST2007, MSTW2008

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  - Choice of initial scale
  - Choice of initial parametrisation
  - Order of the fit (LO, NLO, NNLO)
  - Data selection (*e.g. cuts, old vs. new data*)
  - Heavy-flavour treatment
  - Computation of PDFs uncertainties
  - List of observables (9)

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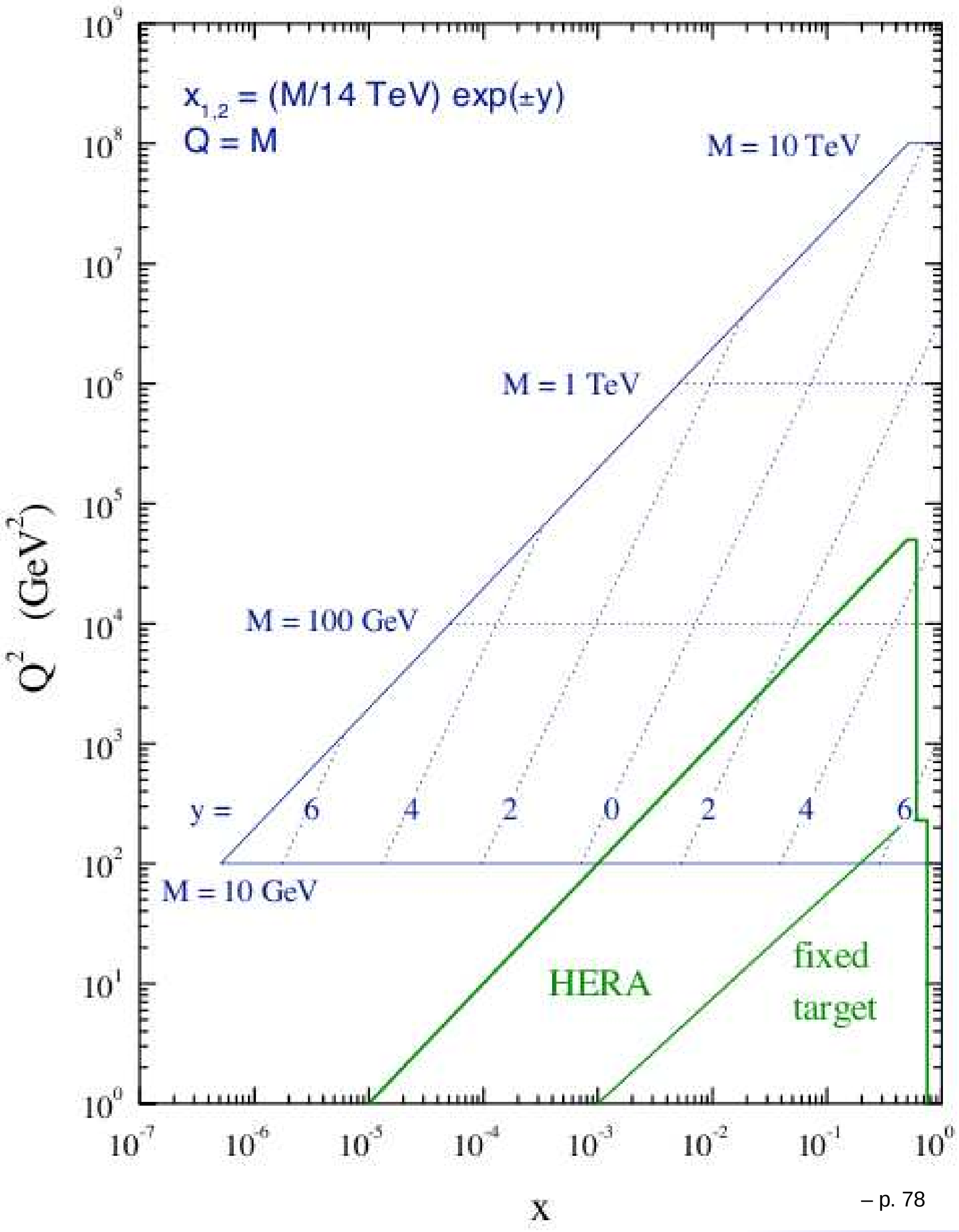
    F_2^p, F_2^d, F_L, F_2^ν, F_3^ν, F_2^c, F_2^b, Drell-Yan, Tev. jets

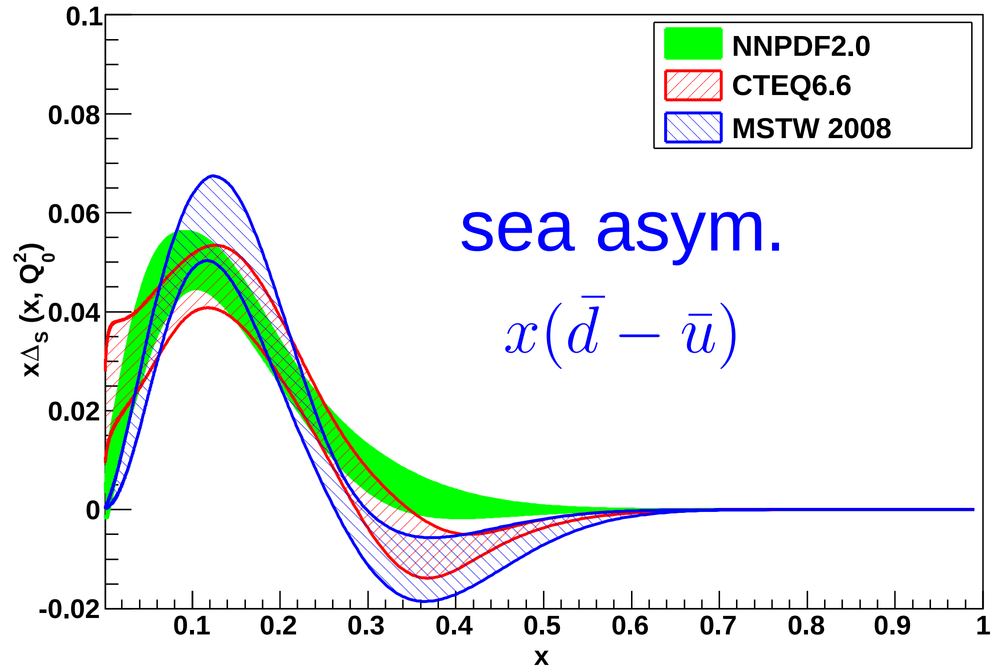
Global fits

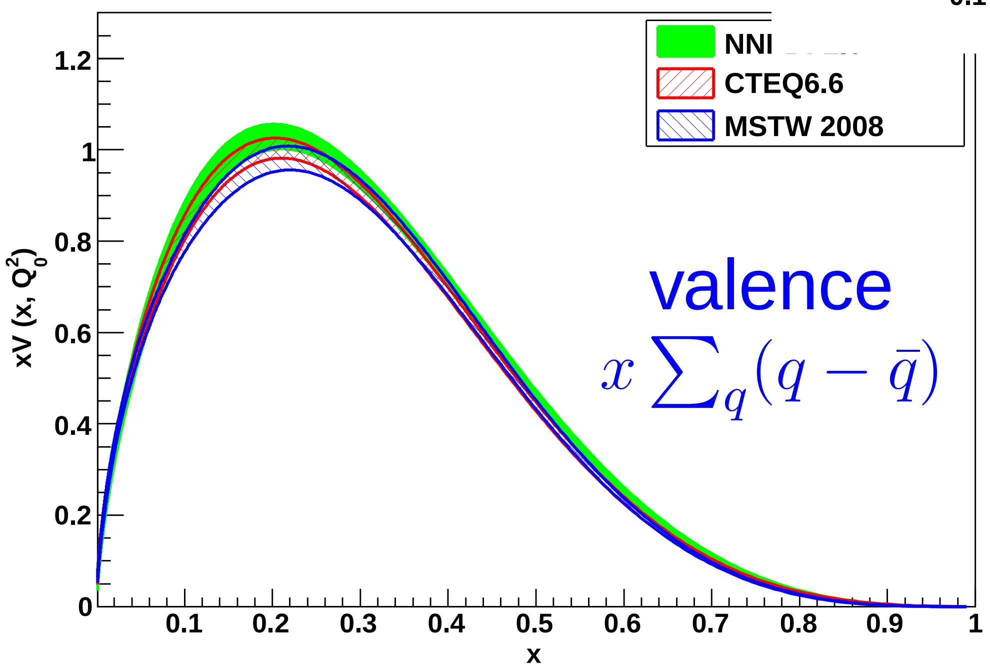
Global fits are important for LHC physics as they affect every perturbative computation



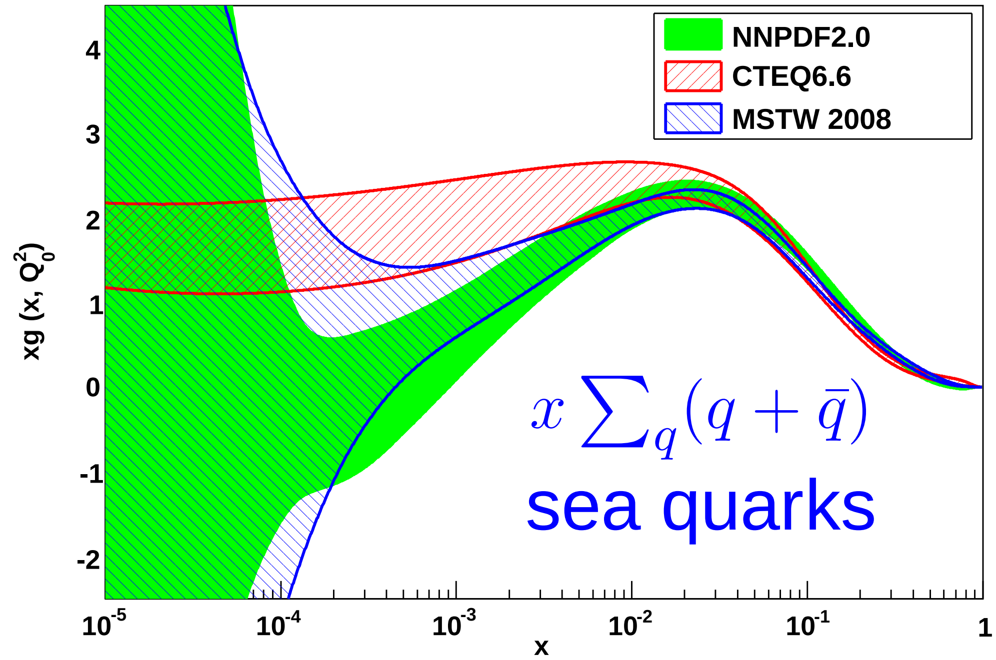
Global fits

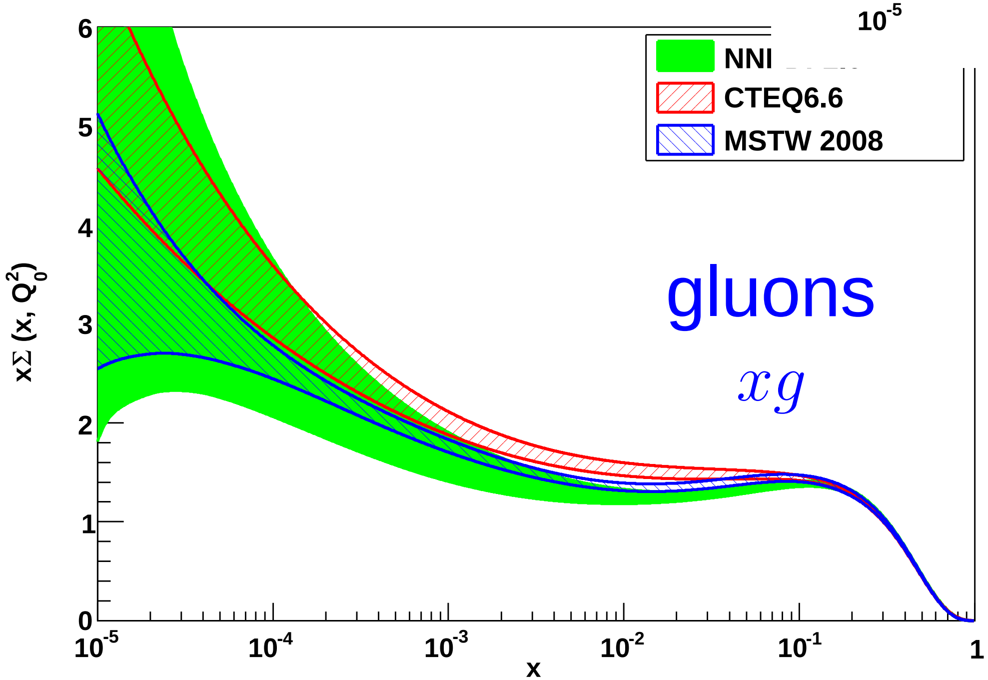
Initial distributions
$Q^2 = \mu^2 = 2$ GeV



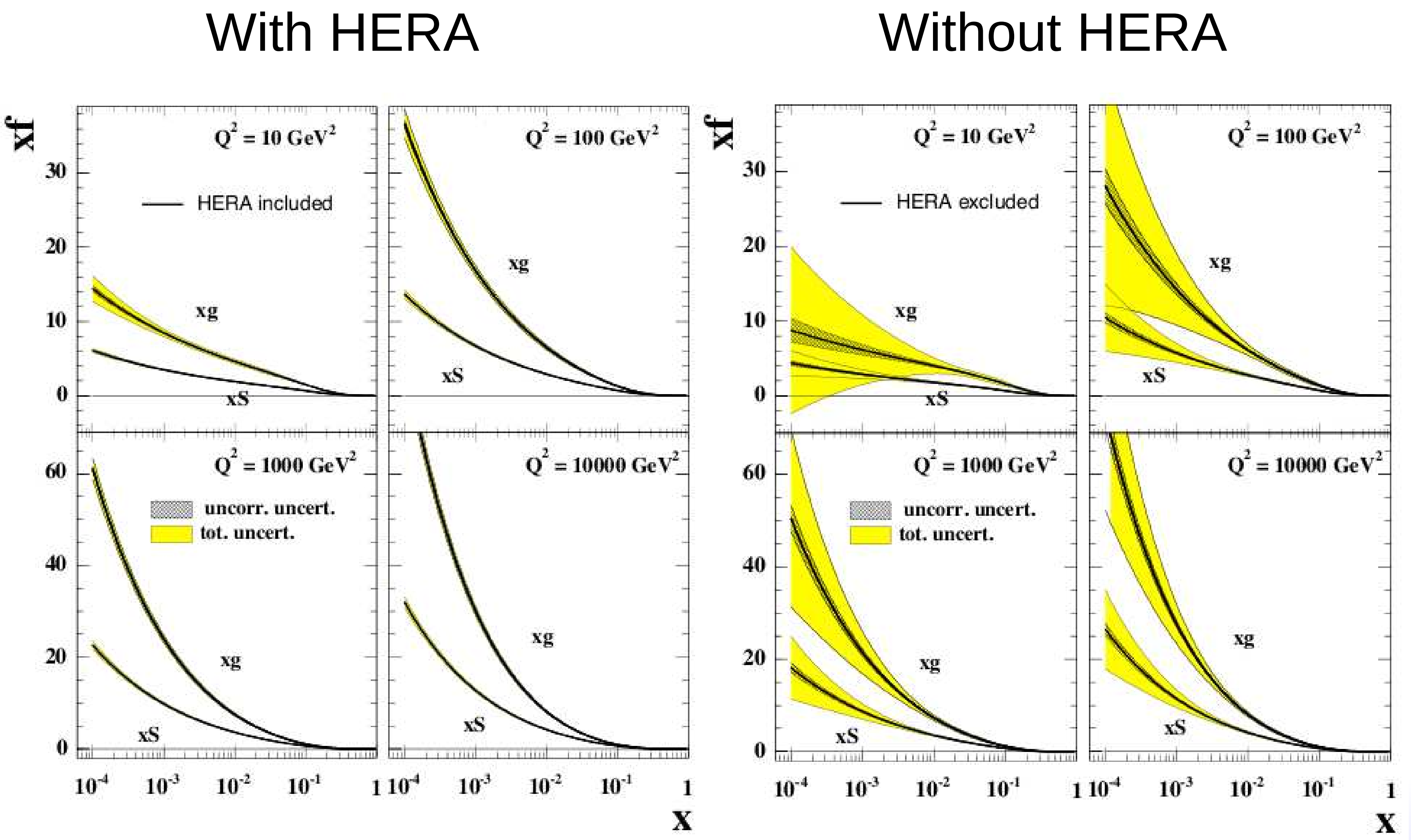


Initial ‘flavour-singlet’ distributions
$Q^2 = \mu^2 = 2$ GeV

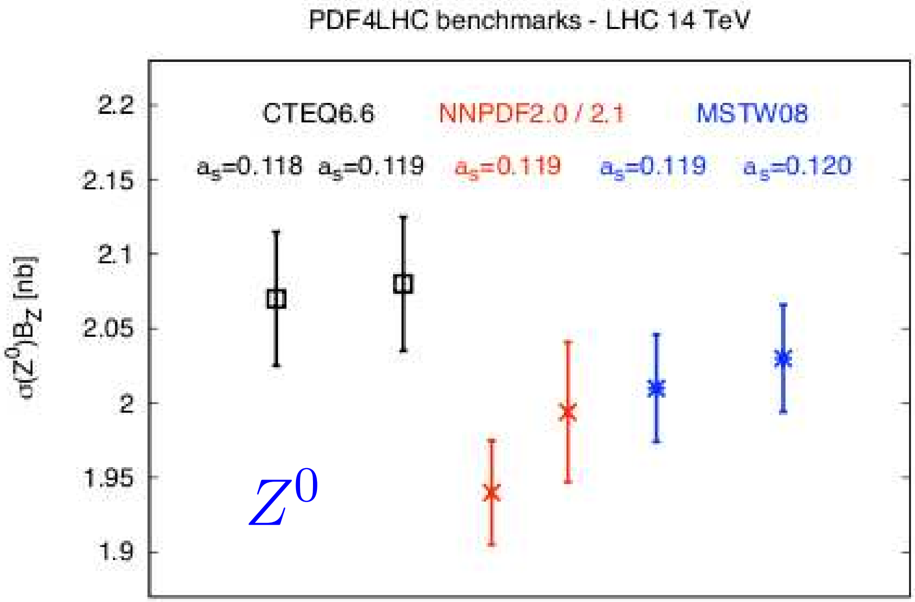


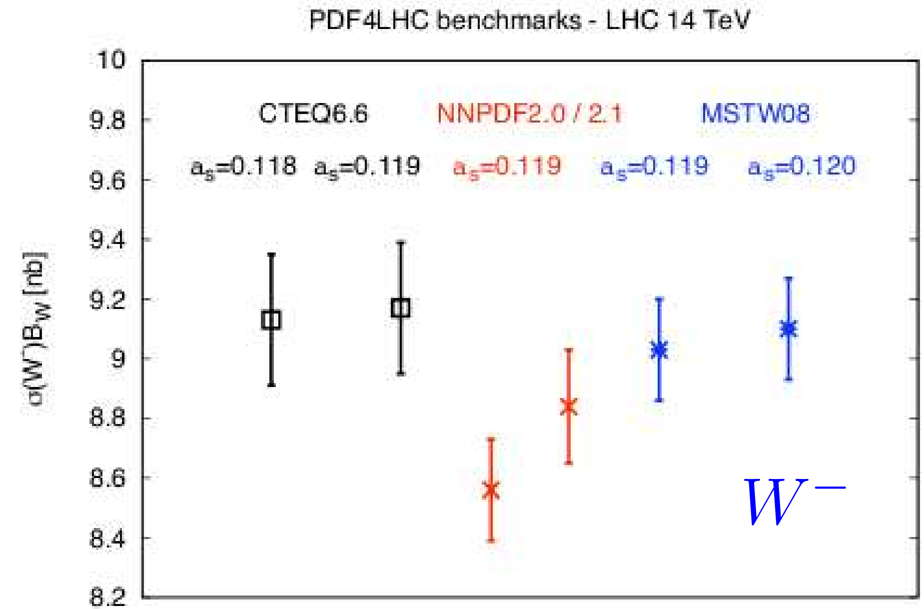


Impact of HERA measurements



Global fits


PDF4LHC benchmarks - LHC 14 TeV
CTEQ6.6
NNPDF2.0 / 2.1
MSTW08
a_s=0.118 a_s=0.119
a_s=0.119
a_s=0.119 a_s=0.120
$\sigma(Z^0)B_Z$ [nb]
2.2
2.15
2.1
2.05
2
1.95
1.9
Z^0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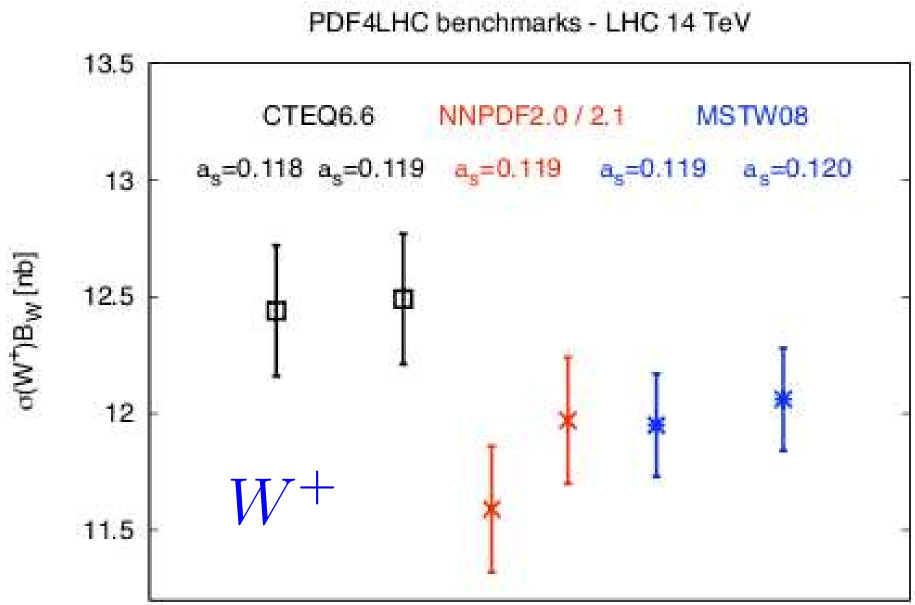
MSTW08
a_s=0.118 a_s=0.119
a_s=0.119
a_s=0.119 a_s=0.120
$\sigma(W^-)B_W$ [nb]
10
9.8
9.6
9.4
9.2
9
8.8
8.6
8.4
8.2
W^-


PDF4LHC benchmarks - LHC 14 TeV
CTEQ6.6
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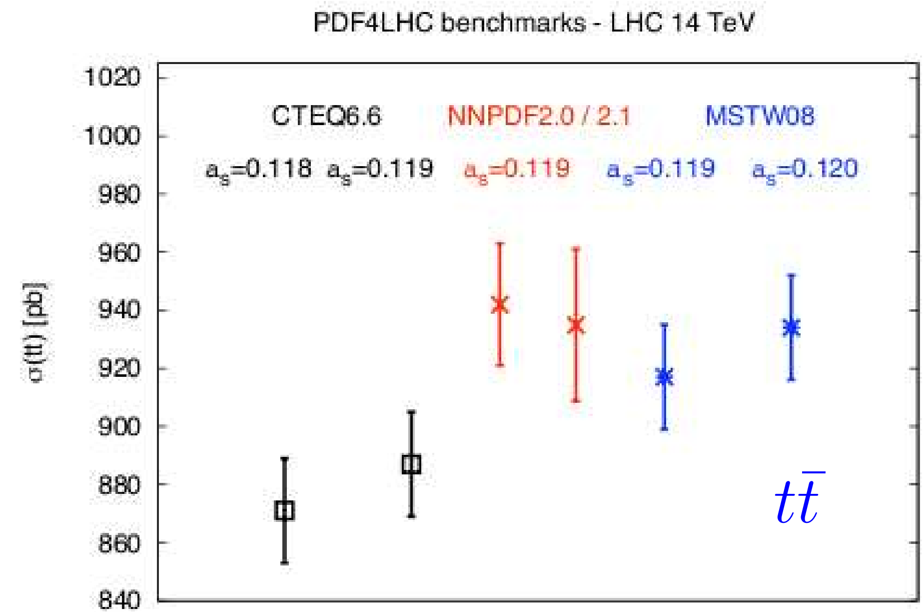
MSTW08
a_s=0.118 a_s=0.119
a_s=0.119
a_s=0.119 a_s=0.120
$\sigma(W^+)B_W$ [nb]
13.5
13
12.5
12
11.5
W^+


PDF4LHC benchmarks - LHC 14 TeV
CTEQ6.6
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MSTW08
a_s=0.118 a_s=0.119
a_s=0.119
a_s=0.119 a_s=0.120
σ(tt) [pb]
1020
1000
980
960
940
920
900
880
860
840
$t\bar{t}$

DIS: summary

DIS: $\gamma^* p$ scattering with highly virtual γ $(Q^2 \gg \Lambda^2_{\mathrm{QCD}})$

- Parton model
  - directly probes partons inside the proton
  - Bjorken scaling

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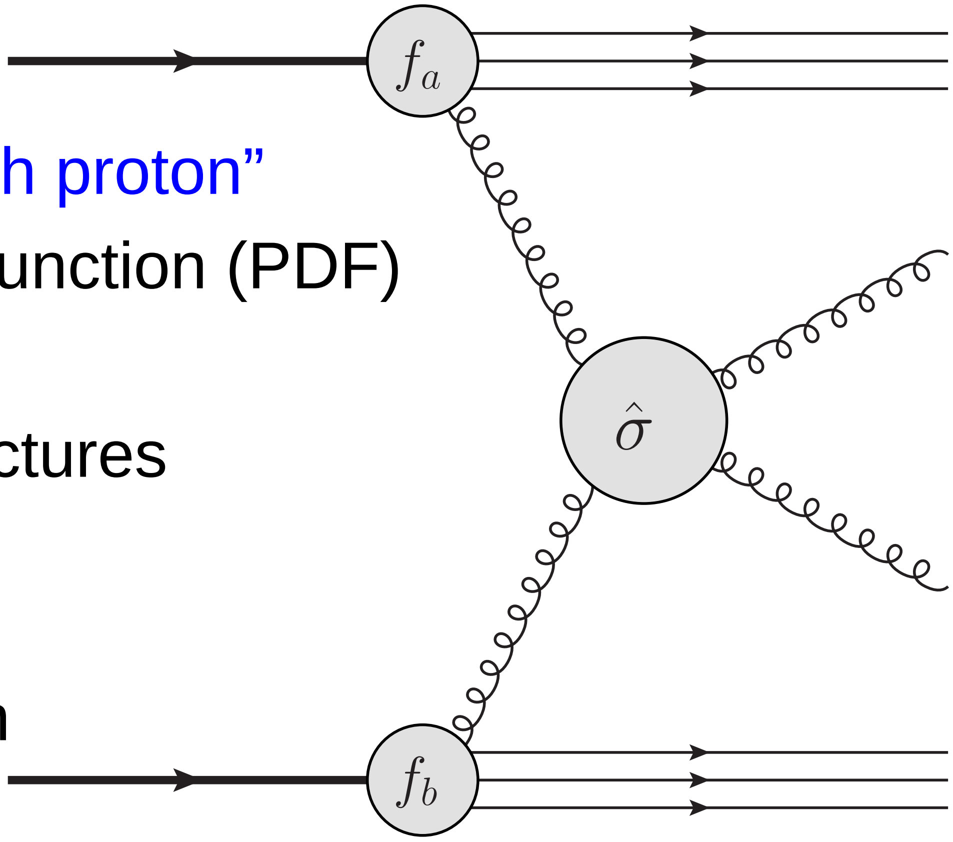
- Parton model
  - directly probes partons inside the proton
  - Bjorken scaling
- QCD collinear divergences
  - Violations of Bjorken scaling
  - Factorisation theorem/DGLAP equation (fundamental result/prediction of QCD)
  - Parton Distribution Functions (PDF)
  - Global fits for the PDF determination of the PDFs: mandatory for precision at the LHC

Time for questions!

pp collisions
(at last!)

The very fundamental collision

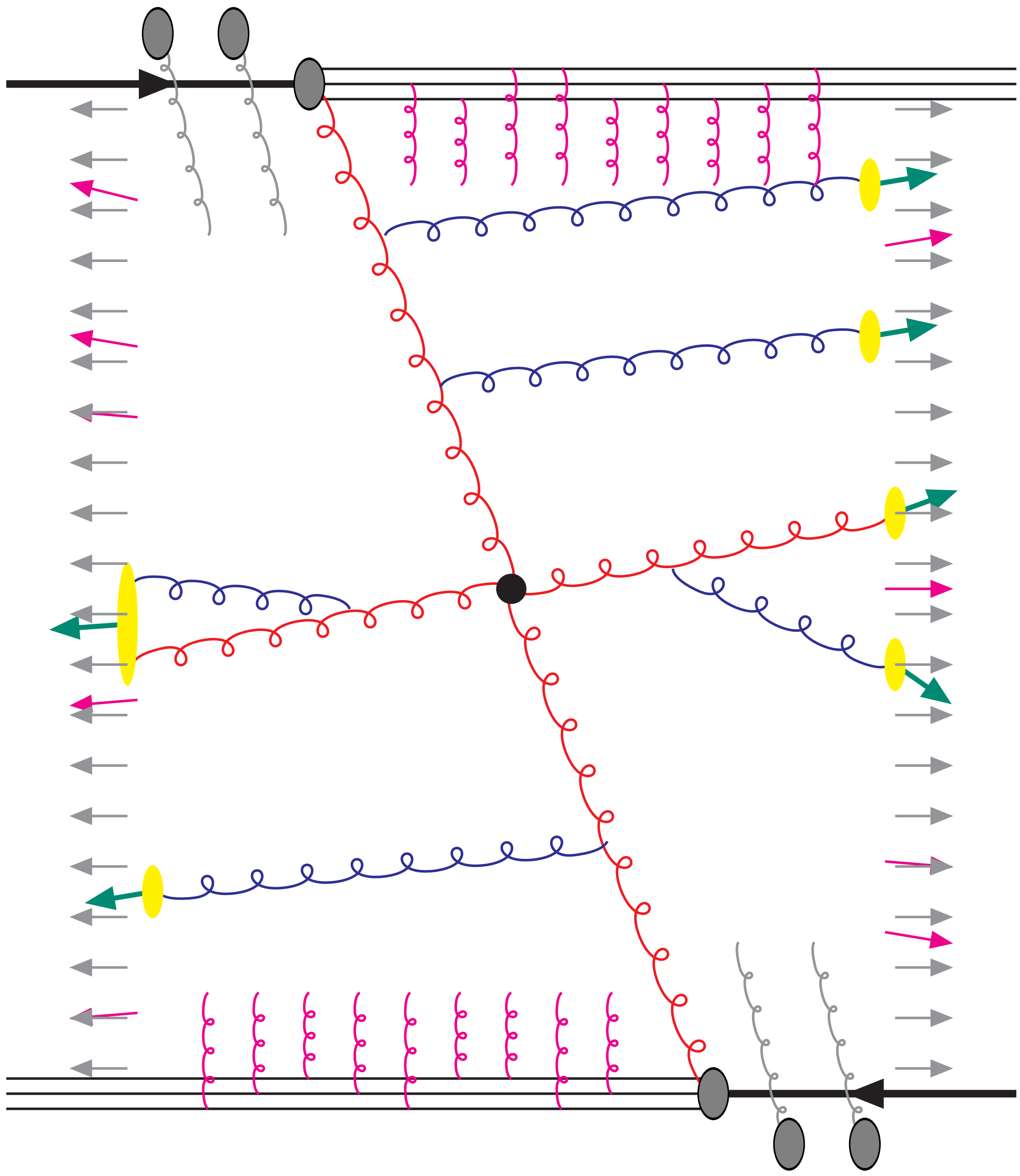
$$\sigma = f_a \otimes f_b \otimes \hat{\sigma}$$



- "take a parton out of each proton"
  $f_a \equiv$ parton distribution function (PDF)
  for quark and gluons
  a big chapter of these lectures

- hard matrix element
  perturbative computation
  Forde-Feynman rules

The more realistic version



- Hard ME

  perturbative
- Parton branching

  initial+final state radiation
- Hadronisation

  $q, g \to$ hadrons
- Multiple interactions

  Underlying event (UE)
- Pile-up

  $\lesssim$ 25 pp at the LHC

Plan

- A few generic considerations
  - kinematics (done)
  - Monte-Carlo

- Processes one-by-one
  - Drell-Yan
  - Jets (done)
  - W/Z (+jets)
  - top
  - H
  - SUSY (?)

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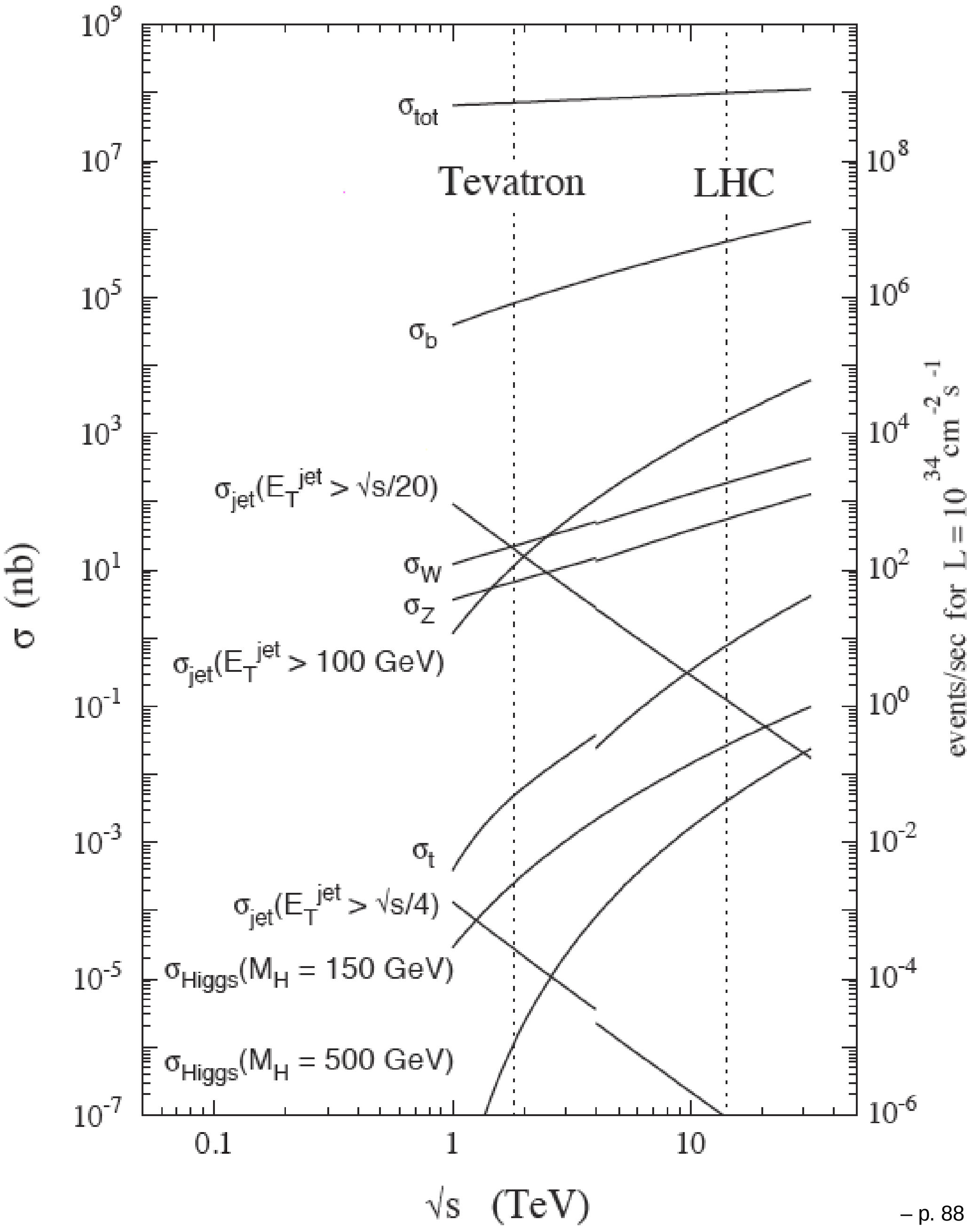
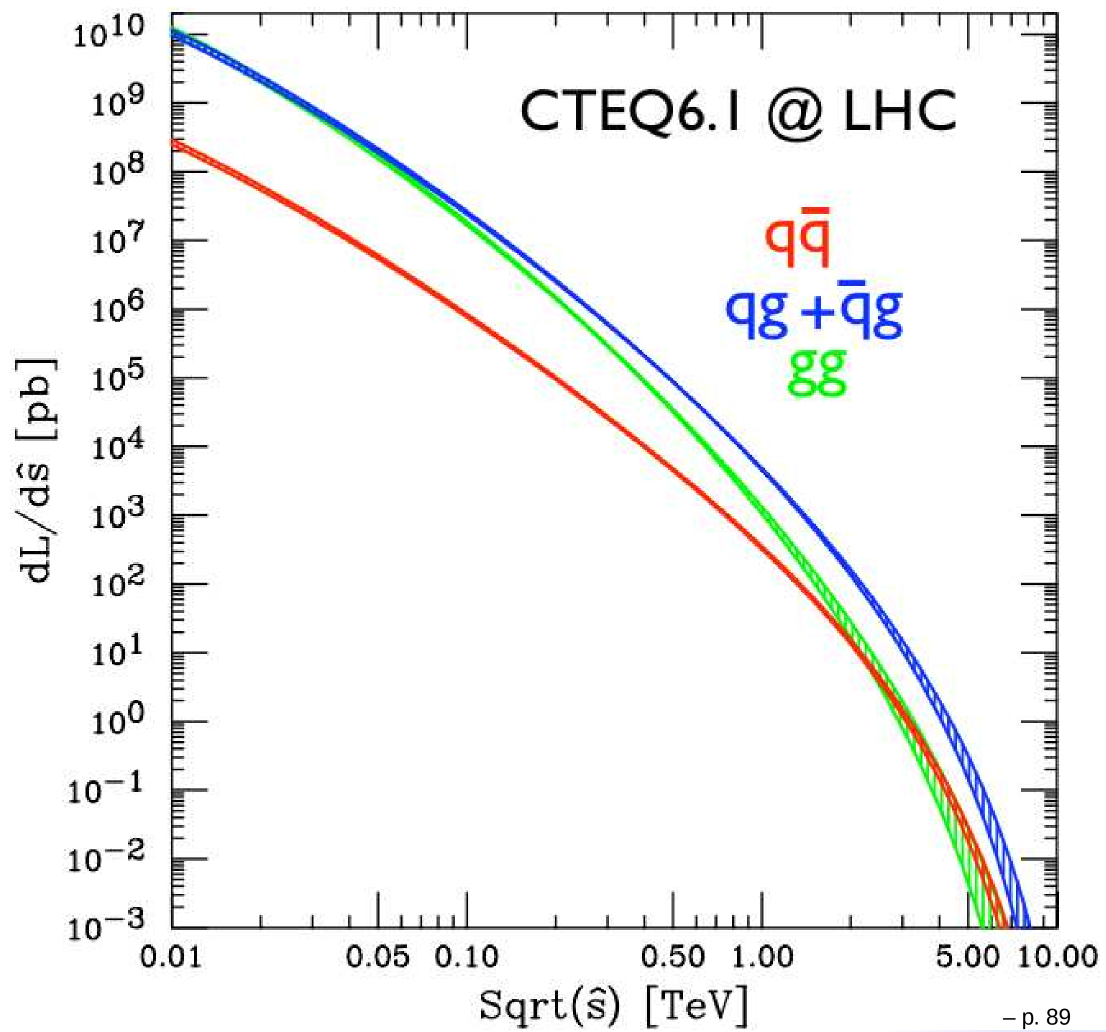


Parton luminosities

Vary $\sqrt{s} \Rightarrow$ same ME, only PDF vary

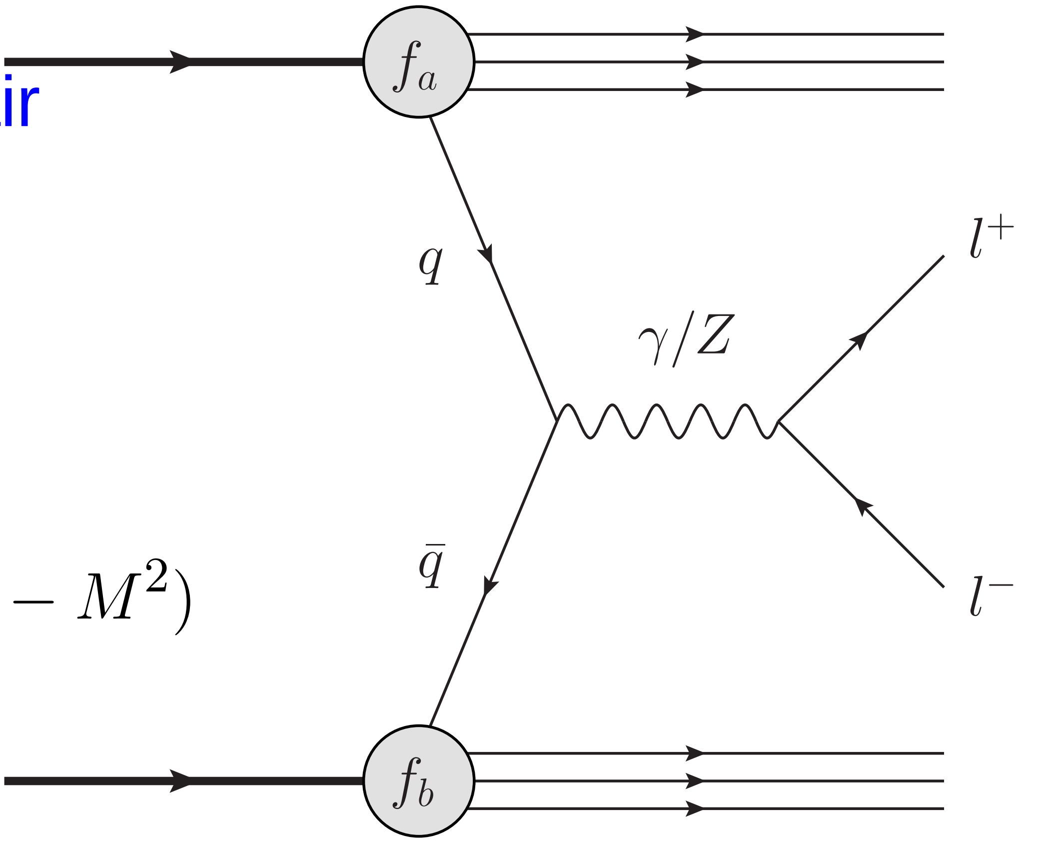
$$\sigma = \sum \int dx_1\, dx_2\, f_a(x_1) f_b(x_2)\, \hat{\sigma}$$
$$= \sum_{ij} \int d\hat{s}\, \frac{dL_{ij}}{d\hat{s}}\, \hat{\sigma}(\hat{s})$$

NB: Tevatron: $p\bar{p}$
LHC: pp



Drell-Yan

Production of a lepton pair
(of mass M)



Hard matrix element:

$$\frac{d\hat{\sigma}}{dM^2} = \frac{e_q^2 N_c}{N_c^2} \frac{4\pi\alpha^2}{3M^2} \delta(x_1 x_2 s - M^2)$$

Lowest order (PDF$_1$⊗PDF$_2$⊗ME)

$$\frac{d\sigma}{dM^2} = \int dx_1\, dx_2 \sum_q [q(x_1, M^2)\bar{q}(x_2, M^2) + (1 \leftrightarrow 2)] \frac{d\hat{\sigma}}{dM^2}$$

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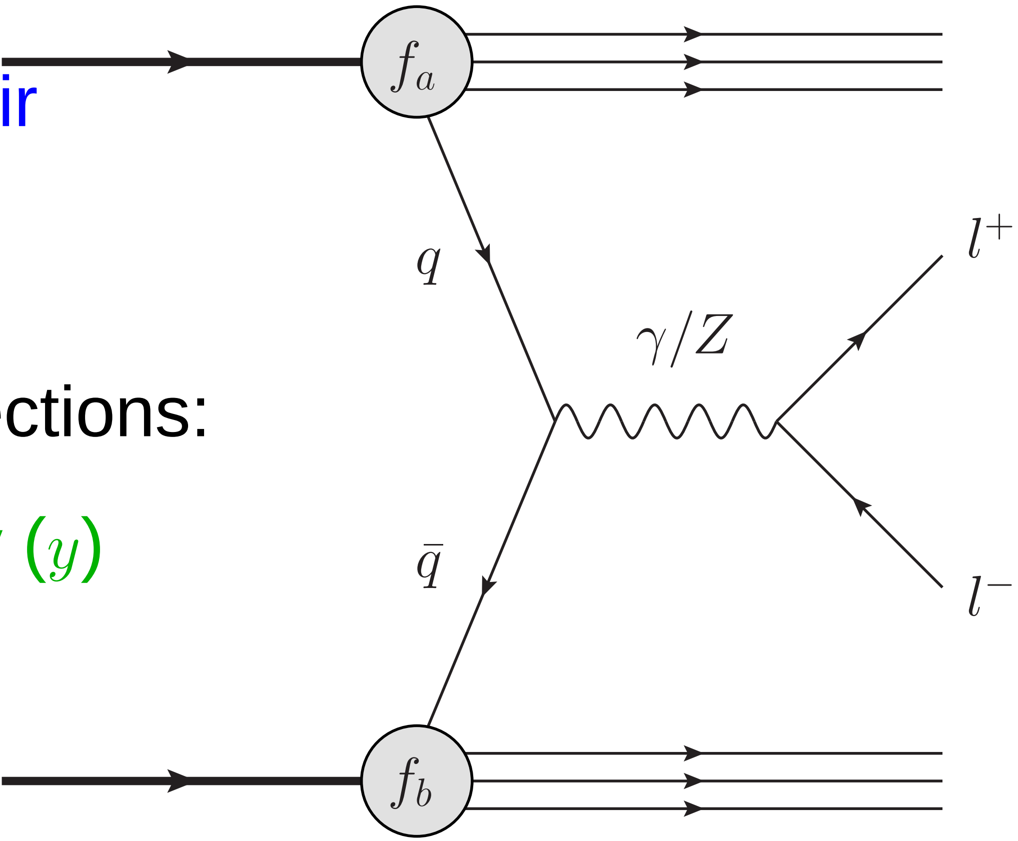


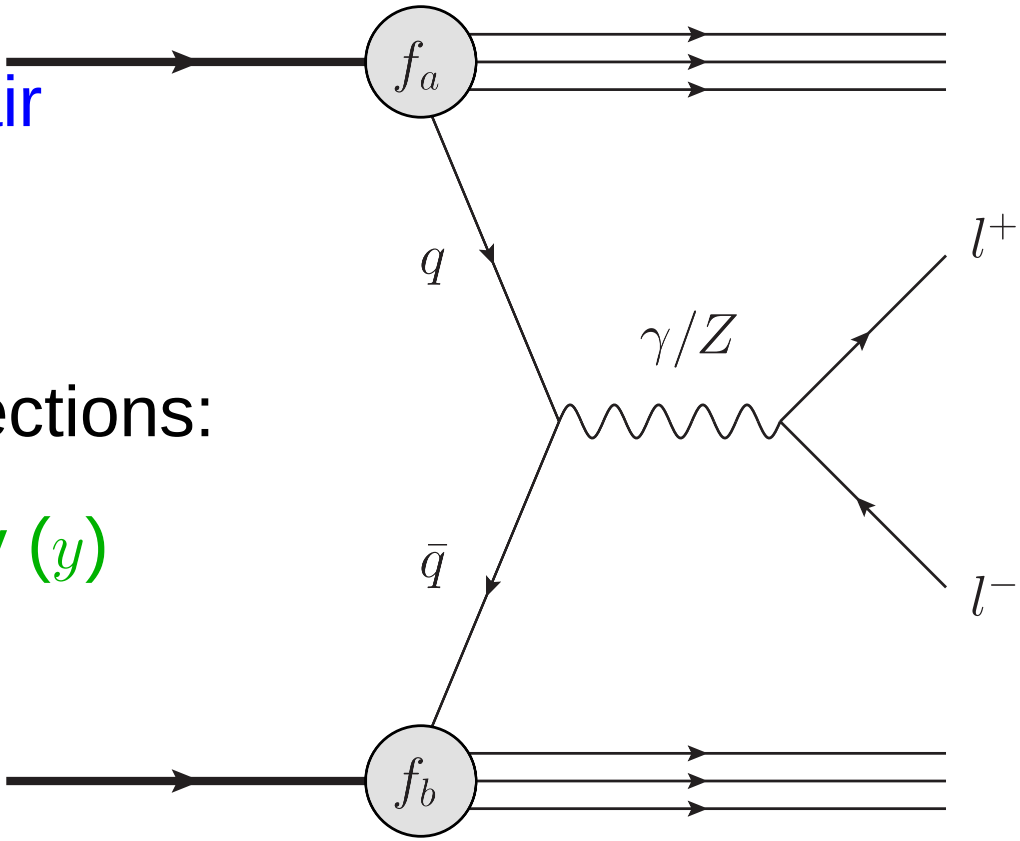
More differential cross-sections:

Ex. 1: lepton-pair rapidity (y)

$$\Rightarrow \delta(x_1 x_2 s - M^2)$$
$$\delta(y - \tfrac{1}{2}\log(x_1/x_2))$$

$$\frac{d^2\sigma}{dM^2 dy} = \sum_q \frac{4\pi e_q^2 \alpha^2}{3N_c M^2 s}\left[q(\frac{M}{\sqrt{s}}e^y, M^2)\bar{q}(\frac{M}{\sqrt{s}}e^{-y}, M^2) + (y \leftrightarrow -y)\right]$$

Drell-Yan



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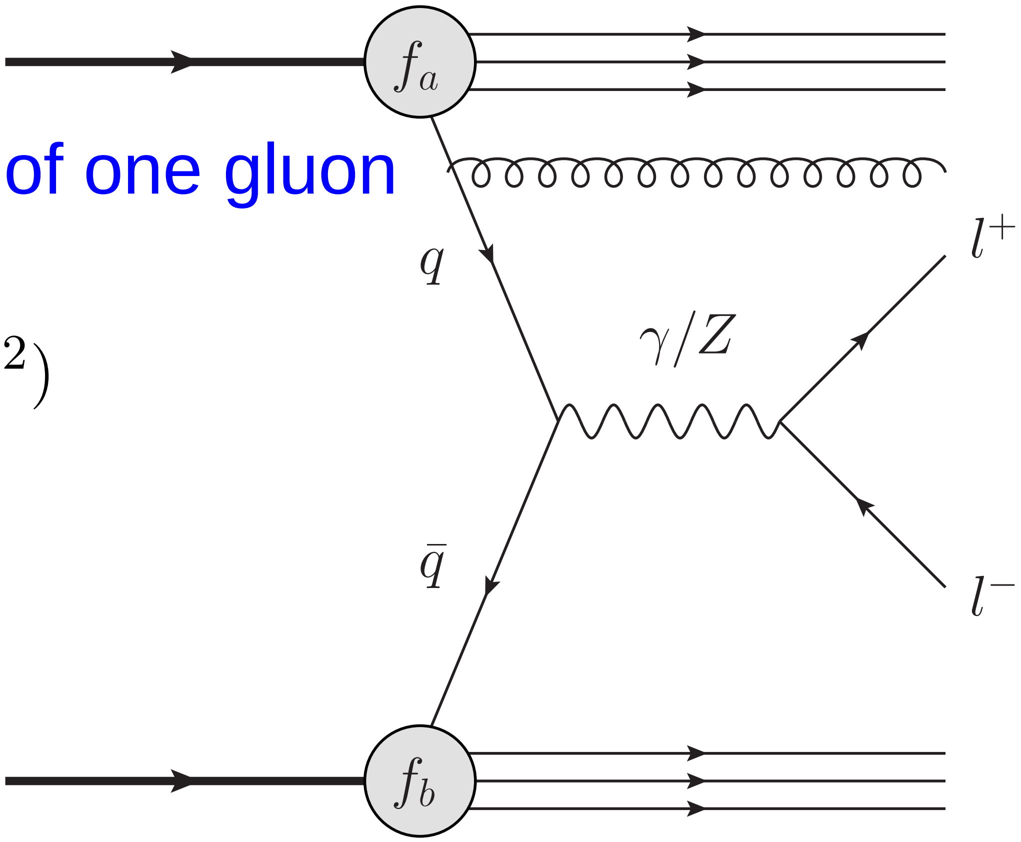
Ex. 2: Feynman x (x_F)

$x_F = \frac{2}{\sqrt{s}}(p_{z,l^+} - p_{z,l^-}) \stackrel{\text{LO}}{=} x_1 - x_2$: also 2 δ's

Drell-Yan

- Next order: emission of one gluon
  - real and virtual
  - depends on $g(x, M^2)$
  - $p_{t,\gamma/Z} \neq 0$



Drell-Yan

- Next order: emission of one gluon
- factorisation proven at ANY order

$$\frac{d\sigma}{dM^2} = \int dx_1\, dx_2\, dz_1\, dz_2$$
$$\sum_f f_a(x_1, M^2) f_b(x_2, M^2) D_{ab}(z_1/x_1, z_2/x_2)$$
$$\frac{d\hat{\sigma}}{dM^2}(z_1, z_2; M^2)$$

Drell-Yan

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- ONLY case where the factorisation $\mathrm{PDF}_1 \otimes \mathrm{PDF}_2 \otimes \mathrm{ME}$ is proven, otherwise it’s just a “reasonable assumption”

Monte-Carlo generators

Parton cascades, hadronisation, Underlying Event, pileup: a realistic event is complicated!

$\Rightarrow$ Use of (Monte-Carlo) event generators to simulate full events

Monte-Carlo generators: fixed order

Perturbative computations are the base of everything

But are often hard/impossible to compute analytically (especially for exclusive measurements)

$\Rightarrow$ use a fixed-order Monte-Carlo genrator

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- Aim: provide signals and backgrounds for LHC studies (usually needed at NLO)
  See the LesHouche list of completed/wanted processes, *e,g,*
  - many jets
  - W+jets
  - H+jets
  - top ($t\bar{t}$ and single top)
  - SUSY

Monte-Carlo generators: fixed order

Perturbative computations are the base of everything
But are often hard/impossible to compute analytically
(especially for exclusive measurements)

$\Rightarrow$ use a fixed-order Monte-Carlo genrator

- Aim: provide signals and backgrounds for LHC studies (usually needed at NLO)
- Generate matrix elements + phase-space
- 2 big categories:
  LO (many legs) or NLO (includes virtual corrections)
- Tendency to automate!
- Plenty of them: Alpgen, MadGraph, NLOJet, MCFM, BlackHat, Golem,...

Monte-Carlo generators: full event

For full-event simulation, Monte-Carlo generators are a cornerstone

- parton cascade: collinear splittings (DGLAP-like)
  As seen in e^+e^-, they have the form

  $$\frac{d^2P}{d\theta dz} = \alpha_s\, P(z) \frac{1}{\theta}$$

  Leading terms ($\alpha_s^n \log^n(1/\theta)$) have angular ordering
  $\theta_1 > \theta_2 > \cdots > \theta_n$

  Watch out: LO collinear branchings!!!
  e.g. Multi-jet processes hardly reliable

  (alternatives like virtuality ordered but always LO

Monte-Carlo generators: full event

For full-event simulation, Monte-Carlo generators are a cornerstone

- parton cascade: collinear splittings (DGLAP-like)
- hadronisation: non-perturbative *per se*!
  e.g. Lund string fragmentations (form strings based on colour connections and fragment them)

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- parton cascade: collinear splittings (DGLAP-like)
- hadronisation: non-perturbative *per se*!
- Multiple interactions/Underlying Event: hadronic beams carry colour *i.e.* interact strongly
  - Modelling
  - Then tuning to Tevatron (and LHC) data

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- Progress towards NLO generator
- Most commonly used: Pythia, Herwig, Sherpa... but others available
- more in the tutorials

W/Z production

- Production:
  - $q\bar{q}' \rightarrow W^{\pm}$
  - $q\bar{q} \rightarrow Z$
  - 14 TeV $\sigma_W \approx 20$ nb *i.e.* 200 $W/$s ($\mathcal{L} = 10^{34}$ cm$^2/$s)

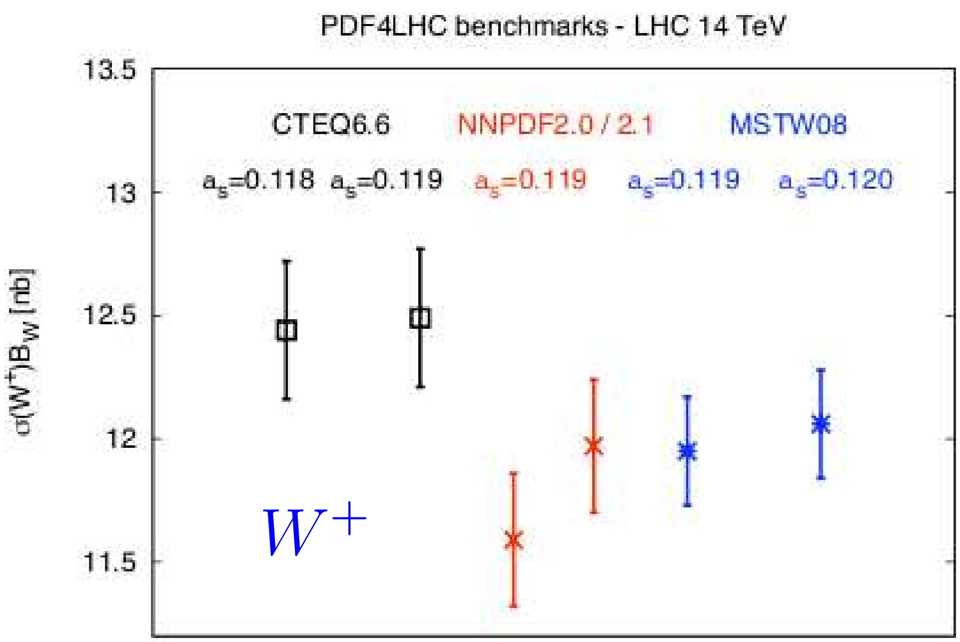
- Decay:
  - $W \rightarrow q\bar{q} \rightarrow 2$ jets (BR$\approx 2/3$)
    $W \rightarrow \ell\nu_\ell$ (BR$\approx 1/3$)
  - $Z \rightarrow q\bar{q} \rightarrow 2$ jets (BR$\approx 70\%$)
    $Z \rightarrow \ell\bar{\ell}$ (BR$\approx 10\%$)
    $Z \rightarrow \nu\bar{\nu}$ (BR$\approx 20\%$)
  - leptonic channel most convenient
    hadronic important for statistics!

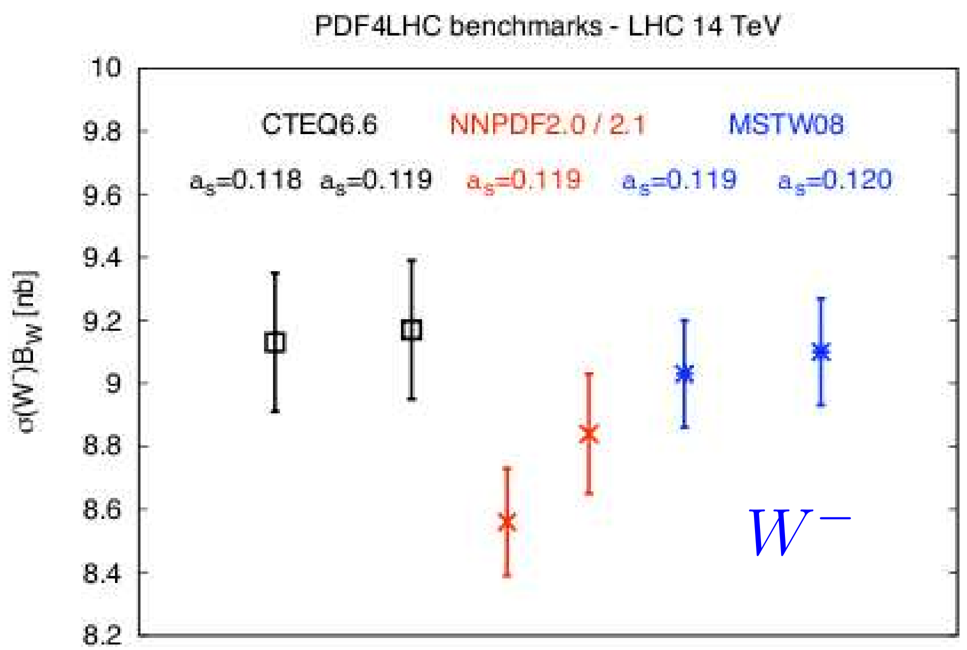
W/Z physics

- not really a discovery channel...
- ... but important in many respects
  - often W/Z+jets
  - standard model tests/MC calibration
  - background to many searches
    e.g. top ($\rightarrow Wb$) or SUSY ($\not{E}_t$)
- W cross-section as a standard candle for luminosity measurements

W for lumi measurement

W cross-section as a standard candle for luminosity measurements





PDF main source of uncertainty

top physics

- Production:
  - Mostly $gg \to t\bar{t}$
  - Tevatron: $\sigma_t \approx 4$ pb: discovery!
  - LHC: $\sigma_t \approx 1$ nb: $\approx 10/$s LHC$\equiv$ top factory
- Decay:
  - Mostly $t \to Wb$
    $t \to q\bar{q}b$ ($\approx 66\%$) or $t \to \ell\nu_\ell b$ ($\approx 33\%$)
  - for $t\bar{t}$: 3 options
    - leptonic: not-so-easy because 2 neutrinos
    - semi-leptonic: ℓ, 4 jets ($2b$) and $\not{E}_t$
      (the most convenient)
    - hadronic: 6 jets *i.e.* technical to reconstruct
      but $\approx$ 45% of the stat!

top physics

top very important at the LHC

- precision mass measurement
- many new physics scenario involve the top (mostly because of its large mass)

$\Rightarrow$ need to reconstruct as many tops as possible

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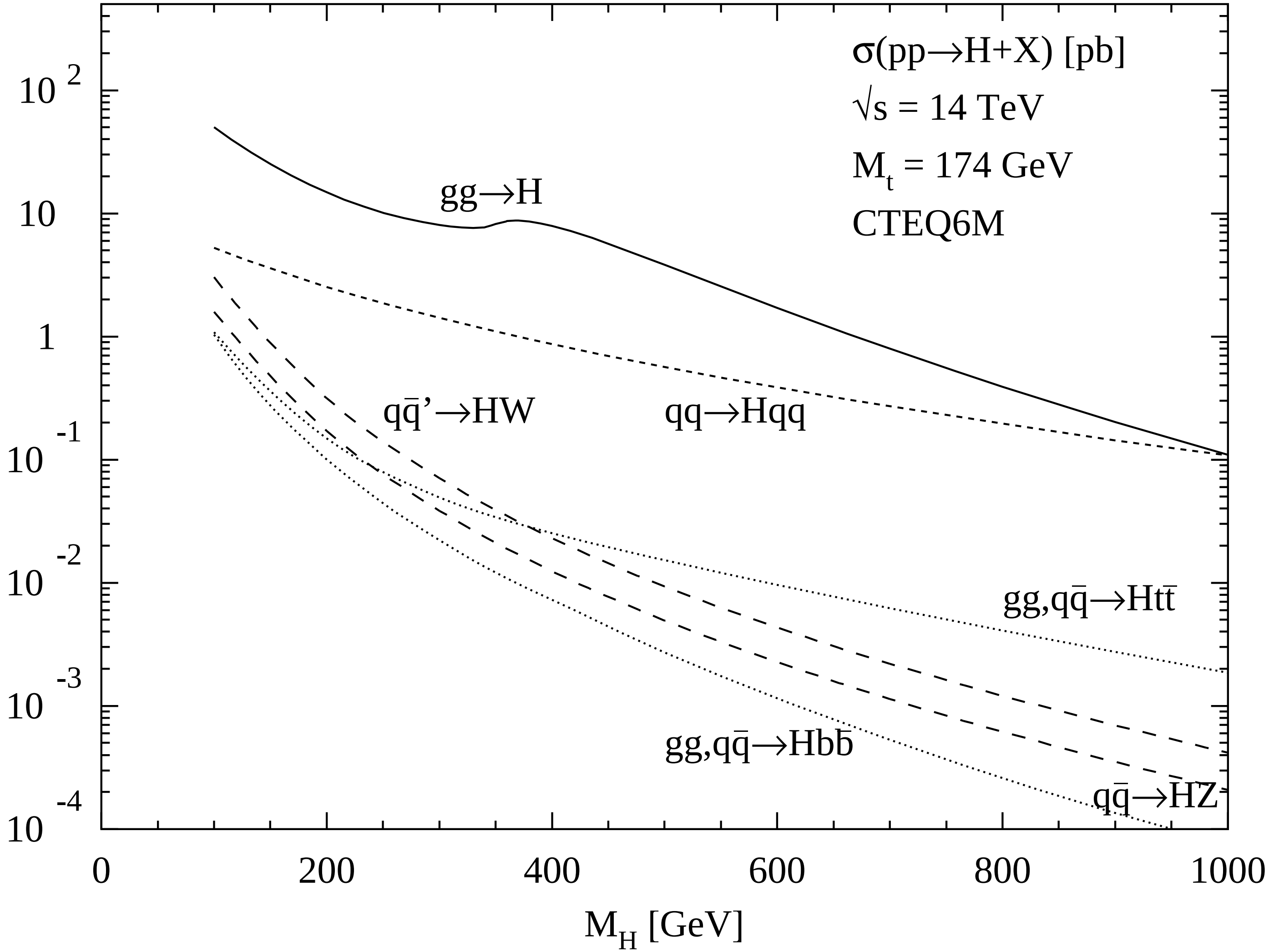
Issues:

- W+jets background
- b mis-tagging
- combinatorial background (especially for full hadr.)
- efforts *e.g.* in boosted-top reconstruction

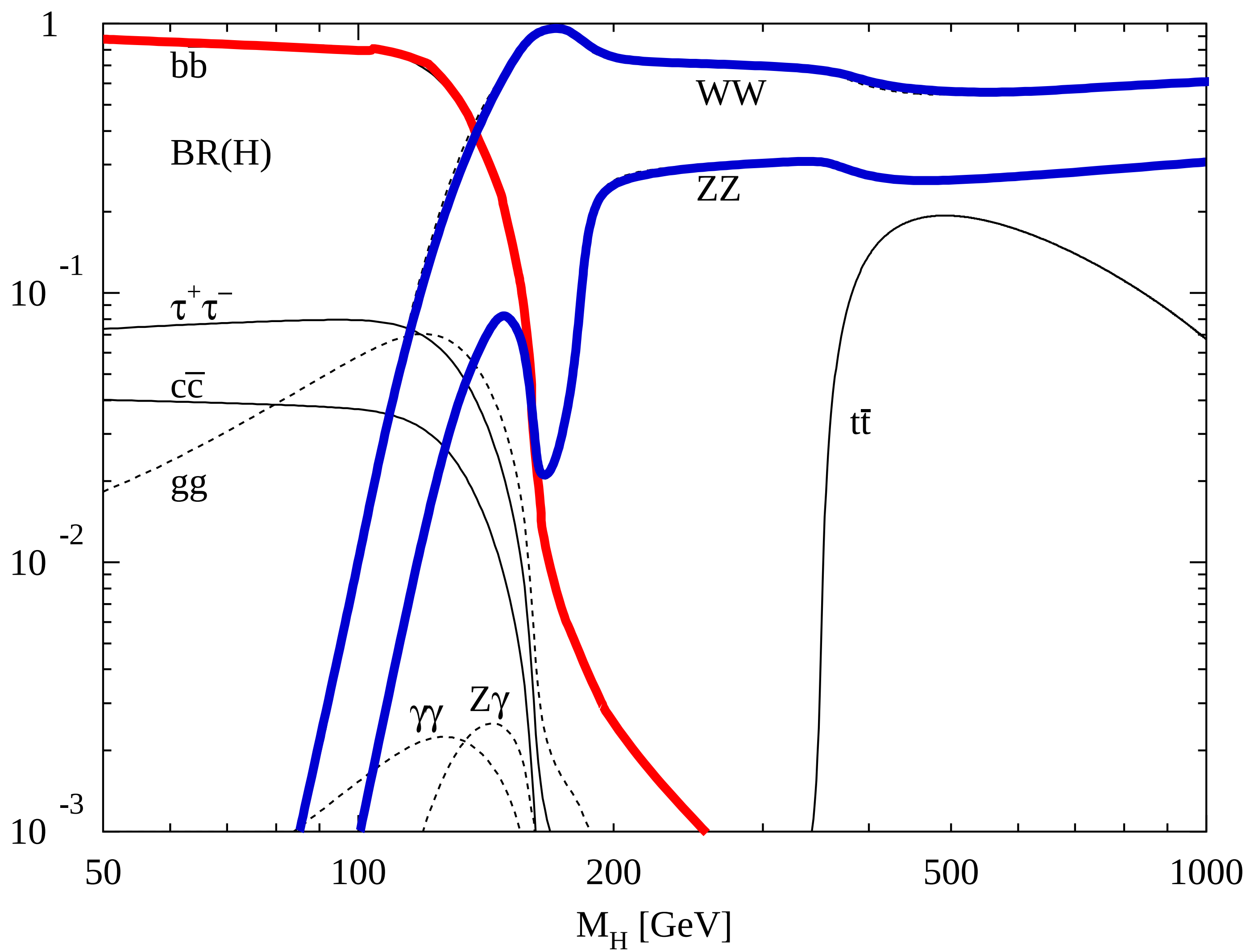
Higgs: production

Production at the LHC: mostly gg fusion (through top loop)



$m_H = 120$ GeV $\Rightarrow \sigma_H^{(\mathrm{L0})} \approx 21$ pb (vs 0.3 at the Tevatron)

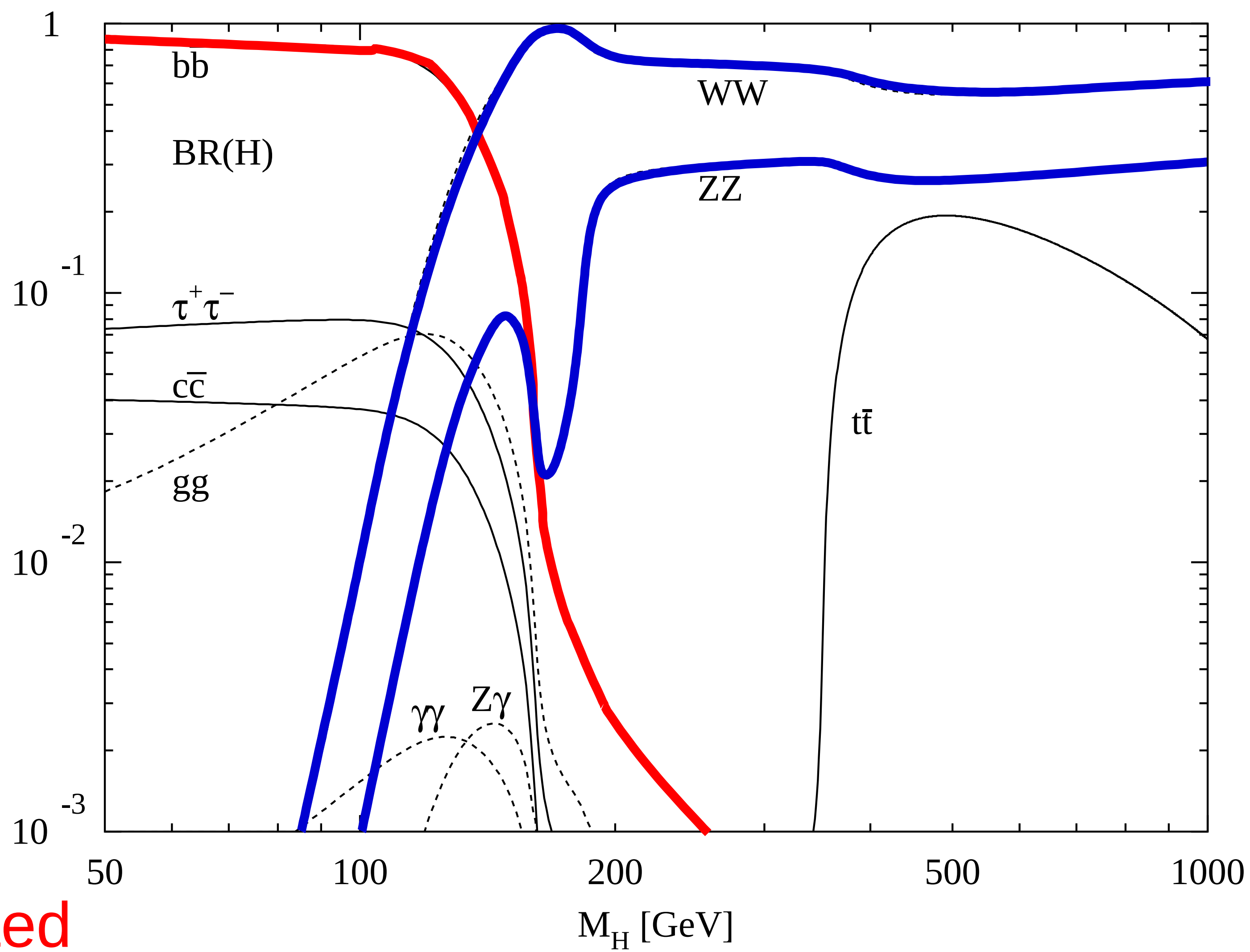
Higgs: decay



Heavy higgs
($m \gtrsim 2m_W$):

mostly $H \to WW^{(*)}$ or $H \to ZZ$
the easiest situation (see *e.g.* Tevatron)

Higgs: decay

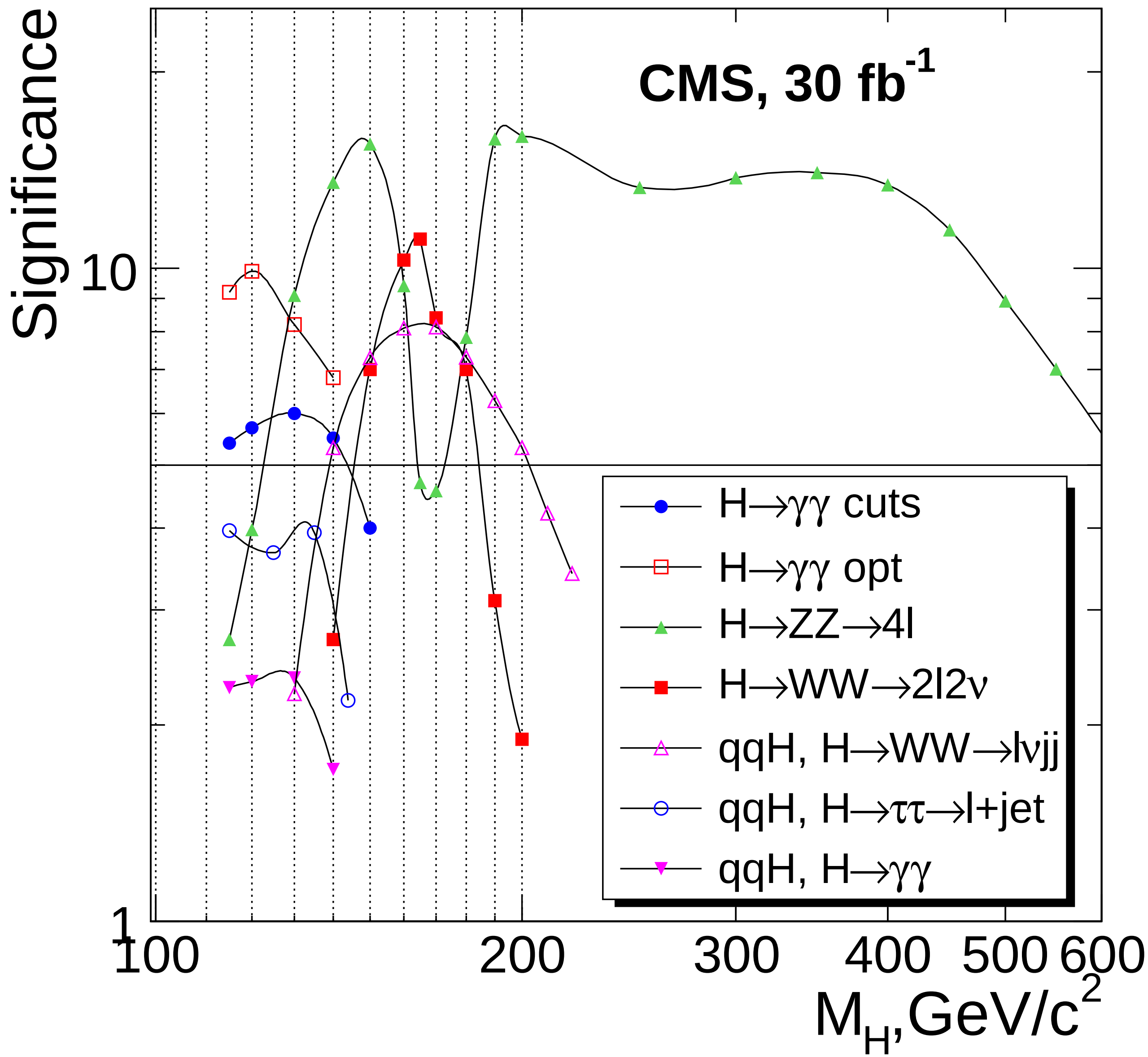


Light higgs
($m < 2m_W$):
more complicated

- bb →jets dominant but buried in the QCD bkgd
- $\gamma\gamma$ clean but only 0.1-0.3% of the events

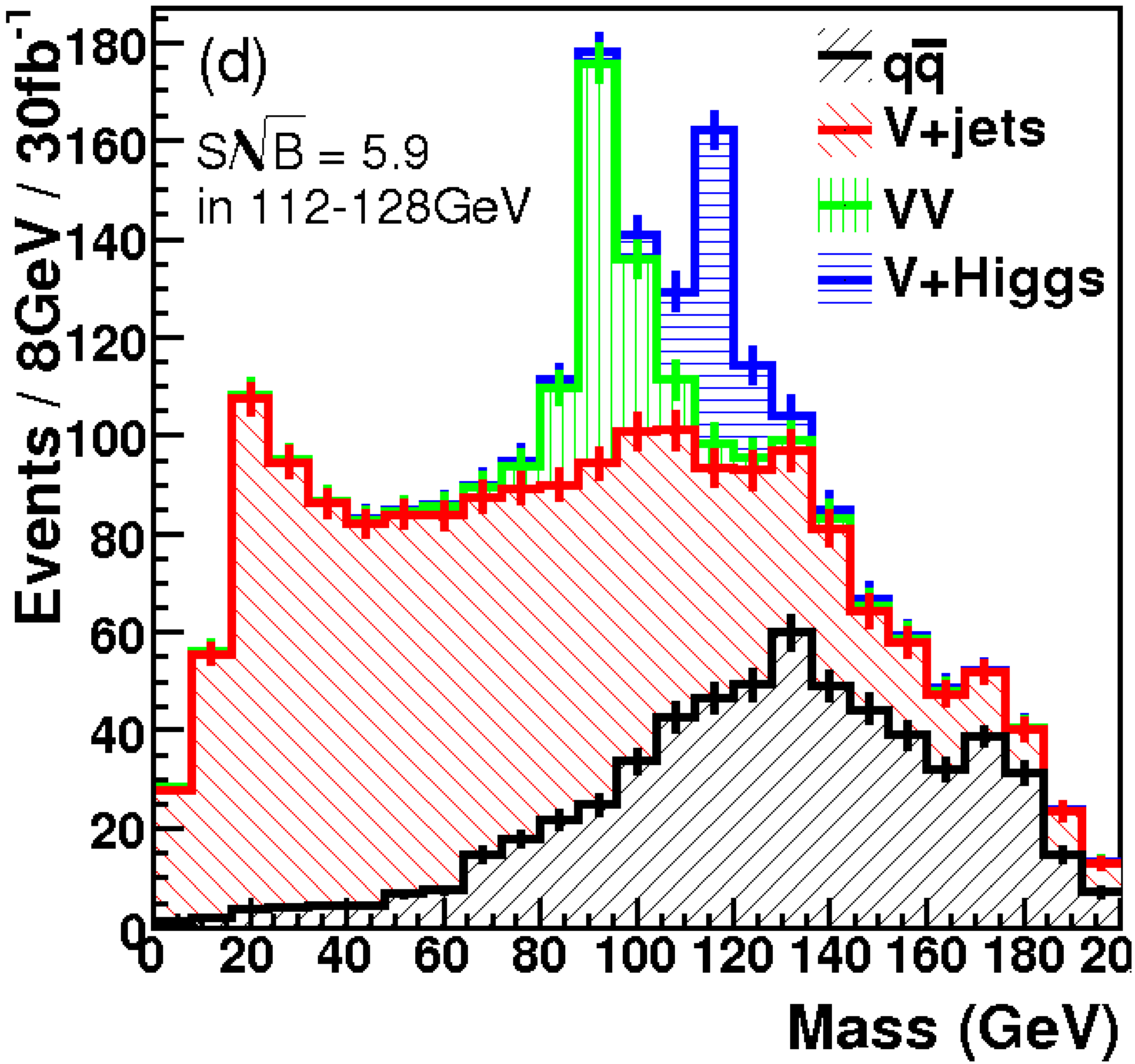
Higgs: discovery

$\sim$ 30 fb^{-1}
needed for
5σ discovery



Higgs: additional comments

- $H \to b\bar{b}$ may be visible/helpful for boosted $H + W/Z$



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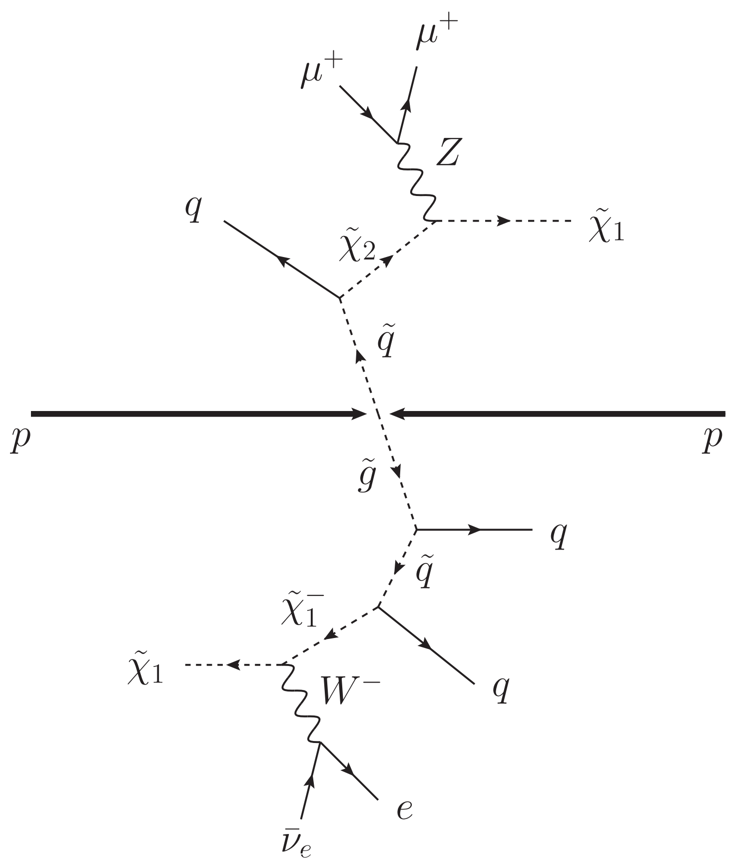
- Not the end of the story:
  also need to verify Higgs properties/couplings.
  - *e.g.* $t\bar{t}H$ may help
  - need for luminosity!

SUSY

Typical SUSY process:

- production of a pair of supersymmetric particles
- decay: SM particles + lightest SUSY particle (LSP)



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Typical issues

- Need good determination of $\not{E}_t$
- Control the multi-jet background at large p_t

Time for questions!