Phenomenology of hadronic collisions

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You are now experts in computing Feynman diagrams

You (hopefully) want to know how to compute things at hadronic colliders (the LHC in particular)
Disclaimer

The physics of hadronic colliders is a very vast topic:
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- **ATLAS TDR (Detector and Physics Performance):** 1852 pages
- **CMS TDR (2 volumes):** 1317 pages

A good coverage of “basic” topics in collider physics:

- **QCD and Collider Physics, R. K. Ellis, W. J. Stirling and B. R. Webber** (447 pages)
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I won’t be able to cover all that in 6+2 hours!
How to describe a collision between 2 hadrons?
The very fundamental collision

\[ \sigma = f_a \otimes f_b \otimes \hat{\sigma} \]

- “take a parton out of each proton”
  \( f_a \equiv \text{parton distribution function (PDF)} \)
  for quark and gluons
  a big chapter of these lectures

- hard matrix element
  perturbative computation
  Forde-Feynman rules
The more realistic version

Hard ME perturbative
The more realistic version

- Hard ME perturbative
- Parton branching
- initial+final state radiation
The more realistic version

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- Parton branching
  initial+final state radiation
- Hadronisation
  \( q, g \rightarrow \text{hadrons} \)
The more realistic version

- Hard ME perturbative
- Parton branching
  initial+final state radiation
- Hadronisation
  $q, g \rightarrow \text{hadrons}$
- Multiple interactions
  Underlying event (UE)
The more realistic version

- Hard ME perturbative
- Parton branching
  initial+final state radiation
- Hadronisation
  \[ q, g \rightarrow \text{hadrons} \]
- Multiple interactions
  Underlying event (UE)
- Pile-up
  \[ \lesssim 25 \text{ } pp \text{ at the LHC} \]
We shall investigate those effects one by one:

- $e^+e^-$ collisions for QCD final state (and hadronisation)

- $ep$ collisions \textit{aka} Deep Inelastic scattering (DIS) for the Parton Distribution Functions

- $pp$ collisions: put everything together
  - kinematics
  - Monte-Carlo
  - jets + various processes ($W/Z$, Higgs, top, ...)
Step by step...

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- $e^+e^-$ collisions for QCD final state (and hadronisation)

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  - kinematics
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  - jets + various processes ($W/Z$, Higgs, top, ...)

- p. 7
The plan is to play with Pythia 8 (the C++ version) and FastJet. You can get them (and a few sample codes) from the link at

http://soyez.fastjet.fr
$e^+ e^- \text{ collisions}$
$e^+e^-$ collisions give QCD final state without initial-state/beam contamination

Useful for many QCD studies

Intermediate state can be $\gamma$ or $Z$, we only consider $\gamma$ for simplicity
QCD final state: basic QCD

\[ p_1 \equiv \frac{\sqrt{s}}{2}(0, 0, 1, 1) \]
\[ p_2 \equiv \frac{\sqrt{s}}{2}(0, 0, -1, 1) \]
\[ k_1 \equiv \frac{\sqrt{s}}{2} (\sin(\theta), 0, \cos(\theta), 1) \]
\[ k_2 \equiv \frac{\sqrt{s}}{2} (-\sin(\theta), 0, -\cos(\theta), 1) \]
QCD final state: basic QCD

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\[ \frac{d\sigma}{d\cos(\theta)} = e_q^2 N_c \frac{\pi \alpha_e^2}{2s} [1 + \cos^2(\theta)] \]

\[ \sigma(e^+e^- \rightarrow q\bar{q}) = N_c \left( \sum_q e_q^2 \right) \sigma_0 \]

\[ \sigma_0 = \frac{4\pi \alpha_e^2}{3s} \]
QCD final state: basic QCD

\[
\frac{d\sigma}{d\cos(\theta)} = e_q^2 N_c \frac{\pi \alpha_e^2}{2s} [1 + \cos^2(\theta)]
\]

\[
\sigma(e^+e^- \rightarrow q\bar{q}) = N_c \left( \sum_q e_q^2 \right) \sigma_0
\]

\[
\sigma(e^+e^- \rightarrow \mu^+\mu^-) = \sigma_0
\]

\[
\sigma_0 = \frac{4\pi \alpha_e^2}{3s}
\]
QCD final state: basic QCD

\[ R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \approx N_c \left( \sum_q e_q^2 \right) \]

- \( u, d, s \): \( R = 3 \left( \frac{4}{9} + \frac{1}{9} + \frac{1}{9} \right) = 2 \)
- \( u, d, s, c \): \( R = 3 \left( \frac{4}{9} + \frac{1}{9} + \frac{1}{9} + \frac{4}{9} \right) = \frac{10}{3} \)
- \( u, d, s, c, b \): \( R = 3 \left( \frac{4}{9} + \frac{1}{9} + \frac{1}{9} + \frac{4}{9} + \frac{4}{9} \right) = \frac{14}{3} \)

Test of
- The 3 colours in QCD \((N_c = 3)\)
- The number of quark flavours
QCD final state: basic QCD

\[ R = \sigma(e^+ e^- \rightarrow \text{hadrons}) / \sigma(e^+ e^- \rightarrow \mu^+ \mu^-) \approx N_c (\sum q e^2 q) \]

- For \( u, d, s \): \( R = 3 \left( \frac{4}{9} + \frac{1}{9} + \frac{1}{9} \right) = 2 \)
- For \( u, d, s, c \): \( R = 3 \left( \frac{4}{9} + \frac{1}{9} + \frac{1}{9} + \frac{4}{9} \right) = 10 \)
- For \( u, d, s, c, b \): \( R = 3 \left( \frac{4}{9} + \frac{1}{9} + \frac{1}{9} + \frac{4}{9} + \frac{4}{9} \right) = 14 \)

Test of

- The 3 colours in QCD (\( N_c = 3 \))
- The number of quark flavours

\( \sqrt{s} \) [GeV]
QCD final state: basic QCD

\[ R = \frac{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}{\sigma(e^+e^- \rightarrow e^+e^-)} \approx N_c \left( \sum q_e^2 \right) \]

- \( u, d, s \): \( R = 3 \)
- \( u, d, s, c \): \( R = 3 + \frac{4}{9} + 1 + 1 + 4 \)
- \( u, d, s, c, b \): \( R = 3 + \frac{4}{9} + 1 + 1 + 4 + 4 \)

Test of
- The 3
- The n

Q: why \( \sigma(e^+e^- \rightarrow \mu^+\mu^-) \) and not \( \sigma(e^+e^- \rightarrow e^+e^-) \)?
QCD final state: QCD dynamics

\[ 3 \times (4 - 1) - 4 = 5 \text{ d.o.f.} \]

- 3 Euler angles
- \( x_i = 2E_i/\sqrt{s}, \) \( x_1 + x_2 + x_3 = 2 \)
- or \( \theta_{13}, \theta_{23} \)

\[
\int d\Phi_3 = \prod_{i=1}^{3} \frac{d^3 k_i}{(2\pi)^3 2E_i} (2\pi)^4 \delta^{(4)}(p_1 + p_2 - k_1 - k_2 - k_3)
\]

\[
= \frac{s}{32(2\pi)^5} \int d\alpha d\cos \beta d\gamma dx_1 dx_2
\]

\[
\cos(\theta_{13}) = -\frac{x_1^2 + x_3^2 - x_2^2}{2x_1 x_3}
\]

\[
\cos(\theta_{23}) = -\frac{x_2^2 + x_3^2 - x_1^2}{2x_2 x_3}
\]
\[\sum |M|^2 = 4(4\pi)^3 \alpha_e^2 \alpha_s C_F N_c \]

\[
\frac{(p_1.k_1)^2 + (p_1.k_2)^2 + (p_2.k_1)^2 + (p_2.k_2)^2}{s(k_1.k_3)(k_2.k_3)}
\]

\[
\frac{d^2 \sigma}{dx_1 \, dx_2} = e_q^2 N_c \sigma_0 \frac{\alpha_s C_F}{2\pi} \frac{x_1^2 + x_2^2}{(1 - x_1)(1 - x_2)}
\]
QCD final state: QCD dynamics

\[ e^+ e^- \rightarrow q \bar{q} \]

QCD coupling

QCD colour

\[ \frac{d^2 \sigma}{dx_1 \, dx_2} = e_q^2 \, N_c \, \sigma_0 \frac{\alpha_s \, C_F}{2 \pi} \frac{x_1^2 + x_2^2}{(1 - x_1)(1 - x_2)} \]
QCD final state: QCD dynamics

\[ \left| \mathcal{M} \right|^2 \propto \frac{(p_1 \cdot k_1)^2 + (p_2 \cdot k_2)^2 + (p_2 \cdot k_1)^2 + (p_2 \cdot k_2)^2}{s(k_1 \cdot k_3)(k_2 \cdot k_3)} \]

\[ \frac{d^2 \sigma}{dx_1 \, dx_2} = e_q^2 N_c \sigma_0 \frac{\alpha_s C_F}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} \]

\[ x_i = \frac{2E_i}{\sqrt{s}} \quad x_1 + x_2 + x_3 = 2 \quad c_{\theta_{13}} = -\frac{x_1^2 + x_3^2 - x_2^2}{2x_1 x_3} \]
QCD final state: QCD dynamics

\[ \sum |M|^2 \propto \frac{(p_1.k_1)^2+(p_1.k_2)^2+(p_2.k_1)^2+(p_2.k_2)^2}{s(k_1.k_3)(k_2.k_3)} \]

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Divergent when \( k_1.k_3 \to 0 \) or \( k_2.k_3 \to 0 \)

\[ k_1.k_3 \to 0 \Rightarrow (k_1 + k_3)^2 \to 0 \quad \text{i.e.} \]

parent quark propag = \[ \frac{1}{(k_1 + k_3)^2} \to \infty \]

Physical origin of the divergence!
They are infrared divergences \((k_1 + k_3)^2 \to 0, \text{ not } \infty\)

(one power cancelled by phase-space \(\Rightarrow\) log divergence)
QCD final state: QCD dynamics

\[ \sum |M|^2 \propto \frac{(p_1.k_1)^2 + (p_1.k_2)^2 + (p_2.k_1)^2 + (p_2.k_2)^2}{s(k_1.k_3)(k_2.k_3)} \]

\[ \frac{d^2\sigma}{dx_1 dx_2} = e_q^2 N_c \sigma_0 \frac{\alpha_s C_F}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} \]

\[ x_i = \frac{2E_i}{\sqrt{s}} \quad x_1 + x_2 + x_3 = 2 \quad c_{\theta_13} = -\frac{x_1^2 + x_2^2 - x_2^2}{2x_1 x_3} \]

Divergent when \( x_1 \) (or \( x_2 \)) \( \rightarrow 1 \)

\[ 1 - x_2 = \frac{1}{2} x_1 x_3 [1 - \cos(\theta_{13})] \]

- \( \theta_{13} \rightarrow 0 \) (or \( \theta_{23} \)): collinear divergence divergence
- \( x_3 \rightarrow 0 \) (i.e. \( E_g \rightarrow 0 \)): soft divergence
Collinear and soft divergences

- fundamental/omnipresent in QCD! (also in QED)
we will meet them often through these lectures
Collinear and soft divergences

- fundamental/omnipresent in QCD! (also in QED)
- also present for $g \rightarrow gg$ ($\neq$ QED; $C_F \rightarrow C_A$)
QCD final state: coll and soft divergences

Collinear and soft divergences

- fundamental/omnipresent in QCD! (also in QED)
- also present for $g \rightarrow gg$ ($\neq$ QED; $C_F \rightarrow C_A$)
- cancelled by virtual corrections

![Diagram of real and virtual processes]
QCD final state: coll and soft divergences

Collinear and soft divergences

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- also present for \( g \rightarrow gg \) \((\neq \text{QED}; C_F \rightarrow C_A)\)
- cancelled by virtual corrections

Dimensional regularisation \( d = 4 - 2\varepsilon \):

\[
\sigma^{(q\bar{q}g)}_{\text{real}} = e_q^2 N_c \sigma_0 \frac{\alpha_s C_F}{2\pi} T(\varepsilon) \left[ \frac{2}{\varepsilon^2} + \frac{3}{\varepsilon} + \frac{19}{2} + \mathcal{O}(\varepsilon) \right]
\]

\[
\sigma^{(q\bar{g}g)}_{\text{virt}} = e_q^2 N_c \sigma_0 \frac{\alpha_s C_F}{2\pi} T(\varepsilon) \left[ \frac{-2}{\varepsilon^2} - \frac{3}{\varepsilon} - 8 + \mathcal{O}(\varepsilon) \right]
\]

\[
\sigma^{(q\bar{q}g)}_{\mathcal{O}(\alpha_s)} = e_q^2 N_c \sigma_0 \frac{3\alpha_s C_F}{4\pi} = e_q^2 N_c \sigma_0 \frac{\alpha_s}{\pi}
\]
Collinear and soft divergences
- fundamental/omnipresent in QCD! (also in QED)
- also present for $g \rightarrow gg$ ($\neq$ QED; $C_F \rightarrow C_A$)
- cancelled by virtual corrections
- cancellation order-by-order in perturbation theory
  Block-Nordsieck, Kinoshita-Lee-Nauenberg theorems
Collinear and soft divergences

- fundamental/omnipresent in QCD! (also in QED)
- also present for \( g \rightarrow gg (\neq \text{QED}; \ C_F \rightarrow C_A) \)
- cancelled by virtual corrections
- cancellation order-by-order in perturbation theory
  
  Block-Nordsieck, Kinoshita-Lee-Nauenberg theorems

- Terminology issue: ’soft’ divergence sometimes called ’infrared’ divergence (though both soft and coll are infrared)
QCD final state: IRC safety

Cancellation of divergence not true for any observable

Example: “number of partons in the final state”, $dP/dn$

- **LO ($O(\alpha_s^0)$):** $dP/dn = \delta(n - 2)$
- **NLO ($O(\alpha_s^1)$):**
  - (i) real emission: $n = 3$
  - (ii) virtual correction: $n = 2$
  \[ dP/dn = [1 - \infty \alpha_s] \delta(n - 2) + \infty \alpha_s \delta(n - 3) \]
Cancellation of divergence not true for any observable

Example: “number of partons in the final state”, $dP/dn$

- LO ($O(\alpha_s^0)$): $dP/dn = \delta(n - 2)$
- NLO ($O(\alpha_s^1)$):
  1. real emission: $n = 3$
  2. virtual correction: $n = 2$

$\Rightarrow dP/dn = [1 - \infty \alpha_s] \delta(n - 2) + \infty \alpha_s \delta(n - 3)$

Observables for which cancellation happens are called **INFRARED-AND-COLLINEAR SAFE**

Necessary for perturbative QCD computation to make sense!!
Observable $\mathcal{O}$:

$$\mathcal{O} = \sum_{n=0}^{\infty} \int \frac{d\Psi_n(k_1, \ldots, k_n)}{\text{phasespace}} \frac{d\sigma}{d\Psi_n}(k_1, \ldots, k_n) \mathcal{O}_n(k_1, \ldots, k_n)$$

- IR safety: “adding a soft particle does not change $\mathcal{O}$”

$$\mathcal{O}_{n+1}(k_1, \ldots, k_n, k_{n+1}) \xrightarrow{k_{n+1} \to 0} \mathcal{O}_n(k_1, \ldots, k_n)$$

- Collinear safety: “a collinear splitting does not change $\mathcal{O}$”

$$\mathcal{O}_{n+1}(k_1, \ldots, \lambda k_n, (1 - \lambda) k_n) = \mathcal{O}_n(k_1, \ldots, k_n)$$

for $0 < \lambda < 1$
Example #1: event-shapes in $e^+e^-$

*thrust, sphericity, thrust major, thrust minor, ...*

**Thrust:**

$$T_n = \max_{|\vec{u}|=1} \frac{\sum_{i=0}^{n} |\vec{k}_i . \vec{u}|}{\sum_{i=0}^{n} |\vec{k}_i|}$$

- **pencil-like:** $T \lesssim 1$
- **spherical:** $T \gtrsim 1/2$
**QCD final state: IRC safety**

**Example #1:** event-shapes in $e^+e^-$

*thrust, sphericity, thrust major, thrust minor, ...*

**Thrust:**

$$T_n = \max_{|\vec{u}|=1} \frac{\sum_{i=0}^{n} |\vec{k}_i \cdot \vec{u}|}{\sum_{i=0}^{n} |\vec{k}_i|}$$

- **the thrust is infrared safe:** for $k_{n+1} \to 0$

$$T_{n+1} = \max_{|\vec{u}|=1} \frac{\sum_{i=0}^{n+1} |\vec{k}_i \cdot \vec{u}|}{\sum_{i=0}^{n+1} |\vec{k}_i|} = \max_{|\vec{u}|=1} \frac{\sum_{i=0}^{n} |\vec{k}_i \cdot \vec{u}|}{\sum_{i=0}^{n} |\vec{k}_i|} = T_n$$

- **the thrust is collinear safe**

$$0 < \lambda < 1 \implies \begin{cases} |\vec{u} \cdot (\lambda \vec{k} + (1 - \lambda) \vec{k})| = |\vec{u} \cdot \vec{k}| \\ |\lambda \vec{k} + (1 - \lambda) \vec{k}| = |\vec{k}| \end{cases}$$
Example #1: event-shapes in $e^+e^-$

*thrust, sphericity, thrust major, thrust minor, ...*

**Thrust:** \[ T_n = \max_{|\vec{u}|=1} \frac{\sum_{i=0}^{n} |\vec{k}_i \cdot \vec{u}|}{\sum_{i=0}^{n} |\vec{k}_i|} \]

Computation in perturbative QCD (from the matrix element given earlier)

\[
\frac{1}{\sigma} \frac{d\sigma}{dT} = \frac{\alpha_s C_F}{2\pi} \left[ \frac{2(2 - 3T + 3T^2)}{T(1 - T)} \log \left( \frac{2T - 1}{1 - T} \right) - \frac{3(2 - T)(3T - 2)}{1 - T} \right]
\]

- Allows for test of QCD (e.g. at LEP)
- “log” is a reminiscence from the soft and collinear divergence
$Q = M_Z$

$\alpha_s (M_Z) = 0.1189$

comparison with LEP data: peaked at $T = 1$
Typical behaviour of divergences:

**Collinear limit:**

\[
\frac{1}{\sigma_0} \frac{d\sigma}{d\theta} \approx \frac{\alpha_s}{2\pi} \frac{1 + (1 - z)^2}{z} \frac{d\theta^2}{\theta^2}
\]

splitting proba  \hspace{1cm} coll.div

For different situations (different parton types), the branching probability changes but the \( d\theta/\theta \) is generic!
Typical behaviour of divergences:

- **Collinear limit:**

\[
\frac{1}{\sigma_0} \frac{d\sigma}{d\theta^2} \approx \frac{1}{2\pi} \frac{\alpha_s}{z} \frac{1 + (1 - z)^2}{\theta^2}
\]

- **Soft limit:**

\[
d\sigma_{q\bar{q}g} = d\sigma_{q\bar{q}} \frac{\alpha_s C_F}{\pi^2} \frac{(k_1.k_2)}{(k_1.k_3)(k_2.k_3)} d^4k_3 \delta(k^2) \propto \frac{dE_3}{E_3} \propto \frac{dz}{z}
\]

Antenna formula — soft-gluon emission
Frequent appearance in computations:

Both **soft** and **collinear** divergences are logarithmic

⇒ the emission of a gluon comes with a factor $\alpha_s \log$

Example:

soft emissions for the thrust: $\alpha_s \log(1 - T)$

At some point, $\alpha_s \log \sim 1$ i.e. NLO~LO in the perturbative series

⇒ At order $n$, we will have $\alpha_s^n \log^n$ all of the same order

⇒ ALL have to be considered: **resummation**
Other interests in $e^+e^-$ collisions

- Fragmentation functions
  “parton $\rightarrow$ hadron transition”, $D_{p/\pi}(z, p_t)$

- Hadronisation
  *e.g.* Lund strings

- Jets
  Collinear divergence $\rightarrow$ a parton develops into a bunch of collimated particles

We will postpone (part of) this to the “hadronic collisions” chapter
$e^+e^- : \textbf{Summary}$

- $e^+e^-$ collisions: good framework to test QCD (final state)

- emission of a gluon has 2 divergences: \textbf{soft} and \textbf{collinear}
  - cancel between “real” and “virtual” diagrams
  - ... provided the observable is IRC safe
  - give rise to “logarithms” in perturbative computations
  - ... resummed to all orders when $\alpha_s \log \sim 1$
  - ... done analytically or by parton cascade MC

- collinear divergence+parton branching $\rightarrow$ jets
Time for questions!
<interlude hadronic collisions>

kinematics

jets
The very fundamental collision

\[ \sigma = f_a \otimes f_b \otimes \hat{\sigma} \]

- “take a parton out of each proton”
- \( f_a \equiv \) parton distribution function (PDF)
  for quark and gluons
- hard matrix element
- perturbative computation
- Forde-Feynman rules
Kinematics

Incoming partons:

\[ p_1 \equiv x_1 \frac{\sqrt{s}}{2} (0, 0, 1, 1) \]
\[ p_2 \equiv x_2 \frac{\sqrt{s}}{2} (0, 0, -1, 1) \]

- carry a fraction of the beam’s (longitudinal) momentum

- Energy\(^2\) in the hard collision: \((p_1 + p_2)^2 = x_1 x_2 s \leq s\)

- the partonic centre-of-mass is shifted/boosted compared to the lab/\(pp\) centre-of-mass
  \(\Rightarrow\) need variables (longitudinally) boost-invariant
Final-state particles: commonly-used variables

\[ k \equiv (k_x, k_y, k_z, E) \equiv E(\sin(\theta) \cos(\phi), \sin(\theta) \sin(\phi), \cos(\theta), 1) \]

- \( E \) and \( \theta \) are not suited!
Kinematics

Final-state particles: commonly-used variables

- Transverse plane
  - azimuthal angle $\phi$
  - transverse momentum $p_t = \sqrt{p_x^2 + p_y^2}$
Kinematics

Final-state particles: commonly-used variables

- **Transverse plane**
  - azimuthal angle $\phi$
  - transverse momentum $p_t = \sqrt{p_x^2 + p_y^2}$

- **Longitudinal variable**
  - Rapidity: $y = \frac{1}{2} \log \left( \frac{E+p_z}{E-p_z} \right)$

Boost: $y \rightarrow \frac{1}{2} \log \left( \frac{\gamma(E - \beta p_z) + \gamma(p_z - \beta E)}{\gamma(E - \beta p_z) - \gamma(p_z - \beta E)} \right)$

$$= \frac{1}{2} \log \left( \frac{\gamma(1 - \beta)(E + p_z)}{\gamma(1 + \beta)(E - p_z)} \right) = y + \frac{1}{2} \log \left( \frac{(1 - \beta)}{(1 + \beta)} \right)$$

not boost-invariant itself but $\Delta y = y_2 - y_1$ is ($\Delta \theta$ is not)
Final-state particles: commonly-used variables

- **Transverse plane**
  - azimuthal angle $\phi$
  - transverse momentum $p_t = \sqrt{p_x^2 + p_y^2}$

- **Longitudinal variable**
  - Rapidity: $y = \frac{1}{2} \log \left( \frac{E+p_z}{E-p_z} \right)$
  - $k \equiv (k_t \cos(\phi), k_t \sin(\phi), m_t \sinh(y), m_t \cosh(y))$
  - Transverse mass: $m_t^2 = k_t^2 + m^2$
  - Pseudo-rapidity: $\eta = \frac{1}{2} \log (\tan(\theta/2))$
  - $\Delta \eta$ boost-invariant if massless
  - For massless particles: $y = \eta$
We have seen in the $e^+e^-$ studies (thrust) that the final state is pencil-like.

Consequence of the collinear divergence: QCD branchings are most likely collinear

$$(dP/d\theta \propto \alpha_s/\theta)$$
We have seen in the $e^+e^-$ studies (thrust) that the final state is pencil-like

Consequence of the collinear divergence
QCD branchings are most likely collinear

\[
\frac{dP}{d\theta} \propto \alpha_s/\theta
\]

“Jets” ≡ bunch of collimated particles \(\sim\) hard partons
Jets

We have seen in the $e^+e^-$ studies (thrust) that the final state is pencil-like

Consequence of the collinear divergence
QCD branchings are most likely collinear

$$(dP/d\theta \propto \alpha_s/\theta)$$

"Jets" $\equiv$ bunch of collimated particles $\sim$ hard partons
“Jets” ≡ bunch of collimated particles ≈ hard partons

obviously 2 jets
“Jets” ≡ bunch of collimated particles ≈ hard partons

3 jets
“Jets” ≡ bunch of collimated particles ≈ hard partons

3 jets... or 4?

“collinear” is arbitrary
“Jets” ≡ bunch of collimated particles ≈ hard partons

3 jets... or 4?

- “collinear” is arbitrary
- “parton” concept strictly valid only at LO
Jets

Partons/Particles/Calorimeter towers/Tracks

Jet definition

Jet algorithm

Parameters

Recomb. scheme

Jets
A jet definition is supposed to be (as) consistent (as possible) across different views of an event.

- LO partons
- NLO partons
- Parton shower
- Hadron level
Several important properties that should be met by a jet definition are [3]:

1. Simple to implement in an experimental analysis;
2. Simple to implement in the theoretical calculation;
3. Defined at any order of perturbation theory;
4. Yields finite cross section at any order of perturbation theory;
5. Yields a cross section that is relatively insensitive to hadronization.
Jet definitions: constraints

SNOWMASS accords (FermiLab, 1990)

Several important properties that should be met by a jet definition are [3]:

1. Simple to implement in an experimental analysis;
2. Simple to implement in the theoretical calculation;
3. Defined at any order of perturbation theory;
4. Yields finite cross section at any order of perturbation theory;
5. Yields a cross section that is relatively insensitive to hadronization.

30 years later, these are only recently satisfied!!!
Jet definitions: cone

Cone algorithm

- Concept of *stable cone* as a direction of energy flow
  - “cone”: circle of fixed radius $R$ in the $(y, \phi)$ plane
  - “stable”: sum of the particles (4-mom.) inside the cone points in the direction of its centre
**Jet definitions: cone**

**Cone algorithm**

- Concept of *stable cone* as a direction of energy flow
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- Iterative stable-cone search (*aka seeded cone*):
  - start from an initial direction (*seed*) for the cone centre
  - the sum of particles in the cone gives a new direction
  - iterate until stable
Jet definitions: cone

Cone algorithm

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- Iterative stable-cone search (**aka seeded cone**):
  - start from an initial direction (**seed**) for the cone centre
  - the sum of particles in the cone gives a new direction
  - iterate until stable

- Stable cones $\equiv$ jets ... up to overlaps!
Jet definitions: cone with SM

Cone algorithm: (1) cone with split–merge

- **Step 1**: find the stable cones with the seeds
  1. input particles (over a seed threshold)
  2. midpoints of the stable cones found above

- **Step 2**: split–merge (with threshold $f$)

  \[ p_{t,\text{common}} > f \ p_{t,\text{hard}} \]

  \[ p_{t,\text{common}} < f \ p_{t,\text{hard}} \]
Jet definitions: cone with SM

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Examples: main algorithm at the Tevatron

- CDF JetClu (1)
- CDF MidPoint (1+2)
- D0 Run II Cone (1+2)
- ATLAS Cone (1)
Jet definitions: cone with SM

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Examples: main algorithm at the Tevatron

- CDF JetClu (1) IR unsafe (2 hard+1 soft)
- CDF MidPoint (1+2) IR unsafe (3 hard+1 soft)
- D0 Run II Cone (1+2) IR unsafe (3 hard+1 soft)
- ATLAS Cone (1) IR unsafe (2 hard+1 soft)
IR unsafety of the Midpoint alg

3-particle event — MidPoint clustering
IR unsafety of the Midpoint alg
IR unsafety of the Midpoint alg
IR unsafety of the Midpoint alg

stable; 2nd seed
IR unsafety of the Midpoint alg

\begin{align*}
\text{iterate} & \quad \phi \\
pt & \quad 0 \quad 1 \quad 2 \quad 3
\end{align*}
IR unsafety of the Midpoint alg

stable; 3rd seed
IR unsafety of the Midpoint alg

stable; midpoint seed
IR unsafety of the Midpoint alg
IR unsafety of the Midpoint alg

\begin{align*}
  &\text{iterate} \\
  &\phi \\
  &p_t
\end{align*}
IR unsafety of the Midpoint alg

add an infinitely soft particle
IR unsafety of the Midpoint alg

3 hard seeds + midpoint seed → 2 stable cones
IR unsafety of the Midpoint alg

new seed!
IR unsafety of the Midpoint alg

iterate

\[
p_t
\]

\[
\phi
\]
IR unsafety of the Midpoint alg

Stable cones:

Midpoint: \{1,2\} & \{3\}

Seedless: \{1,2\} & \{3\} & \{2,3\}

Jets: \( f = 0.5 \)
**IR unsafety of the Midpoint alg**

Stable cones:
- **Midpoint:** \{1,2\} & \{3\}
- **Seedless:** \{1,2\} & \{3\} & \{2,3\}

Jets: \(f = 0.5\)
- **Midpoint:** \{1,2\} & \{3\}
- **Seedless:** \{1,2,3\}

Jet coordinates:
- \(p_t\)
IR unsafety of the Midpoint alg

Stable cones:
- Midpoint: {1,2} & {3} & {2,3}
- Seedless: {1,2} & {3} & {2,3}
- Jets: ($f = 0.5$)
  - Midpoint: {1,2} & {3}
  - Seedless: {1,2,3}

\[ \phi \quad pt \]

\[ \begin{array}{c}
\text{Jet} \\
\text{Midpoint:} \\
\quad \{1,2,3\} \\
\text{Seedless:} \\
\quad \{1,2,3\}
\end{array} \]
IR unsafety of the Midpoint alg

Stable cones:
- Midpoint: \{1,2\} & \{3\} & \{1,2,3\}
- Seedless: \{1,2\} & \{3\} & \{2,3\}
- Jets: \(f = 0.5\)
  - Midpoint: \{1,2\} & \{3\} & \{1,2,3\}
  - Seedless: \{1,2,3\}

Stable cone missed \(\rightarrow\) Midpoint is IR unsafe
Jet definitions

Cone algorithm: (1) cone with split–merge

- Step 1: find **ALL** stable cones in a reasonable time
  - **MidPoint**: time $\propto N^3$
  - **All-Naive**: time $\propto 2^N$
  - **SISCones**: time $\propto N^2 \log(N)$

- Step 2: split–merge (with threshold $f$)

Example: **SISCones** Seedless Infrared-Safe Cone

---

2007!!!
Jet definition: cone with PR

Cone algorithm: (2) cone with progressive removal

Recipe:
- start with the hardest particle as a seed
- iterate to find a stable cone
- stable cone $\rightarrow 1^{st}$ jet
- remove its constituents
- continue with the next hardest particle left
Jet definition: cone with PR

Cone algorithm: (2) cone with progressive removal

Recipe:
- start with the hardest particle as a seed
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Benchmark: circular/soft-resilient hard jets
Jet definition: cone with PR

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Benchmark: circular/soft-resilient hard jets

Example: CMS Iterative Cone
**Jet definition: cone with PR**

Cone algorithm: (2) cone with progressive removal

- **Recipe:**
  - start with the hardest particle as a seed
  - iterate to find a stable cone
  - stable cone $\rightarrow$ 1\textsuperscript{st} jet
  - remove its constituents
  - continue with the next hardest particle left

- **Benchmark:** circular/soft-resilient hard jets

- **Example:** CMS Iterative Cone
  BUT Collinear unsafe (3 hard+1 coll.splitting) !!
Jet definition: successive recombinations

Idea: Undo the QCD cascade

- Define an inter-particle distance $d_{ij}$ and a beam distance $d_{iB}$

- Successively
  - Find the minimum of all $d_{ij}, d_{iB}$
  - If $d_{ij}$, recombine $i + j \rightarrow k$ (remove $i$, $j$; add $k$)
  - If $d_{iB}$, call $i$ a jet (remove $i$)

- Until all particles have been clustered
Jet definition: successive recombinations

Typical choice of distances:

\[ d_{ij}^2 = \min(k_{t,i}^{2p}, k_{t,j}^{2p})(\Delta y_{ij}^2 + \Delta \phi_{ij}^2) \]

\[ d_{iB}^2 = k_{t,i}^{2p} R^2 \]

- \( p = 1 \): \( k_t \) algorithm (1993)

- \( p = 0 \): Cambridge-Aachen algorithm (1997)

- \( p = -1 \): anti-\( k_t \) algorithm (2008)

- parameter \( R \) (jet separation)
- trivially IRC-safe
Jet definition: successive recombinations

Typical choice of distances:

\[ d_{ij}^2 = \min(k_{pt,i}^{2p}, k_{pt,j}^{2p})(\Delta y_{ij}^2 + \Delta \phi_{ij}^2) \]
\[ d_{iB}^2 = k_{t,i}^{2p} R^2 \]

- \( p = 1 \): \( k_t \) algorithm (1993) 
  (as close as possible to pQCD)

- \( p = 0 \): Cambridge-Aachen algorithm (1997) 
  (close to pQCD; useful for substructure)

- \( p = -1 \): anti-\( k_t \) algorithm (2008) 
  (circular/soft-resilient jets; replaces it. cone)

Variants for \( e^+e^- \) collisions (+JADE)
Jet definitions: IRC safety matters

As said in $e^+e^-$: **IRC safety matters** if you want to compare to QCD computations

<table>
<thead>
<tr>
<th>Process</th>
<th>Last OK order IR$_{2+1}$</th>
<th>Last OK order IR/Coll$_{3+1}$</th>
<th>safe</th>
<th>today’s pQCD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Incl. jet x-sect</td>
<td>LO</td>
<td>NLO</td>
<td>any</td>
<td>NLO</td>
</tr>
<tr>
<td>W/Z/H+1 jet</td>
<td>LO</td>
<td>NLO</td>
<td>any</td>
<td>NLO</td>
</tr>
<tr>
<td>3-jet x-sect</td>
<td>none</td>
<td>LO</td>
<td>any</td>
<td>NLO</td>
</tr>
<tr>
<td>W/Z/H+2 jet</td>
<td>none</td>
<td>LO</td>
<td>any</td>
<td>NLO</td>
</tr>
<tr>
<td>jet mass in 3-jet</td>
<td>none</td>
<td>none</td>
<td>any</td>
<td>LO</td>
</tr>
</tbody>
</table>
Jet definitions: IRC safety matters

As said in $e^+e^-$: **IRC safety matters** if you want to compare to QCD computations

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</tr>
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<td>none</td>
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</tr>
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<td>W/Z/H+2 jet</td>
<td>none</td>
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<td>any</td>
</tr>
<tr>
<td>jet mass in 3-jet</td>
<td>none</td>
<td>none</td>
<td>any</td>
</tr>
</tbody>
</table>

⇒ Use an IRC-safe algorithm like $k_t$, C/A, anti-$k_t$ or SISConet
**Jet definitions: comparison**

Quick comparison of the algorithms

<table>
<thead>
<tr>
<th></th>
<th>$k_t$</th>
<th>C/A</th>
<th>anti-$k_t$</th>
<th>SIS Cone</th>
</tr>
</thead>
<tbody>
<tr>
<td>pQCD</td>
<td>✓✓✓✓</td>
<td>✓✓✓✓</td>
<td>✓✓</td>
<td>✓✓</td>
</tr>
<tr>
<td>soft (UE)</td>
<td>✗</td>
<td>~ OK</td>
<td>✓✓</td>
<td>✓✓✓✓</td>
</tr>
<tr>
<td>speed</td>
<td>✓✓✓✓</td>
<td>✓✓✓✓</td>
<td>✓✓✓✓</td>
<td>✓</td>
</tr>
<tr>
<td>substruct</td>
<td>✓✓</td>
<td>✓✓✓✓</td>
<td>✗</td>
<td>✗</td>
</tr>
<tr>
<td>calibr.</td>
<td>✓</td>
<td>✓</td>
<td>✓✓✓✓</td>
<td>✓✓</td>
</tr>
</tbody>
</table>
Jet clustering: usage/access

FastJet

- Fast implementation of recomb. algs ($N \log(N)$)
- Plugins for all common algs (SISCones; CDF, D0, ATLAS, CMS algs; $e^+e^-$ algs)
- Other tools (like jet areas)
- More in the tutorial part!

[M.Cacciari, G.Salam, GS]
Jets: experimentally

- **Tevatron**
- Use of IR-unsafe JetClu or MidPoint and sometimes $k_t$

\[ \sqrt{s} = 1.96 \text{ TeV} \]
\[ L = 0.70 \text{ fb}^{-1} \]
\[ R_{\text{cone}} = 0.7 \]

- NLO pQCD + non-perturbative corrections
- CTEQ6.5M \( \mu_R = \mu_F = p_T \)

- Data points:
  - |y|<0.4 (x32)
  - 0.4<|y|<0.8 (x16)
  - 0.8<|y|<1.2 (x8)
  - 1.2<|y|<1.6 (x4)
  - 1.6<|y|<2.0 (x2)
  - 2.0<|y|<2.4

- **Figure:**
  - (a) Z/$\gamma$(-e$^-$e$^+$) + >1 jet inclusive
  - (b) Z/$\gamma$(-e$^-$e$^+$) + 2 jets inclusive
  - (c) Z/$\gamma$(-e$^-$e$^+$) + >1 jet inclusive

- L = 0.7 fb$^{-1}$
- Systematic uncertainties
- NLO MCFM CTEQ6.1M
- Corrected to hadron level
  - $p_T^2 - M_T^2 + p_T^2(Z, R_{\text{cone}}=1.3)$
  - $\mu = 3\sqrt{s}$, $\mu = \sqrt{s}/2$
  - PDF Uncertainties
Jets: experimentally

- Tevatron
  Use of IR-unsafe JetClu or MidPoint and sometimes $k_t$

- LHC: anti-$k_t$ by default
At hadronic colliders, many “contaminations” to a jet:

- radiation from partons in the initial state
- Underlying event/Multiple interactions
  - shift: UE ≈ uniform soft background *i.e.* contamination ∝ jet area ∝ $R^2$
  - smearing: due to UE fluctuations
- typical scale: a few GeV
- Pile-up: many $pp$ interactions in 1 bunch-crossing:
  \[ n \approx \mathcal{L} \Delta t_{\text{bunch}} \sigma_{pp} \approx 10^{34} 25.10^{-9} 100.10^{-27} \approx 25 \]

Again: shift + smearing
Typical scale: 20-30 GeV
Need for subtraction techniques
</interlude>
The very fundamental collision

\[ \sigma = f_a \otimes f_b \otimes \hat{\sigma} \]

- "take a parton out of each proton"
  \( f_a \equiv \) parton distribution function (PDF) for quark and gluons

- hard matrix element
  perturbative computation
  Forde-Feynman rules
Deep Inelastic Scattering
Introduce/Discuss/Study the PDFs
\[
\begin{align*}
\text{Process + kinematics} \\
\text{\( e(k) \rightarrow e'(k') \)} \\
\text{\( p \rightarrow X \)}
\end{align*}
\]

\[
\begin{align*}
s &= (e + p)^2 \\
W^2 &= (q + p)^2 \\
Q^2 &= -q^2 > 0 \\
\nu &= p.q = W^2 + Q^2 \\
x &= Q^2/(2\nu) \\
y &= p.q/p.k = (W^2 + Q^2)/s
\end{align*}
\]

\[
\begin{align*}
ep &\rightarrow eX \quad \text{with } \gamma \text{ exchange} \\
\bullet \quad Z \text{ and } W \text{ also possible as well as } \nu \text{ instead of } e \\
\bullet \quad \text{also more exclusive meas.: } ep \rightarrow ep, eXY, eYp, \text{ e.g. jets, charm, vector-mesons, photons}
\end{align*}
\]
Experimentally: only the outgoing $e$ is needed to reconstruct the kinematics

$$Q^2 = 4EE' \cos^2(\theta_e/2)$$

$$x = \frac{EE' \cos^2(\theta_e/2)}{P[E - E' \sin^2(\theta_e/2)]}$$

$$s = (e + p)^2$$

$$W^2 = (q + p)^2$$

$$Q^2 = -q^2 > 0$$

$$\nu = p.q = W^2 + Q^2$$

$$x = Q^2/(2\nu)$$

$$y = p.q/p.k = (W^2 + Q^2)/s$$
Process + kinematics

\[ s = (e + p)^2 \]
\[ W^2 = (q + p)^2 \]
\[ Q^2 = -q^2 > 0 \]
\[ \nu = p.q = W^2 + Q^2 \]
\[ x = \frac{Q^2}{2\nu} \]
\[ y = \frac{p.q}{p.k} = \frac{(W^2 + Q^2)}{s} \]

**Idea:**
use the photon to probe the proton structure

\( Q^2 \) large ⇒ small distance \( \sim 1/Q \)
Process + kinematics

\[ s = (e + p)^2 \]
\[ W^2 = (q + p)^2 \]
\[ Q^2 = -q^2 > 0 \]
\[ \nu = p.q = W^2 + Q^2 \]
\[ x = Q^2 / (2\nu) \]
\[ y = p.q / p.k = (W^2 + Q^2) / s \]

Experiments:
most important results recently from HERA at DESY
(H1 and ZEUS experiments)
Electroweak unification

Neutral currents
\[ ep \rightarrow eX \text{ via } \gamma, Z \]

Charged currents
\[ ep \rightarrow \nu X \text{ via } W^{\pm} \]

\[ e^{\pm} \text{ total x-sect differential in } Q^2 \]
Factorisation in a leptonic and hadronic part:

\[ |\mathcal{M}|^2 = l_{\mu\nu} W^{\mu\nu} \]
\[ l_{\mu\nu} = 4e^2 (k^\mu k'^\nu + k'^\nu k^\mu - g^{\mu\nu} k \cdot k') \]

\[ \rightarrow \text{study the hadronic tensor } W^{\mu\nu}(W^2, Q^2) \]

(or \( W^{\mu\nu}(x, Q^2) \))
Hadronic tensor

Most generic structure for $W^{\mu\nu}(x, Q^2)$

$$W^{\mu\nu} = A g^{\mu\nu} + B p^\mu p^\nu + C q^\mu q^\nu + D p^\mu q^\nu + E q^\mu p^\nu.$$ 

Constraints:

$$W^{\mu\nu} = W^{\nu\mu} \quad \text{and} \quad q_\mu W^{\mu\nu} = 0 \quad \text{(gauge inv.)}$$

Implying

$$W^{\mu\nu} = - \left( g^{\mu\nu} + \frac{q^\mu q^\nu}{Q^2} \right) F_1 + \frac{2x}{Q^2} \left( p^\mu + \frac{q^\mu}{2x} \right) \left( p^\nu + \frac{q^\nu}{2x} \right) F_2$$

$F_1, F_2(x, Q^2):$ proton structure functions
(inclusive) proton interaction fully parametrised by the 2 structure functions $F_1$ and $F_2(x, Q^2)$

- dimensionless

$$F_L = F_2 - 2xF_1$$ (longitudinally-polarized $\gamma^*$)

- For charged currents: additional $F_3(x, Q^2)$
Useful to consider a frame where the proton is highly boosted \((P \gg 1, p \text{ looks like a pancake})\)

\[
p^\mu \equiv (0, 0, P, P)
\]
\[
n^\mu \equiv (0, 0, -\frac{1}{2P}, \frac{1}{2P}) \quad (n^2 = 0, \ n.p = 1)
\]
\[
q^\mu \equiv q^\mu_\perp + \nu n^\mu \quad (n.q = 0, q^2_\perp = Q^2)
\]

We obtain

\[
F_2 = \nu n^\mu n^\nu W_{\mu\nu}
\]
\[
F_L = \frac{4x^2}{\nu} p^\mu p^\nu W_{\mu\nu}
\]
Bag model

The photon resolves a quark inside the proton

\[ k^\mu = \xi p^\mu + \frac{k^2 + k_\perp^2}{2\xi} n^\mu + k_\perp^\mu \]

\[ W^{\mu\nu} = e_q^2 \int \frac{d^4k}{(2\pi)^4} \text{tr} (\gamma^\mu (k + \phi) \gamma^\nu B(k, p)) \delta ((k + q)^2) \]
**Parton model**

**Bag model**

The photon resolves a quark inside the proton

$$k^\mu = \xi p^\mu + \frac{k^2 + k_\perp^2}{2\xi} n^\mu + k_\perp^\mu$$

$$F_2 = \nu e_q^2 \int \frac{d^4k}{(2\pi)^4} \text{tr} (\gamma (k + \phi) \gamma B(k, p)) \delta ((k + q)^2)$$
Bag model
The photon resolves a quark inside the proton

\[ k^\mu = \xi p^\mu + \frac{k^2 + k_\perp^2}{2\xi} n^\mu + k_\perp^\mu \]

\[ F_2 = \nu e_q^2 \int \frac{d^4 k}{(2\pi)^4} \text{tr} \left( \gamma^\mu (k^\mu + \phi^\mu) \gamma B(k, p) \right) \delta \left( (k + q)^2 \right) \]

\[ \text{tr} \left( \gamma^\mu (k^\mu + \phi^\mu) \gamma B(k, p) \right) = 2\xi \text{tr}(\gamma B(k, p)) \]
**Bag model**

The photon resolves a quark inside the proton

\[ k^\mu = \xi p^\mu + \frac{k^2 + k_\perp^2}{2\xi} n^\mu + k_\perp^\mu \]

\[ F_2 = \nu e_q^2 \int \frac{d^4k}{(2\pi)^4} \text{tr} \left( \gamma (k + q) \gamma B(k, p) \right) \delta \left( (k + q)^2 \right) \]

\[ \delta \left( (k + q)^2 \right) = \delta \left( k^2 - Q^2 + 2\xi \nu - 2k_\perp^2 \cdot q_\perp^2 \right) \]

\[ Q^2 \gg \delta(2\nu \xi - Q^2) \approx \frac{1}{2\nu} \delta(\xi - x) \]
Putting everything together:

\[ F_2 = x e_q^2 \int \frac{d^4k}{(2\pi)^4} \text{tr} (\eta \ B(k, p)) \delta(x - \xi) \]

i.e.

\[ F_2 = xe_q^2 q(x) \quad \text{with} \quad q(x) = \int \frac{d^4k}{(2\pi)^4} \text{tr} (\eta \ B(k, p)) \delta(x - \xi) \]

with a sum over flavours

\[ F_2 = \sum_q x e_q^2 [q(x) + \bar{q}(x)] \]

\( q(x) \): parton distribution function (PDF)
Parton model

\[ F_2 = \sum_q x e_q^2 [q(x) + \bar{q}(x)] \]

\[ q(x) \equiv \text{PDF} \]

interpreted as the probability density to find a quark carrying a fraction \( x \) of the proton's momentum (universal!!)

\[ F_2(x, Q^2) = F_2(x): Q^2\text{-independent.} \quad \text{Bjorken scaling} \]

\[ F_L \text{ suppressed by } 1/Q^2 \text{ compared to } F_2 \]

\[ F_2 = 2x F_1. \quad \text{Calan-Gross relation: spin 1/2 for q} \]

charged currents: different quark combinations
Bjorken scaling

$F_2$ from BCDMS, SLAC, NMC, H1 and ZEUS ($\sim 1990$)
Bjorken scaling violations

HERA measurements (∼ 1993 – 2007)

Scaling violations!!!
Bjorken scaling violations

A closer look for 3 bins in $x$

decrease at large $x$  (strong) rise at small $x$
Can we describe the scaling violations in QCD?
Can we describe the scaling violations in QCD?

**Idea:** quarks can radiate gluons
One-gluon emission

4 graphs to compute

Work in an axial gauge $n.A = 0$ (recall $n^2 = 0$, $n.p = 1$, $n.q = 0$):

gluon of mom $k^\mu$ has propagator

$$d^{\mu\nu}(k) = \left(-g^{\mu\nu} + \frac{n^\mu k^\nu + k^\mu n^\nu}{n.k}\right) \frac{1}{k^2}$$
One-gluon emission

\[ k^\mu = \xi p^\mu + \frac{k_\perp^2 - |k^2|}{2\xi} n^\nu + k_\perp^\mu \]

\[ p \equiv (0, 0, P, P) \]

\[ n^\mu n^\nu \sum |M|^2 = \frac{1}{2N_c} e_q^2 g^2 \text{tr}(t_a t^a) \frac{1}{k^4} \text{tr} \left( \eta \gamma^\nu (k + q) \eta \gamma^\mu \gamma^\alpha \gamma_\beta \gamma^\gamma k \right) \]

\[ \left[ -g^{\alpha \beta} + \frac{n^\alpha (p - k)^\beta + (p - k)^\alpha n^\beta}{n \cdot (p - k)} \right] \]

\[ = 32\pi e_q^2 \alpha_s \frac{\xi P(\xi)}{|k^2|} \]

\[ P(\xi) = C_F \frac{1 + \xi^2}{1 - \xi} \]
One-gluon emission

\[ k^\mu = \xi p^\mu + \frac{k_\perp^2 - |k^2|}{2\xi} n^\nu + k_\perp^\mu \]

\[ P(\xi) = C_F \frac{1 + \xi^2}{1 - \xi} \]

\[ \hat{F}_2 = e_q^2 \frac{\alpha_s}{4\pi^2} \int d\xi \xi P(\xi) \int \frac{d|k^2|}{|k^2|} d k_\perp^2 d\theta \delta \left((p - k)^2\right) \delta \left((k + q)^2\right) \]
\[ k^\mu = \xi p^\mu + \frac{k^2_\perp - |k^2|}{2\xi} n^\nu + k^\mu_\perp \]

\[ P(\xi) = C_F \frac{1 + \xi^2}{1 - \xi} \]

\[ \hat{F}_2 = e^2 q \frac{\alpha_s}{4\pi^2} \int d\xi \, \xi P(\xi) \int \frac{d|k^2|}{|k^2|} dk^{2\perp} d\theta \delta \left( (p - k)^2 \right) \delta \left( (k + q)^2 \right) \]

\[ = e^2 q \frac{\alpha_s}{2\pi^2} \int^2_0 \frac{d|k^2|}{|k^2|} \int^{\xi_+}_{\xi_-} d\xi \frac{\xi P(\xi)}{\sqrt{(\xi_+ - \xi)(\xi - \xi_-)}} \]

with \( \xi_{\pm} = x \pm O(|k^2|/Q^2) \)
One-gluon emission

\[ \hat{F}_2 = e_q^2 \frac{\alpha_s}{2\pi^2} x P(x) \int_0^{2\nu} \frac{d|k^2|}{|k^2|} = e_q^2 \frac{\alpha_s}{2\pi^2} x P(x) \int_0^{Q^2} \frac{d|k^2|}{|k^2|} \]

- other diagrams suppressed by powers of \( Q \)
- only kept the leading terms in \( Q \)
- \( |k^2| \) integration DIVERGENT!!
\[
\hat{F}_2 = e_q^2 \frac{\alpha_s}{2\pi^2} x P(x) \int_0^{2\nu} \frac{d|k^2|}{|k^2|} = e_q^2 \frac{\alpha_s}{2\pi^2} x P(x) \int_0^{Q^2} \frac{d|k^2|}{|k^2|}
\]

- other diagrams suppressed by powers of \( Q \)
- \(|k^2|\) integration DIVERGENT!!

From \( \delta((p - k)^2) \) we get \( \vec{k}^2 = (1 - \xi)|k^2| \)

Thus, \(|k^2| \to 0 \Rightarrow \vec{k} \to 0\)

This is thus a **collinear divergence**! The same as we already encountered in \( e^+e^- \) collisions.
One-gluon emission

\[
\hat{F}_2 = e_q^2 \frac{\alpha_s}{2\pi^2} x P(x) \int_0^{2\nu} \frac{d|k^2|}{|k^2|} = e_q^2 \frac{\alpha_s}{2\pi^2} x P(x) \int_0^{Q^2} \frac{d|k^2|}{|k^2|}
\]

- Other diagrams suppressed by powers of \( Q \)
- \(|k^2|\) integration DIVERGENT!!

From \( \delta((p - k)^2) \) we get \( \vec{k}_\perp^2 = (1 - \xi)|k^2| \)
Thus, \(|k^2| \to 0 \Rightarrow \vec{k}_\perp \to 0 \)

This is thus a \textbf{collinear divergence}! The same as we already encountered in \(e^+e^-\) collisions.

Not cancelled by virtual corrections
Here: technique similar to renormalisation
Recall: renormalisation

Vertex correction in QED

\[ q \xrightarrow{\alpha_{\text{bare}}} + q \xrightarrow{\alpha_{\text{bare}}} \rightarrow q \xrightarrow{\alpha(q)} \]

\[
\alpha + \beta_0 \alpha^2 \int_0^{q^2} \frac{dk^2}{k^2} = \alpha + \beta_0 \alpha^2 \int_0^{\mu^2} \frac{dk^2}{k^2} + \text{beta} \alpha^2 \int_{\mu^2}^{q^2} \frac{dk^2}{k^2}
\]

\[
\rightarrow \alpha(\mu^2) + \beta_0 \alpha^2 \int_{\mu^2}^{q^2} \frac{dk^2}{k^2}
\]

\[
\rightarrow \alpha(\mu^2) + \beta_0 \alpha^2 (\mu^2) \int_{\mu^2}^{q^2} \frac{dk^2}{k^2}
\]

\[
\rightarrow \alpha(q^2)
\]
Recall: renormalisation

Vertex correction in QED

We have defined a scale-dependent coupling

\[ \alpha(\mu^2) = \alpha + \beta_0 \alpha^2 \int_0^{\mu^2} \frac{dk^2}{k^2} \]

\[ \alpha_{\text{bare}} \quad + \quad \alpha_{\text{bare}} \quad \rightarrow \quad \alpha(q) \]
Recall: renormalisation

Vertex correction in QED

\[ \alpha(\mu^2) = \alpha + \beta_0 \cdot \alpha^2 \int_0^{\mu^2} \frac{dk^2}{k^2} \]

\( \mu^2 \) is arbitrary \( i.e. \) physics should not depend on it

\[ \mu^2 \partial_{\mu^2} \alpha(\mu^2) = \beta_0 \alpha^2(\mu^2) \]

renormalisation group equation
Reabsorption of the collinear divergence

\[ \frac{d^2 \sigma}{d q^2 \, d \eta} = \frac{1}{Q^2} \frac{d \sigma}{d q^2 \, d \eta} + \frac{1}{Q^2} \frac{d \tilde{\sigma}}{d \tilde{q}^2 \, d \tilde{\eta}} \]
Reabsorption of the collinear divergence

\[ Q^2 \not\rightarrow q_{\text{bare}}(x) + Q^2 \not\rightarrow q_{\text{bare}}(\xi) = Q^2 \not\rightarrow q_{\text{bare}}(x) + Q^2 \not\rightarrow q_{\text{bare}}(\xi) + [0 : \mu^2] + [\mu : Q^2] \]
Reabsorption of the collinear divergence

\[ Q^2 \frac{d^2 \sigma}{d^2 p} = q(x, \mu^2) \]

\[ q_{\text{bare}}(x) + q_{\text{bare}}(\xi) \]

\[ [0 : Q^2] + [0 : \mu^2] + [\mu : Q^2] \]
Reabsorption of the collinear divergence

\[ Q^2 \rightarrow x q_{\text{bare}}(x) + \xi q_{\text{bare}}(\xi) \]

\[ Q^2 \rightarrow x q(x, \mu^2) + \xi q(\xi, \mu^2) \]
Reabsorption of the collinear divergence

\[ F_2(x, Q^2) = x e_q^2 \int \frac{d\xi}{\xi} \left[ \delta \left( 1 - \frac{x}{\xi} \right) + P \left( \frac{x}{\xi} \right) \int_0^{Q^2} \frac{d|k^2|}{|k^2|} \right] q_{\text{bare}}(\xi) \]

\[ = x e_q^2 \int \frac{d\xi}{\xi} \left[ \delta \left( 1 - \frac{x}{\xi} \right) + P \left( \frac{x}{\xi} \right) \int_0^{\mu^2} \frac{d|k^2|}{|k^2|} \right] q_{\text{bare}}(\xi) \]

\[ + x e_q^2 \int \frac{d\xi}{\xi} P \left( \frac{x}{\xi} \right) \int_\mu^2 \frac{d|k^2|}{|k^2|} q_{\text{bare}}(\xi) \]

\[ = x e_q^2 \int \frac{d\xi}{\xi} \left[ \delta \left( 1 - \frac{x}{\xi} \right) + P \left( \frac{x}{\xi} \right) \int_\mu^2 \frac{d|k^2|}{|k^2|} \right] q(\xi, \mu^2) \]

\[ = x e_q^2 q(\xi, Q^2) \]

\[ P(x) = \frac{\alpha_s}{2\pi} C_F \frac{1+x^2}{1-x} \]
We have defined

\[ q(x, \mu^2) = q_{\text{bare}}(x) + \int_x^1 \frac{d\xi}{\xi} P \left( \frac{x}{\xi} \right) \int_0^{\mu^2} \frac{d|k^2|}{|k^2|} q_{\text{bare}}(\xi) \]
Reabsorption of the collinear divergence

We have defined

\[ q(x, \mu^2) = q_{\text{bare}}(x) + \int_x^1 \frac{d\xi}{\xi} P \left( \frac{x}{\xi} \right) \int_0^{\mu^2} \frac{d|k^2|}{|k^2|} q_{\text{bare}}(\xi) \]

Physics independent of the choice for \( \mu^2 \)

\[ \mu^2 \partial_{\mu^2} q(x, \mu^2) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} P \left( \frac{x}{\xi} \right) q(\xi, \mu^2) \]

DGLAP equation
The DGLAP equation

\[ Q^2 \partial_{Q^2} q(x, Q^2) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} P \left( \frac{x}{\xi} \right) q(\xi, Q^2) \]

- DGLAP: Dokshitzer-Gribov-Lipatov-Altarelli-Parisi
The DGLAP equation

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- DGLAP: Dokshitzer-Gribov-Lipatov-Altarelli-Parisi
- the PDFs get some dependence on \( Q^2 \)
- Bjorken scaling violations
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- Leading order computation in \( \alpha_s \log(Q^2/\mu^2) \)
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\[ Q^2 \partial_{Q^2} q(x, Q^2) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} P \left( \frac{x}{\xi} \right) q(\xi, Q^2) \]

- DGLAP: Dokshitzer-Gribov-Lipatov-Altarelli-Parisi
- the PDFs get some dependence on \( Q^2 \)
- Bjorken scaling violations
- \( \mu \) called the factorisation scale
- Leading order computation in \( \alpha_s \log(Q^2/\mu^2) \)
- Actually resums all terms \( \alpha_s^n \log^n(Q^2/\mu^2) \)
  (recall: \( \alpha_s \log(Q^2/\mu^2) \sim 1 \Rightarrow \) compute at all orders)
The DGLAP equation: resummation

\[ Q^2 \partial_{Q^2} q(x, Q^2) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} P \left( \frac{x}{\xi} \right) q(\xi, Q^2) \]
The DGLAP equation: resummation

\[ Q^2 \partial_{Q^2} q(x, Q^2) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} P \left( \frac{x}{\xi} \right) q(\xi, Q^2) \]
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The DGLAP equation: resummation

\[ Q^2 \partial_{Q^2} q(x, Q^2) = \frac{\alpha_s}{2\pi} \int \frac{d\xi}{\xi} P \left( \frac{x}{\xi} \right) q(\xi, Q^2) \]

Resumming (leading) contributions: \( \alpha_s^n \log^n \left( \frac{Q^2}{Q_0^2} \right) \)
The DGLAP equation: splitting function

\[ Q^2 \partial_{Q^2} q(x, Q^2) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} P \left( \frac{x}{\xi} \right) q(\xi, Q^2) \]

\( P(\xi) \) called the splitting function:

transition from a quark of longitudinal momentum \( xP \) to a quark of momentum \( x\xi P \) with emission of a gluon
The DGLAP equation: splitting function

\[
Q^2 \frac{\partial}{\partial Q^2} q(x, Q^2) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} P \left( \frac{x}{\xi} \right) q(\xi, Q^2)
\]

\(P(\xi)\) called the splitting function:

transition from a quark of longitudinal momentum \(xP\) to a quark of momentum \(x\xi P\) with emission of a gluon

Correction due to virtual-gluon emission:

\[
P(x) = C_F \left[ \frac{1 + x^2}{1 - x} \right]_+
\]

NB: the \(1/(1 - x)\) behaviour is the soft QCD divergence
The DGLAP equation: splitting function

\[ Q^2 \partial Q^2 \begin{pmatrix} q(x, Q^2) \\ g(x, Q^2) \end{pmatrix} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} \begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix} \begin{pmatrix} x \\ \frac{1}{\xi} \end{pmatrix} \begin{pmatrix} q(\xi, Q^2) \\ g(\xi, Q^2) \end{pmatrix} \]

\( P_{ab}(\xi) \) called the splitting function:

\( P_{qq} \) \( P_{gq} \) \( P_{qg} \) \( P_{gg} \)

\( P_{ab}(x) \) is the probability to obtain a parton of type \( a \) carrying a fraction \( x \) of the longitudinal momentum of a parent parton of type \( b \)
DGLAP and the factorisation theorem

The result is more general: it holds at any order in perturbation theory

\[ Q^2 \partial_{Q^2} q(x, Q^2) = \int_x^1 \frac{d\xi}{\xi} P \left( \frac{x}{\xi} \right) q(\xi, Q^2) \]

with

\[ P(x) = \left( \frac{\alpha_s}{2\pi} \right) P^{(0)}(x) + \left( \frac{\alpha_s}{2\pi} \right)^2 P^{(1)}(x) + \left( \frac{\alpha_s}{2\pi} \right)^3 P^{(2)}(x) + \ldots \]
DGLAP and the factorisation theorem

The result is more general: it holds at any order in perturbation theory

$$Q^2 \partial Q^2 q(x, Q^2) = \int_x^1 \frac{d\xi}{\xi} P \left( \frac{x}{\xi} \right) q(\xi, Q^2)$$

with

$$P(x) = \left( \frac{\alpha_s}{2\pi} \right) P^{(0)}(x) + \left( \frac{\alpha_s}{2\pi} \right)^2 P^{(1)}(x) + \left( \frac{\alpha_s}{2\pi} \right)^3 P^{(2)}(x) + \ldots$$

- LO resums $\alpha_s^n \log^n (Q^2/\mu^2)$ (leading logarithms)
- NLO resums $\alpha_s^n \log^n (Q^2/\mu^2)$ and $\alpha_s^{n+1} \log^n (Q^2/\mu^2)$

Note: order refers to $P$; includes diagrams at all orders
Note: known up to NNLO since 2004 (Moch, Vermaseren, Vogt)
The result is more general: it holds at any order in perturbation theory

\[ Q^2 \partial Q^2 q(x, Q^2) = \int_x^1 \frac{d\xi}{\xi} P \left( \frac{x}{\xi} \right) q(\xi, Q^2) \]

with

\[ P(x) = \left( \frac{\alpha_s}{2\pi} \right) P^{(0)}(x) + \left( \frac{\alpha_s}{2\pi} \right)^2 P^{(1)}(x) + \left( \frac{\alpha_s}{2\pi} \right)^3 P^{(2)}(x) + \ldots \]

Fundamental result in QCD know as the factorisation theorem

Collinear divergences can be reabsorbed in the definition of the PDFs at all orders!
Very nice description of the $Q^2$-dependence observed in the data.
DGLAP vs. data

DGLAP only gives the $Q^2$ evolution of the PDFs
One still needs an initial condition $f_a(x, \mu^2)$

Global PDF fit:

- Parametrise $q$ and $g$ at an initial scale $\mu^2$
  
  \[ q(x, \mu^2) = x^\lambda (1 - x)^\beta (A + B\sqrt{x} + Cx) \]

- Obtain the PDFs $f_a(x, Q^2)$ at all $Q^2$ using DGLAP

- Compute a series of observables (e.g. $F_2$)

- Fit the experimental measurements ($\chi^2$ minimisation)
Many teams: MSTW/MRST, CTEQ, NNPDF, HERA, H1, ZEUS, Alekhin, GRV

Each with many updates

- e.g. CTEQ4l, CTEQ4m, CTEQ5l, CTEQ5m, CTEQ6, CTEQ6l, CTEQ6m, CTEQ61, CTEQ65, CTEQ66
DGLAP vs. data

- Many teams: MSTW/MRST, CTEQ, NNPDF, HERA, H1, ZEUS, Alekhin, GRV
- Each with many updates
- Points of difference (7):
DGLAP vs. data

Many teams: MSTW/MRST, CTEQ, NNPDF, HERA, H1, ZEUS, Alekhin, GRV

Each with many updates

Points of difference (7):
- Choice of initial scale
- Choice of initial parametrisation
- Order of the fit (LO, NLO, NNLO)
- Data selection (e.g. cuts, old vs. new data)
- Heavy-flavour treatment
- Computation of PDFs uncertainties
- List of observables (9)
DGLAP vs. data

- Many teams: MSTW/MRST, CTEQ, NNPDF, HERA, H1, ZEUS, Alekhin, GRV
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- Points of difference (7):
  - Choice of initial scale
  - Choice of initial parametrisation
  - Order of the fit (LO, NLO, NNLO)
  - Data selection (e.g. cuts, old vs. new data)
  - Heavy-flavour treatment
  - Computation of PDFs uncertainties
  - List of observables (9)
    \[ F_2^p, F_2^d, F_L, F_2^\nu, F_3^\nu, F_2^c, F_2^b, \text{Drell-Yan, Tev. jets} \]
Global fits are important for LHC physics as they affect every perturbative computation.

\[ x_{1,2} = (M/14 \text{ TeV}) \exp(\pm y) \]

\[ Q = M \]
Global fits

Initial distributions

\[ Q^2 = \mu^2 = 2 \text{ GeV} \]

- sea asym. \[ x(\bar{d} - \bar{u}) \]
- valence \[ x \sum_q (q - \bar{q}) \]
Initial ‘flavour-singlet’ distributions

\[ Q^2 = \mu^2 = 2 \text{ GeV} \]
Impact of HERA measurements

With HERA

Without HERA
Global fits

PDF4LHC benchmarks - LHC 14 TeV

CTEQ6.6  NNPDF2.0 / 2.1  MSTW08

$a_s=0.118$  $a_s=0.119$  $a_s=0.119$  $a_s=0.120$

$Z^0$

$W^-$

$W^+$

$t\bar{t}$
DIS: summary

DIS: $\gamma^* p$ scattering with highly virtual $\gamma \ (Q^2 \gg \Lambda_{QCD}^2)$

- Parton model
  - directly probes partons inside the proton
  - Bjorken scaling
DIS: \( \gamma^* p \) scattering with highly virtual \( \gamma \) \((Q^2 \gg \Lambda^2_{QCD})\)

- **Parton model**
  - directly probes partons inside the proton
  - Bjorken scaling

- **QCD collinear divergences**
  - Violations of Bjorken scaling
  - Factorisation theorem/DGLAP equation (fundamental result/prediction of QCD)
  - Parton Distribution Functions (PDF)
  - Global fits for the PDF determination of the PDFs: mandatory for precision at the LHC
Time for questions!
$pp$ collisions
(at last!)
\[ \sigma = f_a \otimes f_b \otimes \hat{\sigma} \]

- “take a parton out of each proton”
  \( f_a \equiv \) parton distribution function (PDF)
  for quark and gluons
  a big chapter of these lectures

- hard matrix element
  perturbative computation
  Forde-Feynman rules
The more realistic version

- Hard ME perturbative
- Parton branching
  initial+final state radiation
- Hadronisation
  \( q, g \rightarrow \text{hadrons} \)
- Multiple interactions
  Underlying event (UE)
- Pile-up
  \( \lesssim 25 \text{ pp} \) at the LHC
Plan

- A few generic considerations
  - kinematics (done)
  - Monte-Carlo

- Processes one-by-one
  - Drell-Yan
  - Jets (done)
  - $W/Z$ (+jets)
  - top
  - $H$
  - SUSY (?)
Plan

- A few generic considerations
  - kinematics (done)
  - Monte-Carlo

- Processes one-by-one
  - Drell-Yan
  - Jets (done)
  - $W/Z$ (+jets)
  - $t\bar{t}$
  - $H$
  - SUSY (?)
Parton luminosities

Vary $\sqrt{s} \Rightarrow$ same ME, only PDF vary

$$\sigma = \sum \int dx_1 dx_2 f_a(x_1) f_b(x_2) \hat{\sigma}$$

$$= \sum_{ij} \int d\hat{s} \frac{dL_{ij}}{d\hat{s}} \hat{\sigma}(\hat{s})$$

NB: Tevatron: $p\bar{p}$
LHC: $pp$
Drell-Yan

Production of a lepton pair
(of mass $M$)

Hard matrix element:

$$\frac{d\hat{\sigma}}{dM^2} = e_q^2 \frac{N_c}{N_c^2} \frac{4\pi \alpha^2}{3M^2} \delta(x_1 x_2 s - M^2)$$

Lowest order (PDF$_1 \otimes$PDF$_2 \otimes$ME)

$$\frac{d\sigma}{dM^2} = \int dx_1 \, dx_2 \, \sum_q [q(x_1, M^2) \bar{q}(x_2, M^2) + (1 \leftrightarrow 2)] \frac{d\hat{\sigma}}{dM^2}$$
Production of a lepton pair (of mass $M$)

More differential cross-sections:

**Ex. 1: lepton-pair rapidity ($y$)**

\[
\delta(x_1 x_2 s - M^2) \quad \delta(y - \frac{1}{2} \log(x_1 / x_2))
\]

\[
\frac{d^2 \sigma}{dM^2 dy} = \sum_q \frac{4\pi e_q^2 \alpha^2}{3 N_c M^2 s} \left[ q\left(\frac{M}{\sqrt{s}} e^y , M^2\right) \bar{q}\left(\frac{M}{\sqrt{s}} e^{-y} , M^2\right) + (y \leftrightarrow -y) \right]
\]
Production of a lepton pair (of mass $M$)

More differential cross-sections:

**Ex. 1:** lepton-pair rapidity ($y$)

$$\Rightarrow \delta(x_1 x_2 s - M^2)$$

$$\delta(y - \frac{1}{2} \log(x_1/x_2))$$

**Ex. 2:** Feynman $x$ ($x_F$)

$$x_F = \frac{2}{\sqrt{s}} (p_{z,l^+} - p_{z,l^-}) \overset{\text{LO}}{=} x_1 - x_2: \text{ also } 2 \delta\text{'s}$$
Next order: emission of one gluon

- real and virtual
- depends on $g(x, M^2)$
- $p_{t,\gamma/Z} \neq 0$
Drell-Yan

- Next order: emission of one gluon
- factorisation proven at ANY order

\[
\frac{d\sigma}{dM^2} = \int dx_1 \, dx_2 \, dz_1 \, dz_2 \sum_{f} f_a(x_1, M^2) f_b(x_2, M^2) D_{ab}(z_1/x_1, z_2/x_2)
\]

\[
\frac{d\hat{\sigma}}{dM^2}(z_1, z_2; M^2)
\]
Drell-Yan

- Next order: emission of one gluon
- Factorisation proven at ANY order

\[
\frac{d\sigma}{dM^2} = \int dx_1 \, dx_2 \, d\frac{z_1}{x_1} \, d\frac{z_2}{x_2} \sum \limits_f f_a(x_1, M^2) f_b(x_2, M^2) D_{ab}(\frac{z_1}{x_1}, \frac{z_2}{x_2})
\]

- ONLY case where the factorisation PDF_1 \otimes PDF_2 \otimes ME is proven,
otherwise it’s just a “reasonable assumption”
Monte-Carlo generators

Parton cascades, hadronisation, Underlying Event, pileup: a realistic event is complicated!

⇒ Use of (Monte-Carlo) event generators to simulate full events
Monte-Carlo generators: fixed order

Perturbative computations are the base of everything
But are often hard/impossible to compute analytically (especially for exclusive measurements)
⇒ use a fixed-order Monte-Carlo generator
Monte-Carlo generators: fixed order

Perturbative computations are the base of everything
But are often hard/impossible to compute analytically
(especially for exclusive measurements)
⇒ use a fixed-order Monte-Carlo generator

Aim: provide signals and backgrounds for LHC studies (usually needed at NLO)
See the LesHouche list of completed/wanted processes, e.g,
- many jets
- $W$+jets
- $H$+jets
- top ($t\bar{t}$ and single top)
- SUSY
Monte-Carlo generators: fixed order

Perturbative computations are the base of everything
But are often hard/impossible to compute analytically
(especially for exclusive measurements)
⇒ use a fixed-order Monte-Carlo generator

- Aim: provide signals and backgrounds for LHC studies (usually needed at NLO)
- Generate matrix elements + phase-space
- 2 big categories:
  LO (many legs) or NLO (includes virtual corrections)
- Tendency to automate!
- Plenty of them: Alpgen, MadGraph, NLOJet, MCFM, BlackHat, Golem,...
Monte-Carlo generators: full event

For full-event simulation, Monte-Carlo generators are a cornerstone

- **parton cascade**: collinear splittings (DGLAP-like)
  As seen in $e^+e^-$, they have the form
  \[
  \frac{d^2P}{d\theta dz} = \alpha_s P(z) \frac{1}{\theta}
  \]

  **Leading terms** ($\alpha_s^n \log^n (1/\theta)$) have **angular ordering**
  $\theta_1 > \theta_2 > \cdots > \theta_n$

  Watch out: LO collinear branchings!!!
  *e.g.* Multi-jet processes hardly reliable
  (alternatives like virtuality ordered but always LO)
Monte-Carlo generators: full event

For full-event simulation, Monte-Carlo generators are a cornerstone

- **parton cascade**: collinear splittings (DGLAP-like)
- **hadronisation**: non-perturbative *per se*
  e.g. Lund string fragmentations (form strings based on colour connections and fragment them)
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- **Multiple interactions/Underlying Event**: hadronic beams carry colour *i.e.* interact strongly
  - Modelling
  - Then **tuning to Tevatron (and LHC) data**
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- **Progress towards NLO generator**
- Most commonly used: *Pythia, Herwig, Sherpa...* but others available
- **more in the tutorials**
**W/Z production**

- **Production:**
  - \( q\bar{q}' \rightarrow W^\pm \)
  - \( q\bar{q} \rightarrow Z \)
  - 14 TeV \( \sigma_W \approx 20 \text{ nb} \) *i.e.* 200 \( W/\text{s} \) (\( \mathcal{L} = 10^{34} \text{ cm}^2/\text{s} \))

- **Decay:**
  - \( W \rightarrow q\bar{q} \rightarrow 2 \text{ jets} \) (BR\( \approx 2/3 \))
    - \( W \rightarrow \ell\nu\ell \) (BR\( \approx 1/3 \))
  - \( Z \rightarrow q\bar{q} \rightarrow 2 \text{ jets} \) (BR\( \approx 70\% \))
    - \( Z \rightarrow \ell\bar{\ell} \) (BR\( \approx 10\% \))
    - \( Z \rightarrow \nu\bar{\nu} \) (BR\( \approx 20\% \))

- leptonic channel most convenient
  hadronic important for statistics!
W/Z physics

- not really a discovery channel...

- ... but important in many respects
  - often W/Z+jets
  - standard model tests/MC calibration
  - background to many searches
    e.g. top (→ Wb) or SUSY (E_t)

- W cross-section as a standard candle for luminosity measurements
$W$ cross-section as a standard candle for luminosity measurements

PDF main source of uncertainty
**top physics**

- **Production:**
  - Mostly $gg \rightarrow t\bar{t}$
  - Tevatron: $\sigma_t \approx 4$ pb: discovery!
  - LHC: $\sigma_t \approx 1$ nb: $\approx 10$/s LHC $\equiv$ top factory

- **Decay:**
  - Mostly $t \rightarrow Wb$
    $t \rightarrow q\bar{q}b \ (\approx 66\%)$ or $t \rightarrow \ell\nu\bar{\ell}b \ (\approx 33\%)$
  - for $t\bar{t}$: 3 options
    - **leptonic:** not-so-easy because 2 neutrinos
    - **semi-leptonic:** $\ell$, 4 jets ($2b$) and $E_t$
      (the most convenient)
    - **hadronic:** 6 jets *i.e.* technical to reconstruct
      but $\approx 45\%$ of the stat!
top very important at the LHC

- precision mass measurement
- many new physics scenario involve the top (mostly because of its large mass)

⇒ need to reconstruct as many tops as possible
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Issues:

- $W$+jets background
- $b$ mis-tagging
- combinatorial background (especially for full hadr.)
- efforts e.g. in boosted-top reconstruction
Higgs: production

Production at the LHC: mostly $gg$ fusion (through top loop)

- $\sigma(pp\to H+X)$ [pb]
- $\sqrt{s} = 14$ TeV
- $M_t = 174$ GeV
- CTEQ6M

$$m_H = 120 \text{ GeV} \Rightarrow \sigma^{(L0)}_H \approx 21 \text{ pb} \text{ (vs 0.3 at the Tevatron)}$$
Heavy higgs ($m \gtrsim 2m_W$):

mostly $H \to WW^{(*)}$ or $H \to ZZ$

the easiest situation (see e.g. Tevatron)
Light higgs ($m < 2m_W$): more complicated

- $bb \rightarrow$ jets dominant but buried in the QCD bkgd
- $\gamma\gamma$ clean but only 0.1-0.3% of the events
Higgs: discovery

\[ \sim 30 \text{ fb}^{-1} \]
needed for
\( 5\sigma \) discovery
Higgs: additional comments

- $H \rightarrow b \bar{b}$ may be visible/helpful for boosted $H + W/Z$
Higgs: additional comments

- \( H \rightarrow b\bar{b} \) may be visible/helpful for boosted \( H + W/Z \)

- some additional ideas like
  - \( H \rightarrow \tau\tau \)
  - Higgs in SUSY events
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some additional ideas like

- $H \rightarrow \tau\tau$
- Higgs in SUSY events

Not the end of the story:
also need to verify Higgs properties/couplings.

- e.g. $t\bar{t}H$ may help
- need for luminosity!
Typical SUSY process:

- production of a pair of supersymmetric particles
- decay: SM particles + lightest SUSY particle (LSP)
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- missing $E_T$ (from the LSP + neutrinos)
- leptons
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Typical issues
- Need good determination of $E_T$
- Control the multi-jet background at large $p_t$
Time for questions!